A Description of the Advanced Research WRF Version 2

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Chapter 1

Introduction

The development of the Weather Research and Forecasting (WRF) modeling system is a multi-agency effort intended to provide a next-generation mesoscale forecast model and data assimilation system that will advance both the understanding and prediction of mesoscale weather and accelerate the transfer of research advances into operations. The model is being developed as a collaborative effort among the NCAR Mesoscale and Microscale Meteorology (MMM) Division, the National Oceanic and Atmospheric Administration's (NOAA) National Centers for Environmental Prediction (NCEP) and Forecast System Laboratory (FSL), the Department of Defense's Air Force Weather Agency (AFWA) and Naval Research Laboratory (NRL), the Center for Analysis and Prediction of Storms (CAPS) at the University of Oklahoma, and the Federal Aviation Administration (FAA), along with the participation of a number of university scientists.

The WRF model is designed to be a flexible, state-of-the-art, portable code that is efficient in a massively parallel computing environment. A modular single-source code is maintained that can be configured for both research and operations. It offers numerous physics options, thus tapping into the experience of the broad modeling community. Advanced data assimilation systems are being developed and tested in tandem with the model. WRF is maintained and supported as a community model to facilitate wide use, particularly for research and teaching, in the university community. It is suitable for use in a broad spectrum of applications across scales ranging from meters to thousands of kilometers. Such applications include research and operational numerical weather prediction (NWP), data assimilation and parameterized-physics research, downscaling climate simulations, driving air quality models, atmosphere-ocean coupling, and idealized simulations (e.g boundary-layer eddies, convection, baroclinic waves). With WRF as a common tool in the university and operational centers, closer ties will be promoted between these communities, and research advances will have a direct path to operations. These hallmarks make the WRF modeling system unique in the history of NWP in the United States.

The principal components of the WRF system are depicted in Figure 1.1. The WRF Software Framework (WSF) provides the infrastructure that accommodates multiple dynamics solvers, physics packages that plug into the solvers through a standard physics interface, programs for initialization, and the WRF variational data assimilation (WRF-Var) system. As of this writing there are two dynamics solvers in the WSF: the Advanced Research WRF (ARW) solver (originally referred to as the Eulerian mass or "em" solver) developed primarily at NCAR, and the NMM (Nonhydrostatic Mesoscale Model) solver developed at NCEP, which will be documented and supported to the community by the Developmental Testbed Center (DTC).
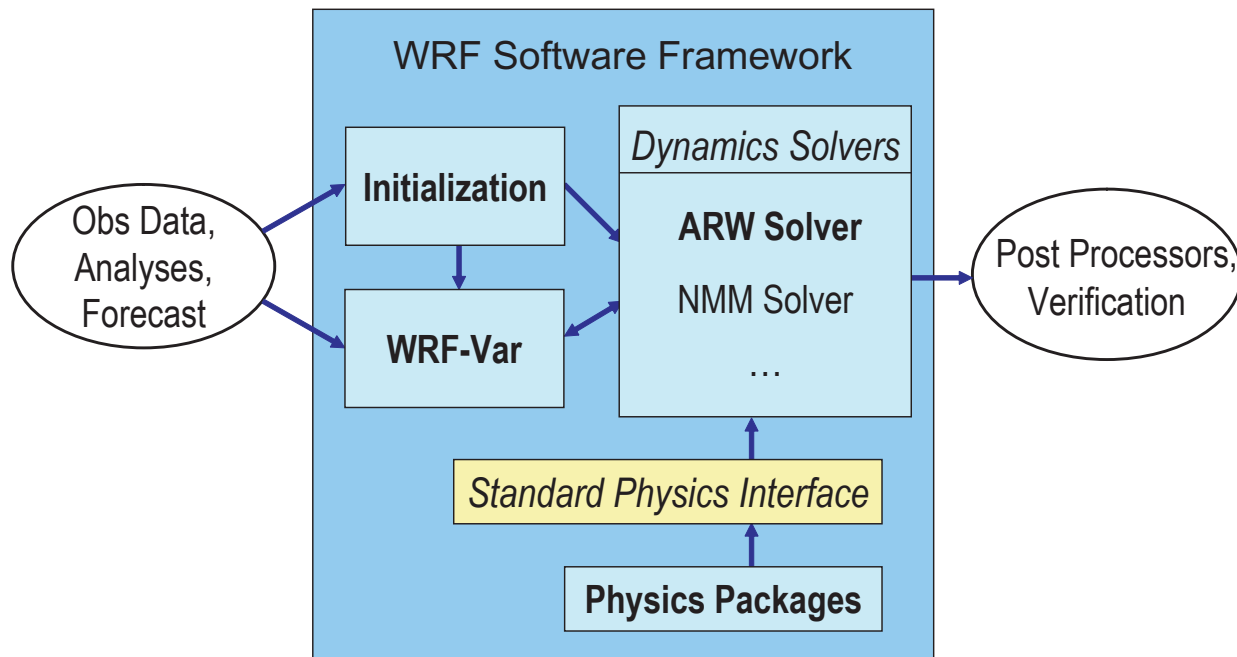
Figure 1.1: WRF system components.

While there are multiple solvers, and while not all physics are available to both solvers, the WSF is common to all components.

1.1   Advanced Research WRF

The ARW system consists of the ARW dynamics solver together with other components of the WRF system needed to produce a simulation. Thus, it also encompasses physics schemes, initialization routines, and a data assimilation package. The ARW shares the WSF, the framework common to all WRF modeling system components, including the NMM solver. Similarly, the physics packages are available to both the ARW and NMM solvers. In this vein, it should be understood that the association of a component of the WRF system with the ARW does not preclude it from being a component of any other WRF configuration. The following section highlights the major features of the ARW system, Version 2, and reflects elements of WRF Version 2, which was first released in May 2004.

   This technical note will focus on the scientific and algorithmic approaches in the ARW. Discussed are the ARW solver, available physics options, initialization capabilities, boundary conditions, and grid-nesting techniques. The WSF provides the software infrastructure for all WRF configurations and is documented separately (Michalakes et al., 1999, 2004). The WRF-Var program, a component of the broader WRF system, has been adapted from MM5 3DVAR (Barker et al., 2004) and is encompassed within the ARW. As a separate document detailing the broader WRF-Var system will be forthcoming, this technical note will focus on a summary of the changes and updates implemented to adapt this data assimilation capability from the MM5 to WRF. For those seeking information on running the ARW modeling system, details on its use are contained in the ARW User's Guide (Wang et al., 2004).

1.2 Major Features of the ARW System

ARW Solver

- *Equations:* Fully compressible, Euler nonhydrostatic with a run-time hydrostatic option available. Conservative for scalar variables.
- *Prognostic Variables:* Velocity components u and v in Cartesian coordinate, vertical velocity w, perturbation potential temperature, perturbation geopotential, and perturbation surface pressure of dry air. Optionally, turbulent kinetic energy and any number of scalars such as water vapor mixing ratio, rain/snow mixing ratio, and cloud water/ice mixing ratio.
- *Vertical Coordinate:* Terrain-following hydrostatic-pressure, with vertical grid stretching permitted. Top of the model is a constant pressure surface.
- *Horizontal Grid:* Arakawa C-grid staggering.
- *Time Integration:* Time-split integration using a 3rd order Runge-Kutta scheme with smaller time step for acoustic and gravity-wave modes.
- *Spatial Discretization:* 2nd to 6th order advection options in horizontal and vertical.
- *Turbulent Mixing and Model Filters:* Sub-grid scale turbulence formulation in both coordinate and physical space. Divergence damping, external-mode filtering, vertically implicit acoustic step off-centering. Explicit filter option also available.
- *Initial Conditions:* Three dimensional for real-data, and one-, two- and three-dimensional using idealized data. A number of test cases are provided.
- *Lateral Boundary Conditions:* Periodic, open, symmetric, and specified options available.
- *Top Boundary Conditions:* Gravity wave absorbing (diffusion or Rayleigh damping). $w = 0$ top boundary condition at constant pressure level.
- *Bottom Boundary Conditions:* Physical or free-slip.
- *Earth's Rotation:* Full Coriolis terms included.
- *Mapping to Sphere:* Three map projections are supported for real-data simulation: polar stereographic, Lambert-conformal, and Mercator. Curvature terms included.
- *Nesting:* One-way, two-way, and moving nests.

Model Physics

- *Microphysics:* Bulk schemes ranging from simplified physics suitable for mesoscale modeling to sophisticated mixed-phase physics suitable for cloud-resolving modeling.
- *Cumulus parameterizations:* Adjustment and mass-flux schemes for mesoscale modeling including NWP.
- *Surface physics:* Multi-layer land surface models ranging from a simple thermal model to full vegetation and soil moisture models, including snow cover and sea ice.
- *Planetary boundary layer physics:* Turbulent kinetic energy prediction or non-local K schemes.
- *Atmospheric radiation physics:* Longwave and shortwave schemes with multiple spectral bands and a simple shortwave scheme. Cloud effects and surface fluxes are included.

WRF-Var System

- Incremental formulation of the model-space cost function.
- Quasi-Newton or conjugate gradient minimization algorithms.
- Analysis increments on un-staggered Arakawa-A grid.
- Representation of the horizontal component of background error $\mathbf{B}$ via recursive filters (regional) or power spectra (global). The vertical component is applied through projection onto climatologically-averaged eigenvectors of vertical error. Horizontal/vertical errors are non-separable (horizontal scales vary with vertical eigenvector).
- Background cost function (J_b) preconditioning via a control variable transform U defined as $\mathbf{B} = \mathrm{UU}^T$.
- Flexible choice of background error model and control variables.
- Climatological background error covariances estimated via either the NMC-method of averaged forecast differences or suitably averaged ensemble perturbations.
- Unified 3D-Var (4D-Var under development), global and regional, multi-model capability.

WRF Software Framework

- Highly modular, single-source code for maintainability.
- Portable across a range of available computing platforms.
- Support for multiple dynamics solvers and physics modules.
- Separation of scientific codes from parallelization and other architecture-specific codes.
- Input/Output Application Program Interface (API) enabling various external packages to be installed with WRF, hence allowing WRF to easily support various data formats.
- Efficient execution on a range of computing platforms (distributed and shared memory, vector and scalar types).
- Use of Earth System Modeling Framework (ESMF) timing package.
- Model coupling API enabling WRF to be coupled with other models such as ocean, and land models.

Chapter 2

Governing Equations

The ARW dynamics solver integrates the compressible, nonhydrostatic Euler equations. The equations are cast in flux form using variables that have conservation properties, following the philosophy of Ooyama (1990). The equations are formulated using a terrain-following mass vertical coordinate (Laprise, 1992). In this chapter we define the vertical coordinate and present the flux form equations in Cartesian space, we extend the equations to include the effects of moisture in the atmosphere, and we further augment the equations to include projections to the sphere.

2.1   Vertical Coordinate and Variables

The ARW equations are formulated using a terrain-following hydrostatic-pressure vertical coordinate denoted by η and defined as

$$\eta = (p_h - p_{ht})/\mu \qquad \text{where} \quad \mu = p_{hs} - p_{ht}. \quad (2.1)$$

p_h is the hydrostatic component of the pressure, and p_{hs} and p_{ht} refer to values along the surface and top boundaries, respectively. The coordinate definition (2.1), proposed by Laprise (1992), is the traditional σ coordinate used in many hydrostatic atmospheric models. η varies from a value of 1 at the surface to 0 at the upper boundary of the model domain (Fig. 2.1). This vertical coordinate is also called a mass vertical coordinate.

Since $\mu(x,y)$ represents the mass per unit area within the column in the model domain at (x,y), the appropriate flux form variables are

$$\mathbf{V} = \mu\mathbf{v} = (U,V,W), \quad \Omega = \mu\dot{\eta}, \quad \Theta = \mu\theta. \quad (2.2)$$

$\mathbf{v} = (u,v,w)$ are the covariant velocities in the two horizontal and vertical directions, respectively, while $\omega = \dot{\eta}$ is the contravariant 'vertical'

Figure 2.1: ARW η coordinate.

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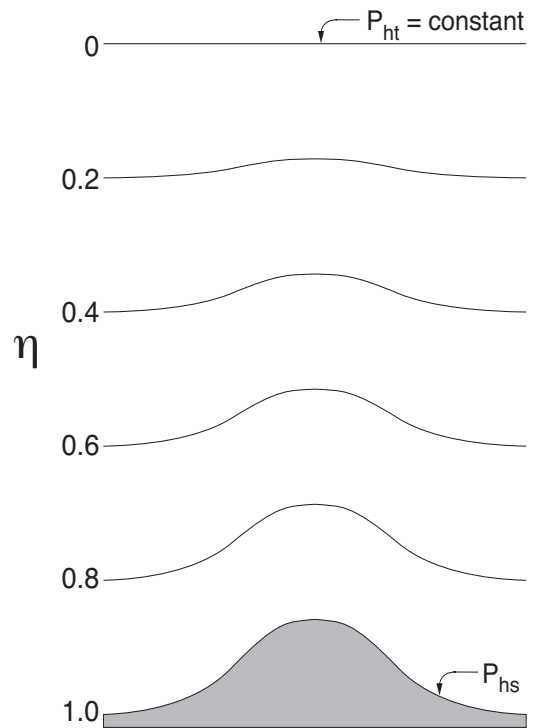
velocity. θ is the potential temperature. Also appearing in the governing equations of the ARW are the non-conserved variables $\phi = gz$ (the geopotential), p (pressure), and $\alpha = 1/\rho$ (the inverse density).

2.2   Flux-Form Euler Equations

Using the variables defined above, the flux-form Euler equations can be written as

$$\partial_t U + (\nabla \cdot \mathbf{V} u) - \partial_x(p\phi_\eta) + \partial_\eta(p\phi_x) = F_U \tag{2.3}$$
$$\partial_t V + (\nabla \cdot \mathbf{V} v) - \partial_y(p\phi_\eta) + \partial_\eta(p\phi_y) = F_V \tag{2.4}$$
$$\partial_t W + (\nabla \cdot \mathbf{V} w) - g(\partial_\eta p - \mu) = F_W \tag{2.5}$$
$$\partial_t \Theta + (\nabla \cdot \mathbf{V} \theta) = F_\Theta \tag{2.6}$$
$$\partial_t \mu + (\nabla \cdot \mathbf{V}) = 0 \tag{2.7}$$
$$\partial_t \phi + \mu^{-1}[(\mathbf{V} \cdot \nabla \phi) - gW] = 0 \tag{2.8}$$

along with the diagnostic relation for the inverse density

$$\partial_\eta \phi = -\alpha\mu, \tag{2.9}$$

and the equation of state

$$p = p_0(R_d\theta/p_0\alpha)^\gamma. \tag{2.10}$$

In $(2.3) - (2.10)$, the subscripts x, y and η denote differentiation,

$$\nabla \cdot \mathbf{V} a = \partial_x(Ua) + \partial_y(Va) + \partial_\eta(\Omega a),$$

and

$$\mathbf{V} \cdot \nabla a = U\partial_x a + V\partial_y a + \Omega\partial_\eta a,$$

where a represents a generic variable. $\gamma = c_p/c_v = 1.4$ is the ratio of the heat capacities for dry air, R_d is the gas constant for dry air, and p_0 is a reference pressure (typically 10^5 Pascals). The right-hand-side (RHS) terms F_U, F_V, F_W, and F_Θ represent forcing terms arising from model physics, turbulent mixing, spherical projections, and the earth's rotation.

The prognostic equations $(2.3) - (2.8)$ are cast in conservative form except for (2.8) which is the material derivative of the definition of the geopotential. (2.8) could be cast in flux form but we find no advantage in doing so since $\mu\phi$ is not a conserved quantity. We could also use a prognostic pressure equation in place of (2.8) (see Laprise, 1992), but pressure is not a conserved variable and we could not use a pressure equation and the conservation equation for Θ (2.6) because they are linearly dependent. Additionally, prognostic pressure equations have the disadvantage of possessing a mass divergence term multiplied by a large coefficient (proportional to the sound speed) which makes spatial and temporal discretization problematic. It should be noted that the relation for the hydrostatic balance (2.9) does not represent a constraint on the solution, rather it is a diagnostic relation that formally is part of the coordinate definition. In the hydrostatic counterpart to the nonhydrostatic equations, (2.9) replaces the vertical momentum equation (2.5) and it becomes a constraint on the solution.

2.3   Inclusion of Moisture

In formulating the moist Euler equations, we retain the coupling of dry air mass to the prognostic variables, and we retain the conservation equation for dry air (2.7), as opposed to coupling the variables to the full (moist) air mass and hence introducing source terms in the mass conservation equation (2.7). Additionally, we define the coordinate with respect to the dry-air mass. Based on these principles, the vertical coordinate can be written as

$$\eta = (p_{dh} - p_{dht})/\mu_d \tag{2.11}$$

where μ_d represents the mass of the dry air in the column and p_{dh} and p_{dht} represent the hydrostatic pressure of the dry atmosphere and the hydrostatic pressure at the top of the dry atmosphere. The coupled variables are defined as

$$\mathbf{V} = \mu_d\mathbf{v}, \quad \Omega = \mu_d\dot{\eta}, \quad \Theta = \mu_d\theta. \tag{2.12}$$

With these definitions, the moist Euler equations can be written as

$$\partial_t U + (\nabla \cdot \mathbf{V}u)_\eta + \mu_d\alpha\partial_x p + (\alpha/\alpha_d)\partial_\eta p\partial_x\phi = F_U \tag{2.13}$$

$$\partial_t V + (\nabla \cdot \mathbf{V}v)_\eta + \mu_d\alpha\partial_y p + (\alpha/\alpha_d)\partial_\eta p\partial_y\phi = F_V \tag{2.14}$$

$$\partial_t W + (\nabla \cdot \mathbf{V}w)_\eta - g[(\alpha/\alpha_d)\partial_\eta p - \mu_d] = F_W \tag{2.15}$$

$$\partial_t \Theta + (\nabla \cdot \mathbf{V}\theta)_\eta = F_\Theta \tag{2.16}$$

$$\partial_t \mu_d + (\nabla \cdot \mathbf{V})_\eta = 0 \tag{2.17}$$

$$\partial_t \phi + \mu_d^{-1}[(\mathbf{V} \cdot \nabla\phi)_\eta - gW] = 0 \tag{2.18}$$

$$\partial_t Q_m + (\mathbf{V} \cdot \nabla q_m)_\eta = F_{Q_m} \tag{2.19}$$

with the diagnostic equation for dry inverse density

$$\partial_\eta\phi = -\alpha_d\mu_d \tag{2.20}$$

and the diagnostic relation for the full pressure (vapor plus dry air)

$$p = p_0(R_d\theta_m/p_0\alpha_d)^\gamma \tag{2.21}$$

In these equations, α_d is the inverse density of the dry air $(1/\rho_d)$ and α is the inverse density taking into account the full parcel density $\alpha = \alpha_d(1 + q_v + q_c + q_r + q_i + ...)^{-1}$ where q_* are the mixing ratios (mass per mass of dry air) for water vapor, cloud, rain, ice, etc. Additionally, $\theta_m = \theta(1 + (R_v/R_d)q_v) \approx \theta(1 + 1.61q_v)$, and $Q_m = \mu_d q_m$; $q_m = q_v, q_c, q_i, ...$.

2.4   Map Projections, Coriolis and Curvature Terms

The ARW solver currently supports three projections to the sphere— the Lambert conformal, polar stereographic, and Mercator projections. These projections are described in Haltiner and Williams (1980). These projections, and the ARW implementation of the map factors, assume that the map factor transformations for x are y are identical at a given point; that is, the

transformation is isotropic. Anisotropic transformations, such as a latitude-longitude grid, can be accommodated by defining separate map factors for the x and y transformations.

In the ARW's computational space, Δx and Δy are constants. Orthogonal projections to the sphere require that the physical distances between grid points in the projection vary with position in the grid. To transform the governing equations, a map scale factor m is defined as the ratio of the distance in computational space to the corresponding distance on the earth's surface:

$$m = \frac{(\Delta x, \Delta y)}{\text{distance on the earth}}. \tag{2.22}$$

The ARW solver includes the map-scale factors in the governing equations by redefining the momentum variables as

$$U = \mu_d u/m, \quad V = \mu_d v/m, \quad W = \mu_d w/m, \quad \Omega = \mu_d \dot{\eta}/m.$$

Using these redefined momentum variables, the governing equations, including map factors and rotational terms, can be written as

$$\partial_t U + m[\partial_x(Uu) + \partial_y(Vu)] + \partial_\eta(\Omega u) + \mu_d \alpha \partial_x p + (\alpha/\alpha_d)\partial_\eta p \partial_x \phi = F_U \tag{2.23}$$

$$\partial_t V + m[\partial_x(Uv) + \partial_y(Vv)] + \partial_\eta(\Omega v) + \mu_d \alpha \partial_y p + (\alpha/\alpha_d)\partial_\eta p \partial_y \phi = F_V \tag{2.24}$$

$$\partial_t W + m[\partial_x(Uw) + \partial_y(Vw)] + \partial_\eta(\Omega w) - m^{-1}g[(\alpha/\alpha_d)\partial_\eta p - \mu_d] = F_W \tag{2.25}$$

$$\partial_t \Theta + m^2[\partial_x(U\theta) + \partial_y(V\theta)] + m\partial_\eta(\Omega\theta) = F_\Theta \tag{2.26}$$

$$\partial_t \mu_d + m^2[U_x + V_y] + m\partial_\eta(\Omega) = 0 \tag{2.27}$$

$$\partial_t \phi + \mu_d^{-1}[m^2(U\phi_x + V\phi_y) + m\Omega\phi_\eta - gW] = 0 \tag{2.28}$$

$$\partial_t Q_m + m^2[\partial_x(Uq_m) + \partial_y(Vq_m)] + m\partial_\eta(\Omega q_m) = F_{Q_m}, \tag{2.29}$$

and, for completeness, the diagnostic relation for the dry inverse density

$$\partial_\eta \phi = -\alpha_d \mu_d, \tag{2.30}$$

and the diagnostic equation for full pressure (vapor plus dry air)

$$p = p_0(R_d \theta_m/p_0 \alpha_d)^\gamma. \tag{2.31}$$

The right-hand-side terms of the momentum equations (2.23) – (2.25) contain the Coriolis and curvature terms along with mixing terms and physical forcings. The Coriolis and curvature terms can be written as follows:

$$F_{U_{cor}} = +\left(f + u\frac{\partial m}{\partial y} - v\frac{\partial m}{\partial x}\right)V - eW\cos\alpha_r - \frac{uW}{r_e} \tag{2.32}$$

$$F_{V_{cor}} = -\left(f + u\frac{\partial m}{\partial y} - v\frac{\partial m}{\partial x}\right)U + eW\sin\alpha_r - \frac{vW}{r_e} \tag{2.33}$$

$$F_{W_{cor}} = +e(U\cos\alpha_r - V\sin\alpha_r) + \left(\frac{uU + vV}{r_e}\right), \tag{2.34}$$

where α_r is the local rotation angle between the y-axis and the meridians, ψ is the latitude, $f = 2\Omega_e \sin\psi$, $e = 2\Omega_e \cos\psi$, Ω_e is the angular rotation rate of the earth, and r_e is the radius of

the earth. In this formulation we have approximated the radial distance from the center of the earth as the mean earth radius r_e, and we have not taken into account the change in horizontal grid distance as a function of the radius. The terms containing m are the horizontal curvature terms, those containing r_e relate to vertical (earth-surface) curvature, and those with e and f are the Coriolis force.

In idealized cases, the map scale factor $m = 1$, f is often taken to be constant, and $e = 0$.

2.5   Perturbation Form of the Governing Equations

Before constructing the discrete solver, it is advantageous to recast the governing equations using perturbation variables so as to reduce truncation errors in the horizontal pressure gradient calculations in the discrete solver, in addition to reducing machine rounding errors in the vertical pressure gradient and buoyancy calculations. For this purpose, new variables are defined as perturbations from a hydrostatically-balanced reference state, and we define reference state variables (denoted by overbars) that are a function of height only and that satisfy the governing equations for an atmosphere at rest. That is, the reference state is in hydrostatic balance and is strictly only a function of z. In this manner, $p = \bar{p}(z) + p'$, $\phi = \bar{\phi}(z) + \phi'$, $\alpha = \bar{\alpha}(z) + \alpha'$, and $\mu_d = \bar{\mu}_d(x, y) + \mu'_d$. Because the η coordinate surfaces are generally not horizontal, the reference profiles $\bar{p}$, $\bar{\phi}$, and $\bar{\alpha}$ are functions of (x, y, η). The hydrostatically balanced portion of the pressure gradients in the reference sounding can be removed without approximation to the equations using these perturbation variables. The momentum equations (2.23) – (2.25) are written as

$$\partial_t U + m[\partial_x(Uu) + \partial_y(Vu)] + \partial_\eta(\Omega u) + (\mu_d \alpha \partial_x p' + \mu_d \alpha' \partial_x \bar{p})$$
$$+ (\alpha/\alpha_d)(\mu_d \partial_x \phi' + \partial_\eta p' \partial_x \phi - \mu'_d \partial_x \phi) = F_U \tag{2.35}$$

$$\partial_t V + m[\partial_x(Uv) + \partial_y(Vv)] + \partial_\eta(\Omega v) + (\mu_d \alpha \partial_y p' + \mu_d \alpha' \partial_y \bar{p})$$
$$+ (\alpha/\alpha_d)(\mu_d \partial_y \phi' + \partial_\eta p' \partial_y \phi - \mu'_d \partial_y \phi) = F_V \tag{2.36}$$

$$\partial_t W + m[\partial_x(Uw) + \partial_y(Vw)] + \partial_\eta(\Omega w)$$
$$- m^{-1} g(\alpha/\alpha_d)[\partial_\eta p' - \bar{\mu}_d(q_v + q_c + q_r)] + m^{-1} \mu'_d g = F_W, \tag{2.37}$$

and the mass conservation equation (2.27) and geopotential equation (2.28) become

$$\partial_t \mu'_d + m^2[\partial_x U + \partial_y V] + m \partial_\eta \Omega = 0 \tag{2.38}$$

$$\partial_t \phi' + \mu_d^{-1}[m^2(U\phi_x + V\phi_y) + m\Omega\phi_\eta - gW] = 0. \tag{2.39}$$

Remaining unchanged are the conservation equations for the potential temperature and scalars

$$\partial_t \Theta + m^2[\partial_x(U\theta) + \partial_y(V\theta)] + m\partial_\eta(\Omega\theta) = F_\Theta \tag{2.40}$$

$$\partial_t Q_m + m^2[\partial_x(Uq_m) + \partial_y(Vq_m)] + m\partial_\eta(\Omega q_m) = F_{Q_m}. \tag{2.41}$$

In the perturbation system the hydrostatic relation (2.30) becomes

$$\partial_\eta \phi' = -\bar{\mu}_d \alpha'_d - \alpha_d \mu'_d. \tag{2.42}$$

Equations (2.35) – (2.41), together with the equation of state (2.21), represent the equations solved in the ARW. The RHS terms in these equations include the Coriolis terms (2.32) –

(2.34), mixing terms (described in Chapter 4), and parameterized physics (described in Chapter 8). Also note that the equation of state (2.21) cannot be written in perturbation form because of the exponent in the expression. For small perturbation simulations, accuracy for perturbation variables can be maintained by linearizing (2.21) for the perturbation variables.

