Improving the Numerical Efficiency of Storm-Scale NWP Models

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Simulation of clouds and convection is a computationally intensive problem and requires numerical methods that are efficient and flexible over a large range of grid spacings and atmospheric flow regimes. In the late-1970's a method to efficiently integrate the fully compressible equations, called time splitting, was introduced by Klemp and Wilhelmson. This method integrates the physically insignificant "fast" sound waves (e.g., the compressible modes) on a smaller time step than the more physically relevant "slower" modes (advection, diffusion, etc.) that are more computationally expensive to evaluate. Time splitting still remains a widely used approach for many atmospheric models today. Computational power has now increased sufficiently to enable the use of the fully compressible equations with convection-permitting resolutions within global models of the deep atmosphere. These models, referred to as Convective Allowing Models (hereafter, CAMs), often employ grids having large aspect ratios i.e., $\frac{\Delta x}{\Delta z} \gg 1$. The model's global time step is then constrained by the vertical Courant number in small regions where convection with large updrafts are occurring. To mitigate this effect, some operational CAMs employ damping methods to reduce updraft speeds and maintain numerical stability for the prescribed time step. This is a form of filtering the solution. As an alternative, a new integration method is presented that extends the Wicker and Skamarock Runge-Kutta time-splitting scheme to improve the computationally accuracy and efficiency for these CAM applications. The results show that time steps can be up to 50 percent larger than the time-split integrator from Wicker and Skamarock (2002). Combined with the decreased cost in physics, due to fewer large time steps, this leads to a reduction in wallclock time of 10-25 percent for many applications.

Baldauf (2008) performed an extensive numerical analysis of the "linear case" Runge-Kutta integration methods (hereafter LC-RK) for the one- and two-dimensional advection equation using spatial operators ranging from first- to sixth-order. The LC-RK schemes with the most optimal characteristics are the WS02 LC-RK3 using upwind-biased approximations (i.e., 3rd or 5th order) for the spatial approximation or the LC-RK4 with a 4th-order spatial discretization. In

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the WRF-ARW model, the 5th order horizontal discretization with 3rd or 5th order discretization is used for NWP applications. In COSMO-DE, the LC-RK4 scheme with 4th order spatial advection has been recently tested (Ogaja and Will 2016). The use of higher-order Runge-Kutta schemes (e.g., N > 3) in atmospheric models has rarely been tested due to the rather large computational cost of evaluating the slow-mode terms, considered by some already to be excessive in the WS02 scheme. In computational aerodynamics the use of five-, six-, or even seven-stage Runge-Kutta schemes are common (Hu et al. 1996, Calvo et al. 2003, Berland et al. 2006). Instead of preserving formal order of the integrator, the extra stages are used to optimize certain aspects of the time truncation errors or stability while preserving a lower formal temporal truncation error (usually at 2nd or 4th order). The most common optimizations increase the maximum stable time step while improving the dispersion and dissipation of the scheme. For example, Allampalli et al. (2009) present several variants of RK-5, RK-6, and RK-7 stage schemes which permit 2.3, 2.8, and 3.28x larger time steps than the maximum time step permitted by the LC-RK3 scheme from WS02 and Baldauf (2008) for inviscid problems. These schemes have 4th-order temporal accuracy, and the extra degrees of freedom associated with stages 5-7 are used to reduce the truncation errors as well as extending region of stability. Therefore for slightly less or slightly more than twice the computational effort, these schemes may permit a time step that is up to 2-3x larger than the current LC-RK3 scheme. Hu et al. (1996, hereafter H96) introduced one of the first optimized Runge-Kutta (hereafter, RK) schemes where the formal order was sub-optimal to stage order. These schemes were designed to be 2nd-order in time and used the extra degrees of freedom to reduce dissipation and dispersion errors as well as increasing the stability time step along the imaginary axis. Table 1 shows the 1D and 2D courant number stability limits for linear advection using 4th-, 5th-, and 6th-order spatial differencing for the standard LC-RK3 scheme and the H96 scheme. The expanded stability from the H96 is evident. The increased stability region

Spatial order	4^{th} order (1D / 2D)	5^{th} order (1D /2D)	6^{th} order (1D / 2D)
LC-RK3	1.5 / 0.75	1.6 / 1.1	1.2 / 0.6
Hu96	2.3 / 1.1	2.1 / 1.05	2.05 / 1.0

Table 1: LC-KR3 and HU96 advection stability limits

offsets the additional cost in the H96 integration from the two extra sub-steps. In a full physics model a larger time step will be even more efficient as the sub-grid parameterizations schemes will be called fewer times, reducing the overall CPU time. *Importantly, because CAMs often use* $\frac{\Delta x}{\Delta z} \gg 1$*, it will be demonstrated that the HU96 scheme can be used as the vertical time integration system while the horizontal time integration remains LC-RK3.* The ability to combine both methods provides even more computational efficiency.

