9.3 Sensitivity of urban canopy and land surface models to input land cover data

Stephen R. Shaffer

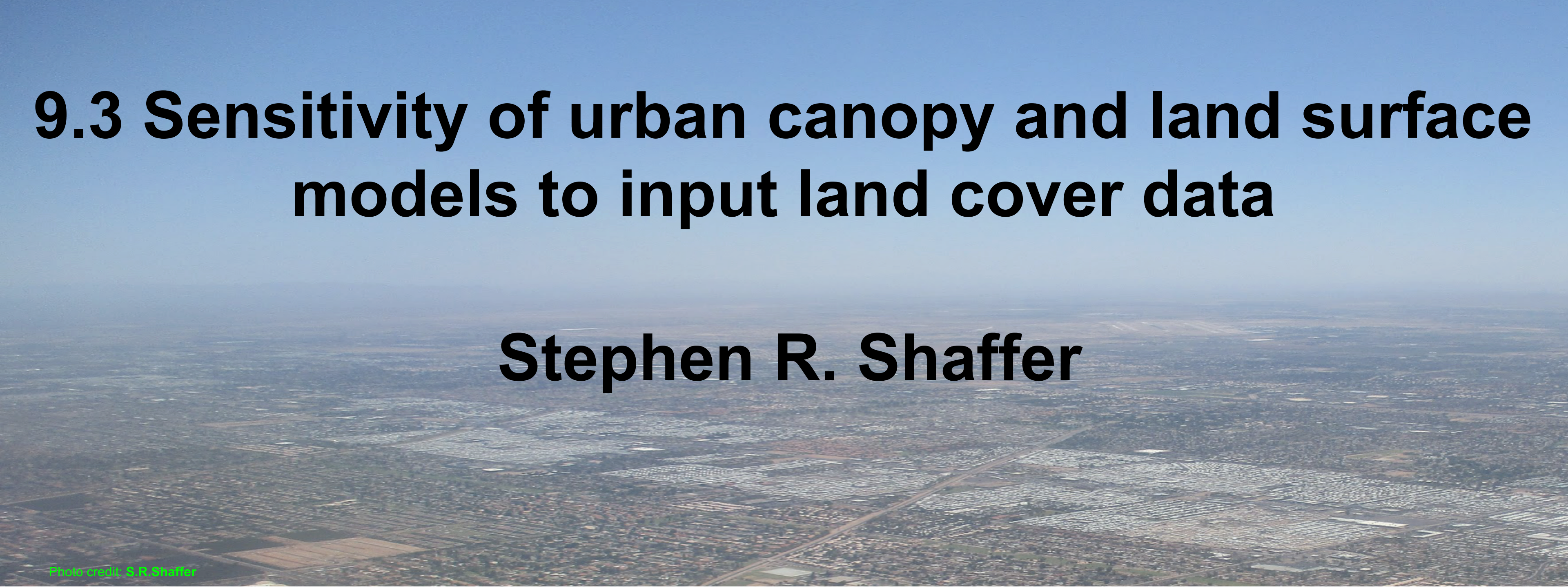


Photo credit: S.R.Shaffer

United States Department of Agriculture
National Institute of Food and Agriculture

Earth System Models-3: Physics-Based Predictive Modeling for Integrated Agricultural and Urban Applications

NSF DMS-1419593
USDA-NIFA 2015-67003-23508

Weather Research & Forecasting

2018 Joint WRF/MPAS Users' Workshop
June 11-15, Boulder, CO

ARIZONA STATE UNIVERSITY

Stephen.Shaffer@asu.edu

WRF Background - canopy layer models & coupling

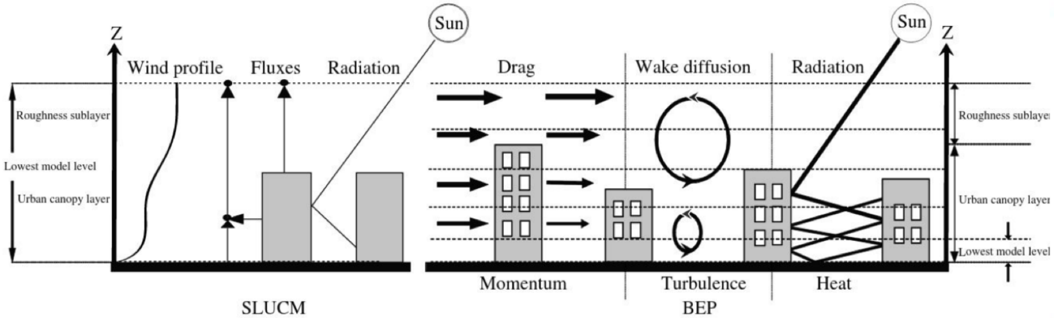
$$V_{total} = f_{urban} V_{urban} + (1 - f_{urban}) V_{natural}$$

"urban" (UCM)

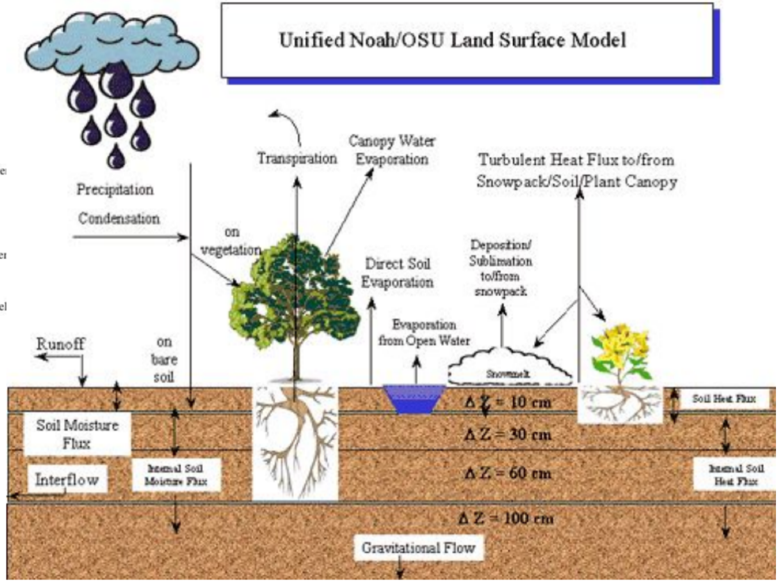
"natural" vegetation (Noah LSM)

Single Layer

Multiple Layers



- Building morphology (shading/trapping), materials (thermal conduction, fluxes)
- Sublayer modifications influence urban boundary layer (momentum, moisture, heat)
- Additional forcings
  - AH, e.g. Sailor and Lu 2004
  - BEM, Salamanca et al. 2010

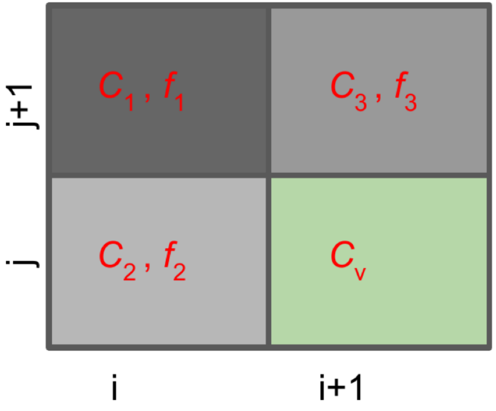


Chen et al. 2011, Chen and Dudhia 2001

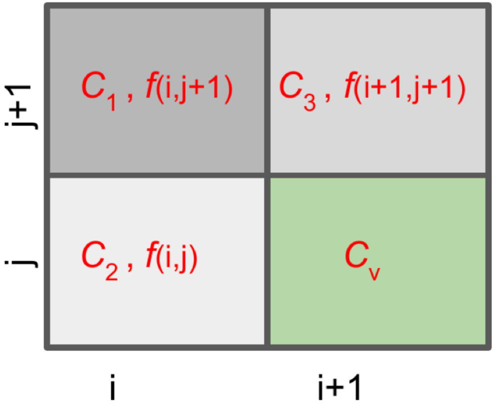
Subgrid Aggregation Methods for urban classes

$$V_{total} = f_{urban} V_{urban} + (1 - f_{urban}) V_{natural}$$

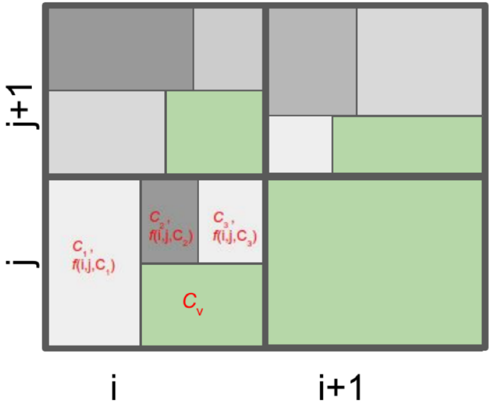
Constant, Dominant



Heterogeneous, Dominant



Heterogeneous, Mosaic



Urban class based on dominant area with f_{urban} as:

domain-wide constant attributed to each class

Spatially heterogeneous drawn from subgrid data

More than one class per grid cell, as “tiles”

Example with n_{tiles}=4

Aggregated by area

f_{urb} as currently employed in WRF (before v4.0)

Default constant f_{urb} (CUF) and
National Urban Database and Portal Access Tool (NUDAPT) approaches:

- CUF, single f_{urb} value for each of 3 classes, for the entire domain (.5,.9,.95)
  - allows for only 3 "types" of city in the entire domain
- NUDAPT, focus on central business districts
  - normalized subgrid areal fractions of each NLCD urban class (α_c) are weighted by assumed fixed urban density w_c = .5,.5,.9,.95,

$$f_{urb} = \sum_c w_c \alpha_c$$

Remaining urban parameter values:

- 250 m and 1 km resolution gridded data (NUDAPT), or other studies
- building morphology (LIDAR) National Building Statistics Database 2 (Burian et al. 2008)
  - **covers only ~17 km^2 for Phoenix**
- NUDAPT building material ratios by class (concrete, wood, steel, brick) from city tax roll (Burian and Ching 2009) - for Houston! (large uncertainties, not all cities!)

