The parametrization of surface fluxes in large-scale models under free convection

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SUMMARY

This paper investigates the idea of formulating free convection in large-scale models as a special case of forced convection. In free convection, the wind speed in the surface transfer law is related to the velocity of the large eddies in the mixed layer and is proportional to the convection velocity scale. The empirical coefficient is estimated with the help of large eddy simulation data by Sykes *et al.* (1993) and the resulting formulation is compared with field data by Stull (1994). This concept is shown to be applicable to smooth ocean surfaces as well as rough land surfaces. It is argued that within this framework, free convection is a natural extension of forced convection and only needs a minor modification of traditional transfer laws.

1. INTRODUCTION

Recently two papers have appeared that address the parametrization of surface fluxes over land in free convection conditions. The paper by Sykes *et al.* (1993) discusses the scaling behaviour of bulk transfer laws in the surface layer over land and uses large eddy simulation (LES) to find the relevant constants. Stull (1994) proposes a new scaling law and derives constants from the BLX83 experiment.

Free convection parametrization over the ocean is rather crucial to the atmospheric circulation as it determines the atmosphere—ocean coupling in the warm-pool area (Miller *et al.* 1992) and therefore also determines to a certain extent the capability of atmospheric models to respond to anomalies in the sea surface temperature (Palmer *et al.* 1992). Over land the problem has received less attention. Data on free convection over rough surfaces is relatively rare as it involves the radiative surface temperature as part of the bulk transfer law.

Although the scaling laws proposed by Sykes *et al.* (1993) and Stull (1994) are perfectly reasonable for free convection and provide a good fit to the data, it is less clear how they fit into the standard transfer laws that are widely used in atmospheric models. In this paper it is shown that the standard Monin–Obukhov scaling works equally well for free convection provided that the free-convection velocity scale is included in the near-surface wind as used in the transfer law. This makes the implementation of free convection in large-scale models rather straightforward.

Sykes *et al.* (1993) develop their scaling from the idea that free convection is a special case of forced convection, where the surface wind is forced by the large-scale eddies in the outer part of the boundary layer. This idea is well accepted now and was used by Deardorff (1970, 1972), Businger (1973) and Schumann (1988) for rough surfaces and by Liu *et al.* (1979), Golitsyn and Gratchov (1986), Miller *et al.* (1992) and Godfrey and Beljaars (1991) for smooth surfaces. However, Sykes *et al.* (1993) develop a special kind of surface layer scaling, which suggests that free convection needs special attention in parametrization schemes. The logarithmic profile is used to relate the free-convection velocity to the friction velocity and an internal boundary layer concept is used to estimate the height of the surface layer. In this paper it is shown that an estimate of the surface layer depth is not needed if stability corrections are applied to the logarithmic profile.

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The reason is that for strong instability (large ratio of height and Obukhov length), the profiles become uniform (or well mixed), so the choice of the height where it is used in a transfer law becomes irrelevant. The implication is that normal Monin–Obukhov similarity can be used in the surface layer without modification for free convection (apart from the use of the free-convection velocity in the surface wind).

Stull (1994) proposes a free-convection bulk transfer law on the basis of similarity arguments and suggests an empirical interpolation with the forced-convection situation. He does not include the surface roughness length as a relevant scaling parameter, which may limit his results to a small range of surface characteristics.

The purpose of this paper is to show that free convection can be handled as a natural extension of Monin–Obukhov similarity theory, by including the free-convection velocity as an extra wind component in the bulk transfer laws for the surface layer. This parametrization of free convection applies to a wide range of surface roughness lengths (ocean and land surfaces) and needs only one parameter in addition to the standard transfer laws for finite wind speeds.