The new HU96 scheme is implemented first in CM1 version 19.3 (Bryan 2002) and compared with the standard LC-RK3 scheme. The Weisman-Klemp quarter circle supercell, a test case provided with the code is using the NSSL microphysics scheme. The vertical grid resolution is increased from 0.5 km to 0.4 km to help test the combined integration scheme. The supercell simulation is integrated for 150 minute to make sure the integration is stable for a long period of time and produces acceptable results. The solution from T = 2 hours will be shown. Figure 1 shows the results from the LC-RK3 integration scheme using a time step $\Delta t = 6\ sec$. A classic supercell with a dominant right moving cell with the typical split left moving cell is seen in the reflectivity field, along with the a teardrop shaped shaped updraft (all figures are from z = 1.8 km). Further tests show that the default LC-RK3 in CM1 can be stably run for this test configuration using a time step of 10 seconds (Fig. 2). There are small differences in the results, but the solutions are similar. Figure 3 shows the same simulation for the combined LC-RK3/HU96 scheme using $\Delta t = 15\ sec$. The differences between the LC-RK3 and HU96 are similar in shape and magnitude when compared to the two LC-RK3 runs with different time steps. *The new scheme has already been implemented and tested in WRF. The results from a real data test will be shown along with timing and computational efficiency estimates will be shown at the workshop.*

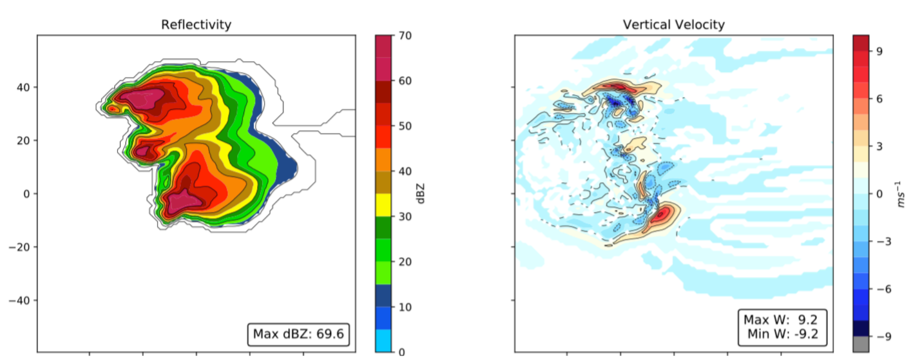

Figure 1: LC-RK3 solution using a time step of 6 sec at T = 2 hours.

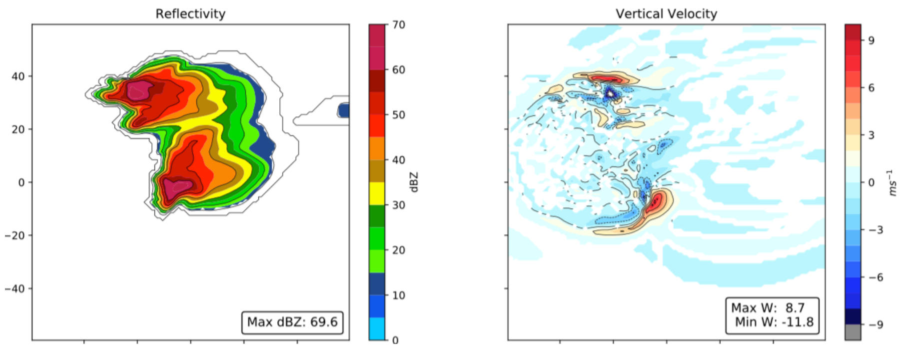

Figure 2: LC-RK3 solution using a time step of 10 sec at T=2 hours.

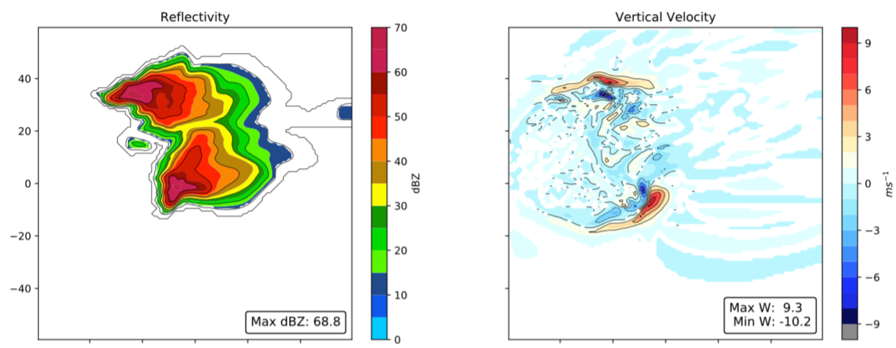

Figure 3: Combined LC-RK3/Hu96 scheme using a time step of 15 sec at T= 2 hours.

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