Shaffer et al. 2016

Analysis of NLCD 2006 data

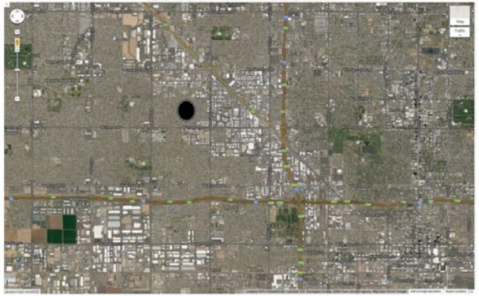
National Land Cover Database derived from Landsat observations

Categorical partitioning scheme:

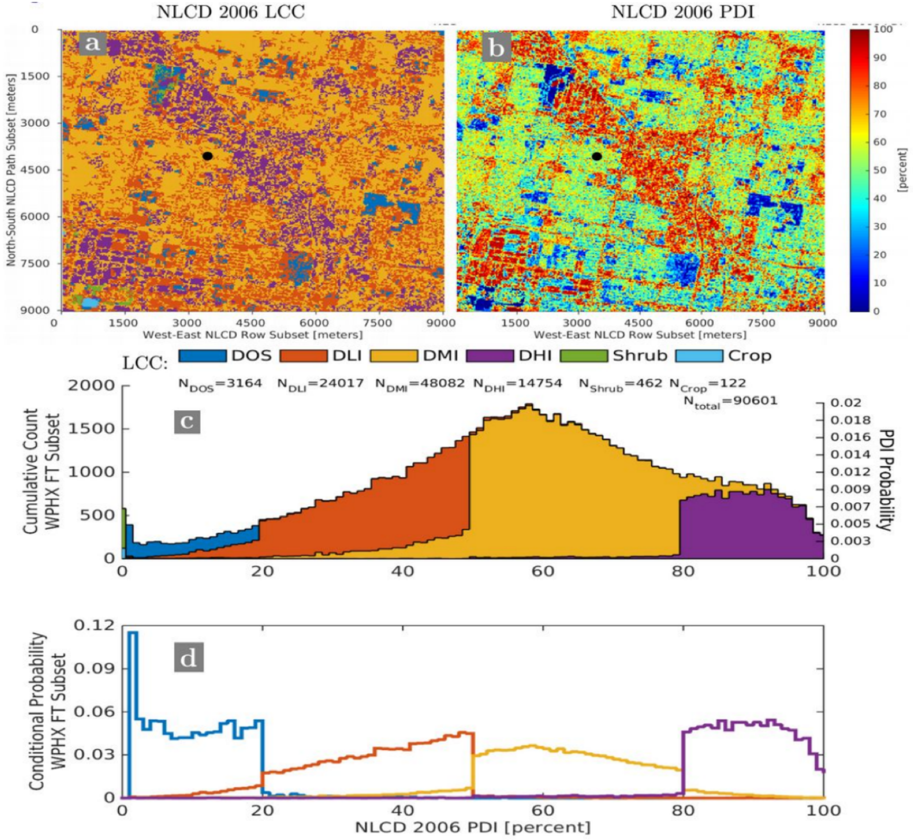
$$\mathcal{C}^{\mathrm{urb}}(\Psi) = \begin{cases} \mathrm{DOS}, & 0 < \Psi < 20, \\ \mathrm{DLI}, & 20 \leq \Psi \leq 49, \\ \mathrm{DMI}, & 50 \leq \Psi \leq 79, \\ \mathrm{DHI}, & 80 \leq \Psi \leq 100. \end{cases}$$

$\Delta_\xi =$ 30 m resolution

Subset of Phoenix



Distribution for a 9 km x 9 km area



Shaffer et al. 2016

Analysis of NLCD 2006 data

National Land Cover Database derived from Landsat observations

Categorical partitioning scheme:

$$\mathcal{C}^{\mathrm{urb}}(\Psi) = \begin{cases} \mathrm{DOS}, & 0 < \Psi < 20, \\ \mathrm{DLI}, & 20 \leq \Psi \leq 49, \\ \mathrm{DMI}, & 50 \leq \Psi \leq 79, \\ \mathrm{DHI}, & 80 \leq \Psi \leq 100. \end{cases}$$

$\Delta_\xi =$ 30 m resolution

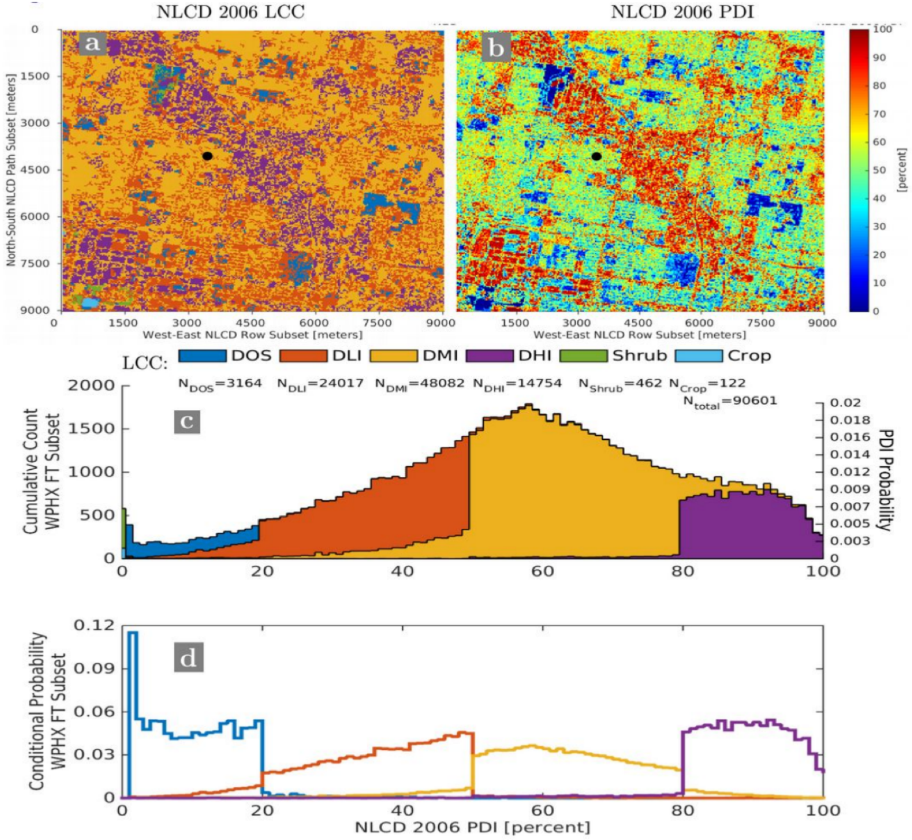
NUDAPT dominant:

$$f_{urb} = \textstyle\sum_c w_c \alpha_c \quad ; \quad w_c = (.5, .5, .9, .95)$$

Proposed method:

$$w(\mathcal{X}, \mathcal{C}^{\mathrm{urb}}) = \max_{\psi \in \Psi} p(\psi(\xi \in \mathcal{X}) | \mathcal{C}^{\mathrm{urb}}(\xi \in \mathcal{X}))$$

Distribution for a 9 km x 9 km area



Shaffer et al. 2016

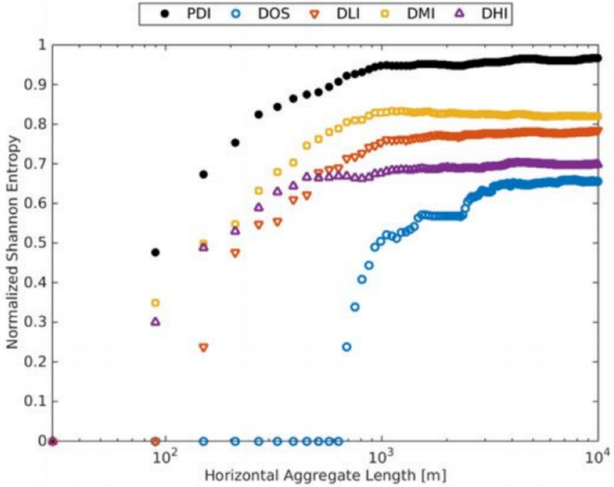
Normalized Shannon Entropy Assessment of f_{urb}

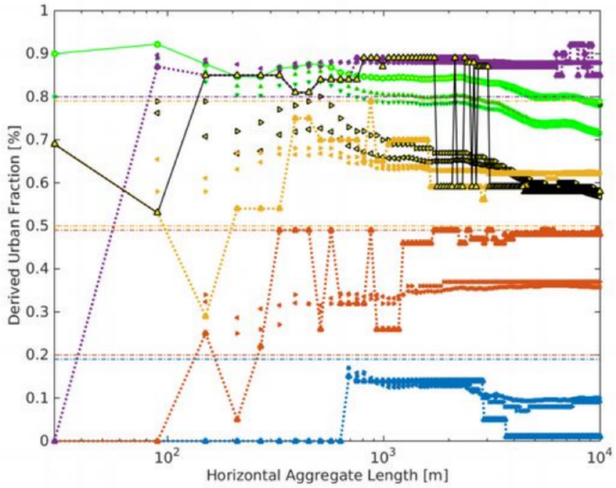
$$h = \frac{-1}{\log N_s}\left[\sum_{\Psi} p(\Psi)\log\left(p(\Psi)\right)\right] \in [0,1]$$

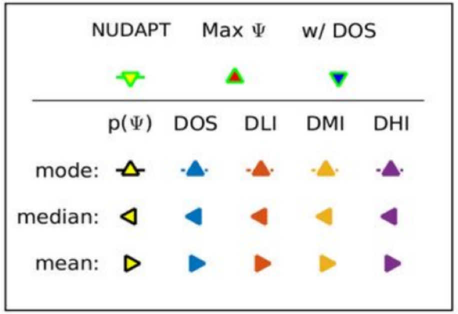
$$f_{\mathrm{urb}} = w(\mathcal{X}, \mathcal{C}^{\mathrm{urb}}) = \max_{\psi\in\Psi} p(\psi(\xi\in\mathcal{X})|\mathcal{C}^{\mathrm{urb}}(\xi\in\mathcal{X}))$$