Chapter 3

Model Discretization

3.1 Temporal Discretization

The ARW solver uses a time-split integration scheme. Generally speaking, slow or low-frequency (meteorologically significant) modes are integrated using a third-order Runge-Kutta (RK3) time integration scheme, while the high-frequency acoustic modes are integrated over smaller time steps to maintain numerical stability. The horizontally propagating acoustic modes (including the external mode present in the mass-coordinate equations using a constant-pressure upper boundary condition) are integrated using a forward-backward time integration scheme, and vertically propagating acoustic modes and buoyancy oscillations are integrated using a vertically implicit scheme (using the acoustic time step). The time-split integration is similar to that first developed by Klemp and Wilhelmson (1978) and analyzed by Skamarock and Klemp (1992). The time-split RK3 scheme is described in general terms in Wicker and Skamarock (2002). The primary differences between the descriptions found in the references and the ARW implementation are associated with our use of the mass vertical coordinate and a flux-form set of equations, along with our use of perturbation variables for the acoustic component of the time-split integration. The acoustic-mode integration is cast in the form of a correction to the RK3 integration.

3.1.1 Runge-Kutta Time Integration Scheme

The RK3 scheme, described in Wicker and Skamarock (2002), integrates a set of ordinary differential equations using a predictor-corrector formulation. Defining the prognostic variables in the ARW solver as $\Phi = (U, V, W, \Theta, \phi', \mu', Q_m)$ and the model equations as $\Phi_t = R(\Phi)$, the RK3 integration takes the form of 3 steps to advance a solution $\Phi(t)$ to $\Phi(t + \Delta t)$:

$$\Phi^* = \Phi^t + \frac{\Delta t}{3} R(\Phi^t) \tag{3.1}$$

$$\Phi^{**} = \Phi^t + \frac{\Delta t}{2} R(\Phi^*) \tag{3.2}$$

$$\Phi^{t+\Delta t} = \Phi^t + \Delta t R(\Phi^{**}) \tag{3.3}$$

where Δt is the time step for the low-frequency modes (*the* model time step). In (3.1) – (3.3), superscripts denote time levels. This scheme is not a true Runge-Kutta scheme *per se* because,

while it is third-order accurate for linear equations, it is only second-order accurate for nonlinear equations. With respect to the ARW equations, the time derivatives Φ_t are the partial time derivatives (the leftmost terms) in equations $(2.35) - (2.41)$, and $R(\Phi)$ are the remaining terms in $(2.35) - (2.41)$.

3.1.2   Acoustic Integration

The high-frequency but meteorologically insignificant acoustic modes would severely limit the RK3 time step Δt in $(3.1) - (3.3)$. To circumvent this time step limitation we use the approach described in Wicker and Skamarock (2002). Additionally, to increase the accuracy of the splitting, we integrate a perturbation form of the governing equations using smaller acoustic time steps within the RK3 large-time-step sequence. To form the perturbation equations for the RK3 time-split acoustic integration, we define small time step variables that are deviations from the most recent RK3 predictor (denoted by the superscript t^* and representing either Φ^t, Φ^*, or Φ^{**} in $(3.1) - (3.3)$):

$$\mathbf{V}'' = \mathbf{V} - \mathbf{V}^{t^*}, \qquad \Omega'' = \Omega - \Omega^{t^*}, \qquad \Theta'' = \Theta - \Theta^{t^*},$$
$$\phi'' = \phi' - {\phi'}^{t^*}, \qquad \alpha_d'' = \alpha_d' - {\alpha_d'}^{t^*}, \qquad \mu_d'' = \mu_d' - {\mu_d'}^{t^*}.$$

The hydrostatic relation (i.e., the vertical coordinate definition) becomes

$$\alpha_d'' = -\frac{1}{\mu_d^{t^*}}\left(\partial_\eta \phi'' + \alpha_d^{t^*}\mu_d''\right). \tag{3.4}$$

Additionally, we also introduce a version of the equation of state that is linearized about t^*,

$$p'' = \frac{c_s^2}{\alpha_d^{t^*}}\left(\frac{\Theta''}{\Theta^{t^*}} - \frac{\alpha_d''}{\alpha_d^{t^*}} - \frac{\mu_d''}{\mu_d^{t^*}}\right), \tag{3.5}$$

where $c_s^2 = \gamma p^{t^*}\alpha_d^{t^*}$ is the square of the sound speed. The linearized state equation (3.5) and the vertical coordinate definition (3.4) are used to cast the vertical pressure gradient in (2.37) in terms of the model's prognostic variables. By combining (3.5) and (3.4), the vertical pressure gradient can be expressed as

$$\partial_\eta p'' = \partial_\eta(C\partial_\eta \phi'') + \partial_\eta\left(\frac{c_s^2}{\alpha_d^{t^*}}\frac{\Theta''}{\Theta^{t^*}}\right), \tag{3.6}$$

where $C = c_s^2/\mu^{t^*}{\alpha_d^{t^*}}^2$. This linearization about the most recent large time step should be highly accurate over the time interval of the several small time steps.

These variables along with (3.6) are substituted into the prognostic equations $(2.35) - (2.41)$

and lead to the acoustic time-step equations:

$$\delta_\tau U'' + \mu^{t^*}\alpha^{t^*}\partial_x p''^\tau + (\mu^{t^*}\partial_x\bar{p})\alpha''^\tau + (\alpha/\alpha_d)[\mu^{t^*}\partial_x\phi''^\tau + (\partial_x\phi^{t^*})(\partial_\eta p'' - \mu'')^\tau] = R_U^{t^*} \tag{3.7}$$

$$\delta_\tau V'' + \mu^{t^*}\alpha^{t^*}\partial_y p''^\tau + (\mu^{t^*}\partial_y\bar{p})\alpha''^\tau + (\alpha/\alpha_d)[\mu^{t^*}\partial_y\phi''^\tau + (\partial_y\phi^{t^*})(\partial_\eta p'' - \mu'')^\tau] = R_V^{t^*} \tag{3.8}$$

$$\delta_\tau\mu_d'' + m^2[\partial_x U'' + \partial_y V'']^{\tau+\Delta\tau} + m\partial_\eta\Omega''^{\tau+\Delta\tau} = R_\mu^{t^*} \tag{3.9}$$

$$\delta_\tau\Theta'' + m^2[\partial_x(U''\theta^{t^*}) + \partial_y(V''\theta^{t^*})]^{\tau+\Delta\tau} + m\partial_\eta(\Omega''^{\tau+\Delta\tau}\theta^{t^*}) = R_\Theta^{t^*} \tag{3.10}$$

$$\delta_\tau W'' - m^{-1}g\overline{\left[(\alpha/\alpha_d)^{t^*}\left[\partial_\eta(C\partial_\eta\phi'') + \partial_\eta\left(\frac{c_s^2}{\alpha^{t^*}}\frac{\Theta''}{\Theta^{t^*}}\right)\right] - \mu_d''\right]}^\tau = R_W^{t^*} \tag{3.11}$$

$$\delta_\tau\phi'' + \frac{1}{\mu_d^{t^*}}[m\Omega''^{\tau+\Delta\tau}\phi_\eta^{t^*} - \overline{gW''}^\tau] = R_\phi^{t^*}. \tag{3.12}$$

The RHS terms in (3.7) – (3.12) are fixed for the acoustic steps that comprise the time integration of each RK3 sub-step (i.e., (3.1) – (3.3)), and are given by

$$\begin{aligned}R_U^{t^*} = &- m[\partial_x(Uu) + \partial_y(Vu)] - \partial_\eta(\Omega u) - (\mu_d\alpha\partial_x p' - \mu_d\alpha'\partial_x\bar{p})\\&- (\alpha/\alpha_d)(\mu_d\partial_x\phi' - \partial_\eta p'\partial_x\phi + \mu_d'\partial_x\phi) + F_U\end{aligned} \tag{3.13}$$

$$\begin{aligned}R_V^{t^*} = &- m[\partial_x(Uv) + \partial_y(Vv)] - \partial_\eta(\Omega v) - (\mu_d\alpha\partial_y p' - \mu_d\alpha'\partial_y\bar{p})\\&- (\alpha/\alpha_d)(\mu_d\partial_y\phi' - \partial_\eta p'\partial_y\phi + \mu_d'\partial_y\phi) + F_V\end{aligned} \tag{3.14}$$

$$R_{\mu_d}^{t^*} = - m^2[\partial_x U + \partial_y V] - m\partial_\eta\Omega \tag{3.15}$$

$$R_\Theta^{t^*} = - m^2[\partial_x(U\theta) + \partial_y(V\theta)] - m\partial_\eta(\Omega\theta) + F_\Theta \tag{3.16}$$

$$\begin{aligned}R_W^{t^*} = &- m[\partial_x(Uw) + \partial_y(Vw)] - \partial_\eta(\Omega w)\\&+ m^{-1}g(\alpha/\alpha_d)[\partial_\eta p' - \bar{\mu}_d(q_v + q_c + q_r)] - m^{-1}\mu_d'g + F_W\end{aligned} \tag{3.17}$$

$$R_\phi^{t^*} = - \mu_d^{-1}[m^2(U\phi_x + V\phi_y) + m\Omega\phi_\eta - gW], \tag{3.18}$$

where all variables in (3.13) – (3.18) are evaluated at time t^* (i.e., using Φ^t, Φ^*, or Φ^{**} for the appropriate RK3 sub-step in (3.1) – (3.3)). Equations (3.7) – (3.12) utilize the discrete acoustic time-step operator

$$\delta_\tau a = \frac{a^{\tau+\Delta\tau} - a^\tau}{\Delta\tau},$$

where $\Delta\tau$ is the acoustic time step, and an acoustic time-step averaging operator

$$\bar{a}^\tau = \frac{1+\beta}{2}a^{\tau+\Delta\tau} + \frac{1-\beta}{2}a^\tau, \tag{3.19}$$

where β is a user-specified parameter (see Section 4.2.3).

The integration over the acoustic time steps proceeds as follows. Beginning with the small time-step variables at time τ, (3.7) and (3.8) are stepped forward to obtain $U''^{\tau+\Delta\tau}$ and $V''^{\tau+\Delta\tau}$. Both $\mu''^{\tau+\Delta\tau}$ and $\Omega''^{\tau+\Delta\tau}$ are then calculated from (3.9). This is accomplished by first integrating (3.9) vertically from the surface to the material surface at the top of the domain, which removes the $\partial_\eta\Omega''$ term such that

$$\delta_\tau\mu_d = m^2\int_1^0[\partial_x U'' + \partial_y V'']^{\tau+\Delta\tau}d\eta. \tag{3.20}$$

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After computing $\mu_d''^{\tau+\Delta\tau}$ from (3.20), $\Omega''^{\tau+\Delta\tau}$ is recovered by using (3.9) to integrate the $\partial_\eta \Omega''$ term vertically using the lower boundary condition $\Omega'' = 0$ at the surface. Equation (3.10) is then stepped forward to calculate $\Theta''^{\tau+\Delta\tau}$. Equations (3.11) and (3.12) are combined to form a vertically implicit equation that is solved for $W''^{\tau+\Delta\tau}$ subject to the boundary condition $\Omega = \Omega'' = 0$ at the surface ($z = h(x,y)$) and $p' = 0$ along the model top. $\phi''^{\tau+\Delta\tau}$ is then obtained from (3.12), and $p''^{\tau+\Delta\tau}$ and $\alpha_d''^{\tau+\Delta\tau}$ are recovered from (3.5) and (3.4).

3.1.3 Full Time-Split Integration Sequence

The time-split RK3 integration technique is summarized below. It consists of two primary loops— an outer loop for the large-time-step Runge-Kutta integration, and an inner loop for the acoustic mode integration.

Begin Time Step

    Begin RK3 Loop: Steps 1, 2, and 3

        (1) If RK3 step 1, compute and store F_Φ
            (i.e., physics tendencies for RK3 step, including mixing).

        (2) Compute $R_\Phi^{t^*}$, (3.13)–(3.18)

        Begin Acoustic Step Loop: Steps $1 \to n$,
          RK3 step 1, $n = 1$, $\Delta\tau = \Delta t/3$;
          RK3 step 2, $n = n_s/2$, $\Delta\tau = \Delta t/n_s$;
          RK3 step 3, $n = n_s$, $\Delta\tau = \Delta t/n_s$.

          (3) Advance horizontal momentum, (3.7) and (3.8)
          (4) Advance μ_d (3.9) and compute $\Omega''^{\tau+\Delta\tau}$ then advance Θ (3.10)
          (5) Advance W and ϕ (3.11) and (3.12)
          (6) Diagnose p'' and α'' using (3.5) and (3.4)

        End Acoustic Step Loop

        (7) Scalar transport: Advance scalars (2.41)
          over RK3 substep (3.1), (3.2) or (3.3)
          (using mass fluxes U, V and Ω time-averaged over the acoustic steps).

        (8) Compute p' and α' using updated prognostic variables in (2.31) and (2.42)

    End RK3 Loop

    (9) Compute non-RK3 physics (currently microphysics), advance variables.

End Time Step

Figure 3.1: Time step integration sequence. Here n represents the number of acoustic time steps for a given substep of the RK3 integration, and n_s is the ratio of the RK3 time step to the acoustic time step for the second and third RK3 substeps.

   In the RK3 scheme, physics can be integrated within the RK3 time integration (using a time

forward step, i.e., step (1) in Fig. 3.1, or the RK3 time integration if higher temporal accuracy is desired, i.e., in step (2)— implying a physics evaluation every RK3 substep) or external to it using additive timesplitting, i.e., step (9).

Within the acoustic integration, the acoustic time step $\Delta\tau$ is specified by the user through the choice of n_s (see Section 3.3.2). Within the first RK3 substep, however, a single acoustic time step is used to advance the solution regardless of n_s. Within the full RK3-acoustic timesplit integration, this modified acoustic time step does not impose any additional stability constraints (see Wicker and Skamarock, 2002).

The major costs in the model arise from the evaluation of the right hand side terms R^{t^*} in (3.7) – (3.12). The efficiency of the RK3 timesplit scheme arises from the fact that the RK3 time step Δt is much larger than the acoustic time step $\Delta\tau$, hence the most costly evaluations are only performed in the less-frequent RK3 steps.

3.1.4   Diabatic Forcing

Within the RK3 integration sequence outlined in Fig. 3.1, the RHS term $R_\Theta^{t^*}$ in the thermodynamic equation (3.10) contains contributions from the diabatic physics tendencies that are computed in step (1) at the beginning of the first RK3 step. This diabatic forcing is integrated within the acoustic steps (specifically, in step 4 in the time integration sequence shown in Fig. 3.1). Additional diabatic contributions are integrated in an additive-time-split manner in step (9) after the RK3 update is complete. Thus, the diabatic forcing computed in step (9) (the microphysics in the current release of the ARW) does not appear in $R_\Theta^{t^*}$ from (3.10) in the acoustic integration. We have discovered that this time splitting can excite acoustic waves and can give rise to noise in the solutions for some applications. Note that the non-RK3 physics are integrated in step (9) because balances produced in the physics are required at the end of the time step (e.g., the saturation adjustment in the microphysics). So while moving these non-RK3 physics into step (1) would eliminate the noise, the balances produced by these physics would be altered.

We have found that the excitation of the acoustic modes can be circumvented while leaving the non-RK3 physics in step (9) by using the following procedure that is implemented in the ARW. In step (1) of the integration procedure (Fig. 3.1), an estimate of the diabatic forcing of Θ arising from the non-RK3 physics in step (9) is included in the diabatic forcing term $R_\Theta^{t^*}$ in (3.10) (which is advanced in step 4). This estimated diabatic forcing is then removed from the updated Θ after the RK3 integration is complete and before the evaluation of the non-RK3 physics in step (9). We use the diabatic forcing from the previous time step as the estimated forcing; hence this procedure results in few additional computations outside of saving the diabatic forcing between time steps.

3.1.5   Hydrostatic Option

A hydrostatic option is available in the ARW solver. The time-split RK3 integration technique summarized in Fig. 3.1 is retained, including the acoustic step loop. Steps (5) and (6) in the acoustic-step loop, where W and ϕ are advanced and p'' and α'' are diagnosed, are replaced by

(1), the diagnosis of the hydrostatic pressure using the definition of the vertical coordinate

$$\delta_\eta p_h = \frac{\alpha_d}{\alpha}\mu_d = \Big(1 + \sum q_m\Big)\mu_d,$$

followed by (2), the diagnosis of α_d using the equation of state (2.31) and the prognosed θ, and (3), the diagnosis of the geopotential using the hydrostatic equation

$$\delta_\eta \phi' = -\bar{\mu}_d \alpha'_d - \mu'_d \alpha_d.$$

The vertical velocity w can be diagnosed from the geopotential equation, but it is not needed in the solution procedure. The acoustic step loop advances gravity waves, including the external mode, when the hydrostatic option is used; there are no horizontally propagating acoustic modes in this hydrostatic system.

3.2 Spatial Discretization

The spatial discretization in the ARW solver uses a C grid staggering for the variables as shown in Fig. 3.2. That is, normal velocities are staggered one-half grid length from the thermodynamic variables. The variable indices, (i,j) for the horizontal plane and (i,k) for the vertical plane, indicate variable locations where $(x,y,\eta) = (i\Delta x, j\Delta y, k\Delta\eta)$. We will denote the points where θ is located as being *mass* points, and likewise we will denote locations where u, v, and w are defined as u points, v points, and w points, respectively. Not shown in Fig. 3.2 are the column mass μ, defined at the (i,j) points (mass points) on the discrete grid, the geopotential ϕ that is defined at the w points, and the moisture variables q_m are defined at the mass points. The diagnostic variables used in the model, the pressure p and inverse density α, are computed at mass points. The grid lengths Δx and Δy are constants in the model formulation; changes in the physical grid lengths associated with the various projections to the sphere are accounted for using the map factors introduced in Section 2.4. The vertical grid length $\Delta\eta$ is not a fixed constant; it is specified in the initialization. The user is free to specify the η values of the model levels subject to the constraint that $\eta = 1$ at the surface, $\eta = 0$ at the model top, and η decreases monotonically between the surface and model top. Using these grid and variable definitions, we can define the spatial discretization for the ARW solver.

3.2.1 Acoustic Step Equations

We begin by defining the column-mass-coupled variables relative to the uncoupled variables. The vertical velocity is staggered only in k, so it can be coupled directly to the column mass with no averaging or interpolation. The horizontal velocities are horizontally staggered relative to the column mass such that the continuous variables are represented discretely as

$$U = \frac{\mu_d u}{m} \rightarrow \frac{\overline{\mu_d}^x u}{\overline{m}^x}, \quad V = \frac{\mu_d v}{m} \rightarrow \frac{\overline{\mu_d}^y v}{\overline{m}^y},$$

where the discrete operator $\bar{a}^x$ denotes a linear interpolation operator. The grid lengths Δx and Δy are constant, hence in this case the operator reduces to $\bar{a}^x = (a_{i+1/2} + a_{i-1/2})/2$.

y

$v_{i,j+3/2}$ $v_{i+1,j+3/2}$

$u_{i-1/2,j+1}$ $\theta_{i,j+1}$ $u_{i+1/2,j+1}$ $\theta_{i+1,j+1}$ $u_{i+3/2,j+1}$

$v_{i,j+1/2}$ $v_{i+1,j+1/2}$

$u_{i-1/2,j}$ $\theta_{i,j}$ $u_{i+1/2,j}$ $\theta_{i+1,j}$ $u_{i+3/2,j}$

Δy

$v_{i,j-1/2}$ $v_{i+1,j-1/2}$

x

Δx

horizontal grid

η

$w_{i,k+3/2}$ $w_{i+1,k+3/2}$

$u_{i-1/2,k+1}$ $\theta_{i,k+1}$ $u_{i+1/2,k+1}$ $\theta_{i+1,k+1}$ $u_{i+3/2}$

$\Delta\eta_{k+1/2}$ $w_{i,k+1/2}$ $w_{i+1,k+1/2}$

$u_{i-1/2,k}$ $\theta_{i,k}$ $u_{i+1/2,k}$ $\theta_{i+1,k}$ $u_{i+3/2}$

$\Delta\eta_k$

$w_{i,k-1/2}$ $w_{i+1,k-1/2}$

Δx
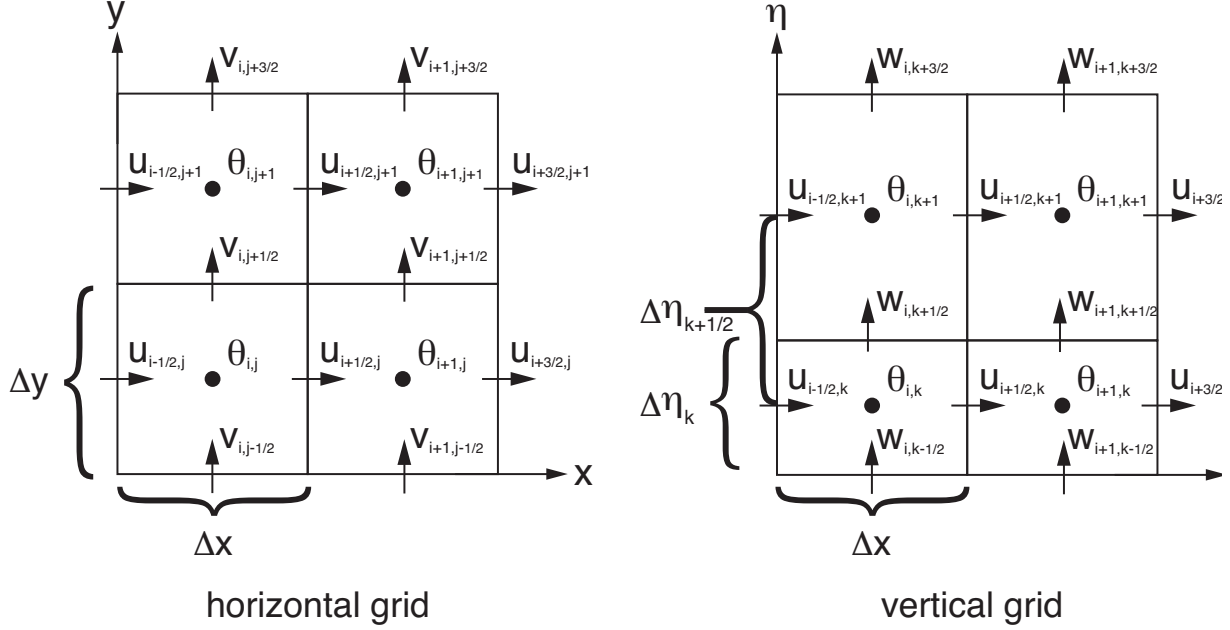
vertical grid

Figure 3.2: Horizontal and vertical grids of the ARW

Using these definitions, we can write the spatially discrete acoustic step equations (3.7) – (3.12) as

$$\delta_\tau U'' + \overline{\mu^{t*}}^x \overline{\alpha^{t*}}^x \delta_x p''^\tau + (\overline{\mu^{t*}}^x \delta_x \bar{p}) \overline{\alpha''^\tau}^x$$
$$+ \overline{(\alpha/\alpha_d)}^x [\overline{\mu^{t*}}^x \delta_x \overline{\phi''^\tau}^\eta + (\delta_x \overline{\phi^{t*}}^\eta)(\delta_\eta \overline{\overline{p''}^x}^\eta - \overline{\mu''}^x)^\tau] = R_U^{t*} \tag{3.21}$$

$$\delta_\tau V'' + \overline{\mu^{t*}}^y \overline{\alpha^{t*}}^y \delta_y p''^\tau + (\overline{\mu^{t*}}^y \delta_y \bar{p}) \overline{\alpha''^\tau}^y$$
$$+ \overline{(\alpha/\alpha_d)}^y [\overline{\mu^{t*}}^y \delta_y \overline{\phi''^\tau}^\eta + (\delta_y \overline{\phi^{t*}}^\eta)(\delta_\eta \overline{\overline{p''}^y}^\eta - \overline{\mu''}^y)^\tau] = R_V^{t*} \tag{3.22}$$

$$\delta_\tau \mu_d'' + m^2 [\delta_x U'' + \delta_y V'']^{\tau+\Delta\tau} + m\delta_\eta \Omega''^{\tau+\Delta\tau} = R_\mu^{t*} \tag{3.23}$$

$$\delta_\tau \Theta'' + m^2 [\delta_x(U'' \overline{\theta^{t*}}^x) + \delta_y(V'' \overline{\theta^{t*}}^y)]^{\tau+\Delta\tau} + m\delta_\eta(\Omega''^{\tau+\Delta\tau} \overline{\theta^{t*}}^\eta) = R_\Theta^{t*} \tag{3.24}$$

$$\delta_\tau W'' - m^{-1} g \left[\overline{(\alpha/\alpha_d)^{t*}}^\eta \delta_\eta(C\delta_\eta \phi'') + \delta_\eta \left(\frac{c_s^2}{\alpha^{t*}} \frac{\Theta''}{\Theta^{t*}} \right) - \mu_d'' \right]^\tau = R_W^{t*} \tag{3.25}$$

$$\delta_\tau \phi'' + \frac{1}{\mu_d^{t*}} [m\Omega''^{\tau+\Delta\tau} \delta_\eta \phi^{t*} - \overline{gW''}^\tau] = R_\phi^{t*}, \tag{3.26}$$

where the discrete operator

$$\delta_x a = \Delta x^{-1}(a_{i+1/2} - a_{i-1/2}) \tag{3.27}$$

with the operators δ_y and δ_η similarily defined. Additionally, the operator $\bar{a}^\eta$ is a vertical interpolation operator. Using the notation given for the vertically stretched grid depicted in Fig. 3.2, it is defined as

$$\bar{a}^\eta|_{k+1/2} = \frac{1}{2} \left(\frac{\Delta\eta_k}{\Delta\eta_{k+1/2}} a_{k+1} + \frac{\Delta\eta_{k+1}}{\Delta\eta_{k+1/2}} a_k \right). \tag{3.28}$$

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The operator vertically interpolates variables on mass levels k to the w levels $(k+\frac{1}{2})$. It should be noted that the vertical grid is defined such that vertical interpolation from w levels to mass levels reduces to $\overline{a}_k^\eta = (a_{k+1/2} + a_{k-1/2})/2$ (see Fig. 3.2).