2. PARAMETRIZATION OF SURFACE FLUXES

Here we follow the standard approach based on Monin–Obukhov similarity theory, to express the surface flux into differences between surface quantities and parameters at the lowest model level which is assumed to be in the surface layer (e.g. Stull 1988). The effect of moisture on buoyancy is included through the concept of virtual temperature.

$$\overline{w'\theta'} = C_{\rm h} |U|(\theta_{\rm s} - \theta) \tag{1}$$

$$\overline{w'q'} = C_q |U|(q_s - q) \tag{2}$$

$$u_*^2 = C_{\rm m} |U|^2 \tag{3}$$

$$C_{\rm h} = k^2 \left/ \left\{ \ln\left(\frac{z + z_{\rm 0m}}{z_{\rm 0m}}\right) - \Psi_{\rm m}\left(\frac{z + z_{\rm 0m}}{L}\right) + \Psi_{\rm m}\left(\frac{z_{\rm 0m}}{L}\right) \right\} \times \left\{ \ln\left(\frac{z + z_{\rm 0m}}{z_{\rm 0h}}\right) - \Psi_{\rm h}\left(\frac{z + z_{\rm 0m}}{L}\right) + \Psi_{\rm h}\left(\frac{z_{\rm 0h}}{L}\right) \right\}$$
(4)

$$C_{q} = k^{2} / \left\{ \ln\left(\frac{z + z_{0m}}{z_{0m}}\right) - \Psi_{m}\left(\frac{z + z_{0m}}{L}\right) + \Psi_{m}\left(\frac{z_{0m}}{L}\right) \right\} \times \left\{ \ln\left(\frac{z + z_{0m}}{z_{0q}}\right) - \Psi_{h}\left(\frac{z + z_{0m}}{L}\right) + \Psi_{h}\left(\frac{z_{0q}}{L}\right) \right\}$$
(5)

$$C_{\rm m} = k^2 \left/ \left\{ \ln\left(\frac{z + z_{0\rm m}}{z_{0\rm m}}\right) - \Psi_{\rm m}\left(\frac{z + z_{0\rm m}}{L}\right) + \Psi_{\rm m}\left(\frac{z_{0\rm m}}{L}\right) \right\}^2$$
(6)

$$|U|^{2} = U^{2} + V^{2} + (\beta w_{*})^{2}$$
⁽⁷⁾

$$w_* = \left\{ z_i \frac{g}{T} \overline{w' \theta'_v} \right\}^{1/3}, \qquad L = \frac{-u_*^3}{k \frac{g}{T} \overline{w' \theta'_v}}$$
(8)

$$\overline{w'\theta'_{v}} = \overline{w'\theta'} + 0.61 \times 10^{-3}\theta_{v}\overline{w'q'}, \qquad \theta_{*} = -\overline{w'\theta'}/u_{*}$$
(9)

where $\overline{w'\theta'}$ is the kinematic heat flux, $\overline{w'\theta'_v}$ is the virtual temperature flux, $\overline{w'q'}$ is the moisture flux, $C_{m,h,q}$ are the transfer coefficients for momentum, heat and moisture, k is the Von Karman constant (0.4), z the reference height (e.g. in a model, the height of the lowest model level; in experiments, the observation height), L the Obukhov length, $z_{0m,h,q}$ the roughness lengths for momentum, heat and moisture, $\Psi_{m,h}$ the stability profile functions for momentum and heat (the functions for heat and moisture are assumed to be the same), |U| the mean length of the horizontal wind vector (as measured by a cup anemometer, including the convection induced large-eddy motion) at height z, U and Vare the large-scale velocity components (component averages as used in models, which are zero in free convection), θ is the potential temperature at height z, θ_s the potential temperature at the surface, u_* the friction velocity, w_* the free convection velocity scale, g/T the buoyancy parameter and z, the boundary layer height. Parameter β is an empirical constant of order 1 to be determined later; in free convection it expresses the relation between the mean length of the horizontal wind vector |U| and w_* (i.e. how much wind is measured by a cup anenometer in free convection with convection velocity w_*). The transfer coefficients depend on surface roughness length, and stability. The stability part can in principle be expressed in a bulk Richardson number (Ri_b) , but in general the relation between z/L and Ri_b has to be solved iteratively (see Beljaars and Holtslag 1991). The use of the roughness length for momentum as a displacement height is irrelevant for large z/z_{0m} but is included here for completeness.