$h = 1 \Rightarrow$
$p(x)$=constant

$h = 0 \Rightarrow$
$p(x)=\delta(x-x_0)$





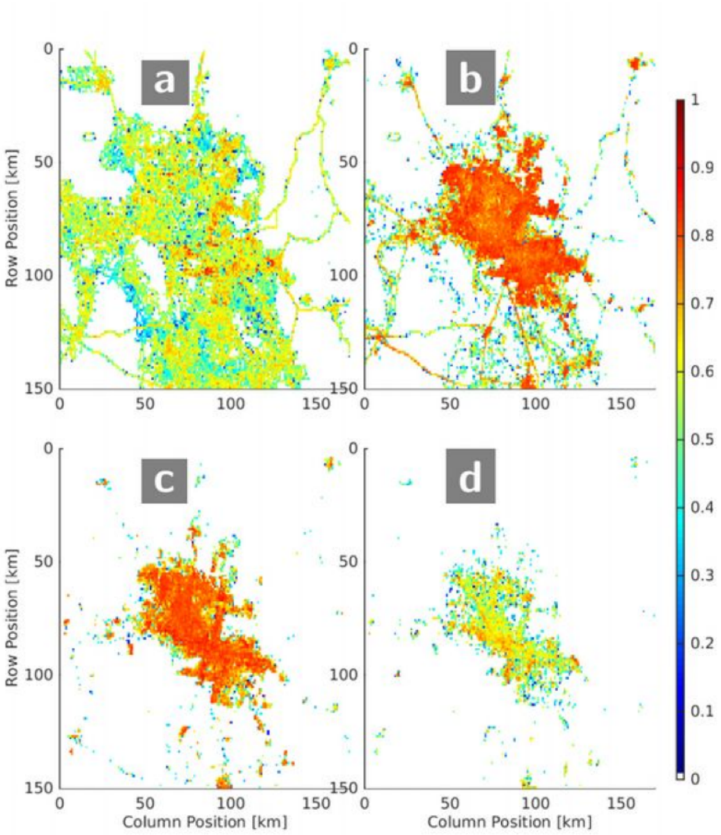


- Ξ centered at flux tower, $(2n-1)\times 30$ m, $n = 1, 2, \ldots$
- "asymtotic" h for $\Xi > 1$ km ($\Xi > 4$ km for DOS)
- $h_{\mathcal{C}^{\mathrm{urb}}}$ has different large Ξ limits for each $\mathcal{C}^{\mathrm{urb}}$
- f_{urb} depends upon Ξ
- f_{urb} by NUDAPT overestimates $\mu(p(\Psi))$,
- f_{urb} decrease with $\Xi > 500m$ for single class method
- f_{urb} for mosaic "asymptotes" with skewed distribution (for this Ξ_0)

Scale-aware (good for MPAS)

Shaffer et al. 2016

Map of $h_{\mathcal{C}^{\text{urb}}}$ at Δ_{Ξ}=990 m for Phoenix Metro Area



$h_{\mathcal{C}^{\text{urb}}}(p(\Psi|\mathcal{C}^{\text{urb}}))$

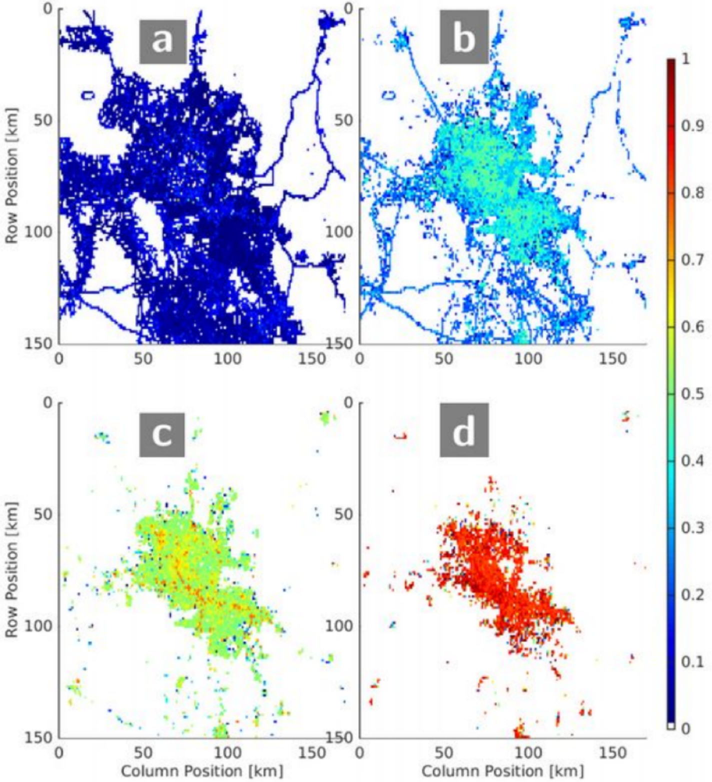
with $\mathcal{C}^{\text{urb}}$ being:

(a) DOS
(b) DLI
(c) DMI
(d) DHI

- Lower $h_{\mathcal{C}^{\text{urb}}}$ in a and d
- $h_{\mathcal{C}^{\text{urb}}}$ higher in urban cores, roadways, where $\mathcal{C}$ and Ψ more variable

Shaffer et al. 2016

Map of $f^{\mathrm{H}}_{\mathrm{urb}}$ at Δ_{Ξ}=990 m for Phoenix Metro Area



$f^{\mathrm{H}}_{\mathrm{urb}}(\Xi, \mathcal{C}^{\mathrm{urb}})$

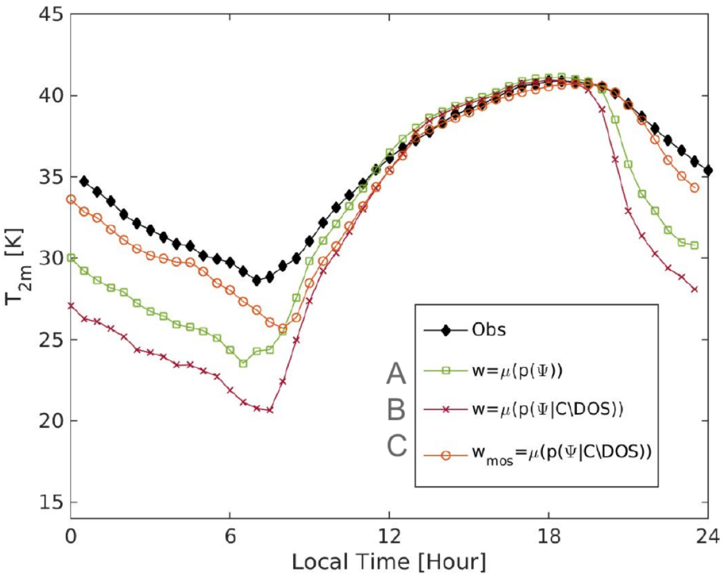
with $\mathcal{C}^{\mathrm{urb}}$ being:

(a) DOS
(b) DLI
(c) DMI
(d) DHI

- Different geographic footprints
- Different range of $f^{\mathrm{H}}_{\mathrm{urb}}$ by $\mathcal{C}^{\mathrm{urb}}$

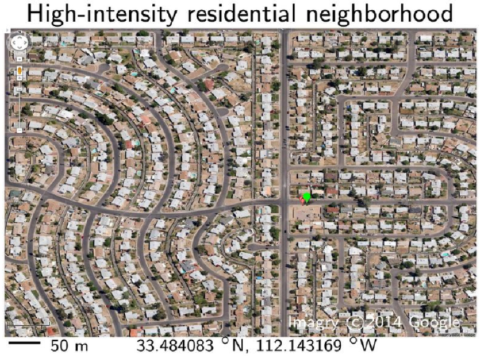
Shaffer et al. 2016

Evaluation of diurnal time-series



Observations from WPHX FT

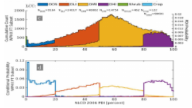
Study period: 17-20 June 2012
1 km resolution domain
Default URBPARM.TBL



cases:

A. Heterogeneous dominant, $f_{urb} = w\,\alpha$
w = mode of $p(\Psi)$
α from $\sum C \in \{DOS, DLI, DMI, DHI\}$
class from max α_C

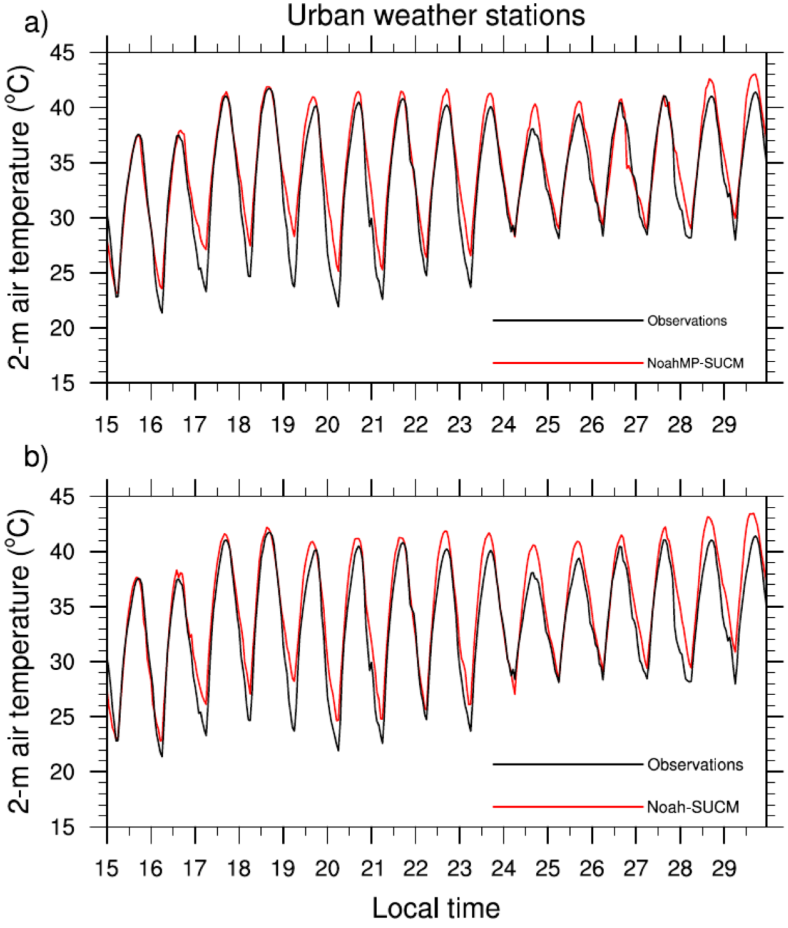
B. Heterogeneous dominant, $f_{urb} = w\,\alpha$
w = mode of $p(\Psi|class)$
α from $\sum C \in \{class\}$
class from max α_C



C. Heterogeneous mosaic
w = mode of $p(\Psi|C\backslash DOS)$
α from $\sum C \in \{class\}$
3 classes (tiles)

Mosaic with urban fraction determined from NLCD 2006
PDI shows good agreement with observations