The RHS terms in the discrete acoustic step equations for momentum (3.21), (3.22) and (3.25) are discretized as

$$R_U^{t^*} = -\,(\overline{\mu_d}^x\overline{\alpha}^x\delta_x p' - \overline{\mu_d}^x\overline{\alpha'}^x\delta_x\overline{p}) - \overline{(\alpha/\alpha_d)}^x(\overline{\mu_d}^x\delta_x\overline{\phi'}^\eta - \delta_\eta\overline{\overline{p'}^x}^\eta\delta_x\overline{\phi}^\eta + \overline{\mu_d'}^x\delta_x\overline{\phi}^\eta)$$
$$+\, F_{U_{cor}} + \text{advection} + \text{mixing} + \text{physics} \tag{3.29}$$

$$R_V^{t^*} = -\,(\overline{\mu_d}^y\overline{\alpha}^y\delta_y p' - \overline{\mu_d}^y\overline{\alpha'}^y\delta_y\overline{p}) - \overline{(\alpha/\alpha_d)}^y(\overline{\mu_d}^y\delta_y\overline{\phi'}^\eta - \delta_\eta\overline{\overline{p'}^y}^\eta\delta_y\overline{\phi}^\eta + \overline{\mu_d'}^y\delta_y\overline{\phi}^\eta)$$
$$+\, F_{V_{cor}} + \text{advection} + \text{mixing} + \text{physics} \tag{3.30}$$

$$R_W^{t^*} = m^{-1}g\overline{(\alpha/\alpha_d)}^\eta[\delta_\eta p' + \bar{\mu}_d\overline{q_m}^\eta] - m^{-1}\mu_d'g$$
$$+\, F_{W_{cor}} + \text{advection} + \text{mixing} + \text{buoyancy} + \text{physics}. \tag{3.31}$$

3.2.2 Coriolis and Curvature Terms

The terms $F_{U_{cor}}$, $F_{V_{cor}}$, and $F_{W_{cor}}$ in (3.29) – (3.31) represent Coriolis and curvature effects in the equations. These terms in continuous form are given in (2.32) – (2.34). Their spatial discretization is

$$F_{U_{cor}} = +\left(\overline{f}^x + \overline{\overline{u}^x\delta_y m - \overline{v}^y\delta_x m}^x\right)\overline{V}^{xy} - \overline{e}^x\overline{W}^{x\eta}\overline{\cos\alpha_r}^x - \frac{u\overline{W}^{x\eta}}{r_e}$$

$$F_{V_{cor}} = -\left(\overline{f}^y + \overline{\overline{u}^x\delta_y m - \overline{v}^y\delta_x m}^y\right)\overline{U}^{xy} + \overline{e}^y\overline{W}^{y\eta}\overline{\sin\alpha_r}^y - \frac{v\overline{W}^{y\eta}}{r_e}$$

$$F_{W_{cor}} = +e(\overline{U}^{x\eta}\cos\alpha_r - \overline{V}^{y\eta}\sin\alpha_r) + \left(\frac{\overline{u}^{x\eta}\overline{U}^{x\eta} + \overline{v}^{y\eta}\overline{V}^{y\eta}}{r_e}\right).$$

Here the operators $\overline{()}^{xy} = \overline{\overline{()}^x}^y$, and likewise for $\overline{()}^{x\eta}$ and $\overline{()}^{y\eta}$.

3.2.3 Advection

The advection terms in the ARW solver are in the form of a flux divergence and are a subset of the RHS terms in equations (3.13) – (3.18):

$$R_{U_{adv}}^{t^*} = -\,m[\partial_x(Uu) + \partial_y(Vu)] + \partial_\eta(\Omega u) \tag{3.32}$$
$$R_{V_{adv}}^{t^*} = -\,m[\partial_x(Uv) + \partial_y(Vv)] + \partial_\eta(\Omega v) \tag{3.33}$$
$$R_{\mu_{adv}}^{t^*} = -\,m^2[U_x + V_y] + m\Omega_\eta \tag{3.34}$$
$$R_{\Theta_{adv}}^{t^*} = -\,m^2[\partial_x(U\theta) + \partial_y(V\theta)] - m\partial_\eta(\Omega\theta) \tag{3.35}$$
$$R_{W_{adv}}^{t^*} = -\,m[\partial_x(Uw) + \partial_y(Vw)] + \partial_\eta(\Omega w) \tag{3.36}$$
$$R_{\phi_{adv}}^{t^*} = -\,\mu_d^{-1}[m^2(U\phi_x + V\phi_y) + m\Omega\phi_\eta]. \tag{3.37}$$

For the mass conservation equation, the flux divergence is discretized using a 2nd-order centered approximation:

$$R_{\mu_{adv}}^{t^*} = -m^2[\delta_x U + \delta_y V]^{t^*} + m\delta_\eta\Omega^{t^*}. \tag{3.38}$$

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In the current version of the ARW, the advection of vector quantities (momentum) and scalars is performed using the RK3 time integration as outlined in Fig. 3.1. The spatial discretization used in this approach is outlined in the next section. For many applications it is desirable to use positive definite or monotonic advection schemes for scalar transport. In the next major release of the ARW we will be including a forward-in-time scheme for scalar transport that has positive definite and monotonic options. We describe that scheme in the section following the description of the RK3 advection.

RK3 Advection

2nd through 6th order accurate spatial discretizations of the flux divergence are available in the ARW for momentum, scalars and geopotential using the RK3 time-integration scheme (scalar advection option 1, step 7 in the time-split integration sequence in Fig. 3.1). The discrete operators can be illustrated by considering the flux divergence equation for a scalar q in its discrete form:

$$R_{q_{adv}}^{t^*} = -m^2[\delta_x(U\overline{q}^{x_{adv}}) + \delta_y(V\overline{q}^{y_{adv}})] - m\delta_\eta(\Omega\overline{q}^{\eta_{adv}}). \tag{3.39}$$

As in the pressure gradient discretization, the discrete operator is defined as

$$\delta_x(U\overline{q}^{x_{adv}}) = \Delta x^{-1}\big[(U\overline{q}^{x_{adv}})_{i+1/2} - (U\overline{q}^{x_{adv}})_{i-1/2}\big]. \tag{3.40}$$

The different order advection schemes correspond to different definitions for the operator $\overline{q}^{x_{adv}}$. The even order operators (2nd, 4th, and 6th) are

$$2^{nd}\ \text{order:} \quad (\overline{q}^{x_{adv}})_{i-1/2} = \frac{1}{2}(q_i + q_{i-1})$$

$$4^{th}\ \text{order:} \quad (\overline{q}^{x_{adv}})_{i-1/2} = \frac{7}{12}(q_i + q_{i-1}) - \frac{1}{12}(q_{i+1} + q_{i-2})$$

$$6^{th}\ \text{order:} \quad (\overline{q}^{x_{adv}})_{i-1/2} = \frac{37}{60}(q_i + q_{i-1}) - \frac{2}{15}(q_{i+1} + q_{i-2}) + \frac{1}{60}(q_{i+2} + q_{i-3}),$$

and the odd order operators (3rd and 5th) are

$$3^{rd}\ \text{order:} \quad (\overline{q}^{x_{adv}})_{i-1/2} = (\overline{q}^{x_{adv}})_{i-1/2}^{4^{th}}$$
$$+ \text{sign}(U)\frac{1}{12}\big[(q_{i+1} - q_{i-2}) - 3(q_i - q_{i-1})\big]$$

$$5^{th}\ \text{order:} \quad (\overline{q}^{x_{adv}})_{i-1/2} = (\overline{q}^{x_{adv}})_{i-1/2}^{6^{th}}$$
$$- \text{sign}(U)\frac{1}{60}\big[(q_{i+2} - q_{i-3}) - 5(q_{i+1} - q_{i-2}) + 10(q_i - q_{i-1})\big].$$

The even-order advection operators are spatially centered and thus contain no implicit diffusion outside of the diffusion inherent in the RK3 time integration. The odd-order schemes are upwind-biased, and the spatial discretization is inherently diffusive. As is evident in their formulation, the odd-order schemes are comprised of the next higher (even) order centered scheme plus an upwind term that, for a constant transport mass flux, is a diffusion term of that next higher (even) order with a hyper-viscosity proportional to the Courant number (Cr). Further details concerning RK3 advection can be found in Wicker and Skamarock (2002)

Forward-In-Time Scalar Advection

A forward-in-time scalar advection scheme having monotonic and positive definite options will be available in the next major release of the ARW. This option entails bypassing step (7) in the time-split integration sequence in Fig. 3.1, and adding a single advection evaluation before step (9), after the end of the RK3 loop in the integration sequence. The new advection algorithm is patterned after Easter (1993) and can be written as follows

$$(\mu_d q)^* = (\mu_d q)^t + m^2 F_x(q^t) \tag{3.41}$$

$$\mu_d^* = \mu_d^t + m^2 F_x(I) \tag{3.42}$$

$$q^* = (\mu_d q)^*/(\mu_d)^* \tag{3.43}$$

$$(\mu_d q)^{**} = (\mu_d q)^* + m^2 F_y(q^*) \tag{3.44}$$

$$\mu_d^{**} = \mu_d^* + m^2 F_y(I) \tag{3.45}$$

$$q^{**} = (\mu_d q)^{**}/(\mu_d)^{**} \tag{3.46}$$

$$(\mu_d q)^{t+\Delta t} = (\mu_d q)^{**} + m F_\eta(q^{**}) \tag{3.47}$$

$$\mu_d^{t+\Delta t} = \mu_d^{**} + m F_\eta(I) \tag{3.48}$$

$$q^{t+\Delta t} = (\mu_d q)^{t+\Delta t}/\mu_d^{t+\Delta t}. \tag{3.49}$$

In (3.41) - (3.49) the operator

$$F_x(q) = -\Delta t \Delta x^{-1}((\overline{U}^t q)_{i+1/2} - (\overline{U}^t q)_{i-1/2}), \tag{3.50}$$

with similar definitions for F_y and F_η, and I is a vector with all values equal to 1. The discrete mass continuity equation ((3.42)+(3.45)+(3.48)) can be written as

$$\delta_\tau \mu_d = -m^2[\delta_x \overline{U}^t + \delta_y \overline{V}^t] - m \delta_\eta \overline{\Omega}^t. \tag{3.51}$$

It can easily be seen that the scheme (3.41) - (3.49) collapses to (3.51) for $q = I$, and hence it is consistent. The mass fluxes $\overline{U}^t$, $\overline{V}^t$, and $\overline{\Omega}^t$, represent time-averaged values where the averaging is performed over the final RK3 small-time-step cycle. Hence, the mass conservation equation (3.51) produces the mass conservation equation integrated within the RK3 scheme. Equation (3.51) is re-integrated within the time-split transport scheme only because a consistent column mass μ_d is needed on the sub-steps to retrieve q from the prognostic variable $\mu_d q$.

Any forward-in-time flux operator that is stable for $Cr \leq 1$ can be used to evaluate the operators $F_x(q)$, $F_y(q)$, or $F_\eta(q)$. A scheme based on the Piecewise Parabolic Method (PPM) has been implemented in the ARW. It is the non-monotonized PPM advection described in Carpenter et al. (1990), and it provides the fluxes $(\overline{U}^t q)_{i\pm\frac{1}{2}}$ used in (3.50) in the flux divergence operators. Defining q_i as the control-volume average mixing ratio for cell i, zone edge values q^{adv} can be defined as

$$q_{i+\frac{1}{2}}^{adv} = \big(7(q_{i+1} + q_i) - (q_{i+2} + q_{i-1})\big)/12. \tag{3.52}$$

The flux through the $i + \frac{1}{2}$ face can be written as

$$(\overline{U}^t q)_{i+\frac{1}{2}} = \overline{U}^t_{i+\frac{1}{2}} \big[q_{i+\frac{1}{2}}^{adv} - Cr(q_{i+\frac{1}{2}}^{adv} - q_i) - Cr(1 - Cr)(q_{i-\frac{1}{2}}^{adv} - 2q_i + q_{i+\frac{1}{2}}^{adv})\big] \tag{3.53}$$

for $Cr > 0$, and

$$(\overline{U}^t q)_{i+\frac{1}{2}} = \overline{U}^t_{i+\frac{1}{2}} \left[q^{adv}_{i+\frac{1}{2}} - Cr(q^{adv}_{i+\frac{1}{2}} - q_{i+1}) + Cr(1 + Cr)(q^{adv}_{i+\frac{1}{2}} - 2q_{i+1} + q^{adv}_{i+\frac{3}{2}}) \right] \qquad (3.54)$$

for $Cr < 0$, where $Cr = u_{i+\frac{1}{2}} \Delta t / \Delta x$. In addition, to maintain second-order accuracy for the time-split forward-in-time advection scheme, the sequence $(3.41) - (3.49)$ is reversed every other time step; that is, the flux divergence in η is computed first followed by y and then x. Finally, the scheme has also been augmented for Courant numbers greater than one using the 1D "semi-Lagrangian" flux calculation described in Lin and Rood (1996). More information about this scheme and a description of the limiters can be found in Skamarock (2005).

3.3   Stability Constraints

There are two time steps that a user must specify when running the ARW: the model time step (the time step used by the RK3 scheme, see Section 3.1.1) and the acoustic time step (used in the acoustic sub-steps of the time-split integration procedure, see Section 3.1.2). Both are limited by Courant numbers. In the following sections we describe how to choose time steps for applications.

3.3.1   RK3 Time Step Constraint

The RK3 time step is limited by the advective Courant number $u\Delta t/\Delta x$ and the user's choice of advection schemes— users can choose 2nd through 6th order discretizations for the advection terms. The time-step limitations for 1D advection in the RK3 scheme using these advection schemes is given in Wicker and Skamarock (2002), and is reproduced here.

Time Scheme	Spatial order			
	3rd	4th	5th	6th
Leapfrog	*Unstable*	0.72	*Unstable*	0.62
RK2	0.88	*Unstable*	0.30	*Unstable*
RK3	1.61	1.26	1.42	1.08

Table 3.1: Maximum stable Courant numbers for one-dimensional linear advection. From Wicker and Skamarock (2002).

As is indicated in the table, the maximum stable Courant numbers for advection in the RK3 scheme are almost a factor of two greater than those for the leapfrog time-integration scheme. For advection in three spatial dimensions, the maximum stable Courant number is $1/\sqrt{3}$ times the Courant numbers given in Table 3.1. For stability, the time step used in the ARW should produce a maximum Courant number less than that given by theory. Thus, for 3D applications, the time step should satisfy the following equation:

$$\Delta t_{max} < \frac{Cr_{theory}}{\sqrt{3}} \cdot \frac{\Delta x}{u_{max}}, \qquad (3.55)$$

where Cr_{theory} is the Courant number taken from the RK3 entry in Table 3.1 and u_{max} is the maximum velocity expected in the simulation. For example in real-data applications, where jet stream winds may reach as high as 100 ms^{-1}, the maximum time step would be approximately 80 s on a $\Delta x = 10$ km grid using 5th order advection. Given additional constraint from the time splitting, and to provide a safety buffer, we usually choose a time step that is approximately 25% less than that given by (3.55). This time step is typically a factor of two greater than that used in leapfrog-based models. For those users familiar with the MM5 model, the rule of thumb for choosing a time step is that the time step, in seconds, should be approximately 3 times the horizontal grid distance, in kilometers. For the ARW, the time step (in seconds) should be approximately 6 times the grid distance (in kilometers).

3.3.2 Acoustic Time Step Constraint

The forward-backward time integration scheme used in the ARW's 2D explicit acoustic step integration allows a maximum Courant number $Cr_{max} = c_s\Delta\tau/\Delta x < 1/\sqrt{2}$, where c_s is the speed of sound. We typically use a more conservative estimate for this by replacing the limiting value $1/\sqrt{2}$ with $1/2$. Thus, the acoustic time step used in the model is

$$\Delta\tau < 2 \cdot \frac{\Delta x}{c_s}. \tag{3.56}$$

For example, on a $\Delta x = 10$ km grid, using a sound speed $c_s = 300$ ms^{-1}, the acoustic time step given in (3.56) is approximately 17 s. In the ARW, the ratio of the RK3 time step to the acoustic time step must be an even integer. For our example using a $\Delta x = 10$ km grid in a real-data simulation, we would specify the RK3 time step $\Delta t = 60s$ (i.e., 25% less than the 80 s step given by (3.55)), and an acoustic time step $\Delta\tau = 15$ s (i.e., 1/4 of the RK3 step, rounding down the $\Delta\tau = 17$ s step given by (3.56)). Note that it is the ratio of the RK3 time step to the acoustic time step that is the required input in the ARW.

Chapter 4

Turbulent Mixing and Model Filters

A number of formulations for turbulent mixing and filtering are available in the ARW solver. Some of these filters are used for numerical reasons. For example, divergence damping is used to filter acoustic modes from the solution. Other filters are meant to represent sub-grid turbulence processes that cannot be resolved on the chosen grid. These filters remove energy from the solution and are formulated in part on turbulence theory and observations, or represent energy sink terms in some approximation to the Euler equation. In this section, we begin by outlining the formulation and discretization of turbulent mixing processes in the ARW solver commonly associated with sub-gridscale turbulence as parameterized in cloud-scale models— the second-order horizontal and vertical mixing. In large-scale models and most NWP models, vertical mixing is parameterized within the planetary boundary layer (PBL) physics. Vertical mixing parameterized within the PBL physics is described later in Chapter 8. Here we note that, when a PBL parameterization is used, all other vertical mixing is disabled. Following the outline of turbulent mixing parameterizations in this chapter, other numerical filters available in the ARW solver are described.

4.1 Explicit Spatial Diffusion

The ARW solver has two formulations for spatial dissipation— diffusion along coordinate surfaces and diffusion in physical (x, y, z) space. In the following sections we present the diffusion operators for these two formulations, followed by the four separate formulations that can be used to compute the eddy viscosities. We conclude with a description of the prognostic turbulent kinetic energy (TKE) equation used in one set of these formulations.

4.1.1 Horizontal and Vertical Diffusion on Coordinate Surfaces

For any model variable, horizontal and vertical second order spatial filtering on model coordinate surfaces is considered part of the RHS terms in the continuous equations $(2.35) - (2.41)$ and can be expressed as follows for a model variable a:

$$\partial_t(\mu_d a) = \ldots \ + \mu_d\big[m\partial_x(mK_h\partial_x a) + m\partial_y(mK_h\partial_y a)\big] + g^2(\mu_d\alpha)^{-1}\partial_\eta(K_v\alpha^{-1}\partial_\eta a). \qquad (4.1)$$

For the horizontal and vertical momentum equations, (4.1) is spatially discretized as

$$\partial_t U = ... \ + \overline{\mu_d}^x \overline{m}^x \left[\delta_x(mK_h\delta_x u) + \delta_y(\overline{m}^{xy}\overline{K_h}^{xy}\delta_y u) \right] + g^2(\overline{\mu_d}^x\overline{\alpha}^x)^{-1}\delta_\eta(K_v(\overline{\alpha}^{x\eta})^{-1}\delta_\eta u)$$

$$\partial_t V = ... \ + \overline{\mu_d}^y \overline{m}^y \left[\delta_x(\overline{m}^{xy}\overline{K_h}^{xy}\delta_x v) + \delta_y(mK_h\delta_y v) \right] + g^2(\overline{\mu_d}^y\overline{\alpha}^y)^{-1}\delta_\eta(K_v(\overline{\alpha}^{y\eta})^{-1}\delta_\eta v)$$

$$\partial_t W = ... \ + \mu_d m \left[\delta_x(\overline{m}^x\overline{K_h}^{x\eta}\delta_x w) + \delta_y(\overline{m}^y\overline{K_h}^{y\eta}\delta_y w) \right] + g^2(\mu_d\overline{\alpha}^\eta)^{-1}\delta_\eta(K_v\alpha^{-1}\delta_\eta w).$$

The spatial discretization for a scalar q, defined at the mass points, is

$$\partial_t(\mu_d q) = ... \ + \mu_d m \left[\delta_x(\overline{m}^x P_r^{-1}\overline{K_h}^x\delta_x q) + \delta_y(\overline{m}^y P_r^{-1}\overline{K_h}^y\delta_y q) \right] + g^2(\mu_d\alpha)^{-1}\delta_\eta(K_v\alpha^{-1}\delta_\eta q).$$

In the current ARW formulation for mixing on coordinate surfaces, the horizontal eddy viscosity K_h is allowed to vary in space, whereas the vertical eddy viscosity does not vary in space; hence there is no need for any spatial averaging of K_v. Additionally, note that the horizontal eddy viscosity K_h is multiplied by the inverse turbulent Prandtl number P_r^{-1} for horizontal scalar mixing.

4.1.2 Horizontal and Vertical Diffusion in Physical Space

Coordinate Metrics

We use the geometric height coordinate in this physical space formulation. The coordinate metrics are computed using the prognostic geopotential in the ARW solver. At the beginning of each Runge-Kutta time step, the coordinate metrics are evaluated as part of the overall algorithm. The definitions of the metrics are

$$z_x = g^{-1}\delta_x\phi \ \text{ and } \ z_y = g^{-1}\delta_y\phi.$$

These metric terms are defined on w levels, and (z_x, z_y) are horizontally coincident with (u,v) points. Additionally, the vertical diffusion terms are evaluated directly in terms of the geometric height, avoiding the need for metric terms in the vertical.

Continuous Equations

The continuous equations for evaluating diffusion in physical space, using the velocity stress tensor, are as follows for horizontal and vertical momentum:

$$\partial_t U = ... \ - m\left[\partial_x\tau_{11} + \partial_y\tau_{12} - \partial_z(z_x\tau_{11} + z_y\tau_{12})\right] - \partial_z\tau_{13} \tag{4.2}$$

$$\partial_t V = ... \ - m\left[\partial_x\tau_{12} + \partial_y\tau_{22} - \partial_z(z_x\tau_{12} + z_y\tau_{22})\right] - \partial_z\tau_{23} \tag{4.3}$$

$$\partial_t W = ... \ - m\left[\partial_x\tau_{13} + \partial_y\tau_{23} - \partial_z(z_x\tau_{13} + z_y\tau_{23})\right] - \partial_z\tau_{33}. \tag{4.4}$$

The stress tensor τ can be written as follows:

$$\tau_{11} = -\mu_d K_h D_{11}$$
$$\tau_{22} = -\mu_d K_h D_{22}$$
$$\tau_{33} = -\mu_d K_v D_{33}$$
$$\tau_{12} = -\mu_d K_h D_{12}$$
$$\tau_{13} = -\mu_d K_h D_{13}$$
$$\tau_{23} = -\mu_d K_h D_{23}.$$

Symmetry sets the remaining tensor values; $\tau_{21} = \tau_{12}$, $\tau_{31} = \tau_{13}$, and $\tau_{32} = \tau_{23}$. The stress tensor τ is calculated from the deformation tensor D. The continuous deformation tensor is defined as

$$D_{11} = 2\,m^2\left[\partial_x(m^{-1}u) - z_x\partial_z(m^{-1}u)\right]$$
$$D_{22} = 2\,m^2\left[\partial_y(m^{-1}v) - z_y\partial_z(m^{-1}v)\right]$$
$$D_{33} = 2\,\partial_z w$$
$$D_{12} = m^2\left[\partial_y(m^{-1}u) - z_y\partial_z(m^{-1}u) + \partial_x(m^{-1}v) - z_x\partial_z(m^{-1}v)\right]$$
$$D_{13} = m^2\left[\partial_x(m^{-1}w) - z_x\partial_z(m^{-1}w)\right] + \partial_z(u)$$
$$D_{23} = m^2\left[\partial_y(m^{-1}w) - z_y\partial_z(m^{-1}w)\right] + \partial_z(v).$$

The deformation tensor is symmetric, hence $D_{21} = D_{12}$, $D_{31} = D_{13}$, and $D_{32} = D_{23}$.