For the stability functions we select the ones proposed by Paulson (1970), which are widely accepted and give a good fit to the data (see Högström 1988 for a review):

$$\Psi_{\rm m} = 2\ln[(x+1)/2] + \ln[(x^2+1)/2] - 2\tan^{-1}(x) + \pi/2$$
(10)
$$\Psi_{\rm h} = 2\ln[(x^2+1)/2]$$
$$x = (1 - \gamma z/L)^{1/4}, \gamma = 16.$$
(11)

This particular form of the stability functions does not have the 1/3-power law dependence one would expect for large -z/L. It may look contradictory, but the behaviour of the stability functions for large -z/L is less important. This is the regime where the temperature and velocity gradients are very small (well mixed profiles) and this part of the profiles contribute very little to the 'bulk-difference' from level z to the surface. The shape of the profiles close to the surface (i.e. for small -z/L) is much more important and therefore functions that satisfy free convection scaling for large -z/L do not necessarily improve the bulk transfer law.

It is important to realize that the transfer laws as expressed by Eqs. (1) to (11) are consistent with Monin–Obukhov similarity theory and widely accepted. The only addition is the term $(\beta w_*)^2$ in Eq. (7), which represents the near surface wind induced by the large eddies. When the surface is heated, this term guarantees a finite surface wind forcing in the transfer law for vanishing U and V and prevents u_* and L from becoming zero.

The additional term $(\beta w_*)^2$ has two empirical aspects. First of all the constant β (which determines for vanishing U and V the mean horizontal surface wind speed including fluctuations as measured by an anemometer, in contrast to wind from mean components as represented in models), is not known and has to be derived from data. Secondly, it is not obvious how the interpolation between free and forced convection should be done. Here, the intuitive choice of a quadratic interpolation has been made, but this may not be the optimal one.

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3. FREE CONVECTION OVER ROUGH SURFACES

For free convection (U = V = 0) the transfer laws can be simplified. To do so we write asymptotic forms of Eqs. (1) to (7) as suggested by Godfrey and Beljaars (1991). For this simplified form, we neglect the effect of moisture on buoyancy and take $w' \theta'_{v} = w' \theta'$.

$$\Psi_{\rm m} = \ln(-\gamma z/L) - 3\ln 2 - \pi/2 = \ln\left\{\frac{\gamma z}{-38.5L}\right\}$$
(12)

$$\Psi_{\rm h} = \ln(-\gamma z/L) - \ln 4 = \ln\left\{\frac{\gamma z}{-4L}\right\}, \text{ for large } -z/L.$$
(13)

This simplifies the transfer laws and z drops out of the problem. It means that for large z the profiles become constant and approach their mixed layer values ($\theta \rightarrow \theta_m$). For free convection (U = V = 0) further simplification of Eqs. (1) to (6) yields

$$\overline{w'\theta'} = \frac{k^2\beta w_*(\theta_s - \theta_m)}{\left\{\ln\left(\frac{-38.5L}{\gamma z_{0m}}\right) + \psi_m\left(\frac{z_{0m}}{L}\right)\right\}\left\{\ln\left(\frac{-4L}{\gamma z_{0h}}\right) + \Psi_h\left(\frac{z_{0h}}{L}\right)\right\}}$$
(14)

$$u_* = \frac{k\beta w_*}{\ln\left(\frac{-38.5L}{\gamma z_{0m}}\right) + \Psi_m\left(\frac{z_{0m}}{L}\right)}.$$
(15)

With the definition of w_* we derive a relation between z_i/z_{0m} and L/z_{0m} and a heat transfer law:

$$\frac{z_{\rm i}}{z_{\rm 0m}} = \frac{-1}{k^2 \beta^3} \frac{L}{z_{\rm 0m}} \left\{ \ln\left(\frac{-38.5L}{\gamma z_{\rm 0m}}\right) + \Psi_{\rm m}\left(\frac{z_{\rm 0m}}{L}\right) \right\}^3 \tag{16}$$

$$\overline{w'\theta'} = b_{\rm h} \left\{ \frac{g}{T} z_{\rm i} \right\}^{1/2} (\theta_{\rm s} - \theta_{\rm m})^{3/2} \tag{17}$$

where

$$b_{\rm h} = \frac{k^3 \beta^{3/2}}{\left\{ \ln\left(\frac{-38.5L}{\gamma z_{0\rm m}}\right) + \Psi_{\rm m}\left(\frac{z_{0\rm m}}{L}\right) \right\}^{3/2} \left\{ \ln\left(\frac{-4L}{\gamma z_{0\rm h}} + \Psi_{\rm h}\left(\frac{z_{0\rm h}}{L}\right) \right\}^{3/2}}.$$
 (18)