Shaffer et al. 2016



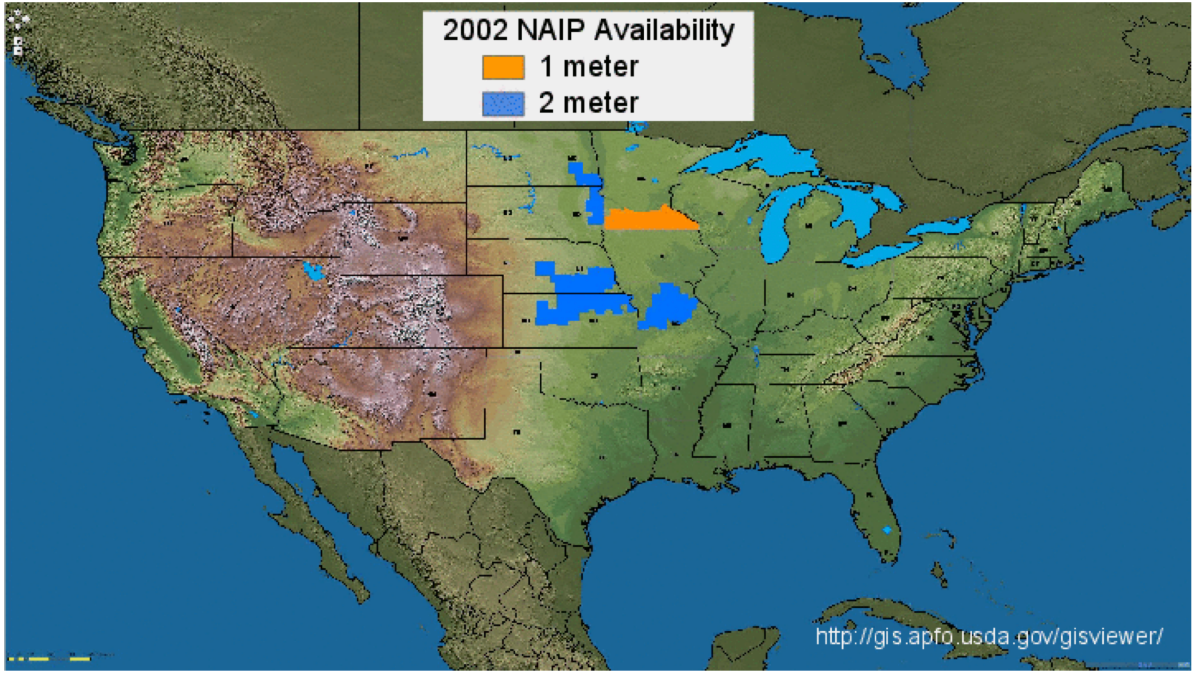
Salamanca et al. 2018 shows similar nighttime temperature bias

- Phoenix 4 urban AZMET stations during 15-day clear-sky June 2012
- Same model configuration as Shaffer et al. 2016 using dominant urban fraction with NLCD data
- Non-urban class set to open shrubland
- Is there bias in the LULC data?

Incorporating high-resolution land cover data to improve mixed agricultural and urban representation

	NAIP	NLCD
resolution	1-2 m	30 m
accuracy	92%	79-84%
availability	1 year	5 years

NAIP = National Agricultural Imagery Program, aerial photos
NLCD = National Land Cover Database, Landsat



Li et al. 2014

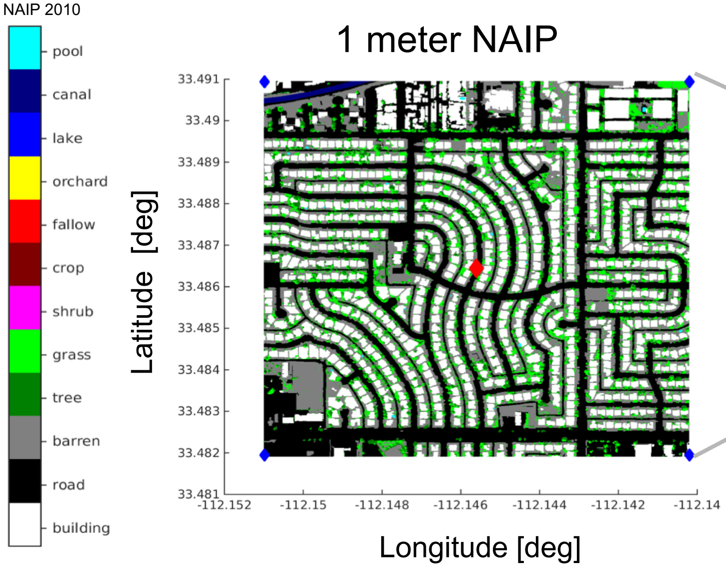
Incorporating high-resolution land cover data to improve mixed agricultural and urban representation

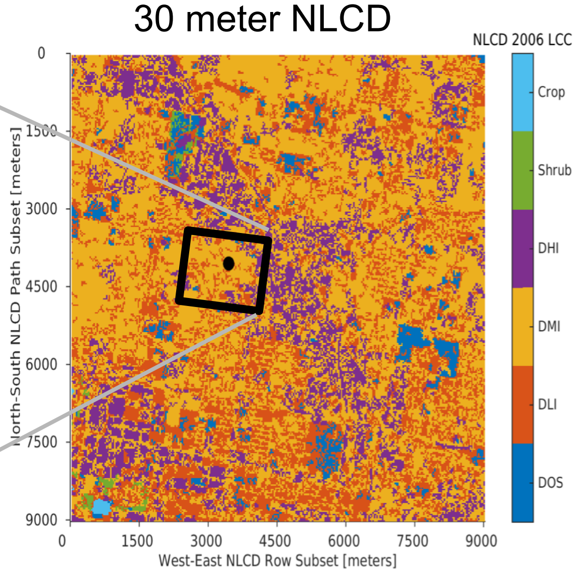
	NAIP	NLCD
resolution	1-2 m	30 m
accuracy	92%	79-84%
availability	1 year	5 years

- **Urban classes in NLCD lose information of non-urban contribution**
- **WRF default ‘natural’ uses ‘cropland/grassland mosaic’, biased**

Q: How to use NAIP within WRF?

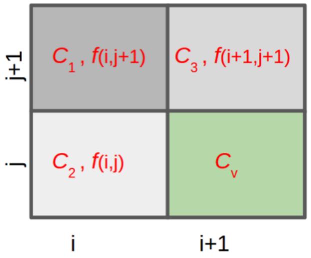
Q: What is model sensitivity to changing input data product?





Shaffer et al. 2016, Shaffer (*Submitted JAMC*), Li et al. 2014

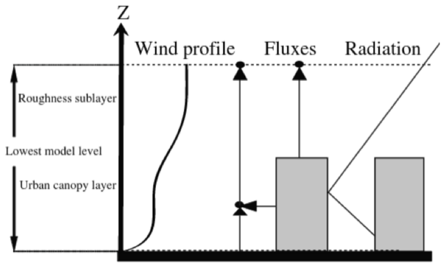
Aggregation in WRF urban canopy and land surface models



$$V_{total} = f_{urban} V_{urban} + (1 - f_{urban}) V_{natural}$$

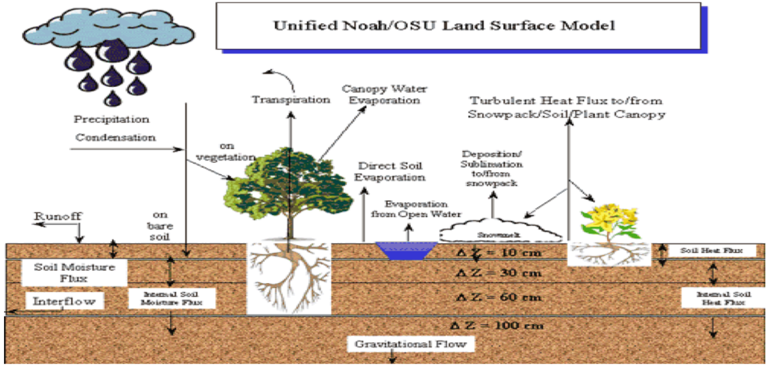
"urban" (SLUCM)

Single Layer Urban Canopy Layer



- Building morphology (shading/trapping), materials (thermal conduction, fluxes)
- Sublayer modifications influence urban boundary layer (momentum, moisture, heat)
- Additional forcings, e.g.,
  - AH, e.g. Sailor and Lu 2004

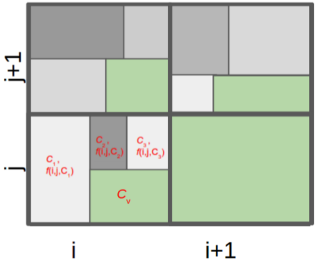
"natural" vegetation (Noah LSM)



- Detailed physical processes of the land surface are linked to observations of land-use and land-cover
- assumes a homogeneous canopy in grid cell/tile

Chen et al. 2011, Chen and Dudhia 2001, Shaffer et al. 2016

Modification to mosaic scheme aggregation



$$V_{total} = f_{urban} V_{urban} + (1 - f_{urban}) V_{natural}$$

Dominant

$$V_{total} = \sum_c \alpha_c V_c$$

Mosaic

The urban class grid-cell total now becomes the urban “tile” contribution.