The diffusion formulation for scalars is

$$\partial_t(\mu_d q) = \dots\ + \left[m\big(\partial_x - \partial_z z_x\big)\big(\mu_d mK(\partial_x - z_x\partial_z)\big) +\right.$$
$$\left.m\big(\partial_y - \partial_z z_y\big)\big(\mu_d mK(\partial_y - z_y\partial_z)\big) + \partial_z\mu_d K\partial_z\right]q. \tag{4.5}$$

Spatial Discretization

Using the definition of the stress tensor, the spatial discretization of the ARW physical-space diffusion operators for the horizontal and vertical momentum equations (4.2) - (4.4) are

$$\partial_t U = \dots\ -\overline{m}^x\left[\delta_x\tau_{11} + \delta_y\tau_{12} - \delta_z(z_x\overline{\tau_{11}}^{x\eta} + \overline{z_y}^{xy}\overline{\tau_{12}}^{y\eta})\right] - \delta_z\tau_{13}$$
$$\partial_t V = \dots\ -\overline{m}^y\left[\delta_y\tau_{22} + \delta_x\tau_{12} - \delta_z(z_y\overline{\tau_{22}}^{y\eta} + \overline{z_x}^{xy}\overline{\tau_{12}}^{x\eta})\right] - \delta_z\tau_{23}$$
$$\partial_t W = \dots\ -m\left[\delta_x\tau_{13} + \delta_y\tau_{23} - \delta_z(\overline{z_x}^{x\eta}\overline{\tau_{13}}^{x\eta} + \overline{z_y}^{y\eta}\overline{\tau_{23}}^{y\eta})\right] - \delta_z\tau_{33}.$$

The discrete forms of the stress tensor and deformation tensor are

$$\tau_{11} = -\mu_d K_h D_{11}$$
$$\tau_{22} = -\mu_d K_h D_{22}$$
$$\tau_{33} = -\mu_d K_v D_{33}$$
$$\tau_{12} = -\overline{\mu_d}^{xy}\overline{K_h}^{xy} D_{12}$$
$$\tau_{13} = -\overline{\mu_d}^{x}\overline{K_h}^{x\eta} D_{13}$$
$$\tau_{23} = -\overline{\mu_d}^{y}\overline{K_h}^{y\eta} D_{23},$$

and

$$D_{11} = 2\,m^2\left[\delta_x(\overline{m^{-1}}^x u) - \overline{z_x}^{x\eta}\delta_z(\overline{m^{-1}}^x u)\right]$$
$$D_{22} = 2\,m^2\left[\delta_y(\overline{m^{-1}}^y v) - \overline{z_y}^{x\eta}\delta_z(\overline{m^{-1}}^y v)\right]$$
$$D_{33} = 2\,\delta_z w$$
$$D_{12} = (\overline{m}^{xy})^2\left[\delta_y(\overline{m^{-1}}^x u) - \overline{\overline{z_y}^{x\eta}\delta_z(\overline{m^{-1}}^x u)}^{y\eta} + \delta_x(\overline{m^{-1}}^y v) - \overline{\overline{z_x}^{y\eta}\delta_z(\overline{m^{-1}}^y v)}^{x\eta}\right]$$
$$D_{13} = m^2\left[\delta_x(m^{-1}w) - z_x\delta_z\overline{(m^{-1}w)}^{x\eta}\right] + \delta_z u$$
$$D_{23} = m^2\left[\delta_y(m^{-1}w) - z_y\delta_z\overline{(m^{-1}w)}^{y\eta}\right] + \delta_z v.$$

The spatial discretization for the scalar diffusion (4.5) is

$$\partial_t(\mu_d q) = ... + m\left[\delta_x\left(\overline{\mu_d}^x H_1(q)\right) - \mu_d\delta_z\left(\overline{\overline{z_x}^x \overline{H_1(q)}^{x\eta}}\right)\right]$$
$$+ m\left[\delta_y\left(\overline{\mu_d}^y H_2(q)\right) - \mu_d\delta_z\left(\overline{\overline{z_y}^y \overline{H_2(q)}^{y\eta}}\right)\right]$$
$$+ \mu_d\delta_z\left(\overline{K_v}^\eta \delta_z q\right),$$

where

$$H_1(q) = \overline{m}^x \overline{K_h}^x\left(\delta_x q - z_x\delta_z(\overline{q}^{x\eta})\right),$$
$$H_2(q) = \overline{m}^y \overline{K_h}^y\left(\delta_y q - z_y\delta_z(\overline{q}^{y\eta})\right).$$

4.1.3 Computation of the Eddy Viscosities

There are four options for determining the eddy viscosities K_h and K_v in the ARW solver.

External specification of K_h and K_v

Constant values for K_h and K_v can be input in the ARW namelist.

K_h determined from the horizontal deformation

The horizontal eddy viscosity K_h can be determined from the horizontal deformation using a Smagorinsky first-order closure approach. In this formulation, the eddy viscosity is defined and discretized as

$$K_h = C_s^2\, l^2 \left[0.25(D_{11} - D_{22})^2 + \overline{D_{12}^2}^{xy}\right]^{\frac{1}{2}}.$$

The deformation tensor components have been defined in the previous section. The length scale $l = (\Delta x \Delta y)^{1/2}$ and C_s is a constant with a typical value $C_s = 0.25$. For scalar mixing, the eddy viscosity is divided by the turbulent Prandtl number P_r that typically has a value of $1/3$ (Deardorff, 1972). This option is most often used with a planetary boundary layer scheme that independently handles the vertical mixing.

3D Smagorinsky Closure

The horizontal and vertical eddy viscosities can be determined using a 3D Smagorinsky turbulence closure. This closure specifies the eddy viscosities as

$$K_{h,v} = C_s^2\, l_{h,v}^2 \max\left[0., D^2 - \left(P_r^{-1}N^2\right)^{1/2}\right], \tag{4.6}$$

where

$$D = \frac{1}{2}\left[D_{11}^2 + D_{22}^2 + D_{33}^2\right] + \left(\overline{D_{12}}^{xy}\right)^2 + \left(\overline{D_{13}}^{x\eta}\right)^2 + \left(\overline{D_{23}}^{y\eta}\right)^2,$$

and N is the Brunt-Väisälä frequency; the computation of N, including moisture effects, is outlined in Section 4.1.4.

If Δx is less than the user-specified critical length scale l_{cr}, then the length scale used for calculating both K_h and K_v in (4.6) is $l_{h,v} = (\Delta x \Delta y \Delta z)^{1/3}$ (and $K_h = K_v = K$). If Δx is greater than an critical length scale l_{cr}, then the length scale $l_h = \sqrt{\Delta x \Delta y}$ in the calculation of the horizontal eddy viscosity K_h using (4.6), and $l_v = \Delta z$ for the calculation of the vertical eddy viscosity K_v using (4.6).

Additionally, the eddy viscosities for scalar mixing are divided by the turbulent Prandtl number $P_r = 1/3$.

Prognostic TKE Closure

For the predicted turbulent kinetic energy option (TKE; see section 4.1.4), the eddy viscosities are computed using

$$K_{h,v} = C_k l_{h,v} \sqrt{e},$$

where e is the turbulent kinetic energy (a prognostic variable in this scheme), C_k is a constant (typically $0.15 < C_k < 0.25$), and l is a length scale.

If the grid spacing Δx is less than the critical length scale l_{cr}, then

$$l_{h,v} = \min\left[(\Delta x \Delta y \Delta z)^{1/3}, 0.76\sqrt{e}/N\right] \qquad \text{for } N^2 > 0,$$
$$l_{h,v} = (\Delta x \Delta y \Delta z)^{1/3} \qquad \text{for } N^2 \leq 0$$

(see section 4.1.4 for the calculation of N^2). Both the horizontal and vertical eddy viscosities are multiplied by an inverse turbulent Prandtl number $P_r^{-1} = 1 + 2l/(\Delta x \Delta y \Delta z)^{1/3}$ for scalar mixing. In this case ($l_{cr} > \Delta x$) the horizontal and vertical eddy viscosities are equivalent.

If the grid spacing Δx is greater than the critical length scale l_{cr}, then $l_h = \sqrt{\Delta x \Delta y}$ for the calculation of K_h. For calculating K_v,

$$l_v = \min\left[\Delta z, 0.76\sqrt{e}/N\right] \qquad \text{for } N^2 > 0,$$
$$l_v = \Delta z \qquad \text{for } N^2 \leq 0.$$

The eddy viscosity used for mixing scalars is divided by a turbulent Prandtl number P_r. The Prandtl number is $1/3$ for the horizontal eddy viscosity K_h, and $P_r^{-1} = 1 + 2l/\Delta z$ for the vertical eddy viscosity K_v.

4.1.4   TKE equation for the 1.5 Order Turbulence Closure

The prognostic equation governing the evolution of the turbulent kinetic energy e is

$$\partial_t(\mu_d e) + (\nabla \cdot \mathbf{V} e)_\eta = \mu_d(\text{ shear production } + \text{ buoyancy } + \text{ dissipation }). \qquad (4.7)$$

The time integration and the transport terms in (4.7) are integrated in the same manner as for other scalars (as described in Chapter 3). The right-hand side source and sink terms for e are given as follows.

Shear Production

The shear production term in (4.7) can be written as

$$\text{shear production} = K_h D_{11}^2 + K_h D_{22}^2 + K_v D_{33}^2 + K_h \overline{D_{12}^2}^{xy} + K_v \overline{D_{13}^2}^{x\eta} + K_v \overline{D_{23}^2}^{y\eta}.$$

Buoyancy

The buoyancy term in the TKE equation (4.7) is written as

$$\text{buoyancy} = -K_v N^2,$$

where the Brunt-Väisälä frequency N is computed using either the formula for a moist saturated or unsaturated environment:

$$N^2 = g\left[A\frac{\partial \theta_e}{\partial z} - \frac{\partial q_w}{\partial z}\right] \qquad \text{if } q_v \geq q_s \text{ or } q_c \geq 0.01 \text{ g/Kg};$$

$$N^2 = g\left[\frac{1}{\theta}\frac{\partial \theta}{\partial z} + 1.61\frac{\partial q_v}{\partial z} - \frac{\partial q_w}{\partial z}\right] \qquad \text{if } q_v < q_s \text{ or } q_c < 0.01 \text{ g/Kg}.$$

The coefficient A is defined as

$$A = \theta^{-1}\frac{1 + \frac{1.61\epsilon L q_v}{R_d T}}{1 + \frac{\epsilon L^2 q_v}{C_p R_v T^2}},$$

where q_w represents the total water (vapor + all liquid species + all ice species), L is the latent heat of condensation and ϵ is the molecular weight of water over the molecular weight of dry air. θ_e is the equivalent potential temperature and is defined as

$$\theta_e = \theta\left(1 + \frac{\epsilon L q_{vs}}{C_p T}\right),$$

where q_{vs} is the saturation vapor mixing ratio.

Dissipation

If Δx is less than the critical length scale l_{cr}, the dissipation term in (4.7) is

$$\text{dissipation} = -\frac{Ce^{3/2}}{l},$$

where

$$C = 1.9\,C_k + \frac{(0.93 - 1.9\,C_k)\,l}{\Delta s},$$

$\Delta s = (\Delta x \Delta y \Delta z)^{1/3}$, and

$$l = min\left[(\Delta x \Delta y \Delta z)^{1/3}, 0.76\sqrt{e}/N\right].$$

If Δx is greater than the critical length scale l_{cr}, the dissipation term in (4.7) is

$$\text{dissipation} = -\frac{2\sqrt{2}}{15}\frac{e^{3/2}}{l},$$

where

$$l = \frac{kz}{1 + kz/l_0},$$

$$l_0 = min\left(\frac{\alpha_b \int_0^{z_i} \sqrt{e}z\,dz}{\int_0^{z_i} \sqrt{e}\,dz}, 80\right),$$

$\alpha_b = 0.2$, and $k = 0.4$ is the von Karman constant.

4.2 Filters for the Time-split RK3 scheme

Three filters are used in the ARW time-split RK3 scheme apart from those in the model physics: three-dimensional divergence damping (an acoustic model filter); an external-mode filter that damps vertically-integrated horizontal divergence; and off-centering of the vertically implicit integration of the vertical momentum equation and geopoential equation. Each of these is described in the following sections.

4.2.1 Divergence Damping

The damping of the full mass divergence is a filter for acoustic modes in the ARW solver. This 3D mass divergence damping is described in Skamarock and Klemp (1992). The filtering is accomplished by forward weighting the pressure update in the acoustic step loop described in Section 3.1.3, step (6). The linearized equation of state (3.5) is used to diagnose the pressure at the new time τ after the U'', V'', μ''_d, and Θ'' have been advanced. Divergence damping consists of using a modified pressure in the computation of the pressure gradient terms in the horizontal momentum equations in the acoustic steps (Equations (3.7) and (3.8)). Denoting the updated value as p^τ, the modified pressure $p^{*\tau}$ used in (3.7) and (3.8) can be written as

$$p^{*\tau} = p^\tau + \gamma_d\big(p^\tau - p^{\tau - \Delta\tau}\big), \tag{4.8}$$

where γ_d is the damping coefficient. This modification is equivalent to inserting a horizontal diffusion term into the equation for the 3D mass divergence, hence the name *divergence damping*. A divergence damping coefficient $\gamma_d = 0.1$ is typically used in the ARW applications, independent of time step or grid size.

4.2.2 External Mode Filter

The external modes in the solution are damped by filtering the vertically-integrated horizontal divergence. This damping is accomplished by adding a term to the horizontal momentum equations. The additional term added to (3.7) and (3.8) are

$$\delta_\tau U'' = ... \ - \gamma_e \delta_x(\delta_{\tau - \Delta\tau}\mu''_d) \tag{4.9}$$

and

$$\delta_\tau V'' = ... \ - \gamma_e \delta_y(\delta_{\tau - \Delta\tau}\mu''_d). \tag{4.10}$$

The quantity $\delta_{\tau - \Delta\tau}\mu$ is the vertically-integrated mass divergence (i.e., (3.20)) from the previous acoustic step (that is computed using the time τ values of U and V), and γ_e is the external mode damping coefficient. An external mode damping coefficient $\gamma_e = 0.01$ is typically used in the ARW applications, independent of time step or grid size.

4.2.3 Semi-Implicit Acoustic Step Off-centering

Forward-in-time weighting of the vertically-implicit acoustic-time-step terms damps instabilities associated vertically-propagating sound waves. The forward weighting also damps instabilities associated with sloping mode levels and horizontally propagating sound waves (see Durran and

Klemp, 1983; Dudhia, 1995). The off-centering is accomplished by using a positive (non-zero) coefficient β (3.19) in the acoustic time-step vertical momentum equation (3.11) and geopotential equation (3.12). An off-centering coefficient $\beta = 0.1$ is typically used in the ARW applications, independent of time step or grid size.

4.3   Other Damping

4.3.1   Gravity Wave Absorbing Layer

A gravity-wave absorbing layer is available in the ARW solver. The absorbing layer increases the second-order horizontal and vertical eddy viscosities in the absorbing layer using the following formulation:

$$K_{dh} = \frac{\Delta x^2}{\Delta t} \gamma_g \cos\left(\frac{z_{top} - z}{z_d} \frac{\pi}{2}\right),$$

and

$$K_{dv} = \frac{\Delta z^2}{\Delta t} \gamma_g \cos\left(\frac{z_{top} - z}{z_d} \frac{\pi}{2}\right).$$

Here γ_g is a user-specified damping coefficient, z_{top} is the height of the model top for a particular grid column, z_d is the depth of the damping layer (from the model top), and K_{dh} and K_{dv} are the horizontal and vertical eddy viscosities for the wave absorbing layer. If other spatial filters are being used, then the eddy viscosities that are used for the second-order horizontal and vertical eddy viscosities are the maximum of (K_h, K_{dh}) and (K_v, K_{dv}). The effect of this filter on gravity waves is discussed in Klemp and Lilly (1978), where guidance on the choice of the damping coefficeint γ_g can also be found.

4.3.2   Rayleigh Damping Layer

A Rayleigh damping layer is also available in the ARW solver. This scheme applies a tendency to u, v, w, and θ to gradually relax the variable back to a predetermined reference state value,

$$
\begin{aligned}
\frac{\partial u}{\partial t} &= \tau(z)\left(u - \overline{u}\right), \\
\frac{\partial v}{\partial t} &= \tau(z)\left(v - \overline{v}\right), \\
\frac{\partial w}{\partial t} &= \tau(z)w, \\
\frac{\partial \theta}{\partial t} &= \tau(z)\left(\theta - \overline{\theta}\right).
\end{aligned}
$$

Overbars indicate the reference state fields, which are functions of z only and are typically defined as the initial horizontally homogeneous fields in idealized simulations. The reference state vertical velocity is assumed to be zero. The variable τ defines the vertical structure of the damping layer, and has a form similar to the Rayleigh damper in Durran and Klemp (1983),

$$
\tau(z) = \begin{cases} -\gamma_r \sin^2\left[\frac{\pi}{2}\left(1 - \frac{z_{top} - z}{z_d}\right)\right] & \text{for } z \geq (z_{top} - z_d); \\ 0 & \text{otherwise,} \end{cases}
$$

where γ_r is a user specified damping coeficient, z_{top} is the height of the model top for a particular grid column, and z_d is the depth of the damping layer (from the model top).

Because the model surfaces change height with time in the ARW solver, the reference state values at each grid point need to be recalculated at every time step. Thus, a linear interpolation scheme is used to calculate updated reference state values based on the height of the model levels at each time step.

The effect of this filter on gravity waves is discussed in Klemp and Lilly (1978), where guidance on the choice of the damping coefficeint γ_r can also be found.

4.3.3   Vertical-Velocity Damping

This is also called w-damping. In semi-operational or operational NWP applications, the model robustness can be improved by detecting locations where the vertical motion approaches the limiting Courant number for stability, and applying a Rayleigh damping term in the vertical momentum equation to stabilize the motion. This term is non-physical and should only be used in the situation where many, or long, model runs are being done, and there is no option for a re-run with a shorter time-step if a failure occurs due to an excessively strong updraft. This might be the case, for example, in an operational setting where real-time forecasts have to be produced on time to be useful. However, if this term activates frequently, consideration should be given to reducing the model time-step.

The term is calculated from

$$Cr = \left| \frac{\Omega dt}{\mu d\eta} \right|.$$

If $Cr > Cr_\beta$, then

$$\partial_t W = ... - \mu_d \, \text{sign}(W) \gamma_w (Cr - Cr_\beta),$$

where γ_w is the damping coefficient (typically 0.3 ms^{-2}), and Cr_β is the activation Courant number (typically 1.0). The ARW outputs the location of this damping when it is active.

Chapter 5

Initial Conditions

The ARW may be run with initial conditions that are defined analytically for idealized simulations, or it may be run using interpolated data from either a large-scale analysis or forecast for real-data cases. Both 2D and 3D tests cases for idealized simulations are provided. Several sample cases for real-data simulations are provided, which rely on pre-processing from an external package that converts the large-scale GriB data into a format suitable for ingest by the ARW's real-data processor.

The programs that generate the specific initial conditions for the selected idealized or real-data case function similarly. They provide the ARW with:

- input data that is on the correct horizontal and vertical staggering;
- hydrostatically balanced reference state and perturbation fields; and
- metadata specifying such information as the date, grid physical characteristics, and projection details.

For neither the idealized nor the real-data cases are the initial conditions enhanced with observations. However, output from the ARW system initial condition programs is suitable as input to the WRF variational assimilation package (see Chapter 9).

5.1   Initialization for Idealized Conditions

The ARW comes with a number of test cases using idealized environments, including mountain waves (em_hill2d_x), squall lines (em_squall2d_x, em_squall2d_y), supercell thunderstorms (em_quarter_ss), gravity currents (em_grav2d_x), and baroclinic waves (em_b_wave). A brief description of these test cases can be found in the README_test_cases file provided in the ARW release. The test cases include examples of atmospheric flows at fine scales (e.g., the gravity current example has a grid-spacing of 100 meters and a time step of 1 second) and examples of flow at large scales (e.g., the baroclinic wave test case uses a grid-spacing of 100 km and a time step of 600 s), in addition to the traditional mesoscale and cloudscale model simulations. The test suite allows an ARW user to easily reproduce these known solutions. The test suite is also the starting point for constructing idealized flow simulations by modifying initializations that closely resemble a desired initialization.

All of these tests use as input a 1D sounding specified as a function of geometric height z (except for the baroclinic wave case that uses a 2D sounding specified in $[y, z]$), and, with the

exception of the baroclinic wave test case, the sounding files are in text format that can be directly edited by the user. The 1D specification of the sounding in these test files requires the surface values of pressure, potential temperature, and water vapor mixing ratio, followed by the potential temperature, vapor mixing ratio, and horizontal wind components at some heights above the surface. The initialization programs for each case assume that this moist sounding represents an atmosphere in hydrostatic balance.

Two sets of thermodynamic fields are needed for the model— the reference state and the perturbation state (see Chapter 2 for further discussion of the equations). The reference state used in the idealized initializations is computed using the input sounding from which the moisture is discarded (because the reference state is dry). The perturbation state is computed using the full moist input sounding. The procedure for computing the hydrostatically-balanced ARW reference and perturbation state variables from the input sounding is as follows. First, density and both a dry and full hydrostatic pressure are computed from the input sounding at the input sounding levels z. This is accomplished by integrating the hydrostatic equation vertically up the column using the surface pressure and potential temperature as the lower boundary condition. The hydrostatic equation is

$$\delta_z p = -\overline{\rho}^z g (1 + (R_d/R_v)\overline{q}_v{}^z), \tag{5.1}$$

where $\overline{\rho}^z$ is a two point average between input sounding levels, and $\delta_z p$ is the difference of the pressure between input sounding levels divided by the height difference. Additionally, the equation of state is needed to close the system:

$$\alpha_d = \frac{1}{\rho_d} = \frac{R_d\theta}{p_o}\left(1 + \frac{R_d}{R_v}q_v\right)\left(\frac{p}{p_o}\right)^{-\frac{c_v}{c_p}}, \tag{5.2}$$

where q_v and θ are given in the input sounding. (5.1) and (5.2) are a coupled set of nonlinear equations for p and ρ in the vertical integration, and they are solved by iteration. The dry pressure on input sounding levels is computed by integrating the hydrostatic relation down from the top, excluding the vapor component.

Having computed the full pressure (dry plus vapor) and dry air pressure on the input sounding levels, the pressure at the model top (p_{dht}) is computed by linear interpolation in height (or possibly extrapolation) given the height of the model top (an input variable). The column mass μ_d is computed by interpolating the dry air pressure to the surface and subtracting it from p_{dht}. Given the column mass, the dry-air pressure at each η level is known from the coordinate definition (2.1), repeated here

$$\eta = (p_{dh} - p_{dht})/\mu_d \qquad \text{where} \quad \mu_d = p_{dhs} - p_{dht},$$

and the pressures p_{dhs} and p_{dht} refer to the dry atmosphere. The potential temperature from the input sounding is interpolated to each of the model pressure levels, and the equation of state (5.2) is used to compute the inverse density α_d. Finally, the ARW's hydrostatic relation (2.9), in discrete form

$$\delta_\eta \phi = -\alpha_d \mu_d$$

is used to compute the geopotential. This procedure is used to compute the reference state (assuming a dry atmosphere) and the full state (using the full moist sounding). The perturbation variables are computed as the difference between the reference and full state. It should also be

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noted that in the nonhydrostatic model integration, the inverse density α_d is diagnosed from the geopotential using this equation of state, and the pressure is diagnosed from the equation of state using the inverse density α_d and the prognostic potential temperature θ. Thus, the ARW's prognostic variables μ_d, θ, and ϕ are in exact hydrostatic balance for the model equations (to machine roundoff).

5.2   Initialization for Real-Data Conditions

The initial conditions for the real-data cases are pre-processed through a separate package called the Standard Initialization (SI, see Fig. 5.1). The output from the SI is passed to the real-data pre-processor in the ARW— program *real*— which generates initial and lateral boundary conditions. This section is primarily about the steps taken to build the initial and the lateral boundary conditions for a real-data case. Even though the SI is outside of the ARW system, a brief description is appropriate to see how the raw meteorological and static terrestrial data are brought into the model for real-data cases.

5.2.1   Use of the Standard Initialization by the ARW

The SI is a set of programs that takes terrestrial and meteorological data (typically in GriB format) and transforms them for input to the ARW pre-processor program for real-data cases (*real*). Figure 5.1 shows the flow of data into and out of the SI system. The first step for the SI is to define a physical grid (including the projection type, location on the globe, number of grid points, nest locations, and grid distances) and to interpolate static fields to the prescribed domain. Independent of the domain configuration, an external analysis or forecast is processed by the SI's GriB decoder, which diagnoses required fields and reformats the GriB data into an internal binary format. With a specified domain, the SI horizontally interpolates the meteorological data onto the projected domain(s), and vertically interpolates the data to the ARW's η coordinate. The output data from the SI supplies a complete 3-dimensional snapshot of the atmosphere on the selected model grid's horizontal and vertical staggering at the selected time slices, which is sent to the ARW pre-processor program for real-data cases.