The coefficient b_h is shown as a function of z_i/z_{0m} in Fig. 1 (with $z_{0m} = z_{0h}$) for three different values of β . This figure suggests that the coefficient in scaling law (17) is strongly dependent on the surface roughness length. The mechanism is that rough surfaces generate more mixing in response to the convective motion than smooth surfaces and therefore have a more efficient coupling with the atmosphere, also in the free convection regime.

4. FREE CONVECTION OVER SMOOTH SURFACES

For smooth surfaces the surface roughness lengths are small and scale with the kinematic viscosity of air ν and the friction velocity (e.g. Hinze 1975):

$$z_{0m} = a_m \nu / u_*, \qquad z_{0h} = a_h \nu / u_*.$$
 (19)

We use $a_m = 0.11$ and $a_h = 0.40$ for air with a Prandtl number (*Pr*) of 0.71 (Brutsaert 1982). In the traditional heat transfer laws for free convection over smooth surfaces, the boundary layer height or the mixed layer height is usually not considered as a parameter and therefore the transfer law is put in non-dimensional form with help of the kinematic viscosity and the Prandtl number (Townsend 1964):

$$\overline{w'\theta'} = C_{\rm s} \left\{ \frac{g}{T} \frac{\nu}{Pr^2} \right\}^{1/3} (\theta_{\rm s} - \theta_{\rm m})^{4/3}.$$
⁽²⁰⁾

 $C_{\rm s}$ can be derived from Eqs. (8), (15), (17) and (18) by using the definition of $z_{0\rm m}$ and $z_{0\rm h}$ for a smooth surface and by neglecting $z_{0\rm m}/L$ and $z_{0\rm h}/L$ (which are small for smooth surfaces):

$$C_{\rm s} = \frac{\beta k^{7/3} P r^{2/3} (z_i u_* / \nu)^{1/3}}{\left\{ \ln \left(\frac{-38.5}{\gamma a_{\rm m}} \frac{L u_*}{\nu} \right) \right\} \left\{ \ln \left(\frac{-4}{\gamma a_{\rm h}} \frac{L u_*}{\nu} \right) \right\}^{4/3}}.$$
 (21)

Similarly to Eq. (16) we have a relation between $z_i u_* / \nu$ and $L u_* / \nu$:

$$\frac{z_{\rm i}u_*}{\nu} = \frac{-1}{k^2\beta^3} \frac{Lu_*}{\nu} \left\{ \ln\left(\frac{-38.5}{\gamma a_{\rm m}} \frac{Lu_*}{\nu}\right) \right\}^3.$$
(22)

The numerical relation between Lu_*/ν and z_iu_*/ν is indicated by the top and bottom horizontal axes in Fig. 2. It gives an *a-postiori* justification to neglect $\Psi_m(z_{0m}/L)$ and $\Psi_h(z_{0h}/L)$, because with $-Lu_*/\nu = 10$ (for the already unrealistically low value of $z_iu_*/\nu = 10^4$), $-z_{0m}/L$, and $-z_{0h}/L$ are 0.011 and 0.04 respectively.



Figure 1. Transfer coefficient for heat over a rough surface as defined by Eq. (18) as a function of z_{0m}/z_i for different values of β (with $z_{0m} = z_{0h}$). The solid line represents the empirical formulation by Sykes *et al.* (1993). The horizontal axis at the top of the graph indicates the values of $-z_{0m}/L$ for $\beta = 1$ in accordance with Eq. (16).