The urban and natural contributions for urban tiles can be separated:

$$\boxed{V_{\mathrm{total}} = \sum_{c \in \mathscr{C}} \alpha_c V_{\mathrm{non\text{-}urban},c} + \sum_{c \in \mathscr{C}^{\mathrm{urb}}} \alpha_c f_{\mathrm{urb},c} V_{\mathrm{urban},c} + \sum_{c \in \mathscr{C}^{\mathrm{urb}}} \alpha_c (1 - f_{\mathrm{urb},c}) V_{\mathrm{non\text{-}urban},c}}$$

The “natural” class can be combined with it’s respective LCC, via an effective areal fraction:

$$\alpha_{\text{"natural"eff},c} = \alpha_{\text{"natural"},c} + \alpha_{urb,c}(1 - f_{urb,c})$$

Shaffer et al. 2016, Shaffer (*Submitted JAMC*)

Proposed Method

1. Bin: [Figures 2 and 4]
1 m categorical to 30 m percent

Define: [Figures 2]
Impervious = Building + Road

2. Classify at 30 m: [Figure 5]
Urban Classes
Use NLCD definition: [Figures 4, 5]

Partition 30 m percent impervious (Ψ) into NLCD developed classes[3], and assign to WRF urban[6,1] classes:

$$\mathcal{C}^{urb}(\Psi) = \begin{cases} \text{DOS}, \ 0 < \Psi < 20 & \text{, Not used} \\ \text{DLI}, \ 20 \le \Psi \le 49 & \text{, LIR} \\ \text{DMI}, \ 50 \le \Psi \le 79 & \text{, HIR} \\ \text{DHI}, \ 80 \le \Psi \le 100 & \text{, CIT} \end{cases}$$

3. Re-classify at 1 m: [Figure 6]
Assign most probable WRF class to each 1 m NAIP grid cell

4. Aggregate to WRF grid: [Figures 8,10]
Direct from 1 m categorical to 1 km areal fraction per class

Step 1, binning

Figure 2

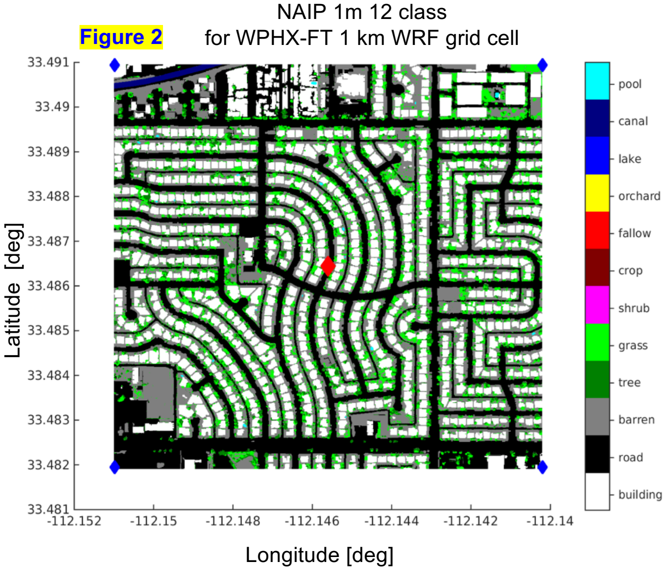
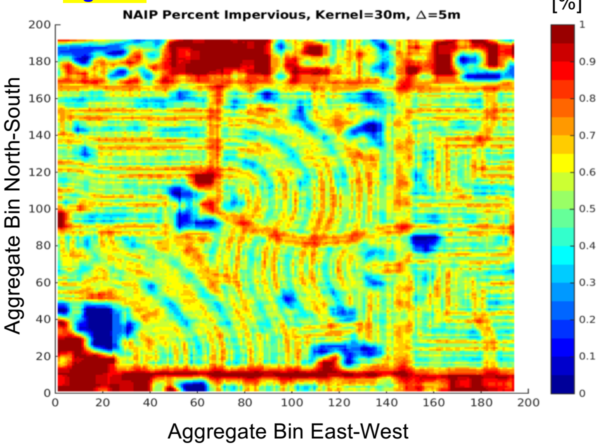


Figure 4



[Figure 4] Percent of NAIP class (here Impervious) at 30 m x 30 m kernel with 5 meter increment (box convolution), 30 m was chosen to mimic NLCD resolution, 5 m increment provides an ensemble for each 1m NAIP grid cell;

Shaffer (*Submitted JAMC*)

Proposed Method

1. **Bin**: [Figures 2 and 4]
   1 m categorical to 30 m percent

   Define: [Figures 2]
   Impervious = Building + Road

2. **Classify at 30 m**: [Figure 5]
   Urban Classes
   Use NLCD definition: [Figures 4, 5]

   Partition 30 m percent impervious (Ψ) into NLCD developed classes[3], and assign to WRF urban[6,1] classes:

$$
c^{urb}(\Psi) = \begin{cases} \text{DOS},\ 0 < \Psi < 20 & \text{, Not used} \\ \text{DLI},\ 20 \le \Psi \le 49 & \text{, LIR} \\ \text{DMI},\ 50 \le \Psi \le 79 & \text{, HIR} \\ \text{DHI},\ 80 \le \Psi \le 100 & \text{, CIT} \end{cases}
$$

3. **Re-classify at 1 m**: [Figure 6]
   Assign most probable WRF class to each 1 m NAIP grid cell

4. **Aggregate to WRF grid**: [Figures 8,10]
   Direct from 1 m categorical to 1 km areal fraction per class

Step 2, Classify at 30 m

Step 3, Re-classify at 1 m

Figure 5

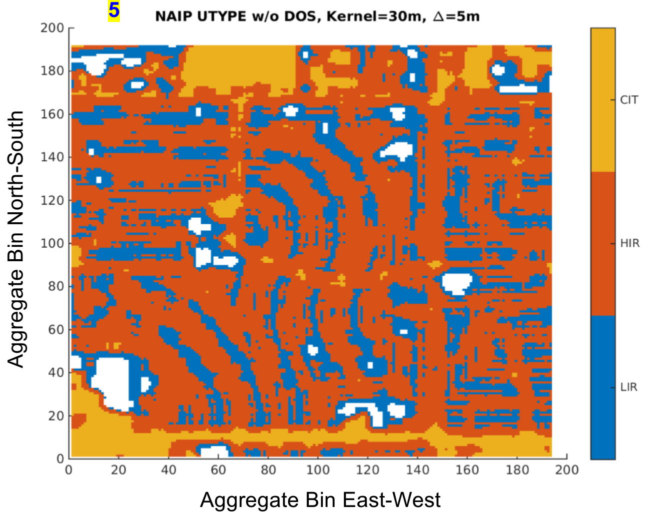
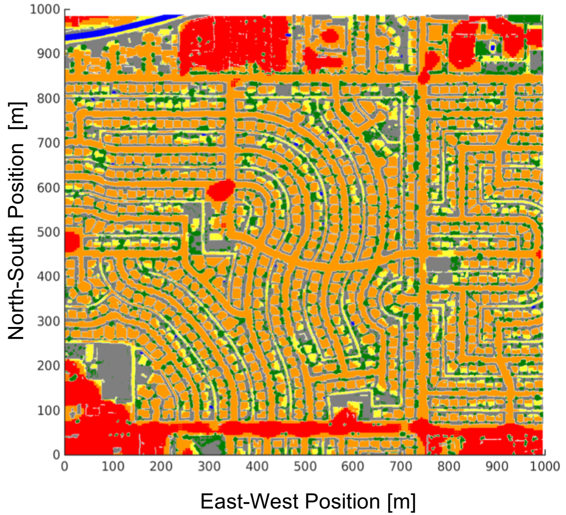


Figure 6

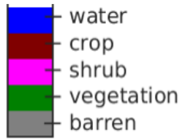


[Figure 5] WRF urban class determined for each aggregate bin in Figure 4, using NLCD definition but not using DOS;

[Figure 6] Identification of WRF urban class for 1m NAIP impervious class (from Figure 3), with remaining NAIP classes merged to lower-level classification[5];

WRF Urban Class

Merged NAIP Class



Shaffer (*Submitted JAMC*)

Proposed Method

Step 4, aggregate to WRF grid

1. Bin: [Figures 2 and 4]
1 m categorical to 30 m percent

Define: [Figures 2]
Impervious = Building + Road

2. Classify at 30 m: [Figure 5]
Urban Classes
Use NLCD definition: [Figures 4, 5]

Partition 30 m percent impervious (Ψ) into NLCD developed classes[3], and assign to WRF urban[6,1] classes:

$$c^{urb}(\Psi) = \begin{cases} \text{DOS}, \; 0 < \Psi < 20 & \text{, Not used} \\ \text{DLI}, \; 20 \le \Psi \le 49 & \text{, LIR} \\ \text{DMI}, \; 50 \le \Psi \le 79 & \text{, HIR} \\ \text{DHI}, \; 80 \le \Psi \le 100 & \text{, CIT} \end{cases}$$

3. Re-classify at 1 m: [Figure 6]
Assign most probable WRF class to each 1 m NAIP grid cell

4. Aggregate to WRF grid: [Figures 8,10]
Direct from 1 m categorical to 1 km areal fraction per class

WRF 1km grid for CAP-LTER study area, zoomed to show Phoenix

[Figure 8] Total (of 3 classes) urban areal fraction with reclassified NAIP.

[Figure 10] Dominant WRF land cover class with urban classes derived from NAIP within the CAP-LTER area. Non-urban classes are derived from 1km MODIS IGBP data.

To evaluate NLCD bias:

- Bin 1m to 30m to emulate NLCD

- Use dominant urban at 30m and bin to 1km

- Prescribe “natural” as open shrubland

- Use effective areal fraction for natural class

Figure 8

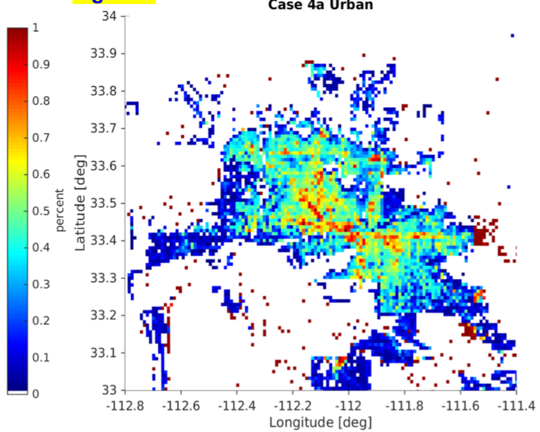
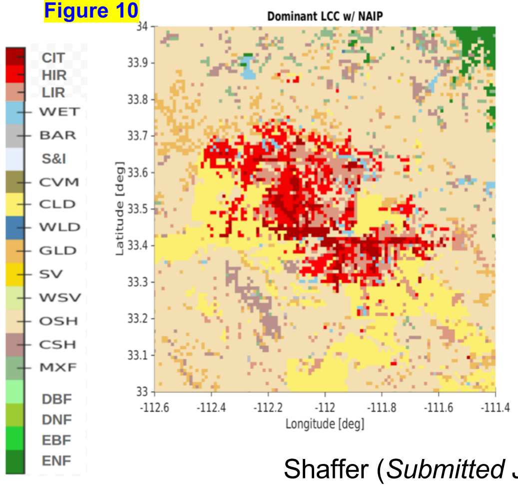


Figure 10



Shaffer (*Submitted JAMC*)

Maps of WRF 1km grid for CAP-LTER study area showing:

[Figure 7 and 8] Total (of 3 classes) urban areal fraction with NLCD, and NAIP, respectively.

[Figure 9 and 10] Dominant WRF land cover class with the NLCD based urban classes, and replaced by NAIP within the CAP-LTER area, respectively. Non-urban classes are derived from 1km MODIS IGBP data.