The input to the ARW real-data processor from the SI contains 3-dimensional fields of potential temperature (K), mixing ratio (kg/kg), and the horizontal components of momentum (m/s, already rotated to the model projection). The 2-dimensional static terrestrial fields include: albedo, Coriolis parameters, terrain elevation, vegetation/land-use type, land/water mask, map scale factors, map rotation angle, soil texture category, vegetation greenness fraction, annual mean temperature, and latitude/longitude. The 2-dimensional time-dependent fields from the external model, after processing by the SI, include: μ_d (Pa), layers of soil temperature (K) and soil moisture (kg/kg, either total moisture, or binned into total and liquid content), snow depth (m), skin temperature (K), and fractional sea ice. All of the fields in the final output from the SI are on the correct horizontal and vertical staggering for the ARW. The input data from the SI is assumed to be hydrostatically balanced.
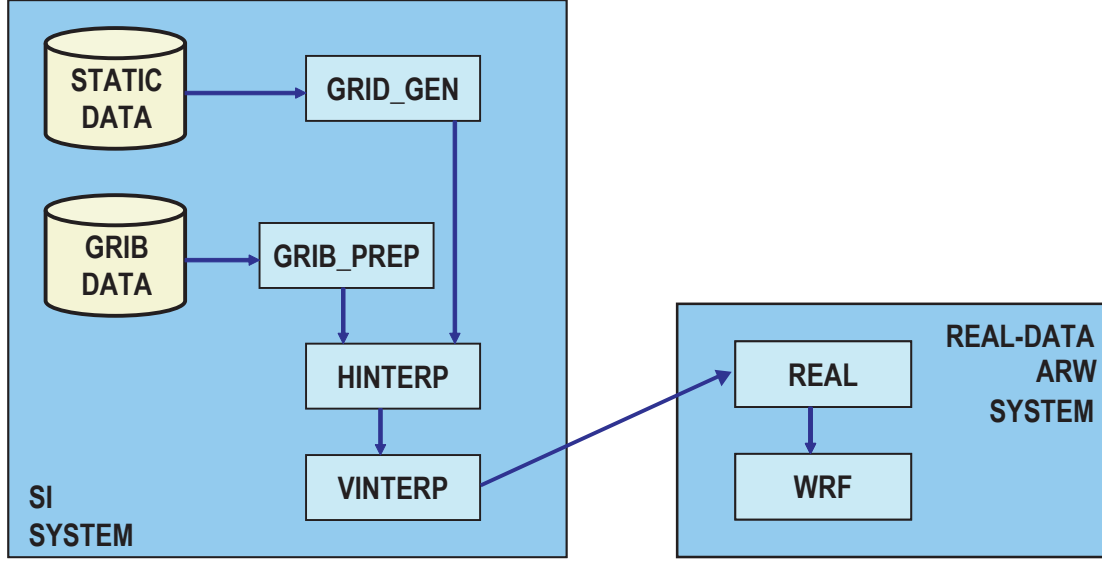
Figure 5.1: Schematic showing the data flow and program components in the SI, and how the SI feeds initial data to the ARW. Letters in the rectangular boxes indicate program names. GRID_GEN: defines the model domain and creates static files of terrestrial data. GRIB_PREP: decodes GriB data. HINTERP: interpolates meteorological data to the model domain. VINTERP: vertically interpolates data to model coordinate.

5.2.2 Reference and Perturbation State

Identical to the idealized initializations, there is a partitioning of some of the meteorological data into reference and perturbation fields. For real-data cases, the reference state is defined by terrain elevation and the following three constants:

- p_0 (10^5 Pa) reference sea level pressure;
- T_0 (usually 270 to 300 K) reference sea level temperature; and
- A (50 K) temperature difference between the pressure levels of p_0 and p_0/e.

Using these parameters, the dry reference state surface pressure is

$$p_{dhs} = p_0 \ exp\left(\frac{-T_0}{A} + \sqrt{\left(\frac{T_0}{A}\right)^2 - \frac{2\phi_{sfc}}{A \ R_d}} \right). \tag{5.3}$$

From (5.3), the 3-dimensional reference pressure is computed as a function of the vertical coordinate η levels and the model top p_{dht} (input provided by SI for real-data cases):

$$\overline{p}_d = \eta \ (p_{dhs} - p_{dht}) + p_{dht}. \tag{5.4}$$

With (5.4), the reference temperature is a straight line on a skew-T plot, defined as

$$T = T_0 + A \ ln\frac{\overline{p}_d}{p_0}.$$

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From the reference temperature and pressure, the reference potential temperature is then defined as

$$\overline{\theta}_d = \left(T_0 + A\ ln\frac{\overline{p}_d}{p_0} \right) \left(\frac{p_0}{\overline{p}_d} \right)^{\frac{R_d}{C_p}}, \tag{5.5}$$

and the reciprocal of the reference density using (5.4) and (5.5) is given by

$$\overline{\alpha}_d = \frac{1}{\overline{\rho}_d} = \frac{R_d\ \overline{\theta}_d}{p_0} \left(\frac{\overline{p}_d}{p_0} \right)^{-\frac{C_v}{C_p}}. \tag{5.6}$$

The base state difference of the dry surface pressure from (5.3) and the model top is given as

$$\overline{\mu}_d = p_{dhs} - p_{dht}. \tag{5.7}$$

From (5.6) and (5.7), the reference state geopotential defined from the hydrostatic relation is

$$\delta_\eta \overline{\phi} = -\overline{\alpha}_d\ \overline{\mu}_d.$$

One of the total fields provided to the real-data cases by the SI is μ_d. The perturbation field given the reference value (5.7) is

$$\mu_d' = \mu_d - \overline{\mu}_d. \tag{5.8}$$

Starting with the reference state fields (5.4, 5.6, and 5.7) and the dry surface pressure perturbation (5.8), the perturbation fields for pressure and inverse density are diagnosed. The pressure perturbation includes moisture and is diagnosed from the hydrostatic equation

$$\delta_\eta p' = \mu_d' \left(1 + \overline{q_v}^\eta \right) + \overline{q_v}^\eta\ \overline{\mu}_d,$$

which is integrated down from at the model top (where $p' = 0$) to recover p'. The total dry inverse density is given as

$$\alpha_d = \frac{R_d}{p_0}\ \theta \left(1 + \frac{R_v}{R_d}q_v \right) \left(\frac{p_d' + \overline{p}_d}{p_0} \right)^{-\frac{C_v}{C_p}},$$

which defines the perturbation field for inverse density

$$\alpha_d' = \alpha_d - \overline{\alpha}_d.$$

The perturbation geopotential is diagnosed from the hydrostatic relation

$$\delta_\eta \phi' = -\left(\mu_d \alpha_d' + \mu_d' \overline{\alpha}_d \right)$$

by upward integration using the terrain elevation as the lower boundary condition. In future versions of the real-data pre-processor, p' will be re-diagnosed consistent with the method used in the model (2.21) as a final step. No modifications to the original μ_d, u, v, q_v, or θ from the SI are performed. The vertical component of velocity is initialized to zero.

5.2.3 Generating Lateral Boundary Data

This section deals with generating the separate lateral boundary condition file used exclusively for the real-data cases. For information on which lateral boundary options are available for both the idealized and real-data cases, see Chapter (6).

The specified lateral boundary condition for the coarse grid for real-data cases is supplied by an external file that is generated by program *real*. This file contains records for the fields u, v, θ, q_v, ϕ', and μ_d' that are used by the ARW to constrain the lateral boundaries (other fields are in the boundary file, but are not used). The lateral boundary file holds one less time period than was processed by the SI. Each of these variables has both a valid value at the initial time of the lateral boundary time and a tendency term to get to the next boundary time period. For example, assuming a 3-hourly availability of data from the SI, the first time period of the lateral boundary file for u would contain data for both coupled u (map scale factor and μ_d interpolated to the variable's staggering) at the 0 h time

$$U_{0h} = \left. \frac{\overline{\mu_d}^x u}{\overline{m}^x} \right|_{0h},$$

and a tendency value defined as

$$U_t = \frac{U_{3h} - U_{0h}}{3h},$$

which would take a grid point from the initial value to the value at the next large-scale time during 3 simulation hours. The horizontal momentum fields are coupled both with μ_d and the inverse map factor. The other 3-dimensional fields (θ, q_v, and ϕ') are coupled only with μ_d. The μ_d' lateral boundary field is not coupled.

Each lateral boundary field is defined along the four sides of the rectangular grid (loosely referred to as the north, south, east, and west sides). The boundary values and tendencies for vertical velocity and the non-vapor moisture species are included in the external lateral boundary file, but act as place-holders for the nested boundary data for the fine grids. The width of the lateral boundary along each of the four sides is user selectable at run-time.

5.2.4 Masking of Surface Fields

Some of the meteorological and static fields are "masked". A masked field is one in which the values are typically defined only over water (e.g., sea surface temperature) or defined only over land (e.g., soil temperature). The need to match all of the masked fields consistently to each other requires additional steps for the real-data cases due to the masked data's presumed use in various physics packages in the soil, at the surface, and in the boundary layer. If the land/water mask for a location is flagged as a water point, then the vegetation and soil categories must also recognize the location as the special water flag for each of their respective categorical indices.

The values for the soil temperature and soil moisture come from the SI on the native levels originally defined for those variables in the large-scale model. The SI does no vertical interpolation for the soil data. While it is typical to try to match the ARW soil scheme with the incoming data, that is not a requirement. Pre-processor *real* will vertically interpolate (linear in depth below the ground) from the incoming levels to the requested soil layers to be used within the model.

Chapter 6

Lateral Boundary Conditions

Several lateral boundary condition options exist for the ARW that are suitable for idealized flows, and a specified lateral boundary condition for real-data simulations is available. These choices are handled via a run-time option in the Fortran namelist file. The coarsest grid of any single simulation is eligible for any of the lateral boundary selections. For example, real-data cases could use combinations of periodic, symmetric, or open lateral boundary conditions instead of the more traditional time-dependent conditions provided by an external boundary file. However, use of the specified time-dependent lateral boundary conditions for one of the idealized simulations is not possible because an external boundary file is not generated. The ARW supports rectangular horizontal grid refinement with integer ratios of the parent and child grid distances and time steps.

For nesting, all fine domains use the nest time-dependent lateral boundary condition where the outer row and column of the fine grid is specified from the parent domain, described in Section 7.3.

6.1 Periodic Lateral Boundary Conditions

Periodic lateral boundary conditions in the ARW can be specified as periodic in x (west-east), y (south-north), or doubly periodic in (x, y). The periodic boundary conditions constrain the solutions to be periodic; that is, a generic model state variable ψ will follow the relation

$$\psi(x + nL_x, y + mL_y) = \psi(x, y)$$

for all integer (n, m). The periodicity lengths (L_x, L_y) are
[(dimension of the domain in x) - 1]Δx and [(dimension of the domain in y) - 1]Δy.

6.2 Open Lateral Boundary Conditions

Open lateral boundary conditions, also referred to as gravity-wave radiating boundary conditions, can be specified for the west, east, north, or south boundary, or any combination thereof. The gravity wave radiation conditions follow the approach of Klemp and Lilly (1978) and Klemp and Wilhelmson (1978).

There are a number of changes in the base numerical algorithm described in Chapter 3 that accompany the imposition of these conditions. First, for the normal horizontal velocities along

a boundary on which the condition is specified, the momentum equation for the horizontal velocity, (3.7) or (3.8), is replaced by

$$\delta_\tau U'' = -U^* \delta_x u,$$

where $U^* = \min(U - c_b, 0)$ at the $x = 0$ (western) boundary, $U^* = \max(U + c_b, 0)$ at the $x = L$ (eastern) boundary, and likewise for the south and north boundaries for the V momentum. The horizontal difference operator δ_x is evaluated in a one-sided manner using the difference between the boundary value and the value one grid-point into the grid from the boundary. c_b is the phase speed of the gravity waves that are to be radiated; it is specified as a model constant (for more details see Klemp and Lilly, 1978; Klemp and Wilhelmson, 1978).

For scalars and non-normal momentum variables, the boundary-perpendicular flux divergence term is replaced with

$$\delta_x(U\psi) = U^* \delta_x \psi + \psi \; \delta_x U,$$

where $U^* = \min(U, 0)$ at the $x = 0 + \Delta x/2$ (western) scalar boundary, $U^* = \max(U, 0)$ at the $x = L - \Delta x/2$ (eastern) boundary, and likewise for the south and north boundaries using V. As was the case for the momentum equations, the horizontal difference operator δ_x is evaluated in a one-sided manner using the difference between the boundary value and the value one grid-point into the grid from the boundary.

6.3   Symmetric Lateral Boundary Conditions

Symmetry lateral boundary conditions can be specified for the west, east, north, or south boundary, or any combination thereof. The symmetry boundaries are located on the normal-velocity planes at the lateral edges of the grids. The normal velocities are zero at these boundaries, and on either side of the boundary the normal velocity satisfies the relation

$$U_\perp(x_b - x) = -U_\perp(x_b + x),$$

where x_b is the location of the symmetry boundary. All other variables satisfy the relation

$$\psi(x_b - x) = \psi(x_b + x).$$

6.4   Specified Lateral Boundary Conditions

Primarily for real-data cases, the specified boundary condition is also referred to as a relaxation, or nudging, boundary condition. There are two uses of the specified boundaries in the ARW: for the outer-most coarse grid or for the time-dependent boundaries supplied to a nested grid. The specified lateral boundary conditions for the nest are automatically selected for all of the fine grids, even if the coarse grid is using combinations of the symmetry, periodic, or open options. If the specified lateral boundary condition is selected for the coarse grid, then all four grid sides (west, east, north, and south) use specified lateral conditions.

The coarse grid specified lateral boundary is comprised of both a specified and a relaxation zone as shown in Fig. 6.1). For the coarse grid, the specified zone is determined entirely by temporal interpolation from an external forecast or analysis (supplied by the SI). The width

Real-Data Lateral Boundary Condition: Location of Specified and Relaxation Zones
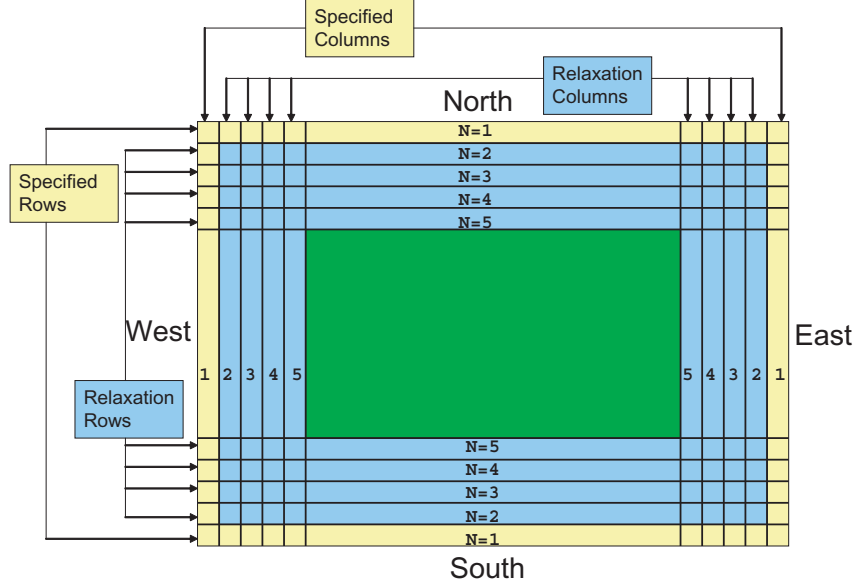
Figure 6.1: Specified and relaxation zones for a grid with a single specified row and column, and four rows and columns for the relaxation zone. These are typical values used for a specified lateral boundary condition for a real-data case.

of the specified zone is run-time configurable, but is typically set to 1 (i.e., the last row and column along the outer edge of the coarse grid). The second region of the lateral boundary for the coarse grid is the relaxation zone. The relaxation zone is where the model is nudged or relaxed towards the large-scale forecast (e.g., rows and columns 2 through 5 in Fig. 6.1). The size of the relaxation zone is a run-time option.

The specified lateral boundary condition for the coarse grid requires an external file, generated during the same pre-processing as the initial condition file. Let ψ be any prognostic value having a lateral boundary entry, after Davies and Turner (1977),

$$\partial_t \psi \big|_n = F_1(\psi_{LS} - \psi) - F_2 \Delta^2 (\psi_{LS} - \psi), \tag{6.1}$$

where n is the number of grid points in from the outer row or column along the boundary ($SpecZone + 1 \leq n \leq SpecZone + RelaxZone - 1$; see Fig. 6.1) and ψ_{LS} is the large-scale value obtained by spatial and temporal interpolation from the analyses. Δ^2 is a 5-point horizontal smoother applied along η-surfaces. The weighting function coefficients F_1 and F_2 in (6.1) are given by

$$F_1 = \frac{1}{10\Delta t} \frac{SpecZone + RelaxZone - n}{RelaxZone - 1},$$

$$F_2 = \frac{1}{50\Delta t} \frac{SpecZone + RelaxZone - n}{RelaxZone - 1},$$

where n extends only through the relaxation zone ($SpecZone + 1 \leq n \leq SpecZone + RelaxZone - 1$). F_1 and F_2 are linear ramping functions with a maximum at the first relaxation row or column nearest the coarse grid boundary (just inside of the specified zone).

On the coarse grid, the specified boundary condition applies to the horizontal wind components, potential temperature, ϕ', μ'_d, and water vapor. The lateral boundary file contains enough information to update the boundary zone values through the entire simulation period. The momentum fields are coupled with μ_d and the inverse map factor (both at the specific variable's horizontal staggering location), and the other 3-dimensional fields are coupled with μ_d. The μ'_d field is not coupled in the lateral boundary file.

Vertical velocity has a zero gradient boundary condition applied in the specified zone (usually the outer-most row and column). Microphysical variables, except vapor, and all other scalars have flow-dependent boundary conditions applied in the specified zone. This boundary condition specifies zero on inflow and zero-gradient on outflow. Since these boundary conditions require only information from the interior of the grid, these variables are not in the specified boundary condition file.

Chapter 7

Nesting

The ARW supports horizontal nesting that allows resolution to be focused over a region of interest by introducing an additional grid (or grids) into the simulation. In the current implementation, only horizontal refinement is available: there is no vertical nesting option. The nested grids are rectangular and are aligned with the parent (coarser) grid within which they are nested. Additionally, the nested grids allow any integer spatial ($\Delta x_{coarse}/\Delta x_{fine}$) and temporal refinements of the parent grid. This nesting implementation is in many ways similar to the implementations in other mesoscale and cloudscale models (e.g. MM5, ARPS, COAMPS). The major improvement in the ARW's nesting infrastruture compared with techniques used in other models is the ability to compute nested simulations efficiently on parallel distributed-memory computer systems, which includes support for moving nested grids. The WRF Software Framework, described in Michalakes et al. (2004), makes these advances possible. In this chapter we describe the various nesting options available in the ARW and the numerical coupling between the grids.

7.1 Overview

1-Way and 2-Way Grid Nesting

Nested grid simulations can be produced using either 1-way nesting or 2-way nesting as outlined in Fig. 7.1. The 1-way and 2-way nesting options refer to how a coarse grid and the fine grid interact. In both the 1-way and 2-way simulation modes, the fine grid boundary conditions (i.e., the lateral boundaries) are interpolated from the coarse grid forecast. In a 1-way nest, this is the only information exchange between the grids (from coarse grid to fine grid). Hence, the name *1-way nesting.* In the 2-way nest integration, the fine grid solution replaces the coarse grid solution for coarse grid points that lie inside the fine grid. This information exchange between the grids is now in both directions (coarse-to-fine and fine-to-coarse). Hence, the name *2-way nesting.*

The 1-way nest option may be run in one of two different methods. One option is to produce the nested simulation as two separate ARW simulations as described in the leftmost box in Fig. 7.1. In this mode, the coarse grid is integrated first. Output from the coarse grid integration is then processed to provide boundary conditions for the nested run (usually at a much lower temporal frequency than the coarse grid time step), and this is followed by the complete time integration of fine (nested) grid. Hence, this 1-way option is equivalent to
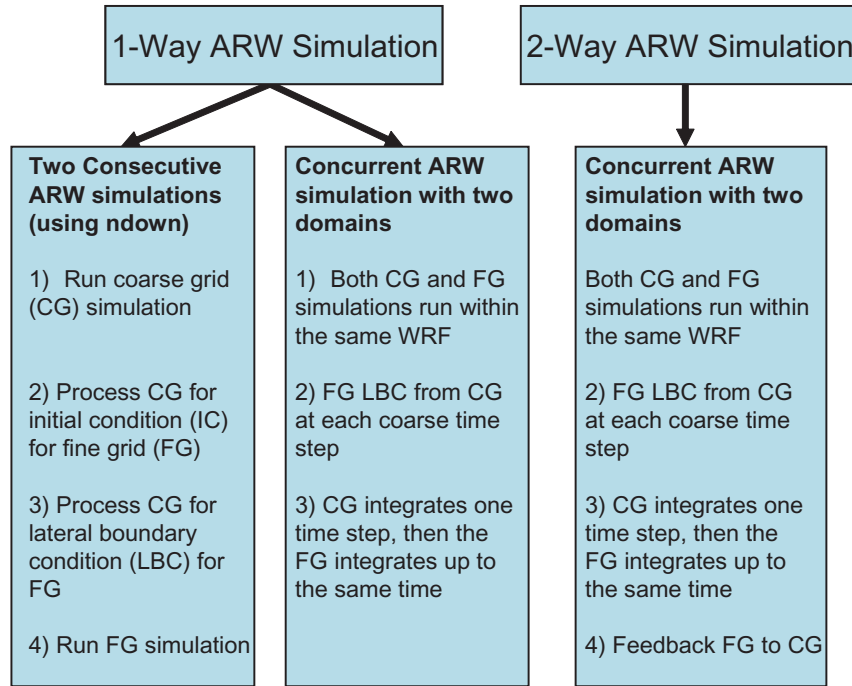
Figure 7.1: 1-way and 2-way nesting options in the ARW.

running two separate simulations with a processing step in between. Also with separate grid simulations, an intermediate re-analysis (such as via 3D-Var, see Section 9) can be used.

The second 1-way option (lockstep with no feedback), depicted in the middle box in Fig. 7.1, is run as a traditional simulation with two (or more) grids integrating concurrently, except with the feedback runtime option shut off. This option provides lateral boundary conditions to the fine grid at each coarse grid time step, which is an advantage of the concurrent 1-way method (no feedback).

Fine Grid Initialization Options

The ARW supports several strategies to refine a coarse-grid simulation with the introduction of a nested grid. When using 1-way and 2-way nesting, several options for initializing the fine grid are provided.

- All of the fine grid variables can be interpolated from the coarse grid.
- All of the fine grid variables can be input from an external file which has high-resolution information for both the meteorological and the terrestrial fields.
- The fine grid can have some of the variables initialized with a high-resolution external data set, while other variables are interpolated from the coarse grid.
- For a moving nest, an external orography file may be used to update the fine grid terrain elevation. This option is not generally available in this release.

These fine grid initialization settings are user specified at run-time, and the ARW allows nested grids to instantiate and cease during any time that the fine grid's parent is still integrating. (The
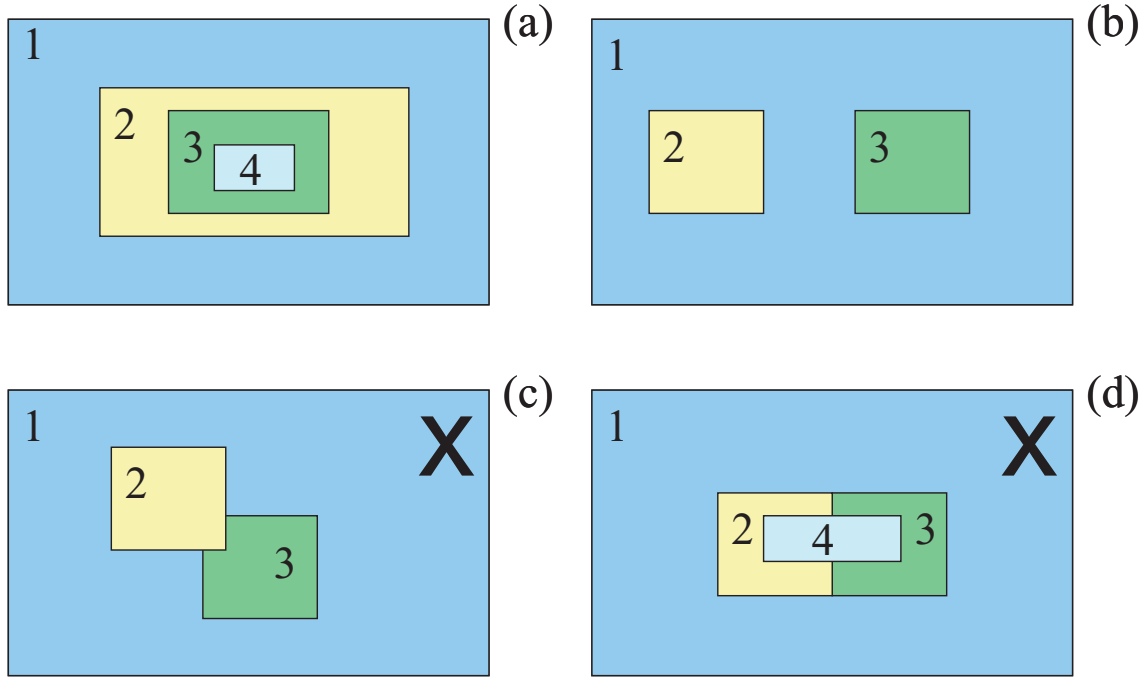
Figure 7.2: Various nest configurations for multiple grids. (a) Telescoping nests. (b) Nests at the same level with respect to a parent grid. (c) Overlapping grids: not allowed (d) Inner-most grid has more than one parent grid: not allowed

system is currently constrained to starting nests at the beginning of the coarse grid simulations if runs require input of nest-resolution terrain or other lower boundary data. This limitation will be addressed in the near future.)

Possible Grid Configurations

A simulation involves one outer grid and may contain multiple inner nested grids. In the ARW, each nested region is entirely contained within a single coarser grid, referred to as the *parent* grid. The finer, nested grids are referred to as *child* grids. Using this terminology, children are also parents when multiple levels of nesting are used. The fine grids may be telescoped to any depth (i.e., a parent grid may contain one or more child grids, each of which in turn may successively contain one or more child grids; Fig. 7.2a), and several fine grids may share the same parent at the same level of nesting (Fig. 7.2b). Any valid fine grid may either be a static domain or it may be a moving nest with prescribed incremental shifts. The ARW does not permit overlapping grids, where a coarse grid point is contained within more than a single child grid (i.e., both of which are at the same nest level with respect to the parent; Fig. 7.2c). In addition, no grid can have more than a single parent (Fig. 7.2d).