Figure 2. Transfer coefficient for heat over a smooth surface as defined by Eq. (21) as a function of $z_i u_* / \nu$ for different values of β . The horizontal axis at the top of the graph indicates the values of $-Lu_* / \nu$ for $\beta = 1$ in accordance with Eq. (22).

Parameter C_s , which is assumed to be constant in traditional free convection scaling laws (e.g. Townsend 1962, 1964) is now a function of $z_i u_*/\nu$ and of Lu_*/ν , but the relation between these two expressed by Eq. (22) enables the computation of C_s as a function of $z_i u_*/\nu$, which can be interpreted as the ratio of boundary layer height and 'surface roughness length' for a smooth surface. The result is shown in Fig. 2 for four different values of β . The remarkable result is that the dependence on $z_i u_*/\nu$ is relatively weak and that the numerical value of C_s is in the same range as for laboratory data. The laboratory data are mainly from tank experiments where a stably stratified water layer with a depth of 0.1 to 1 m are heated. The temperature differences are typically 10 K. Townsend (1964) reports 0.193 for C_s , Deardorff *et al.* (1969) measure a decrease from 0.21 to 0.18 during the increase of the mixed layer depth. However, Deardorff and Willis (1985) observe a decline of C_s from 0.14 to 0.10. Golitsyn and Grachov (1986) report experimental values ranging from 0.135 to 0.15 for multi-component media.

Although the numbers we find (see Fig. 2) are relatively close to the laboratory ones, it should be realized that the parametrization as proposed here does not necessarily apply to the tank experiments. The dimensions of the laboratory tanks are in fact too small to justify a separation of scales between z_0 , L and z_i . However, the tank experiments as well as the mechanism of free convection depicted in this paper, suggest that C_s is in principle expected to be a function of the depth of the mixed layer, although the dependence may be weak in practice.

5. Comparison with large-eddy simulation data for rough surfaces

The heat transfer law as expressed by Eqs. (16), (17) and (18) can be compared with the LES results by Sykes *et al.* (1993). The coefficient β is unknown and has to be determined empirically. Its numerical value is not necessarily the same as found by Sykes *et al.* (1993) as their formulation is different and has two coefficients to be determined. Although not realistic for most natural surfaces, we select $z_{0m} = z_{0h}$ for the comparison with Sykes *et al.* (1993). These roughness lengths are specified in the LES runs with help of the logarithmic profile which is used together with Monin–Obukhov stability functions as a boundary condition at the lowest model level.

Figure 3 shows the ratio u_*/w_* as a function of z_{0m}/z_i for different values of β in comparison with the Sykes *et al.*'s LES data for three different values of β . Parameter β strongly influences the relation between u_* and w_* , so the ratio between them is probably the best parameter to consider when tuning β . The agreement is best with $\beta = 1$ for small ratios of z_{0m}/z_i , but for $z_{0m}/z_i = 0.001$, $\beta = 1.2$ gives a better result. The fit to the data is slightly less accurate than the curve by Sykes *et al.* (1993), but certainly accurate enough for most applications.

Also, the dimensionless temperature profiles $(\theta_s - \theta)w_*/\overline{w'\theta'}$ are compared with those deduced from Sykes *et al.* (1993) (see Fig. 4). The dimensionless coefficient b_h as a function of z_{0m}/z_i is compared with LES data in Fig. 1. From these comparisons, we conclude that the heat transfer from a rough surface in free convection is well represented by the extended Monin–Obukhov similarity, where free convection is seen as a special case of forced convection with the near surface wind determined by the free-convection velocity of the bulk of the boundary layer.



Figure 3. Friction velocity divided by the free convection velocity scale as a function of z_{0m}/z_i . The dashed curves represent the current formulation with $\beta = 0.8$, 1.0 and 1.2, the solid curve represents the empirical curve by Sykes *et al.* (1993) and the symbols represent LES data by Sykes *et al.* The horizontal axis at the top of the graph indicates the values of $-z_{0m}/L$ for $\beta = 1$ in accordance with Eq. (16).



Figure 4. Dimensionless temperature profile (difference with respect to the surface temperature) according to the current formulation with $\beta = 1.2$ (dashed) in comparison with the LES data (solid) by Sykes *et al.* (1993). The surface layer depth z_s is the one defined by Sykes *et al.* to enable comparison with their Fig. 6.