NLCD based

NAIP based

Figure 7

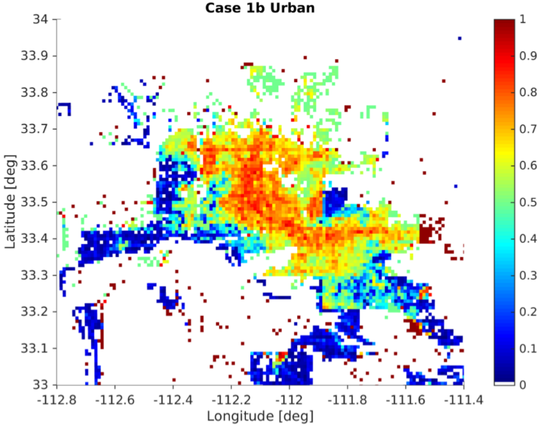


Figure 8

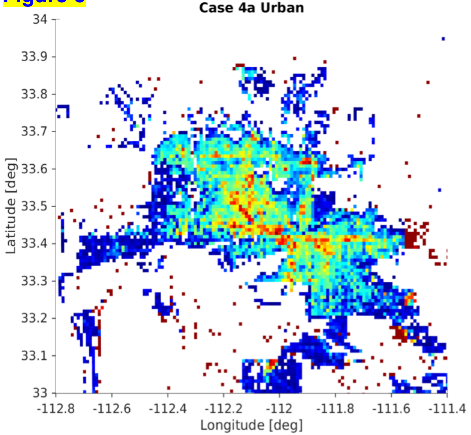


Figure 9

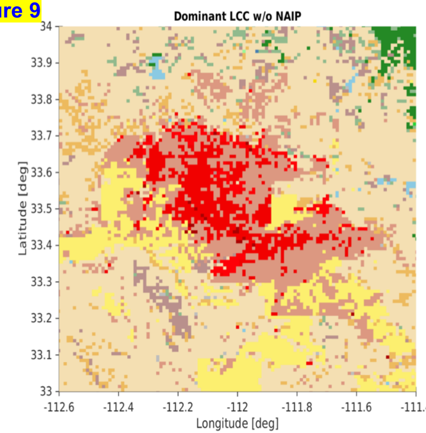
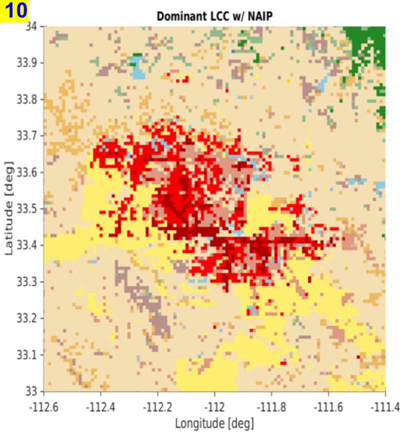


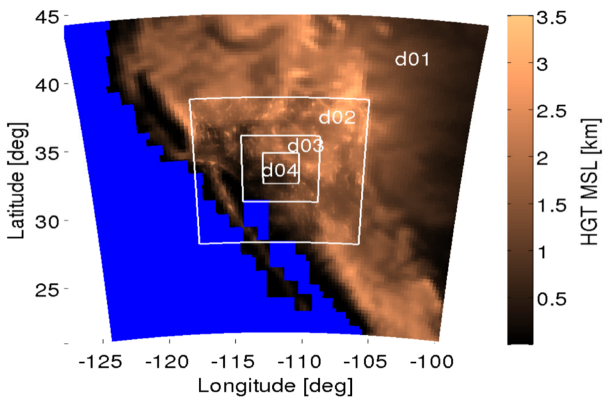
Figure 10

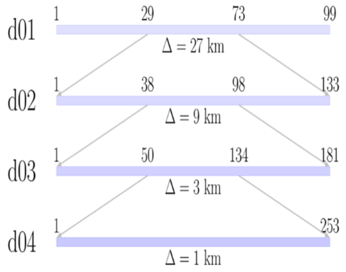


Shaffer (*Submitted JAMC*)

Sensitivity Experiments

WRF-v3.6.1
4 nested domains centered on Phoenix, AZ
d01-d03: Noah + SLUCM 1-way
provide d04 LBC + IC
d04: Noah-mosaic LSM + SLUCM (cases)
Land cover: MODIS + NLCD or NAIP
$f_{urb,c}$= μ(p(Ψ|c)) as per Shaffer et al. 2016





Case	Areal fraction	Urban fraction
1 (1b)	NLCD	WRF defaults
2 (2a)	NLCD	NAIP
3 (3)	NAIP	WRF defaults
4 (4a)	NAIP	NAIP

d01 forcing: NCEP FNL 1°, 6-hr, 26 levels
Microphysics: WSM-3
Radiation: Dudhia + RRTM
PBL-SLS: YSU-MM5

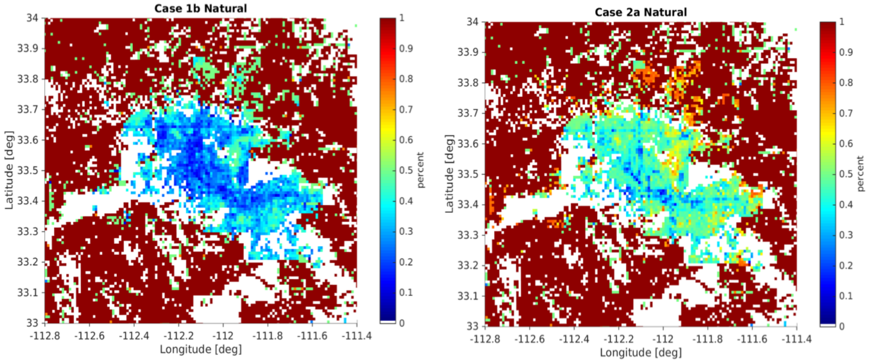
Analysis:
d04 1km
17 June 2012 18Z to 20 June 2012 18Z
5-minute model output
Focus on 2-meter air temperature (for now)
time-averaged for 0000-0500 local time (nighttime)

Shaffer et al. 2015, Shaffer et al. 2016, Shaffer (*Submitted JAMC*)

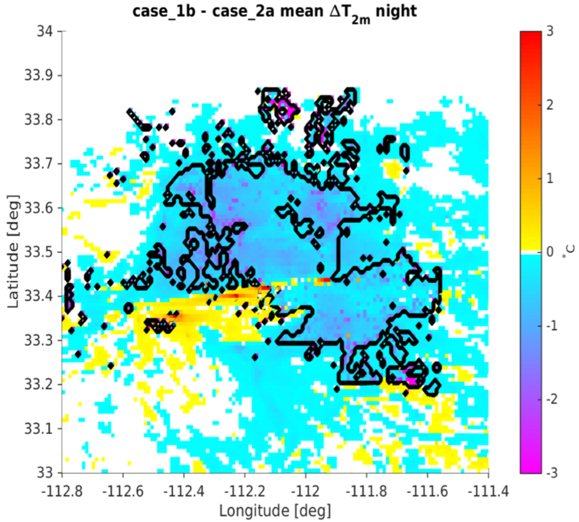
Effect from **urban fraction** derived using NAIP versus default WRF values, with NLCD α

$$\alpha_{\text{"natural"eff},c} = \alpha_{\text{"natural"},c} + \alpha_{urb,c}\underbrace{(1-f_{urb,c})}$$

"natural" class effective areal fraction



Case	Areal fraction	Urban fraction
1 (1b)	NLCD	WRF defaults
2 (2a)	NLCD	NAIP
3 (3)	NAIP	WRF defaults
4 (4a)	NAIP	NAIP

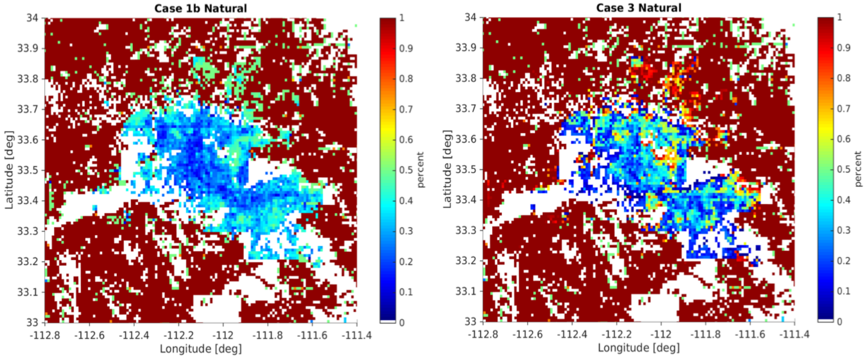


NAIP urban fraction is lower than default WRF, resulting in more "natural" contribution (open shrubland), reducing nighttime temperature by ≈ 1 °C

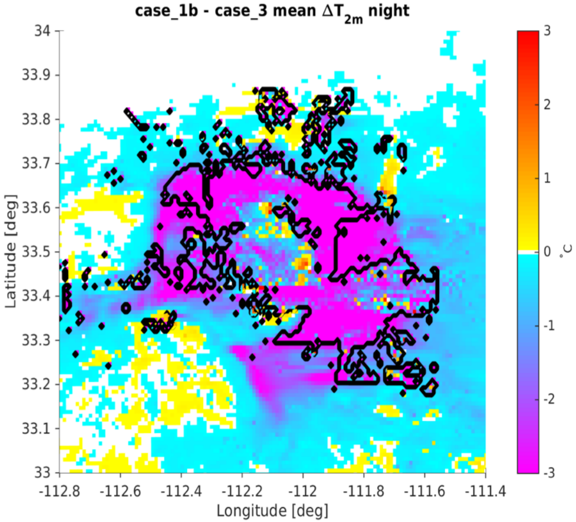
Shaffer (*Submitted JAMC*)

Effect from **areal fraction** derived using NAIP versus NLCD

“natural” class effective areal fraction



$$\alpha_{\text{"natural"eff},c}=\alpha_{\text{"natural"},c}+\underbrace{\alpha_{urb,c}}(1-f_{urb,c})$$



Case	Areal fraction	Urban fraction
1 (1b)	NLCD	WRF defaults
2 (2a)	NLCD	NAIP
3 (3)	NAIP	WRF defaults
4 (4a)	NAIP	NAIP