For 2-way nested grid simulations, the ratio of the parent horizontal grid distance to the child horizontal grid distance (the spatial refinement ratio) must be an integer. This is also true for the time steps (the temporal refinement ratio). The model does allow the time step refinement ratio to differ from the spatial refinement ratio. Also, nested grids on the same level (i.e., children who have the same parent) may have different spatial and temporal refinement
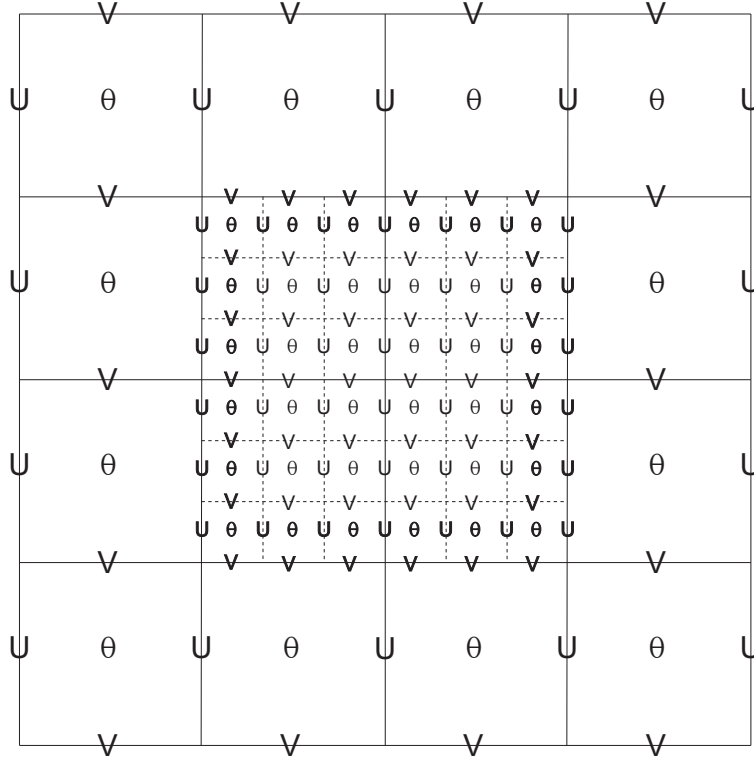
Figure 7.3: Arakawa-C grid staggering for a portion of a parent domain and an imbedded nest domain with a 3:1 grid size ratio. The solid lines denote coarse grid cell boundaries, and the dashed lines are the boundaries for each fine grid cell. The horizontal components of velocity ("U" and "V") are defined along the normal cell face, and the thermodynamic variables ("θ") are defined at the center of the grid cell (each square). The bold typeface variables along the interface between the coarse and the fine grid define the locations where the specified lateral boundaries for the nest are in effect.

ratios.

7.2   Nesting and Staggering

The ARW uses an Arakawa-C grid staggering. As shown in Fig. 7.3, the u and v components of horizontal velocity are normal to the respective faces of the grid cell, and the mass/thermodynamic/scalar variables are located in the center of the cell.

The variable staggering has an additional column of u in the x-direction and an additional row of v in the y-direction because the normal velocity points define the grid boundaries. The horizontal momentum components reflect an average across each cell-face, while each mass/thermodynamic/scalar variable is the representative mean value throughout the cell. Feedback is handled to preserve these mean values: the mass/thermodynamic/scalar fields are fed back with an average from within the entire coarse grid point (Fig. 7.3), and the horizontal momentum variables are averaged along their respective normal coarse grid cell faces.

The horizontal interpolation (to instantiate a grid and to provide time-dependent lateral

boundaries) does not conserve mass. The feedback mechanism, for most of the unmasked fields, uses cell averages (for mass/thermodynamic/scalar quantities) and cell-face averages for the horizontal momentum fields.

The staggering defines the way that the fine grid is situated on top of the coarse grid. For all odd ratios there is a coincident point for each variable: a location that has the coarse grid and the fine grid at the same physical point. The location of this point depends on the variable. In each of the coarse-grid cells with an odd ratio, the middle fine-grid cell is the coincident point with the coarse grid for all of the mass-staggered fields (Fig. 7.3). For the horizontal momentum variables the normal velocity has coincident points along the grid boundaries for odd ratios.

For fields that are averaged back to the coarse grid in the feedback, the mean of the nine mass/thermodynamic/scalar (for example, due to the 3:1 grid-distance ratio in the example shown in Fig. 7.3) fine grid points is fed back to the coarse grid. These fields include most 3D and 2D arrays. For the horizontal momentum fields averaged back to the coarse grid in the feedback, the mean of three (for example, due to the 3:1 grid-distance ratio in the example shown in Fig. 7.3) fine grid points is fed back to the coarse grid from along the coincident cell face. The fields that are masked due to the land/sea category are fed back directly from the coincident points for odd ratios. Only masked fields use the feedback method where a single point from the fine grid is assigned to the coarse grid.

A difference between the odd and even grid-distance ratios is in the feedback from the fine grid to the coarse grid. No coincident points exist for the single point feedback mechanisms for even grid distance ratios (such as used for the land/sea masked 2D fields). For a 2:1 even grid distance ratio, Figure 7.4 shows that each coarse grid point has four fine grid cells that are equally close, and therefore four equally eligible grid points for use as the single fine-grid point that feeds back to the coarse grid. The single-point feedback is arbitrarily chosen as the south-west corner of the four neighboring points. This arbitrary assignment to masked fields implies that even grid distance ratios are more suited for idealized simulations where masked fields are less important.

7.3 Nested Lateral Boundary Conditions

For the fine grid with 2-way nesting or 1-way nesting (using a concurrent ARW simulation, see Fig. 7.1), the boundary conditions are specified by the parent grid at every coarse-grid time step. The nest lateral boundary condition behaves similarly to the specified boundary condition for real-data cases (see Section 6.4), but the relaxation zone is not active. Prognostic variables are entirely specified in the outer row and column of the fine grid through spatial and temporal interpolation from the coarse grid (the coarse grid is stepped forward in time prior to advancement of any child grid of that parent).

7.4 Steps to Generate a Nest Grid

Only the concurrent 1-way nest option or the 2-way nest option are considered in this section. The 1-way nest option (using two consecutive ARW simulations, see Fig. 7.1) is functionally similar to two separate, single-grid simulations and does not fit the following description. For a multiple grid simulation within a single model run, there are some additional infrastructure steps
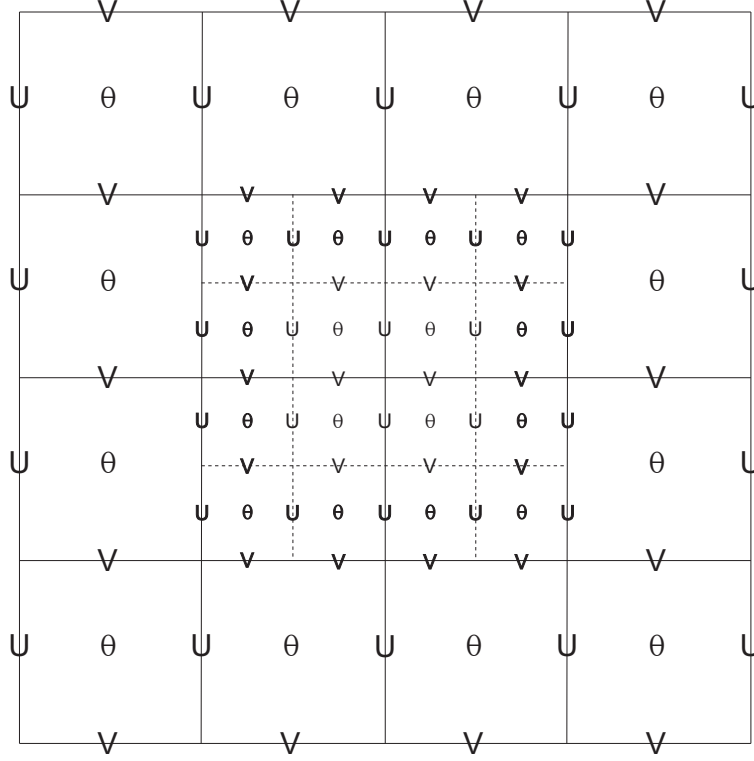
Figure 7.4: Similar to Fig. 7.3, but with a 2:1 grid-distance ratio.

that are required (briefly described in Fig. 7.5). While the following text details a simulation with a single coarse-grid and a single fine-grid, this implies no lack of generality when handling multiple grid levels or multiple grids on the same level.

Nest Instantiation

The fine grid is instantiated as a child of a parent grid at the requested start time. This initialization is within the integration step for the parent grid, so no child grid can begin if the parent is not active. To fill in the correct meteorological fields, an initialization routine is called to horizontally interpolate the coarse-grid data to the fine grid locations using a monotone interpolation scheme (described in Smolarkiewicz and Grell, 1990) for most fields (i.e., the same scheme employed for generating the fine grid lateral boundary conditions) and a simple linear interpolation or averaging scheme for masked or categorical fields. For fields that are masked with the land/sea background (such as land only fields (e.g., snow), or water only fields (e.g., sea ice)), the interpolator needs to know what field defines the template for the masking (such as the land use category). Part of the automatic code generation handles calling each field with its associated interpolator.

```
┌─────────────────────────────────────────────────────────────────────────┐
│  Integrate Parent Grid One Time Step                                      │
│         If Nest Grid Start Time                                           │
│                (1) Horizontally Interpolate Parent to Child Grid          │
│                (2) Optionally Input High-Resolution Child Data            │
│                (3) Compute Child Reference State                          │
│                (4) Feedback Child Initial Data to Parent Grid             │
│                (5) Re-Compute Parent Reference State                      │
│         End If Nest Grid Start Time                                       │
│         Solve Time Step for Parent Grid (see Fig. 3.1)                    │
│         While Existing Nest Grids to Integrate                           │
│                (1) Lateral Forcing from Parent Grid to Child              │
│                (2) Integrate Child Grid to Current Time of Parent Grid    │
│                (3) Feedback Child Grid Information to Parent Grid          │
│         End While Existing Nest Grids to Integrate                       │
│  End Grid Integrate                                                       │
└─────────────────────────────────────────────────────────────────────────┘
```

Figure 7.5: Nest grid integration sequence.

Fine Grid Input

After the horizontal interpolation is completed, a few orographic-based variables are saved so that they may be used to blend the lateral boundaries along the coarse-grid/fine-grid interface. The terrain elevation, $\overline{\mu}_d$, and the reference geopotential ($\overline{\phi}$) are stored for later use. The fields selected as input from the fine grid input file (for the concurrent 1-way and 2-way forecast methods shown in Fig. 7.1) are ingested, and they overwrite the arrays that were horizontally interpolated from the coarse grid. No quality control for data consistency is performed for the fine grid input. All such masked checks are completed by the ARW real-data pre-processor *real*.

Interface Blended Orography

When the fine grid data has been input, the previously-saved orographic-based fields are blended across the four outer rows and columns of the fine grid. The blending is a simple linear weighting between the interpolated coarse-grid values (the saved data) and the fine grid values from the input file. The weighting scheme is given as:

- row/column 1: 100% interpolated coarse grid, 0% fine grid,
- row/column 2: 75% interpolated coarse grid, 25% fine grid,
- row/column 3: 50% interpolated coarse grid, 50% fine grid,
- row/column 4: 25% interpolated coarse grid, 75% fine grid, and
- row/column 5: 0% interpolated coarse grid, 100% fine grid,

where the row or column nearest the outer edge takes precedence in ambiguous corner zones. The blended arrays are required to compute the reference state for the fine grid. The first row and column (100% interpolated from the coarse grid) ensures that the reference state for the

coarse grid and fine grid is consistent along the fine grid boundary interface. The blending along the inner rows and columns ramps the coarse grid reference state to the fine grid reference state for a smooth transition between the grids.

Feedback

So that the coarse grid and the fine grid are consistent at coincident points, the fine grid values are fed back to the coarse grid. There are two available options for feedback: either the mean of all fine grid cells contained within each coarse grid cell is fed back (or cell faces in the case of the horizontal momentum fields), or a single-point feedback is selected for the masked or categorical fields.

Subsequent to the feedback step, the coarse grid may be optionally smoothed in the area of the fine grid. Two smoothers are available: a 5-point 1-2-1 smoother and a smoother-desmoother with a similar stencil size. Both the feedback and the smoothers are run one row and column in from the interface row and column of the coarse grid that provides the lateral boundary conditions to the fine grid.

Reference State

The initial feedback when the nest is instantiated ensures that the coarse grid is consistent with the fine grid, particularly with regards to elevation and the reference state fields inside the blended region, and for such terrestrial features as coasts, lakes, and islands. The adjustment of the elevation in the coarse grid forces a base state recalculation. The fine-grid needs an initial base state calculation, and after the terrain feedback, the coarse grid is also in need of a base state recalculation.

Note that with the horizontal interpolation of the coarse grid to the fine grid and the feedback of the fine grid to the coarse grid, the coarse grid base state is recomputed even without a separate fine-grid initial data file, since the coarse grid topography is adjusted.

With the completed base state computations, the routines return back to the integration step for the coarse and fine grids. The fine grid data is now properly initialized for integration and can be advanced forward a time step.

Integration

The integration by grid is recursive. At the end of each grid's time step, a check is made to determine if a child grid exists for that parent and if the current time is bracketed by the child's start/end time. This is shown in Fig. 7.5. The integration process for the nest (step 2 under the while loop) is recursively calling the top step in the overall sequence as a coarse grid itself. This is a "depth first" traversal of the tree of grids. If a child grid does exist, that child grid is integrated up through the current time of the parent grid.

Chapter 8

Physics

This chapter outlines the physics options available in the ARW. The WRF physics options fall into several categories, each containing several options. The physics categories are (1) microphysics, (2) cumulus parameterization, (3) planetary boundary layer (PBL), (4) land-surface model, and (5) radiation. Diffusion, which may also be considered part of the physics, is described in Chapter 4.

The physics section is insulated from the rest of the dynamics solver by the use of physics drivers. These are between solver-dependent routines: a pre-physics preparation and post-physics modifications of the tendencies. The physics preparation involves filling arrays with physics-required variables that include the temperature, pressure, heights, layer thicknesses, and other state variables in MKS units at half-level grid points and on full levels. The velocities are also de-staggered so that the physics part is independent of the dynamical solver's velocity staggering. Physics packages compute tendencies for the velocity components (un-staggered), potential temperature, and moisture fields. The solver-dependent post-physics step will re-stagger these tendencies as necessary, couple tendencies with coordinate metrics, and convert to variables or units appropriate to the dynamics solver.

In the first Runge-Kutta step, prior to the acoustic steps (see Fig. 3.1, step(1)), tendencies are computed for radiation, surface, PBL, and cumulus physics. These tendencies are then held fixed through the Runge-Kutta steps. Microphysics is computed after the last Runge-Kutta step (see Fig. 3.1, step (9)) in order to maintain proper saturation conditions at the end of the time-step.

The initialization of the physics is called prior to the first model step. This initialization may include reading in data files for physics tables or calculating look-up tables of functions. Each physics module includes an initialization routine for this purpose. Often physics packages will have many of their own constants that should also be included in their own module, while common physical constants are passed in from the physics drivers.

8.1 Microphysics

Microphysics includes explicitly resolved water vapor, cloud, and precipitation processes. The model is general enough to accommodate any number of mass mixing-ratio variables, and other moments such as number concentrations. Four-dimensional arrays with three spatial indices and one species index are use to carry such scalars. Memory, i.e., the size of the fourth dimension

Table 8.1: Microphysics Options

Scheme	Number of Variables	Ice-Phase Processes	Mixed-Phase Processes
Kessler	3	N	N
Purdue Lin	6	Y	Y
WSM3	3	Y	N
WSM5	5	Y	N
WSM6	6	Y	Y
Eta GCP	2	Y	Y
Thompson	7	Y	Y

in these arrays, is allocated depending on the needs of the scheme chosen, and advection of the species also applies to all those required by the microphysics option. In the current version of the ARW, microphysics is carried out at the end of the time-step as an adjustment process, and so does not provide tendencies. The rationale for this is that condensation adjustment should be at the end of the time-step to guarantee that the final saturation balance is accurate for the updated temperature and moisture. However, it is also important to have the latent heating forcing for potential temperature during the dynamical sub-steps, and this is done by saving the microphysical heating as an approximation for the next time-step as described in Section 3.1.4.

Currently, the sedimentation process is accounted for in the microphysics, and a smaller time step is allowed to calculate the vertical flux of precipitation to prevent instability. The saturation adjustment is also included inside the microphysics. In the future, however, it might be separated into an individual subroutine to enable the remaining microphysics to be called less frequently than the model's advection step for efficiency.

Table 8.1 shows a summary of the options indicating the number of moisture variables, and whether ice-phase and mixed-phase processes are included. Mixed-phase processes are those that result from the interaction of ice and water particles, such as riming that produces graupel or hail. As a general rule, for grid sizes less than 10 km, where updrafts may be resolved, mixed-phase schemes should be used, particularly in convective or icing situations. For coarser grids the added expense of these schemes is not worth it because riming is not likely to be well resolved.

8.1.1   Kessler scheme

This scheme (Kessler, 1969), which was taken from the COMMAS model (Wicker and Wilhelmson, 1995), is a simple warm cloud scheme that includes water vapor, cloud water, and rain. The microphysical processes included are: the production, fall, and evaporation of rain; the accretion and autoconversion of cloud water; and the production of cloud water from condensation.

8.1.2 Purdue Lin scheme

Six classes of hydrometeors are included: water vapor, cloud water, rain, cloud ice, snow, and graupel. All parameterization production terms are based on Lin et al. (1983) and Rutledge and Hobbs (1984) with some modifications, including saturation adjustment following Tao et al. (1989) and ice sedimentation. This is a relatively sophisticated microphysics scheme in WRF, and it is more suitable for use in research studies. The scheme is taken from the Purdue cloud model, and the details can be found in Chen and Sun (2002).

8.1.3 WRF Single-Moment 3-class (WSM3) scheme

This scheme follows Hong et al. (2004) including ice sedimentation and other new ice-phase parameterizations revised from the older NCEP3 scheme (Hong et al., 1998) that was in WRF Version 1. A major difference from other schemes is that a diagnostic relation is used for ice number concentration that is based on ice mass content rather than temperature. Three categories of hydrometers are included: vapor, cloud water/ice, and rain/snow. As with Dudhia (1989), this is a so-called simple-ice scheme wherein the cloud ice and cloud water are counted as the same category. They are distinguished by temperature: namely, cloud ice can only exist when the temperature is less than or equal to the freezing point; otherwise, cloud water can exist. The same condition is applied to rain and snow. Though the ice phase is included, it is considered efficient enough for using in operational models.

8.1.4 WSM5 scheme

This scheme is similar to the WSM3 simple ice scheme. However, vapor, rain, snow, cloud ice, and cloud water are held in five different arrays. Thus, it allows supercooled water to exist, and a gradual melting of snow as it falls below the melting layer. Details can be found in Hong et al. (2004). It replaces WRF Version 1's NCEP5 scheme (Hong et al., 1998).

8.1.5 WSM6 scheme

The six-class scheme extends the WSM5 scheme to include graupel and its associated processes. Many of these processes are parameterized similarly to Lin et al. (1983), but there are differences for the accretion calculation and in some other parameters. The freezing/melting processes are computed during the fall-term sub-steps to increase accuracy in the vertical heating profile of these processes. The order of the processes is also optimized to decrease the sensitivity of the scheme to the time step of the model. As with WSM3 and WSM5, saturation adjustment follows Dudhia (1989) and Hong et al. (1998) in separately treating ice and water saturation processes, rather than a combined saturation such as the Purdue Lin (above) and Goddard (Tao et al., 1989) schemes.

8.1.6 Eta Grid-scale Cloud and Precipitation (2001) scheme

This is also known as EGCP01 or the Eta Ferrier scheme. The scheme predicts changes in water vapor and condensate in the forms of cloud water, rain, cloud ice, and precipitation ice (snow/graupel/sleet). The individual hydrometeor fields are combined into total condensate,

and it is the water vapor and total condensate that are advected in the model. Local storage arrays retain first-guess information that extract contributions of cloud water, rain, cloud ice, and precipitation ice of variable density in the form of snow, graupel, or sleet. The density of precipitation ice is estimated from a local array that stores information on the total growth of ice by vapor deposition and accretion of liquid water. Sedimentation is treated by partitioning the time-averaged flux of precipitation into a grid box between local storage in the box and fall out through the bottom of the box. This approach, together with modifications in the treatment of rapid microphysical processes, permits large time steps to be used with stable results. The mean size of precipitation ice is assumed to be a function of temperature following the observational results of Ryan (1996). Mixed-phase processes are now considered at temperatures warmer than -30°C (previously -10°C), whereas ice saturation is assumed for cloudy conditions at colder temperatures. Further description of the scheme can be found in Sec. 3.1 of the November 2001 Technical Procedures Bulletin (TPB) at http://www.emc.ncep.noaa.gov/mmb/mmbpll/eta12tpb/ and on the COMET page at http://meted.ucar.edu/nwp/pcu2/etapcp1.htm.

8.1.7 Thompson et al. scheme

The Thompson et al. (2004) microphysical parameterization scheme includes improvements to the earlier bulk scheme of Reisner et al. (1998) and has been extensively tested and compared with both idealized case studies and documented real case studies of mid-latitude wintertime observations. The scheme includes six classes of moisture species plus number concentration for ice as prognostic variables. The scheme was designed to improve the prediction of freezing drizzle events for aircraft safety warnings. Generally microphysical parameterizations have had problems of overpredicting the amount of snow and graupel fields and under predicting the ice in outflow regions and often not accurately predicting freezing drizzle. Key improvements are the following:

- Primary ice nucleation as in Cooper (1986), replaces the Fletcher (1962) curve.
- Auto-conversion as in Walko et al. (1995), replaces the Kessler (1969) scheme.
- A generalized gamma distribution for graupel replaces the exponential distribution.
- The associated intercept parameter depends on mixing ratio instead of remaining constant.
- Riming growth of snow must exceed depositional growth of snow by a factor of 3 before rimmed snow transfers into the graupel category.
- The intercept parameter of the snow size distribution depends on temperature.
- The intercept parameter for the rain size distribution depends on rain mixing ratio, thereby simulating the fall velocity of drizzle drops as well as raindrops.

8.2 Cumulus parameterization

These schemes are responsible for the sub-grid-scale effects of convective and/or shallow clouds. The schemes are intended to represent vertical fluxes due to unresolved updrafts and downdrafts and compensating motion outside the clouds. They operate only on individual columns where the scheme is triggered and provide vertical heating and moistening profiles. Some schemes additionally provide cloud and precipitation field tendencies in the column, and future schemes

Table 8.2: Cumulus Parameterization Options

Scheme	Cloud Detrainment	Type of scheme	Closure
Kain-Fritsch	Y	Mass flux	CAPE removal
Betts-Miller-Janjic	N	Adjustment	Sounding adjustment
Grell-Devenyi	Y	Mass flux	Various

may provide momentum tendencies due to convective transport of momentum. The schemes all provide the convective component of surface rainfall.

Cumulus parameterizations are theoretically only valid for coarser grid sizes, (e.g., greater than 10 km), where they are necessary to properly release latent heat on a realistic time scale in the convective columns. While the assumptions about the convective eddies being entirely sub-grid-scale break down for finer grid sizes, sometimes these schemes have been found to be helpful in triggering convection in 5–10 km grid applications. Generally, they should not be used when the model can resolve the convective eddies itself (e.g., $\leq$ 5 km grid).

Table 8.2 summarizes the basic characteristics of the available cumulus parameterization options in the ARW.

8.2.1 Kain-Fritsch

The modified version of the Kain-Fritsch scheme (KF-Eta) is based on Kain and Fritsch (1990) and Kain and Fritsch (1993), but has been modified based on testing within the Eta model. As with the original KF scheme, it utilizes a simple cloud model with moist updrafts and downdrafts, including the effects of detrainment, entrainment, and relatively simple microphysics. It differs from the original KF scheme in the following ways:

- A minimum entrainment rate is imposed to suppress widespread convection in marginally unstable, relatively dry environments.
- Shallow (non precipitating) convection is allowed for any updraft that does not reach minimum cloud depth for precipitating clouds; this minimum depth varies as a function of cloud-base temperature.
- The entrainment rate is allowed to vary as a function of low-level convergence.
- Downdraft changes:
  - Source layer is the entire 150 – 200 mb deep layer just above cloud base.
  - Mass flux is specified as a fraction of updraft mass flux at cloud base. Fraction is a function of source layer RH rather than wind shear or other parameters, i.e., old precipitation efficiency relationship not used.
  - Detrainment is specified to occur in updraft source layer and below.

8.2.2 Betts-Miller-Janjic

The Betts-Miller-Janjic (BMJ) scheme (Janjic, 1994, 2000) was derived from the Betts-Miller (BM) convective adjustment scheme (Betts, 1986; Betts and Miller, 1986). However, the BMJ scheme differs from the Betts-Miller scheme in several important aspects. The deep convection profiles and the relaxation time are variable and depend on the cloud efficiency, a non-dimensional parameter that characterizes the convective regime (Janjic, 1994). The cloud efficiency depends on the entropy change, precipitation, and mean temperature of the cloud. The shallow convection moisture profile is derived from the requirement that the entropy change be small and nonnegative (Janjic, 1994). The BMJ scheme has been optimized over years of operational application at NCEP, so that, in addition to the described conceptual differences, many details and/or parameter values differ from those recommended in Betts (1986) and Betts and Miller (1986). Recently, attempts have been made to refine the scheme for higher horizontal resolutions, primarily through modifications of the triggering mechanism. In particular:

- A floor value for the entropy change in the cloud is set up below which the deep convection is not triggered;
- In searching for the cloud top, the ascending particle mixes with the environment; and
- The work of the buoyancy force on the ascending particle is required to exceed a prescribed positive threshold.