6. COMPARISON WITH FIELD DATA OVER LAND (BLX83)

Stull (1994) uses aircraft data from the BLX83 experiment in Oklahoma to estimate coefficient b_h in the transfer law of Eq. (17). He argues that this transfer law is universal and most likely to be independent of the surface roughness length. This is difficult to demonstrate on the basis of data with a limited range of z_{0m} values; however the LES data indicate quite a strong dependency on the surface roughness length (see Fig. 1). Here we will demonstrate that the BLX83 data are not in conflict with the scaling laws of section 2.

Before considering free convection, we have to determine z_{0m} and z_{0h} for the experimental location. The roughness lengths are integration constants in the logarithmic profiles and as such part of a neutral concept. So we concentrate on high wind speed data to determine z_{0m} and z_{0h} . Although we will apply stability corrections, the selection of high wind speed data guarantees that the stability corrections do not dominate the results. We use the data in Table 1 of Stull (1994). The table combines surface fluxes measured by aircraft at low levels with mixed layer observations of wind and temperature measured between 0.3 and 0.7 of the mixed layer height. The transfer properties of the layer between the middle of the mixed layer and the surface are considered.

Figure 5 shows the quantity

$$\ln(z_{0m}) = \ln(z) - \frac{k(U^2 + V^2)^{1/2}}{u_*} - \Psi_{\rm m}(z/L),$$
(23)

as a function of wind speed, where all the parameters at the right hand side have been derived from direct observations. The measuring height for the wind speed is assumed to be 50% of the boundary layer height. This is rather high, but it is the only wind information available from the paper by Stull (1994). The results are not very sensitive to the choice of height because we are in the well mixed layer where the gradients are small. This also applies to the asymptotic behaviour of the wind and temperature profiles



Figure 5. $Ln(z_{0m})$ as defined by Eq. (23) for the BLX83 data as a function of wind speed in the mixed layer.

i.e. $\ln(z) - \Psi(z/L)$ becomes constant for large -z/L. The turbulent fluxes needed for the friction velocity and the Obukhov length are taken from the near surface flight leg. Equation (10) has been used for Ψ . Since the aerodynamic roughness length is best defined in situations with considerable surface stress, we limit ourselves to wind speeds that are large compared to the free convection velocity. The average of $\ln(z_{0m})$ for winds above 5 m s⁻¹ is -1.07, corresponding to $z_{0m} = 0.34$ m.

The integration constant in the logarithmic temperature profile, z_{0h} can be estimated in a similar way, by considering

$$\ln(z_{0h}) = \ln(z) - \frac{k(\theta - \theta_s)}{\theta_*} - \Psi_h(z/L).$$
(24)

Figure 6 shows this quantity, with the right hand side of Eq. (24) evaluated from observations. Again we average for data with wind speeds above 5 m s^{-1} to stay away from the free-convection regime ($w_* < (U^2 + V^2)^{1/2}$). The average is -17.2 resulting in $z_{0h} = 3 \times 10^{-8}$ m. This value is rather unusual and reflects low values of the transfer coefficients for heat. For homogeneous vegetation a ratio of 10 for z_{0m}/z_{0h} is more common (Brutsaert 1982). It is worth mentioning that in particular the measurements of the radiative surface temperature have large uncertainties which are reflected in the scatter and perhaps also produce a bias. Beljaars and Holtslag (1991) argue that low values for z_{0h} may be related to inhomogeneous terrain effects, consisting of small height variations, changes in land use, streams, trees etc. More recently Garratt *et al.* (1993) question the value of the concept of z_{0h} as it may depend on atmospheric variables. It was also concluded by Stull (1994) that the transfer coefficient for heat is considerably smaller than the one for momentum. He interprets it as part of the free-convection regime, but it is important to realize that the same difference shows up in the high wind speed regime.