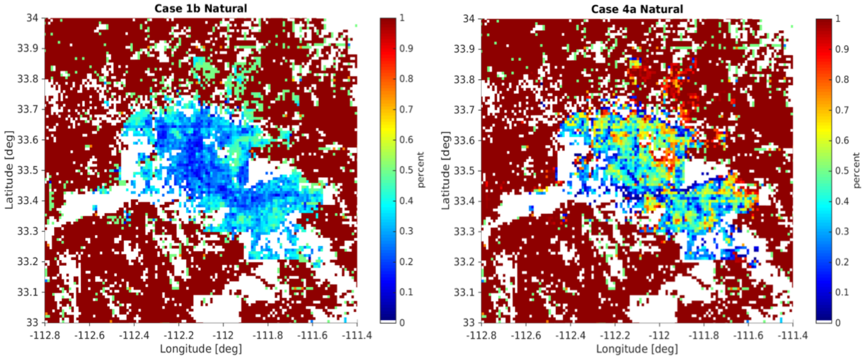
Mixed increase and decrease with NAIP versus NLCD α in urban core has mixed influence on temperature

Lower α rural fringe, shows a ≈1-3 °C decrease

Shaffer (*Submitted JAMC*)

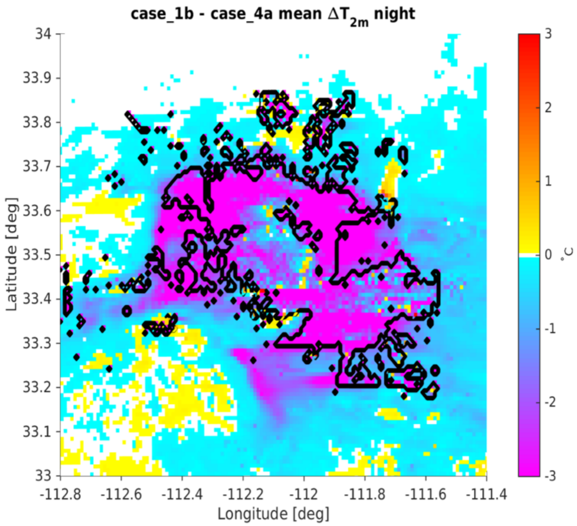
Effect from using NAIP versus NLCD

"natural" class effective areal fraction



Case	Areal fraction	Urban fraction
1 (1b)	NLCD	WRF defaults
2 (2a)	NLCD	NAIP
3 (3)	NAIP	WRF defaults
4 (4a)	NAIP	NAIP

$$\alpha_{\text{"natural"eff},c} = \alpha_{\text{"natural"},c} + \alpha_{urb,c}(1 - f_{urb,c})$$

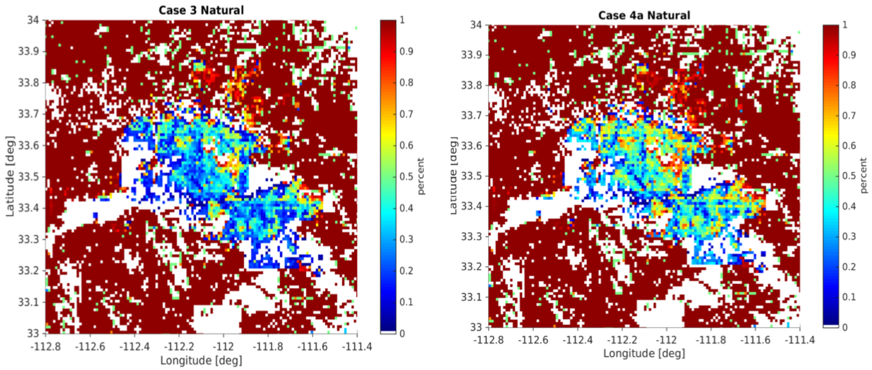


Again, NAIP shows a ≈1-3 °C decrease.

Shaffer (*Submitted JAMC*)

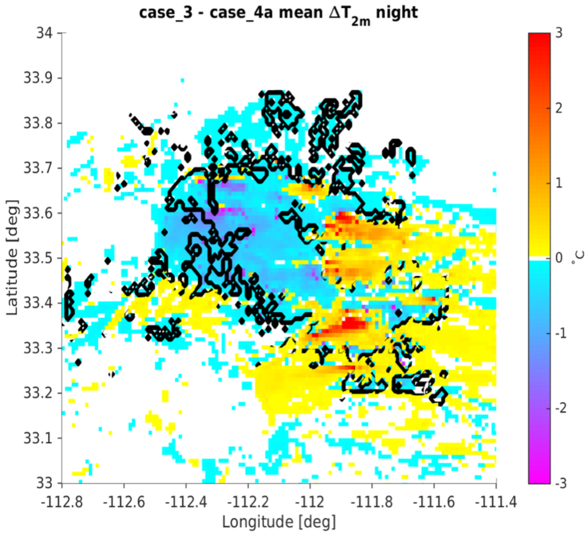
Effect from **urban fraction** derived using NAIP versus default WRF values, with NAIP α

“natural” class effective areal fraction



Case	Areal fraction	Urban fraction
1 (1b)	NLCD	WRF defaults
2 (2a)	NLCD	NAIP
3 (3)	NAIP	WRF defaults
4 (4a)	NAIP	NAIP

$$\alpha_{\text{"natural"eff},c} = \alpha_{\text{"natural"},c} + \alpha_{urb,c}(\underbrace{1-f_{urb,c}})$$



Using WRF derived f_{urb} with NAIP α show ≈ 1-3 °C influence

Shaffer (*Submitted JAMC*)

Conclusions

- The Noah mosaic scheme was adapted for using effective areal fractions

- The proposed method with NAIP allows for direct calculation of areal fractions for the WRF Noah-mosaic land surface model approach, without the need for urban fraction and “natural” class parameters, with NAIP non-urban classes.

- Need to consider appropriateness of NLCD based urban class approach for this resolution data (1 m) within current urban models, where front and back yard may change from LIR to HIR based upon impervious fraction, yet the urban schemes assume unresolved homogeneous canyons.
  - Are the aggregated fluxes correctly parameterized?
  - Are there more appropriate urban class definitions (e.g. Local Climate Zone Stewart and Oke 2012)?

Conclusions

- Pre-monsoon summertime nocturnal 2-meter temperature differences demonstrate sensitivity to input data source, showing that,
    - changing NLCD to NAIP to derive urban classes give a change of ≈ 1 °C owing to smaller urban footprint
    - the reduced urban fraction of NAIP, especially in the rural fringe, shows a ≈1-3 °C change, which remained consistent with changed urban class area
    - using default urban fraction versus from NAIP gives ≈ 1-2 °C change, both increase or decrease depending on development density.

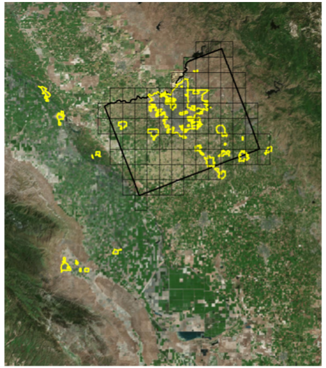
- Comparison with observations will be conducted, in addition to investigating “tuning” of additional urban parameters, and deriving fractional contribution of non-urban classes from NAIP

- In preparation: additional input data products (i.e. Landsat), and classifications (i.e. NLCD40) are being explored. These cases will be evaluated with WRF model predictions versus observations.

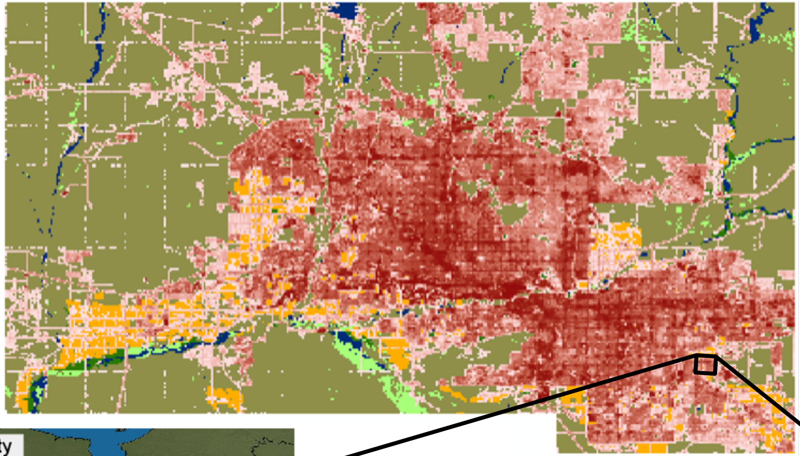
Study areas using National Agricultural Imagery Program data

