The WRF NMM Core

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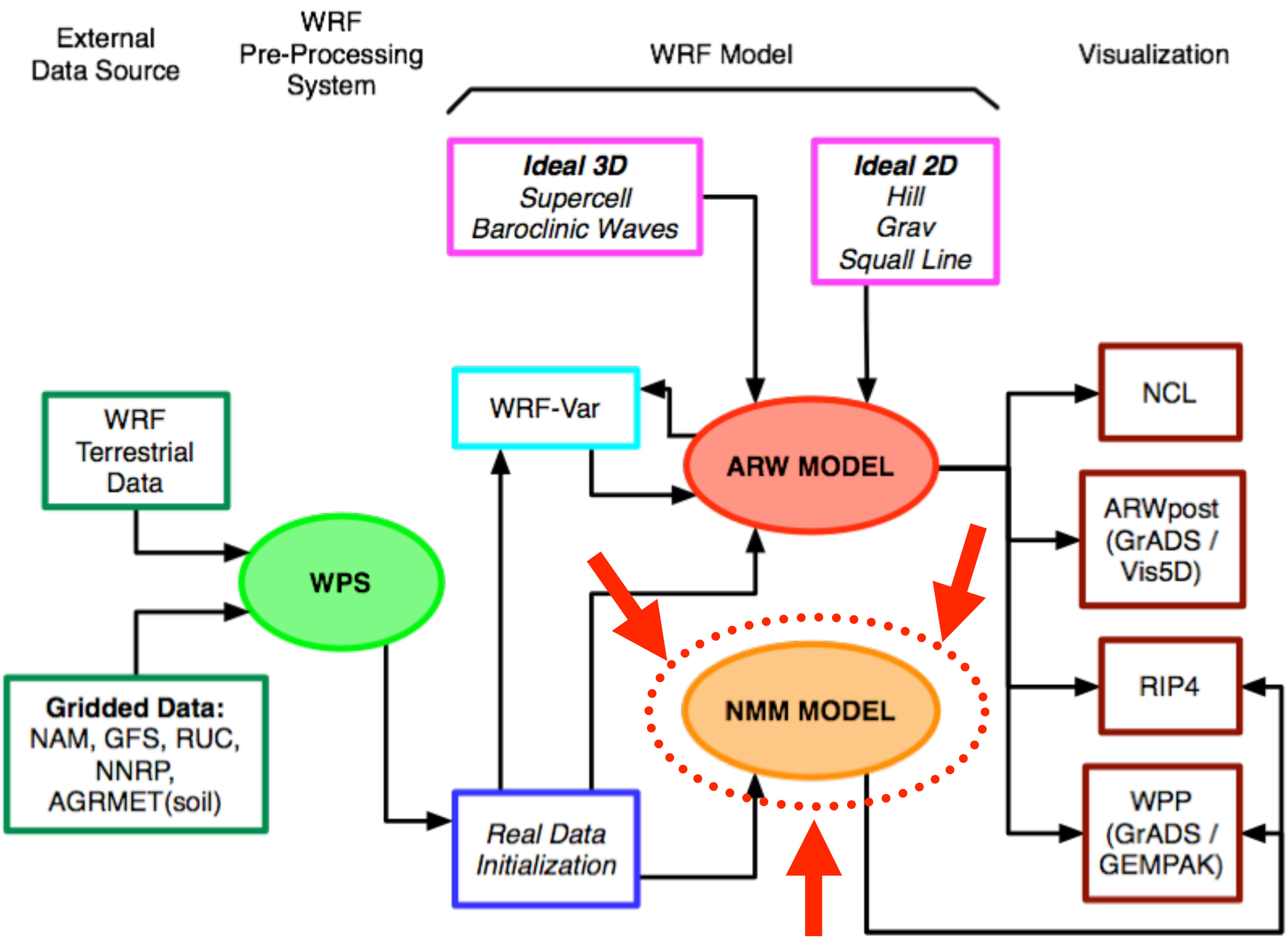
WRF Modeling System Flow Chart (for WRFV2)



NMM Dynamic Solver

- Basic Principles
- Vertical Coordinate
- Equations / Variables
- Time Integration
- Horizontal Grid
- Spatial Discretization
- Vertical Grid
- Boundary Conditions
- Dissipative Processes
- Summary

Basic Principles

- Robust, computationally efficient
- Use modeling principles proven in NWP and regional climate applications
- Use full compressible equations split into hydrostatic and nonhydrostatic contributions
  - Easy comparison of hydro and nonhydro solutions
  - Reduced computational effort at lower resolutions
- Use methods that minimize the generation of small-scale noise