Having defined the surface conditions for high wind speeds we consider now the transfer coefficient for heat over the entire wind speed regime. When we plot the



Figure 6. $Ln(z_{0h})$ as defined by Eq. (24) for the BLX83 data as a function of wind speed in the mixed layer.

conventional transfer coefficient $C'_{\rm h} = \overline{w'\theta'}(U^2 + V^2)^{-1/2}(\theta_{\rm s} - \theta)^{-1}$ and compare with Eq. (4) we see a good correspondence at high wind speeds, because the roughness lengths in Eq. (4) have been tuned to this equation. For low wind speeds the transfer coefficient $C'_{\rm h}$ increases more than Eq. (4). The match is much better when we define the modified transfer coefficient $C_{\rm h} = \overline{w'\theta'} \{(U^2 + V^2 + (\beta w_*)^2\}^{-1/2}(\theta_{\rm s} - \theta)^{-1}$ as discussed before. In fact we include the large scale convective motion as part of the surface wind. The curve in Fig. 7 shows the results for $\beta = 1.2$.

In section 2 we already raised the question how to combine or interpolate from 'surface wind' βw_* in the free-convection regime to $(U^2 + V^2)^{1/2}$ in the high wind speed regime. The rather intuitive quadratic interpolation is not necessarily optimal. To illustrate this we try a simple addition of $(U^2 + V^2)^{1/2}$ and βw_* as suggested by Stull (1994; see Fig. 8). The match is nearly perfect now, but it is extremely difficult to justify the shape of the interpolation between two regimes on the basis of this limited data set. Further study with help of large eddy simulation would be an attractive way of addressing this problem.

7. OCEAN DATA AT LOW WIND SPEEDS

Air-sea transfer coefficients have been extensively studied (e.g. Smith 1988, 1989; Large and Pond 1982), but most observational studies have no data points below 4 m s⁻¹. At low wind speeds, the sea surface becomes aerodynamically smooth and therefore the sea surface roughness-length scales with the kinematic viscosity, rather than with the constant of gravity as in the Charnock relation (e.g. Liu *et al.* 1979; Miller *et al.* 1992). To cover the low as well as the high wind speed regime we use an interpolation between the two regimes for z_{0m} .

$$z_{0m} = a_m \nu / u_* + \alpha u_*^2 / g \tag{25}$$

$$z_{0h} = a_h \nu / u_* \tag{26}$$

$$z_{0q} = a_q \nu / u_* \tag{27}$$

where the chosen constants are $a_{\rm m} = 0.11$, $a_{\rm h} = 0.40$, $a_{\rm q} = 0.62$ (Brutsaert, 1982), and $\alpha = 0.018$. It is not trivial to use the smooth surface parametrization for heat and moisture also for the high wind speed rough regime. An empirical approach has been adopted, in the sense that it has been tried to obtain a reasonable fit to observations at high wind speeds. Initially it was tried to add a constant to the smooth surface part of z_{0h} and z_{0q} (1.4×10^{-5} m and 1.3×10^{-4} m respectively as suggested by Smith 1988, 1989), but the result was that the transfer coefficient for moisture went up to 0.0015 at a wind speed of



Figure 7. Heat transfer coefficient C_h derived from observations as a function of wind speed. Solid line $C'_h = w'\theta'(U^2 + V^2)^{-1/2}(\theta_s - \theta)^{-1}$ solid line $C_h = w'\theta'(U^2 + V^2 + (1.2w_*)^2)^{-1/2}(\theta_s - \theta)^{-1}$, dashed line Eq. (4) with L from observations, $z_{0m} = 0.34$ m and $z_{0h} = 3 \times 10^{-8}$ m.



Figure 8. As Fig. 7 except $C_{\rm h} = \overline{w'\theta'} \{ (U^2 + V^2)^{1/2} + 1.2w_* \}^{-1} (\theta_{\rm s} - \theta)^{-1}$.

5 m s⁻¹ (reference level is 10 m). With the parametrization, described above, the transfer coefficient for moisture has a minimum of about 0.0012 at 4 m s⁻¹ and gradually increases to 0.0015 at 20 m s⁻¹. This is believed to be supported by observations, indicating a value of 0.0012 with very little wind speed dependence (see Smith 1989; DeCosmo 1991).