Multiscale data development for integrated agricultural and urban applications

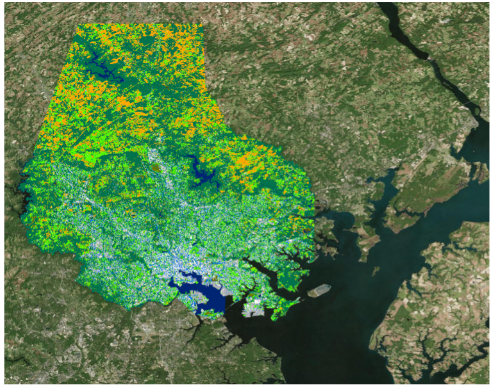
Fresno, CA

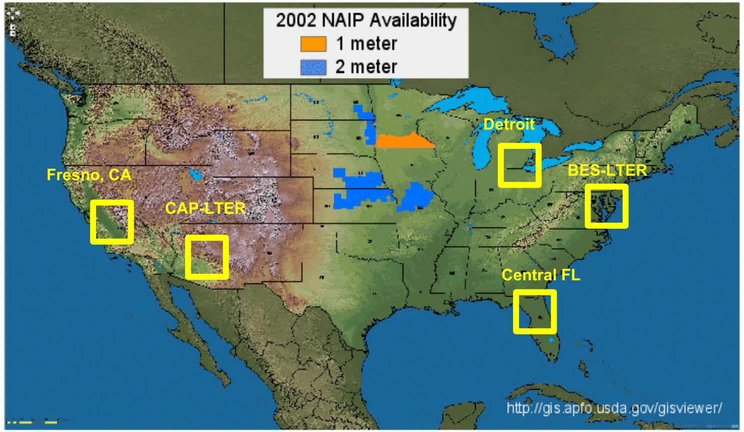


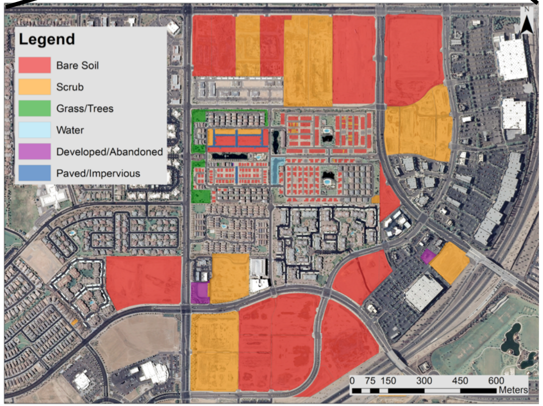
Central Arizona Phoenix LTER



Baltimore Ecosystem Study LTER



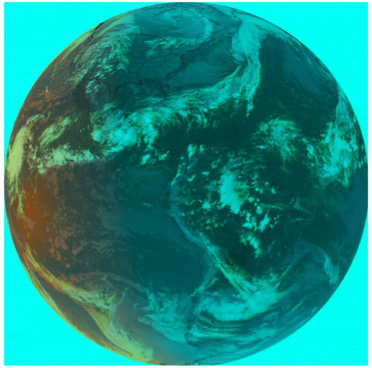


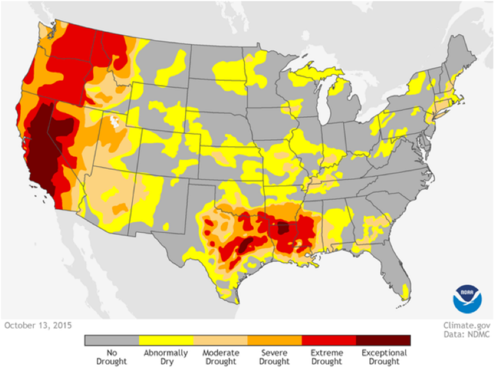


Li et al (*in Prep*), Shaffer and Li (*In Prep*), Smith et al. (2017.), Li et al. (*In Prep*)

Development of the Integrated WRF-Urban-Crop model

- Quantify complex hydro-climate-soil-crop interactions
  - Essential for supporting agricultural management strategies and policy decisions at **multiple scales**:

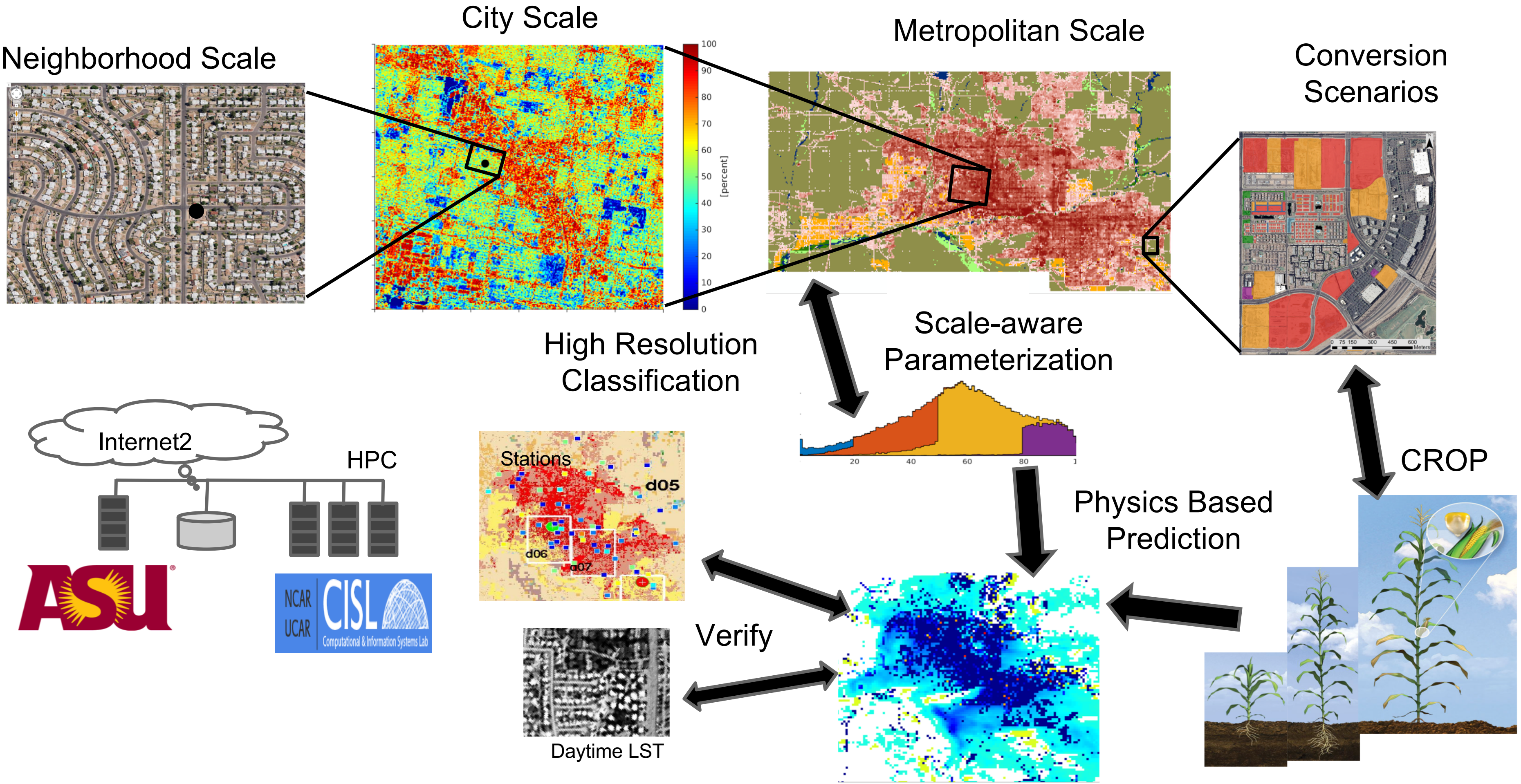




global **continental** **local** farm/urban

- Apply to investigations within mixed developed urban regions.

Image Sources: goes.gsfc.nasa.gov, climate.gov, maps.google.com



Multiscale data development for integrated agricultural and urban applications

Thank you

Photo credit: S.R.Shaffer

Stephen.Shaffer@asu.edu

References

[1] Shaffer et al. 2016, doi:10.1175/JAMC-16-0027.1
[2] Homer et al. 2004, doi:10.14358/PERS.70.7.829.
[3] Yang et al. 2003, doi:10.5589/m02-098.
[4] Shaffer et al. 2015, doi:10.1175/JAMC-D-14-0051.1.
[5] Li et al. 2014, doi:10.1016/j.jag.2014.04.018.
[6] Loridan and Grimmond 2012 doi:10.1002/qj.963
[7] Wickham et al. 2013 doi:10.1016/j.rse.2012.12.001
[8] Friedl et al. 2002 doi:10.1016/S0034-4257(02)00078-0
Shaffer (*Submitted*) Material presented herein

Acknowledgements

We would like to acknowledge high-performance computing support from Cheyenne (doi:10.5065/D6RX99HX) and Yellowstone (ark:/85065/d7wd3xhc) provided by NCAR's Computational and Information Systems Laboratory, sponsored by the National Science Foundation, along with support from ASU Research Computing. This work is supported by grants: NSF DMS-1419593, and USDA-NIFA 2015-67003-23508.

Additional slides

Assessment of Subgrid Land Cover Entropy

- Observations of PDI: $\Psi(\xi)$, ξ fine-grid; $\mathcal{X}$ aggregated grid, with resolution $\Delta_\xi \ll \Delta_\mathcal{X}$ and $\psi \in [1, 2, \ldots, 100]$ having N_s states
- Probability density function, $p_{i,j}(\Psi|_{\xi \in \mathcal{X}_{i,j}})$, of observations $\Psi(\xi)$ within $\mathcal{X}_{i,j}$, i.e. $\forall \xi \in \mathcal{X}_{i,j}$
- Normalized Shannon Entropy h

$$h = \frac{-1}{\log N_s}\left[\sum_{\Psi} p(\Psi) \log\left(p(\Psi)\right)\right] \in [0, 1]. \qquad (1)$$

- h depends explicitly upon $\mathcal{X}$, with $\Delta_\mathcal{X}$ and positioning of $\mathcal{X}$ with respect to ξ as implicit parameters that will influence the partitioning and aggregation of Ψ.
- Small h indicate $\Psi|_{\xi \in \mathcal{X}_{i,j}}$ has a distribution near the mode of $p_{i,j}$, whereas h=1 indicates that each state is equally likely.

Shaffer et al. 2016

Assessment of Subgrid Land Cover Entropy

- Categorical observations $\mathcal{C}^{\mathrm{urb}}(\xi)$
- Conditional density function $p(\Psi|\mathcal{C}^{\mathrm{urb}})$ for each $\mathcal{C}^{\mathrm{urb}}$
- Conditional NSE,

$$h_{\mathcal{C}^{\mathrm{urb}}} = h(p(\Psi|\mathcal{C}^{\mathrm{urb}})). \tag{2}$$

- Use for assessing distributions of Ψ for each $\mathcal{C}^{\mathrm{urb}}$
- In the context of mosaic partitioning of the grid-cell for different ULCC, $\mathcal{C}^{\mathrm{urb}}$, more classes are needed for h near 1.

Shaffer et al. 2016

Proposed $f_{\mathrm{urb}}^{\mathrm{H}}$ approach for mosaic $\mathcal{C}^{\mathrm{urb}}$

Define:

$$f_{\mathrm{urb}}^{\mathrm{H}}(\mathcal{X}_{i,j}, \mathcal{C}^{\mathrm{urb}}) = w(\mathcal{X}_{i,j}, \mathcal{C}^{\mathrm{urb}}), \forall \xi \in \mathcal{X}|_{i,j}$$

where $w(\mathcal{X}, \mathcal{C}^{\mathrm{urb}})$ determined as the most probable value (i.e. mode) of $p(\Psi|\mathcal{C}^{\mathrm{urb}})$:

$$w(\mathcal{X}, \mathcal{C}^{\mathrm{urb}}) = \max_{\psi \in \Psi} p(\psi(\xi \in \mathcal{X})|\mathcal{C}^{\mathrm{urb}}(\xi \in \mathcal{X}))$$

- $f_{\mathrm{urb}}^{\mathrm{H}}$ for single dominant $\mathcal{C}^{\mathrm{urb}}$
- or, mosaic, with $h_{\mathcal{C}^{\mathrm{urb}}}$ reduced from h
- Requires additional categorical data or partitioning scheme

Shaffer et al. 2016