8.2.3 Grell-Devenyi ensemble

Grell and Devenyi (2002) introduced an ensemble cumulus scheme in which effectively multiple cumulus schemes and variants are run within each grid box and then the results are averaged to give the feedback to the model. In principle, the averaging can be weighted to optimize the scheme, but the default is an equal weight. The schemes are all mass-flux type schemes, but with differing updraft and downdraft entrainment and detrainment parameters, and precipitation efficiencies. These differences in static control are combined with differences in dynamic control, which is the method of determining cloud mass flux. The dynamic control closures are based on convective available potential energy (CAPE or cloud work function), low-level vertical velocity, or moisture convergence. Those based on CAPE either balance the rate of change of CAPE or relax the CAPE to a climatological value, or remove the CAPE in a convective time scale. The moisture convergence closure balances the cloud rainfall to the integrated vertical advection of moisture. Another control is the trigger, where the maximum cap strength that permits convection can be varied. These controls typically provide ensembles of 144 members.

8.3 Surface Layer

The surface layer schemes calculate friction velocities and exchange coefficients that enable the calculation of surface heat and moisture fluxes by the land-surface models and surface stress in the planetary boundary layer scheme. Over water surfaces, the surface fluxes and surface diagnostic fields are computed in the surface layer scheme itself. The schemes provide no tendencies, only the stability-dependent information about the surface layer for the land-surface and PBL schemes. Currently, each surface layer option is tied to particular boundary-layer options, but in the future more interchangeability and options may become available.

8.3.1 Similarity theory (MM5)

This scheme uses stability functions from Paulson (1970), Dyer and Hicks (1970), and Webb (1970) to compute surface exchange coefficients for heat, moisture, and momentum. A convective velocity following Beljaars (1994) is used to enhance surface fluxes of heat and moisture. No thermal roughness length parameterization is included in the current version of this scheme. A Charnock relation relates roughness length to friction velocity over water. There are four stability regimes following Zhang and Anthes (1982). This surface layer scheme must be run in conjunction with the MRF or YSU PBL schemes.

8.3.2 Similarity theory (Eta)

The Eta surface layer scheme (Janjic, 1996, 2002) is based on similarity theory (Monin and Obukhov, 1954). The scheme includes parameterizations of a viscous sub-layer. Over water surfaces, the viscous sub-layer is parameterized explicitly following Janjic (1994). Over land, the effects of the viscous sub-layer are taken into account through variable roughness height for temperature and humidity as proposed by Zilitinkevich (1995). The Beljaars (1994) correction is applied in order to avoid singularities in the case of an unstable surface layer and vanishing wind speed. The surface fluxes are computed by an iterative method. This surface layer scheme must be run in conjunction with the Eta (Mellor-Yamada-Janjic) PBL scheme, and is therefore sometimes referred to as the MYJ surface scheme.

8.4 Land-Surface Model

The land-surface models (LSMs) use atmospheric information from the surface layer scheme, radiative forcing from the radiation scheme, and precipitation forcing from the microphysics and convective schemes, together with internal information on the land's state variables and land-surface properties, to provide heat and moisture fluxes over land points and sea-ice points. These fluxes provide a lower boundary condition for the vertical transport done in the PBL schemes (or the vertical diffusion scheme in the case where a PBL scheme is not run, such as in large-eddy mode). [Note that large-eddy mode with interactive surface fluxes is not yet available in the ARW, but is planned for the near future.] The land-surface models have various degrees of sophistication in dealing with thermal and moisture fluxes in multiple layers of the soil and also may handle vegetation, root, and canopy effects and surface snow-cover prediction. The land-surface model provides no tendencies, but does update the land's state variables which include the ground (skin) temperature, soil temperature profile, soil moisture profile, snow cover, and possibly canopy properties. There is no horizontal interaction between neighboring points in the LSM, so it can be regarded as a one-dimensional column model for each WRF land grid-point, and many LSMs can be run in a stand-alone mode. Table 8.3 summarizes the basic features of the land-surface treatments in ARW.

8.4.1 5-layer thermal diffusion

This simple LSM is based on the MM5 5-layer soil temperature model. Layers are 1, 2, 4, 8, and 16 cm thick. Below these layers, the temperature is fixed at a deep-layer average. The energy

Table 8.3: Land Surface Options

Scheme	Vegetation Processes	Soil Variables (Layers)	Snow Scheme
5-layer	N	Temperature (5)	none
Noah	Y	Temperature, Water+Ice, Water (4)	1-layer, fractional
RUC	Y	Temperature, Ice, Water + Ice (6)	multi-layer

budget includes radiation, sensible, and latent heat flux. It also allows for a snow-cover flag, but the snow cover is fixed in time. Soil moisture is also fixed with a landuse- and season-dependent constant value, and there are no explicit vegetation effects.

8.4.2   Noah LSM

The Noah LSM is the successor to the OSU LSM described by Chen and Dudhia (2001). The scheme was developed jointly by NCAR and NCEP, and is a unified code for research and operational purposes, being almost identical to the code used in the NCEP North American Mesoscale Model (NAM). This has the benefit of being consistent with the time-dependent soil fields provided in the analysis datasets. This is a 4-layer soil temperature and moisture model with canopy moisture and snow cover prediction. It includes root zone, evapotranspiration, soil drainage, and runoff, taking into account vegetation categories, monthly vegetation fraction, and soil texture. The scheme provides sensible and latent heat fluxes to the boundary-layer scheme. The Noah LSM additionally predicts soil ice, and fractional snow cover effects, has an improved urban treatment, and considers surface emissivity properties, which are all new since the OSU scheme.

8.4.3   Rapid Update Cycle (RUC) Model LSM

This is a LSM with 6 sub-soil layers and up to two snow layers that is used operationally in the RUC model (Smirnova et al., 1997, 2000). The model considers frozen soil processes, patchy snow, with snow temperature and density variation, vegetation effects, and canopy water.

8.5   Planetary Boundary Layer

The planetary boundary layer (PBL) is responsible for vertical sub-grid-scale fluxes due to eddy transports in the whole atmospheric column, not just the boundary layer. Thus, when a PBL scheme is activated, explicit vertical diffusion is de-activated with the assumption that the PBL scheme will handle this process. The most appropriate horizontal diffusion choices (Section 4.1.3) are those based on horizontal deformation or constant K_h values where horizontal and vertical mixing are treated independently. The surface fluxes are provided by the surface layer

Table 8.4: Planetary Boundary Layer Options

Scheme	Unstable PBL Mixing	Entrainment treatment	PBL Top
MRF	K profile + countergradient term	part of PBL mixing	from critical bulk Ri
YSU	K profile + countergradient term	explicit term	from buoyancy profile
MYJ	K from prognostic TKE	part of PBL mixing	from TKE

and land-surface schemes. The PBL schemes determine the flux profiles within the well-mixed boundary layer and the stable layer, and thus provide atmospheric tendencies of temperature, moisture (including clouds), and horizontal momentum in the entire atmospheric column. Most PBL schemes consider dry mixing, but can also include saturation effects in the vertical stability that determines the mixing. The schemes are one-dimensional, and assume that there is a clear scale separation between sub-grid eddies and resolved eddies. This assumption will become less clear at grid sizes below a few hundred meters, where boundary layer eddies may start to be resolved, and in these situations the scheme should be replaced by a fully three-dimensional local sub-grid turbulence scheme such as the TKE diffusion scheme (Section 4.1.4.) Table 8.4 summarizes the basic features of the PBL schemes in ARW.

8.5.1 Medium Range Forecast Model (MRF) PBL

The scheme is described by Hong and Pan (1996). This PBL scheme employs a so-called counter-gradient flux for heat and moisture in unstable conditions. It uses enhanced vertical flux coefficients in the PBL, and the PBL height is determined from a critical bulk Richardson number. It handles vertical diffusion with an implicit local scheme, and it is based on local Ri in the free atmosphere.

8.5.2 Yonsei University (YSU) PBL

The Yonsei University PBL is the next generation of the MRF PBL, also using the counter-gradient terms to represent fluxes due to non-local gradients. This adds to the MRF PBL an explicit treatment of the entrainment layer at the PBL top. The entrainment is made proportional to the surface buoyancy flux in line with results from studies with large-eddy models. The PBL top is defined using a critical bulk Richardson number of zero (compared to 0.5 in the MRF PBL), so is effectively only dependent on the buoyancy profile which, in general, lowers the calculated PBL top compared to MRF.

8.5.3 Mellor-Yamada-Janjic (MYJ) PBL

This parameterization of turbulence in the PBL and in the free atmosphere (Janjic, 1990, 1996, 2002) represents a nonsingular implementation of the Mellor-Yamada Level 2.5 turbulence clo-

Table 8.5: Radiation Options

Scheme	Longwave/ Shortwave	Spectral Bands	CO_2, O_3, clouds
RRTM	LW	16	CO_2, O_3, clouds
GFDL LW	LW	14	CO_2, O_3, clouds
GFDL SW	SW	12	CO_2, O_3, clouds
MM5 SW	SW	1	clouds
Goddard	SW	11	CO_2, O_3, clouds

sure model (Mellor and Yamada, 1982) through the full range of atmospheric turbulent regimes. In this implementation, an upper limit is imposed on the master length scale. This upper limit depends on the TKE as well as the buoyancy and shear of the driving flow. In the unstable range, the functional form of the upper limit is derived from the requirement that the TKE production be nonsingular in the case of growing turbulence. In the stable range, the upper limit is derived from the requirement that the ratio of the variance of the vertical velocity deviation and TKE cannot be smaller than that corresponding to the regime of vanishing turbulence. The TKE production/dissipation differential equation is solved iteratively. The empirical constants have been revised as well (Janjic, 1996, 2002).

8.6   Atmospheric Radiation

The radiation schemes provide atmospheric heating due to radiative flux divergence and surface downward longwave and shortwave radiation for the ground heat budget. Longwave radiation includes infrared or thermal radiation absorbed and emitted by gases and surfaces. Upward longwave radiative flux from the ground is determined by the surface emissivity that in turn depends upon land-use type, as well as the ground (skin) temperature. Shortwave radiation includes visible and surrounding wavelengths that make up the solar spectrum. Hence, the only source is the Sun, but processes include absorption, reflection, and scattering in the atmosphere and at surfaces. For shortwave radiation, the upward flux is the reflection due to surface albedo. Within the atmosphere the radiation responds to model-predicted cloud and water vapor distributions, as well as specified carbon dioxide, ozone, and (optionally) trace gas concentrations. All the radiation schemes in WRF currently are column (one-dimensional) schemes, so each column is treated independently, and the fluxes correspond to those in infinite horizontally uniform planes, which is a good approximation if the vertical thickness of the model layers is much less than the horizontal grid length. This assumption would become less accurate at high horizontal resolution. Table 8.5 summarizes the basic features of the radiation schemes in the ARW.

8.6.1 Rapid Radiative Transfer Model (RRTM) Longwave

This RRTM, which is taken from MM5, is based on Mlawer et al. (1997) and is a spectral-band scheme using the correlated-k method. It uses pre-set tables to accurately represent longwave processes due to water vapor, ozone, CO_2, and trace gases (if present), as well as accounting for cloud optical depth.

8.6.2 Eta Geophysical Fluid Dynamics Laboratory (GFDL) Longwave

This longwave radiation scheme is from GFDL. It follows the simplified exchange method of Fels and Schwarzkopf (1975) and Schwarzkopf and Fels (1991), with calculation over spectral bands associated with carbon dioxide, water vapor, and ozone. Included are Schwarzkopf and Fels (1985) transmission coefficients for carbon dioxide, a Roberts et al. (1976) water vapor continuum, and the effects of water vapor-carbon dioxide overlap and of a Voigt line-shape correction. The Rodgers (1968) formulation is adopted for ozone absorption. Clouds are randomly overlapped. This scheme is implemented to conduct comparisons with the operational Eta model.

8.6.3 Eta Geophysical Fluid Dynamics Laboratory (GFDL) Shortwave

This shortwave radiation is a GFDL version of the Lacis and Hansen (1974) parameterization. Effects of atmospheric water vapor, ozone (both from Lacis and Hansen, 1974), and carbon dioxide (Sasamori et al., 1972) are employed. Clouds are randomly overlapped. Shortwave calculations are made using a daylight-mean cosine solar zenith angle over the time interval (given by the radiation call frequency).

8.6.4 MM5 (Dudhia) Shortwave

This scheme is base on Dudhia (1989) and is taken from MM5. It has a simple downward integration of solar flux, accounting for clear-air scattering, water vapor absorption (Lacis and Hansen, 1974), and cloud albedo and absorption. It uses look-up tables for clouds from Stephens (1978).

8.6.5 Goddard Shortwave

This scheme is based on Chou and Suarez (1994). It has a total of 11 spectral bands and considers diffuse and direct solar radiation components in a two-stream approach that accounts for scattered and reflected components. Ozone is considered with several climatological profiles available.

Table 8.6: Physics Interactions. Columns correspond to model physical processes: radiation (Rad), microphysics (MP), cumulus parameterization (CP), planetary boundary layer/vertical diffusion (PBL), and surface physics (Sfc). Rows corresponds to model variables where i and o indicate whether a variable is input or output (updated) by a physical process.

		Rad	MP	CP	PBL	Sfc
Atmospheric State or Tendencies	Momentum			i	io	
	Pot. Temp.	io	io	io	io	
	Water Vapor	i	io	io	io	
	Cloud	i	io	o	io	
	Precip	i	io	o		
Surface Fluxes	Longwave Up	i				o
	Longwave Down	o				i
	Shortwave Up	i				o
	Shortwave Down	o				i
	Sfc Convective Rain			o		i
	Sfc Resolved Rain		o			i
	Heat Flux				i	o
	Moisture Flux				i	o
	Surface Stress				i	o

8.7 Physics Interactions

While the model physics parameterizations are categorized in a modular way, it should be noted that there are many interactions between them via the model state variables (potential temperature, moisture, wind, etc.) and their tendencies, and via the surface fluxes. Table 8.6 summarizes how the various physics processes interact in the model. In the table, i indicates that the state variable or flux is required input for the physics scheme, and o indicates that the tendency or flux is a probable output of the scheme. It can be seen that all the physical schemes interact in some way with the surface physics (land-surface models, and, potentially, coupled ocean models). The surface physics, while not explicitly producing tendencies of atmospheric state variables, is responsible for updating the land-state variables.

Note also that, as mentioned, the microphysics does not output tendencies, but updates the atmospheric state at the end of the model time-step. However, the rest of the o's in the upper half of the table are representative of the physical tendencies of these variables in the model.

The radiation, cumulus parameterization, and boundary-layer schemes all output tendencies, but the tendencies are not added until later in the solver, so from this perspective the order of call is not important. Moreover, these physics schemes do not have to be called at the same frequency as each other or the model time step. When lower frequencies are used, their tendencies are kept constant between calls. This is typically done for the radiation schemes, which are too expensive to call every time, and for the cumulus schemes, for which it is also not necessary. However, the surface/boundary-layer schemes are normally called every step in the ARW because this is likely to give the best results.

The radiation is called first because of the required radiative fluxes that are input to the land-surface scheme. The land-surface also requires rainfall from the microphysics and cumulus schemes, but that is from the previous time-step. The boundary-layer scheme is necessarily after the land-surface scheme because it requires the heat and moisture fluxes.

Chapter 9

Variational Data Assimilation

An introduction to the basic ideas of variational data assimilation and the WRF-Var system is given in this chapter, followed by a brief overview of recent major improvements to WRF-Var. This overview supplements the original description of the three-dimensional variational (3D-Var) algorithm found in Barker et al. (2004). One of the most important additions to WRF-Var is a new utility *gen_be*, used to calculate background error covariances for a user's own application; it is discussed in the latter half of this chapter. The WRF-Var system is evolving rapidly, and a future technical note will accompany the general release of the 4D-Var component of WRF-Var. That technical note will include an extensive description of the entire WRF-Var system.

9.1   Introduction

The basic goal of any variational data assimilation system is to produce an optimal estimate of the true atmospheric state at analysis time through iterative solution of a prescribed cost-function (Ide et al., 1997):

$$J(\mathbf{x}) = J_b(\mathbf{x}) + J_o(\mathbf{x}) = \frac{1}{2}(\mathbf{x} - \mathbf{x}^b)^T \mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}^b) + \frac{1}{2}(\mathbf{y} - \mathbf{y}^o)^T(\mathbf{E} + \mathbf{F})^{-1}(\mathbf{y} - \mathbf{y}^o). \qquad (9.1)$$

The variational problem can be summarized as the iterative minimization of (9.1) to find the analysis state $\mathbf{x}$ that minimizes $J(\mathbf{x})$. This solution represents the *a posteriori* maximum likelihood (minimum variance) estimate of the true state of the atmosphere given the two sources of *a priori* data: the first guess (or background) $\mathbf{x^b}$ and observations $\mathbf{y^o}$ (Lorenc, 1986). The fit to individual data points is weighted by estimates of their errors: $\mathbf{B}$, $\mathbf{E}$, and $\mathbf{F}$ are the background, observation (instrumental), and representivity error covariance matrices, respectively. Representivity error is an estimate of inaccuracies introduced in the observation operator H used to transform the gridded analysis $\mathbf{x}$ to observation space $\mathbf{y} = H(\mathbf{x})$ for comparison against observations. This error will be resolution dependent and may also include a contribution from approximations (e.g., linearizations) in H.

As described in Barker et al. (2004), the particular variational data assimilation algorithm adopted in WRF-Var is a model-space, incremental formulation of the variational problem. In this approach, observations, previous forecasts, their errors, and physical laws are combined to produce analysis increments $\mathbf{x^{a'}}$, which are added to the first guess $\mathbf{x^b}$ to provide an updated analysis.
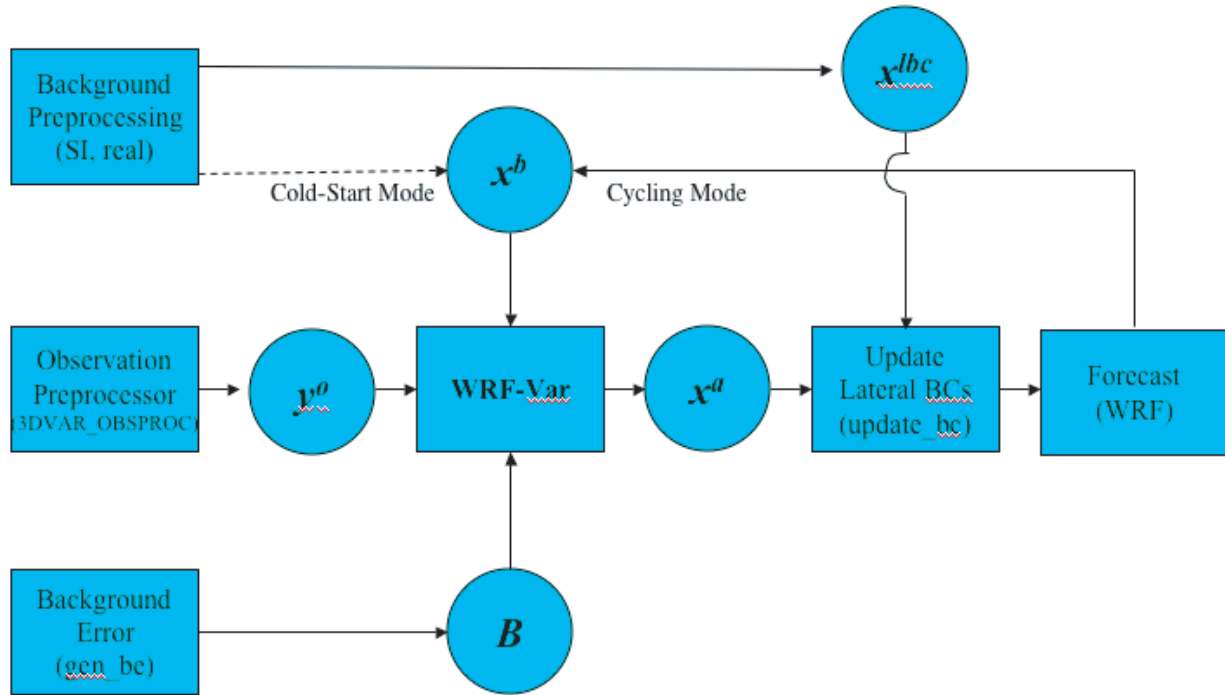
Figure 9.1: Sketch showing the relationship between datasets (circles), and algorithms (rectangles) of the ARW system.

Figure 9.1 illustrates the relationship between WRF-Var, the various datasets, and the other components of a typical NWP system (here ARW). The WRF-Var assimilation proceeds as described in Barker et al. (2004). A number of recent upgrades to the WRF-Var algorithm will be described in Section 9.2.

The three inputs to WRF-Var are:

a) First guess $\mathbf{x^b}$— In cold-start mode, this is typically a forecast/analysis from another model interpolated to the ARW grid (and variables) via the WRF SI and *real* programs. In cycling mode, the first guess is a short-range (typically 1–6 hour) ARW forecast.

b) Observations $\mathbf{y^o}$— In the current version of WRF-Var, observations may be supplied either in a text (MM5 3D-Var) format or BUFR format (but not a combination of the two). An observation preprocessor (3DVAR_OBSPROC) is supplied with the code release to perform basic quality control, assign observation errors, and reformat observations from the MM5 *little_r* text format into 3D-Var's own text format. Details can be found in Barker et al. (2003, 2004).

c) Background error covariances $\mathbf{B}$— used to define the spatial and multivariate response of the analysis to an observation. In variational systems, these covariances are typically calculated off-line, and significant tuning is required to optimize performance for a particular application (e.g., Ingleby (2001); Wu et al. (2002)). The amount of work required to do this satisfactorily is significant, and should not be underestimated. In order to assist the user, WRF developers

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supply the following: i) a default set of statistics used for the initial set up of a domain; ii) a utility *gen_be* (described in Section 9.3) to process ensembles of forecasts into the appropriate control variable space; and iii) diagnostic routines to assess the accuracy of observation and background error statistics. These routines include both innovation vector-based approaches (Hollingsworth and Lonnberg, 1986) and variational tuning approaches (Desroziers and Ivanov, 2001).

Following assimilation of all data, an analysis $\mathbf{x^a}$ is produced that must be merged with the existing lateral boundary conditions $\mathbf{x^{lbc}}$ (described in Barker et al. (2003)). Note: In cycling mode, only the *wrfbdy* lateral boundary condition files ($\mathbf{x^{lbc}}$) output of SI/real are used, and not the *wrfinput* initial condition files ($\mathbf{x^b}$). In cold-start mode, both are required.

9.2 Improvements to the WRF-Var Algorithm

9.2.1 Improved vertical interpolation

The original WRF 3D-Var system described in Barker et al. (2004) used height interpolation for all observation operators. If an observation is reported as a function of pressure, then height is approximated using the hydrostatic relation. This step introduces an unnecessary source of error. The new WRF-Var system performs vertical interpolation in terms of the original observed coordinate, i.e., pressure or height.

9.2.2 Improved minimization and "outer loop"

The default WRF-Var cost function minimization uses a modified version of the limited memory Quasi-Newton Method (QNM). Recently, an alternative Conjugate Gradient Method (CGM) has been implemented. Unlike the QNM technique, the CGM method restricts 3D-Var's inner loop to be completely linear. This limitation is dealt with through the inclusion of an outer loop in WRF-Var, the purpose of which is to iterate towards nonlinear solutions (e.g., observation operators, balance constraints, and the forecast itself in 4D-Var) using the WRF-Var analysis from the previous iteration as new background. The outer loop is also used as a form of variational quality control as follows: observations are rejected if their O-B values are outside a prescribed range (typically several times the observation error standard deviation). This *errormax* test implicitly assumes the rejected large O-B values are due to a bad observation (O) rather than poor background (B). However, if it is the background B that is incorrect then the system will reject the most useful observations available to the assimilation system, i.e., those in areas where the first-guess is poor. The outer loop alleviates this effect by allowing observations rejected in previous iterations to be accepted if their new O-B falls within the required range in subsequent outer loops. The assimilation of nearby observations in previous iterations essentially provides a "buddy check" to the observation in question.

9.2.3 Flexible choice of control variables

In practical variational data assimilation schemes, the background error covariance matrix $\mathbf{B}$ is computed not in model space $\mathbf{x'} : u, v, T, q, p_s$, but in a control variable space $\mathbf{v}$ related to model

space via the control variable transform U, i.e.,

$$\mathbf{x}' = \mathrm{U}\mathbf{v} = \mathrm{U}_p\mathrm{U}_v\mathrm{U}_h\mathbf{v}. \tag{9.2}$$

The expansion $\mathrm{U} = \mathrm{U}_p\mathrm{U}_v\mathrm{U}_h$ represents the various stages of covariance modeling: horizontal correlations U_h, vertical covariances U_v, and multivariate covariances U_p.

The components of $\mathbf{v}$ are chosen so that their error cross-correlations are negligible, thus permitting the matrix $\mathbf{B}$ to be block-diagonalized. The many varying applications (high/low resolution, polar/tropical, etc.) of WRF-Var require a flexible choice of background error model. This is achieved via a namelist option "cv_options" as defined in Table 9.1.

cv_options		2 (original MM5)	3 (NCEP)	4 (Global)	5 (Regional)
Analysis Increment	$\mathbf{x}'$	$u',v',T',q',p_s'(i,j,k)$			
Change of Variable	U_p	ψ',χ',p_u',q'	$\psi',\chi_u',T_u',r',p_{su}'$		
Vertical Covariances	U_v	$\mathbf{B}=\mathbf{E}\Lambda\mathbf{E}^T$	RF	$\mathbf{B}=\mathbf{E}\Lambda\mathbf{E}^T$	
Horizontal Correlations	U_h	RF		Spectral	RF
Control Variables	$\mathbf{v}$	$\mathbf{v}(i,j,m)$	$\mathbf{v}(i,j,k)$	$\mathbf{v}(l,n,m)$	$\mathbf{v}(i,j,m)$

Table 9.1: The definitions of the various stages of the control variable transform given by (9.2) for the unified global/regional WRF-Var system. Indices (i, j, k) refer to grid-point space, index m to vertical mode, and l, n to global spectral mode. The variables are: u, v: velocity components; T: temperature; q: mixing ratio; p_s: surface pressure; ψ: streamfunction; χ: velocity potential; r: relative humidity. The subscript u indicates an unbalanced field. The acronym RF stands for recursive filter.