With the specification of the sea surface 'roughness' lengths (Eqs. (25) to (27) and Eqs. (1) to (9)) we can compute all the surface fluxes when wind speed, temperature and specific humidity at a reference height (e.g. 10 m for observations) and the sea surface temperature are known. The results for typical temperature differences are shown in Fig. 9. There is a potential interpretation problem as far as experimental data is concerned, because wind speeds are not always averaged in the same way. If the horizontal wind speed is averaged, the βw_* term is in fact already included in the measurement, and the resulting transfer coefficients are the ones defined in Eqs. (4) to (6). If the components are averaged, then we will have transfer coefficients defined as e.g. for heat $\overline{w'\theta'}(U^2 + V^2)^{-1/2}(\theta_s - \theta)^{-1}$. The results are shown in Fig. 9 according to both definitions. The results are in fact very similar, except that the averaging of the horizontal wind will never give wind speeds below 0.5 m s^{-1} for these temperature differences whereas the component averaging may produce zero wind speed. The reassuring aspect is that we do not really have to know how data has been gathered, to make a comparison with observations.

Data published by Bradley *et al.* (1991) is shown in Fig. 10 in comparison with the parametrization presented here. The scatter in the data is large, but there is a clear indication that the transfer coefficients increase at low wind speeds, implying that the fluxes of heat and moisture remain finite in the limit for free convection.

8. CONCLUSIONS

We have shown that free convection can be parametrized as a special case of forced convection. The near surface wind forcing is produced by the large eddies in the mixed layer. Asymptotic transfer laws have been derived for smooth and rough surfaces. The resulting transfer laws for free convection are in reasonable agreement with data from tank experiments (e.g. smooth surface data by Deardorff and Willis 1985) and with results from large-eddy simulation for rough surfaces (Sykes *et al.* 1993). The empirical constant β , which relates the free-convection velocity to the surface wind forcing is estimated to be about 1.2 on the basis of the LES data by Sykes *et al.* (1993).

Atmospheric data for free convection data are virtually non-existent; most data will have a non-negligible large-scale wind component. It is therefore necessary to use an interpolation between free and forced convection. Such a formulation is proposed and compared with BLX83 data (Stull 1994). It is concluded that the formulation works very well, but it is difficult to decide on the exact form of the interpolation on the basis of this data set.

The proposed formulation has a few advantages over the free-convection formulations proposed by Sykes *et al.* (1993) and Stull (1994):

- The formulation is extremely simple to implement in an atmospheric model as it only involves an adjustment of the surface wind forcing in the traditional transfer laws.
- The same formulation works for a wide range of surface conditions (for smooth as well as rough surfaces).
- Particular surface conditions that affect high wind speeds (e.g. differences in roughness lengths for momentum and heat or inhomogeneous terrain effects) are automatically taken care of in free convection, once they are dealt with at high wind speed.



Figure 9. Air-sea transfer coefficient for (a) moisture and (b) heat, as a function of wind speed, according to the parametrization described in this paper (reference level is 10 m). Two different ocean-air temperature differences are used, 1 and 2 K and the ocean-air specific humidity difference is chosen 7 g kg⁻¹. The curves are represented in two different ways: as in experiments where the averaged horizontal velocity |U| is measured (solid line) and for experiments where the vector average $(U^2 + V^2)^{1/2}$ is measured (dashed line).



Figure 10. Air-sea transfer coefficients as a function of wind speed in comparison with the ocean data by Bradley *et al.* (1991). The reference level is 10 m. (a) C_q and (b) C_h .

In this paper, the effect of dry convective motion on the air-surface interaction has been discussed. The dry convective boundary layer (mixed layer) is relatively well documented. It is not clear however, how the near surface wind forcing should be handled in the transition regime from free to forced convection. Large-eddy simulation seems an attractive tool to study this problem. It is even less clear how the air-surface interaction is influenced by other types of motion that are subgrid to large scale models. We can think of shallow clouds, organized convection, precipitating convection and meso-scale activity, all enhancing the averaged near surface wind but not resolved by large-scale models.

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