Table 9.1 indicates that the only difference between global (cv_options=4) and WRF regional (cv_options=5) versions of the WRF-Var control variable transform is in the horizontal error correlations U_h. Note also, the only difference between the old MM5 background error model (cv_options=2) and WRF regional (cv_options=5) is in the U_p transform. The former imposes a dynamical balance constraint via an unbalanced pressure control variable (Barker et al., 2004), whereas in the new regional covariance model, balance is imposed via statistical regression (see Section 9.3 for details). This choice of control variables is considered more appropriate for the mass-based ARW solver.

9.2.4 First Guess at Appropriate Time (FGAT)

A First Guess at Appropriate Time (FGAT) procedure has been implemented in WRF-Var (Lee et al., 2004). The FGAT capability results in a more accurate calculation of innovation vectors (observation minus first guess difference), and hence a better use of observations when their valid time differs from that of the analysis. FGAT is most effective for the analysis of observations from asynoptic, moving platforms (e.g., aircraft and satellite data). Surface observations with high temporal resolution also benefit from the use of FGAT.

9.2.5 Radar Data Assimilation

Numerous modifications have been made in order to assimilate Doppler radar radial velocity and reflectivity observations. Firstly, vertical velocity increments are included in WRF-Var via the "Richarson balance equation" that combines the continuity equation, adiabatic thermodynamic equation, and hydrostatic relation. Linear and adjoint codes of Richardson's equation have been incorporated into WRF-Var. In order to develop a capability for Doppler reflectivity assimilation, we use the total water as a control variable, requiring a partitioning of the moisture and water hydrometeor increments. A warm-rain parameterization is also included, which includes condensation of water vapor into cloud, accretion of cloud by rain, automatic conversion of cloud to rain, and evaporation of rain to water vapor. Finally, the observation operators for Doppler radial velocity and reflectivity are included in WRF-Var. Further details and results of the radial velocity work can be found in Xiao et al. (2005). The radar reflectivity approach will be described in a future paper.

9.2.6 Unified Regional/Global 3D-Var Assimilation

There are many benefits to having a single data assimilation system for regional and global applications. The majority of the code is common to both (observation operators, minimization, background error preconditioning, interpolation, etc.). Technically, the main changes required to extend the regional application to global are related to the presence of a) the polar singularity, and b) periodic East-West boundary conditions. Of course, there are also scientific questions concerning the optimal mix of observations required for global/regional models, and the choice of control variables and balance constraints. A unified global/regional 3D-Var system should therefore be flexible to a variety of thinning/quality-control algorithms and also to alternative formulations of the background error covariance matrix. This flexibility has been a key design requirement throughout the WRF-Var project.

The major difference between regional and global options in WRF-Var is in the definition of horizontal background error covariances. In regional mode, recursive filters (Purser et al., 2003) are used to project observation information away from the observation location. In global mode, a spectral decomposition is applied. The main benefits of the spectral technique are a) the isotropic and homogeneous covariances that are implied neatly solve the problems associated with the pole (the pole is not a special point in spectral space), and b) horizontal correlations are defined in terms of a single function— the power spectrum (a function of total wavenumber). However, the isotropy of correlation defined in spectral space is also a weakness— anisotropies need to be defined in an alternative manner. One solution to this problem is to replace the

spectral correlations with grid-point correlations (e.g., in the Gridpoint Statistical Interpolation scheme under development at NCEP). An alternative technique is to supplement the isotropic spectral correlations with an anisotropic component derived via grid transformations, additional control variables or 4D-Var. Research using the latter techniques is underway using the WRF-Var system.

The WRF-Var code has been adapted to perform assimilation on a global, regular latitude-longitude grid. The major modifications are as follows.

a) Spectral to grid-point transformations (and their adjoints) have been included in 3D-Var to represent the horizontal component (U_h) of the background error covariance model.

b) A new global WRF-Var registry was created to accommodate the output of global forecast models (currently only the Korean Meteorological Administration's (KMA) global model has been tested). The final analysis increments are written in binary format and added back to the global first guess to provide the global analysis in the native model format.

c) Changes have been made to allow for periodic boundary conditions in the East–West direction.

d) A number of minor changes have been made to treat the polar rows as special points (e.g., in the grid-point ψ, χ to u, v wind conversion in the U_p transform and the observation operators for polar winds).

9.3   Background Error Covariances

Forecast ("first guess" or "background") error covariances are a vital input to variational data assimilation systems. They influence the analysis fit to observations and also completely define the analysis response away from observations. The latter impact is particularly important in data-sparse areas of the globe. Unlike ensemble filter data assimilation techniques (e.g., the Ensemble Adjustment Kalman Filter, the Ensemble Transform Kalman Filter), 3/4D-Var systems do not implicitly evolve forecast error covariances in real-time. Instead, climatologic statistics are usually estimated offline. The "NMC-method", in which forecast error covariances are approximated using forecast difference (e.g., T+48 minus T+24) statistics, is a commonly used approach (Parrish and Derber, 1992). Experiments at ECMWF (Fisher, 2003) indicate superior statistics may be obtained using a cycling analysis/forecast ensemble prediction system based on perturbed observations/physics.

Recent advances permit the use of flow-dependent forecast error covariances in 3D/4D-Var through, for example, grid transformations (Desroziers, 1997), anisotropic recursive filters (Wu et al., 2002; Purser et al., 2003), or observation-space formulations of the variational problem (Daley and Barker, 2001). Flow-dependence may be enhanced in 4D-Var through the use of the forecast model to provide dynamical consistency to the evolving forecast state during 4D-Var's time window (Rabier et al., 1998). Still, the practical effort required to specify and implement flow-dependent error covariances in 3/4D-Var is significant.

The NMC-method code developed for MM5 3D-Var (Barker et al., 2004) is nearing the end

of its useful life. The development of a unified global/regional WRF-Var system, and its application to a variety of models (e.g., ARW, MM5, KMA global model, Taiwan's Nonhydrostatic Forecast System [NFS]) has required a new, efficient, portable forecast background error covariance calculation code to be written. There is also a demand for such a capability to be available and supported for the wider 3/4D-Var research community for application to their own geographic areas of interest (the default statistics supplied with the WRF-Var release are designed only as a starting point). In this section, the new *gen_be* code developed by NCAR/MMM to generate forecast error statistics for use with the WRF-Var system is described.

The background error covariance matrix is defined as

$$\mathbf{B} = \overline{\epsilon\epsilon^T} \simeq \overline{\mathbf{x}'\mathbf{x}'^T}, \tag{9.3}$$

where the overbar denotes an average over time and/or geographical area. The true background error ϵ is not known in reality, but is assumed to be statistically well-represented by a model state perturbation $\mathbf{x}'$. In the standard NMC-method (Parrish and Derber, 1992), the perturbation $\mathbf{x}'$ is given by the difference between two forecasts (e.g., 24 hour minus 12 hour) verifying at the same time. Climatological estimates of background error may then be obtained by averaging such forecast differences over a period of time (e.g., one month). An alternative strategy proposed by (Fisher, 2003) makes use of ensemble forecast output, defining the $\mathbf{x}'$ vectors as ensemble perturbations (ensemble minus ensemble mean). In either approach, the end result is an ensemble of model perturbation vectors from which estimates of background error may be derived. The new *gen_be* utility has been designed to work with either forecast difference, or ensemble-based, perturbations.

As described above, the WRF-Var background error covariances are specified not in model space $\mathbf{x}'$, but in a control variable space $\mathbf{v}$, which is related to the model variables (e.g., wind components, temperature, humidity, and surface pressure) via the control variable transform defined in (9.2). Both (9.2) and its adjoint are required in WRF-Var. In contrast, the background error code performs the inverse control variable transform $\mathbf{v} = U_h^{-1}U_v^{-1}U_p^{-1}\mathbf{x}'$ in order to accumulate statistics for each component of the control vector $\mathbf{v}$.

Using the NMC-method, $\mathbf{x}' = \mathbf{x_{T2}} - \mathbf{x_{T1}}$ where $T2$ and $T1$ are the forecast difference times (e.g., 48h minus 24h for global, 24h minus 12h for regional). Alternatively, for an ensemble-based approach, $\mathbf{x_k}' = \mathbf{x_k} - \bar{\mathbf{x}}$, where the overbar is an average over ensemble members $k = 1, n_e$. The total number of binary files produced by this stage is $n_f \times n_e$ where n_f is the number of forecast times used (e.g., for 30 days with forecast every 12 hours, $n_f = 60$). Using the NMC-method, $n_e = 1$ (1 forecast difference per time). For ensemble-based statistics, n_e is the number of ensemble members.

The background error covariance generation code *gen_be* is designed to process data from a variety of regional/global models (e.g., ARW, MM5, KMA global model, NFS, etc.), and process it in order to provide error covariance statistics for use in variational data assimilation systems. The initial, model-dependent "stage 0" is illustrated in Fig. 9.2.

Alternative models use different grids, variables, data formats, etc., and so initial converters are required to transform model output into a set of standard perturbation fields (and metadata), and to output them in a standard binary format for further, model-independent processing. The standard grid-point fields are as follows.

- Perturbations: Streamfunction $\psi'(i, j, k)$, velocity potential $\chi'(i, j, k)$, temperature $T'(i, j, k)$,

BE Generation: Stage 0
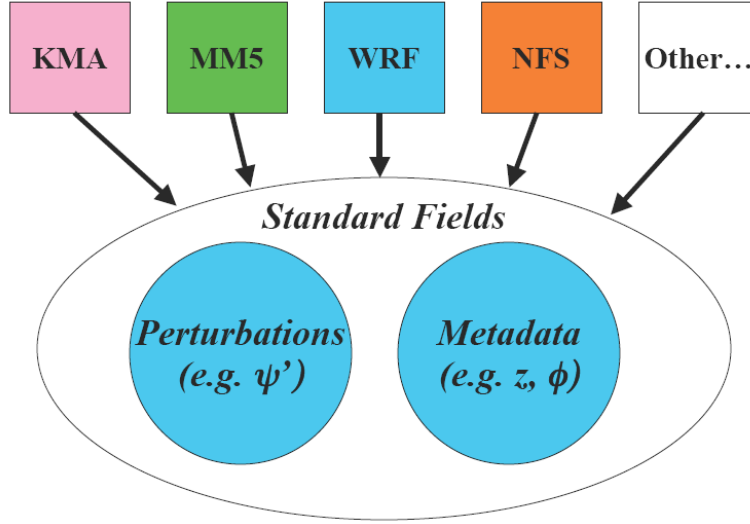
Figure 9.2: Sketch of the role of Stage 0 converters in transforming model-specific data (e.g., ARW, KMA global model, etc.) to standard perturbation fields and relevant metadata (e.g., latitude, height, land/sea, etc.).

    relative humidity $r'(i, j, k)$, surface pressure $p'_s(i, j)$.

- Full-fields: height $z(i, j, k)$, latitude $\phi(i, j)$. (These are required for the production of background error statistics stored in terms of physics variables, rather than tied to a specific grid. This flexibility is included in *gen_be* through a namelist option to define the bins over which data is averaged in a variety of ways (e.g., latitude height, grid points). Land-sea and orographic effects may also be represented in this way.

In general, the *stage_0* converters are developed and maintained by those supporting individual models. Only the WRF-to-standard-fields converter *gen_be_stage0_wrf* is maintained and supported by the ARW effort.

9.3.1 Removal of time-mean

In order to calculate covariances between fields, the average value must first be removed. This is performed in the first stage utility *gen_be_stage1*.

9.3.2 Multivariate Covariances: Regression coefficients and unbalanced variables

The WRF-Var system permits a variety of background error covariance models to be employed, as described in Section 9.2.3 above. The utility *gen_be* is used to provide background error statistics only for cv_options 4 and 5.

The second stage of *gen_be (gen_be_stage2)* provides statistics for the unbalanced fields χ_u, T_u, and P_{su} used as control variables in WRF-Var. The unbalanced control variables are defined as the difference between full and balanced (or correlated) components of the field. In this stage of the calculation of background errors, the balanced component of particular fields is modeled via a regression analysis of the field using specified predictor fields (e.g., streamfunction; see Wu et al. (2002) for further details). The resulting regression coefficients are output for use in WRF-Var's U_p transform. Currently, three regression analyses are performed resulting in three sets of regression coefficients (Note: The perturbation notation has been dropped for the remainder of this chapter for clarity.):

- Velocity potential/streamfunction regression: $\chi_b = c\psi$;
- Temperature/streamfunction regression: $T_{b,k1} = \sum_{k2} G_{k1,k2}\psi_{k2}$; and
- Surface pressure/streamfunction regression: $p_{sb} = \sum_k kW_k\psi_k$.

Data is read from all $n_f \times n_e$ files and sorted into bins defined via the namelist option *bin_type*. Regression coefficients $G(k1,k2)$ and $W(k)$ are computed individually for each bin (bin_type=1 is used here, representing latitudinal dependence) in order to allow representation of differences between, for example, polar, mid-latitude, and tropical dynamical and physical processes. In addition, the scalar coefficient c used to estimate velocity potential errors from those of streamfunction is calculated as a function of height to represent, for example, the impact of boundary-layer physics. Latitudinal/height smoothing of the resulting coefficients may be optionally performed to avoid artificial discontinuities at the edges of latitude/height boxes.

Having computed regression coefficients, the unbalanced components of the fields are calculated as $\chi_u = \chi - c\psi$, $T_{u,k1} = T_{k1} - \sum_{k2} G_{k1,k2}\psi_{k2}$, and $p_{su} = p_s - \sum_k W_k\psi_k$. These fields are output for the subsequent calculation of the spatial covariances as described below.

9.3.3 Vertical Covariances: Eigenvectors/eigenvalues and control variable projections

The third stage (*gen_be_stage3*) of *gen_be* calculates the statistics required for the vertical component of the control variable transform. This calculation involves the projection of 3D fields on model-levels onto empirical orthogonal functions (EOFs) of the vertical component of background error covariances (Barker et al., 2004). For each 3D control variable (ψ, χ_u, T_u, and r), the vertical component of **B**, is calculated and an eigenvector decomposition performed. The resulting eigenvectors **E** and eigenvalues Λ are saved for use in WRF-Var.

The *gen_be* code calculates both domain-averaged and local values of the vertical component of the background error covariance matrix. The definition of local again depends on the value of the namelist variable bin_type chosen. For example, for bin_type=1, a $kz \times kz$ (where kz is the number of vertical levels) vertical component of **B** is produced at every latitude (data is averaged over time and longitude) for each control variable. Eigendecomposition of the resulting climatological vertical error covariances $\mathbf{B} = \mathbf{E}\Lambda\mathbf{E}^T$ results in both domain-averaged and local eigenvectors **E** and eigenvalues Λ. Both sets of statistics are included in the dataset supplied to WRF-Var, allowing the choice between homogeneous (domain-averaged) or local (inhomogeneous) background error variances and vertical correlations to be chosen at run time

(Barker et al., 2004). Having calculated and stored eigenvectors and eigenvalues, the final part of *gen_be_stage3* is to project the entire sequence of 3D control variable fields into EOF space $\mathbf{v_v} = U_v^{-1}\mathbf{v_p} = \Lambda^{-1/2}\mathbf{E}^T\mathbf{v_p}$.

9.3.4 Horizontal Covariances: Recursive filter lengthscale (regional), or power spectra (global)

The last aspect of the climatological component of background error covariance data required for WRF-Var is the horizontal error correlations, the representation of which forms the largest difference between running WRF-Var in regional and global mode. (It is however, still a fairly local change.)

In a global application (*gen_be_stage4_global*), power spectra are computed for each of the kz vertical modes of the 3D control variables ψ, χ_u, T_u, and r, and for the 2D control variable p_{su} data. In contrast, in regional mode, horizontal correlations are computed between grid-points of each 2D field, binned as a function of distance. A Gaussian curve is then fitted to the data as described in Barker et al. (2004) to provide correlation lengthscales for use in the recursive filter algorithm.

Appendix A

Physical Constants

The following is a list of physical constants used in the model.

$$
\begin{array}{rcll}
\pi & = & 3.1415926 & \text{Pi} \\
k & = & 0.4 & \text{Von Karman constant} \\
r_e & = & 6.370 \times 10^6 \;\; \text{m} & \text{Radius of earth} \\
g & = & 9.81 \;\; \text{m s}^{-2} & \text{Acceleration due to gravity} \\
\Omega_e & = & 7.2921 \times 10^{-5} \;\; \text{s}^{-1} & \text{Angular rotation rate of the earth} \\
\sigma_B & = & 5.67051 \times 10^{-8} \;\; \text{W m}^{-2}\,\text{K}^{-4} & \text{Stefan} - \text{Boltzmann constant} \\
R_d & = & 287 \;\; \text{J kg}^{-1}\,\text{K}^{-1} & \text{Gas constant for dry air} \\
R_v & = & 461.6 \;\; \text{J kg}^{-1}\,\text{K}^{-1} & \text{Gas constant for water vapor} \\
c_p & = & 7 \times R_d/2 \;\; \text{J kg}^{-1}\,\text{K}^{-1} & \text{Specific heat of dry air at constant pressure} \\
c_v & = & c_p - R_d \;\; \text{J kg}^{-1}\,\text{K}^{-1} & \text{Specific heat of dry air at constant volume} \\
c_{pv} & = & 4 \times R_v \;\; \text{J kg}^{-1}\,\text{K}^{-1} & \text{Specific heat of water vapor at constant pressure} \\
c_{vv} & = & c_{pv} - R_v \;\; \text{J kg}^{-1}\,\text{K}^{-1} & \text{Specific heat of water vapor at constant volume} \\
c_{liq} & = & 4190 \;\; \text{J kg}^{-1}\,\text{K}^{-1} & \text{Specific heat capacity of water} \\
c_{ice} & = & 2106 \;\; \text{J kg}^{-1}\,\text{K}^{-1} & \text{Specific heat capacity of ice} \\
L_v & = & 2.5 \times 10^6 \;\; \text{J kg}^{-1} & \text{Latent heat of vaporization} \\
L_s & = & 2.85 \times 10^6 \;\; \text{J kg}^{-1} & \text{Latent heat of sublimation} \\
L_f & = & 3.50 \times 10^5 \;\; \text{J kg}^{-1} & \text{Latent heat of fusion} \\
\rho_w & = & 1.0 \times 10^3 \;\; \text{kg m}^{-3} & \text{Density of liquid water}
\end{array}
$$

Appendix B

List of Symbols

Symbols used in this document are listed in alphabatical order in this appendix.

Symbols	Definition
a	generic variable
A	coefficient (Chapter 4), base-state lapse rate constant (Chapter 5)
$\mathbf{B}$	background error covariance matrix
c	scalar coefficient
c_s	speed of sound
C_k	a constant used in TKE closure
Cr	Courant number
Cr_{max}	maximum Courant number
Cr_{theory}	Courant number from Table 3.1
Cr_β	activation Courant number in vertical velocity damping
C_s	a constant used in eddy viscosity calculation
D	deformation
D_{nm}	deformation tensor, where $n, m = 1$, 2 and 3
e	cosine component of the Coriolis term (Chapters 2, 3); turbulent kinetic energy (Chapter 4)
$\mathbf{E}$	observation error covariance matrix
f	sine component of the Coriolis term
F	forcing terms for U, V, W, Θ and Q_m
$\mathbf{F}$	representivity error covariance matrix
$F_{X_{cor}}$	Coriolis forcing terms for $X = U$, V, and W
$F_{1,2}$	coefficients for weighting functions in specified boundary condition
g	acceleration due to gravity
G_k	regression coefficient
H	observation operator
J	cost function
$K_{dh,dv}$	horizontal and vertical eddy viscosity for gravity wave absorbing layer
$K_{h,v}$	horizontal and vertical eddy viscosities

Symbols *Definition*

l_0	minimum length scale for dissipation
$l_{h,v}$	horizontal and vertical length scales for turbulence
l_{cr}	critical length scale for dissipation
L	latent heat of condensation
$L_{x,y}$	periodicity length in x and y
m	map scale factor
n_s	ratio of the RK3 time step to the acoustic time step
N	Brunt-Väisälä frequency
p	pressure
p'	perturbation pressure
p_0	reference sea-level pressure
p_h	hydrostatic pressure
$p_{ht,hs}$	hydrostatic pressure at the top and surface of the model
$p_{dht,dhs}$	dry hydrostatic pressure at the top and surface of the model
p_s	surface pressure
P_r	Prandtl number
q	generic scalar
$q_{c,i,r,s}$	mixing ratios for cloud water, ice, rain water and snow
q_m	generic mixing ratios for moisture
q_v	mixing ratio for water vapor
q_{vs}	saturation mixing ratio for water vapor
Q_m	generic coupled moisture variable
r	relative humidity
r_e	radius of earth
R	remaining terms in equations
R_d	gas constant for dry air
R_v	gas constant for water vapor
t	time
Δt	a full model time step
T	temperature
T_0	reference sea-level temperature
u	horizontal component of velocity in x-direction
U	coupled horizontal component of velocity in x-direction (Chapters 2, 3, 6, 7); control variable transform (Chapter 9)
U_h	horizontal correlation
U_p	multivariate covariance
U_v	vertical covariance
v	horizontal component of velocity in y-direction
$\mathbf{v}$	three dimensional vector velocity
V	coupled horizontal component of velocity in y-direction
$\mathbf{V}$	three dimensional coupled vector velocity
w	vertical component of velocity
W	coupled vertical component of velocity

Symbols	Definition

W_k	regression coefficient
z	height
z_d	depth of damping layer
z_{top}	height of model top
α	inverse density of air
α'	perturbation inverse density of air
$\bar{\alpha}$	inverse density of air for the reference state
α_d	inverse density of dry air
α_r	local rotation angle between y-axis and the meridian
β	off-centering coefficient for semi-implicit acoustic step
γ	ratio of heat capacities for dry air at constant pressure and volume
γ_d	divergence damping coefficient
γ_e	external mode damping coefficient
γ_g	damping coefficient for upper boundary gravity wave absorbing layer
γ_r	Rayleigh damping coefficient
ϵ	molecular weight of water over the molecular weight of dry air (Chapter 4); true background error (Chapter 9)
η	terrain-following hydrostatic-pressure vertical coordinate
$\dot{\eta}$	contravariant 'vertical' velocity or coordinate velocity
θ	potential temperature
θ_e	equivalent potential temperature
θ_m	moist potential temperature
Θ	coupled potential temperature
μ	hydrostatic pressure difference between surface and top of the model
$\bar{\mu}$	reference state hydrostatic pressure difference between surface and top of the model
μ_d	dry hydrostatic pressure difference between surface and top of the model
τ	acoustic time (Chapter 3), vertical structure function for Rayleigh damping (Chapter 4)
τ_{nm}	stress tensor (Chapter 4) where $n.m = 1$, 2 and 3
$\Delta\tau$	acoustic time step
ϕ	geopotential (Chapters 2, 3, 5); latitude (Chapter 9)
$\bar{\phi}$	geopotential for reference state
ϕ'	perturbation geopotential
Φ	generic prognostic variable (coupled)
ψ	generic variable (Chapter 6)
ψ'	streamfunction increment
χ'	velocity potential increment
ω	same as $\dot{\eta}$
Ω	coupled coordinate velocity
Ω_e	angular rotation rate of the earth

Subscripts/Superscripts	Definition
$()_d$	dry
$()_h$	hydrostatic
$()_0$	base state sea-level constant
$\overline{()}$	reference state
$()'$	perturbation from reference state
$()^{t*}$	full value at a Runge-Kutta step
$()''$	perturbation from Runge-Kutta step value in acoustic steps

Appendix C

Acronyms

AFWA	Air Force Weather Agency
API	Application Program Interface
ARPS	Advanced Regional Prediction System
ARW	Advanced Research WRF
BUFR	Binary Universal Form for the Representation of Meteorological Data
CAPE	Convectively Available Potential Energy
CAPS	Center for the Analysis and Prediction of Storms
CGM	Conjugate Gradient Method
COAMPS	Coupled Ocean / Atmosphere Mesoscale Prediction System
COMET	Cooperative Program for Operational Meteorology, Education, and Training
DTC	Developmental Testbed Center
ECMWF	The European Centre for Medium-Range Weather Forecasts
EOF	Empirical Orthogonal Function
ESMF	Earth System Modeling Framework
FAA	Federal Aviation Administration
FGAT	First Guess at Appropriate Time
FSL	Forecast System Laboratory
GFDL	Geophysical Fluid Dynamics Laboratory
GFS	Global Forecast System
GRIB	Gridded Binary
KMA	Korean Meteorological Administration
LSM	Land Surface Model
MKS	Meter Kilogram Second
MM5	Pennsylvania State / NCAR Mesoscale Model Version 5
MMM	Mesoscale and Microscale Meteorology Division
MRF	Medium Range Forecast Model
NAM	North American Mesoscale Model
NCAR	National Center for Atmospheric Research
NCEP	National Centers for Environmental Prediction
NFS	Non-hydrostatic Forecast System (Central Weather Bureau of Taiwan)
NMM	Nonhydrostatic Mesoscale Model
NOAA	National Oceanographic and Atmospheric Administration
NRL	Navy Research Laboratory

NWP	Numerical Weather Prediction
OSU	Oregon State University
PBL	Planetary Boundary Layer
PPM	Piecewise Parabolic Method
QNM	Quasi Newton Method
RHS	Right Hand Side
RRTM	Rapid Radiative Transfer Model
RUC	Rapid Update Cycle
SI	(WRF) Standard Initialization
TKE	Turbulent Kinetic Energy
UCAR	University Corporation for Atmospheric Research
YSU	Yonsei University (Korea)
VAR	Variational Assimilation
WRF	Weather Research and Forecasting Model
WSF	WRF Software Framework

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