Mass Based Vertical Coordinate

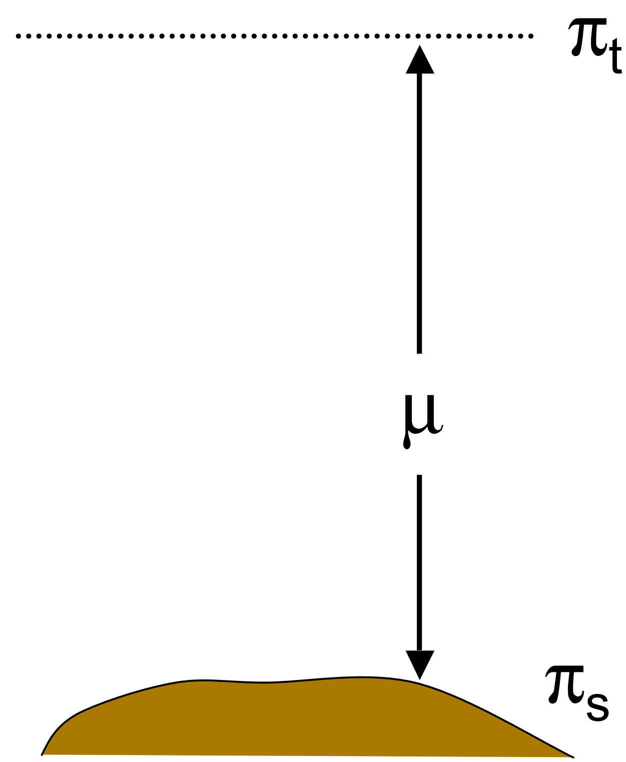
The vertical coordinate is based on hydrostatic pressure (π):

$$\sigma = \frac{\pi - \pi_t}{\mu}$$

$$\mu = \pi_s - \pi_t$$

π_t = model top π

π_s = surface π



Inviscid, adiabatic, sigma (Janjic et al., 2001, MWR)

Analogous to a hydrostatic system, except for *p* and *ε*

Momentum eqn.

$$\frac{\partial \mathbf{v}}{\partial t} = -\mathbf{v}\cdot\nabla_\sigma \mathbf{v} - \dot{\sigma}\frac{\partial \mathbf{v}}{\partial \sigma} - (1+\varepsilon)\nabla_\sigma \Phi - \alpha\nabla_\sigma p + f\mathbf{k}\times\mathbf{v}$$

Thermodynamic eqn.

$$\frac{\partial T}{\partial t} = -\mathbf{v}\cdot\nabla_\sigma T - \dot{\sigma}\frac{\partial T}{\partial \sigma} + \frac{\alpha}{c_p}\left[\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla_\sigma p + \dot{\sigma}\frac{\partial p}{\partial \sigma}\right]$$

Continuity eqn.

$$\frac{\partial \mu}{\partial t} + \nabla_\sigma\cdot(\mu\mathbf{v}) + \frac{\partial(\mu\dot{\sigma})}{\partial \sigma} = 0$$

$$\alpha = RT/p$$

$$\varepsilon \equiv \frac{1}{g}\frac{dw}{dt}$$

π is the hydrostatic pressure,

p is the total (nonhydrostatic) pressure

3rd eqn of motion

$$\frac{\partial p}{\partial \pi} = 1 + \varepsilon$$

Hypsometric eqn.

$$\frac{\partial \Phi}{\partial \sigma} = -\mu \frac{RT}{p}$$

w definition

$$w = \frac{1}{g}\frac{d\Phi}{dt} = \frac{1}{g}\left(\frac{\partial \Phi}{\partial t} + \mathbf{v} \, . \, \nabla_\sigma \Phi + \dot{\sigma}\frac{\partial \Phi}{\partial \sigma}\right)$$

Side note: separation of the thermodynamic equation into two parts

$$\frac{\partial T}{\partial t} = -\mathbf{v} \cdot \nabla_\sigma T - \dot{\sigma}\frac{\partial T}{\partial \sigma} + \frac{\alpha}{c_p}[\omega_1 + \omega_2]$$

$$\left(\frac{\partial T}{\partial t}\right)_1 = -\mathbf{v} \cdot \nabla_\sigma T - \dot{\sigma}\frac{\partial T}{\partial \sigma} + \frac{1}{c_p}\alpha\omega_1$$

$$\omega_1 = (1+\varepsilon)\frac{\partial \pi}{\partial t} + \mathbf{v} \cdot \nabla_\sigma p + (1+\varepsilon)\dot{\sigma}\frac{\partial \pi}{\partial \sigma}$$

Reduces to hydrostatic equation for ε=0

$$\left(\frac{\partial T}{\partial t}\right)_2 = \frac{1}{c_p}\alpha\omega_2$$

$$\omega_2 = \frac{\partial p}{\partial t} - (1+\varepsilon)\frac{\partial \pi}{\partial t}$$

Vanishes for purely hydrostatic flow

Properties of system

- Φ, w, ε are not independent, no independent prognostic equation for w!

- ε<<1 in meso and large scale atmospheric flows

- Impact of nonhydrostatic dynamics becomes detectable at resolutions <10km, important at 1km.

WRF-NMM predictive variables

- Mass variables:
  - PD – column pressure depth (time and space varying component) (Pa)
  - T – sensible temperature (K)
  - Q – specific humidity (g/g)
  - CWM – total cloud water condensate (g/g)
  - Q2 – 2 * turbulent kinetic energy (m^2/s^2)

- Wind variables:
  - U, V – wind components (m/s)

Time Integration

General Philosophy

- Explicit where possible for computational efficiency and coding transparency:
  - horizontal advection of u, v, T
  - passive substance advection of q, cloud water, TKE
- Implicit for very fast processes that would require a restrictively short time step for numerical stability:
  - vertical advection of u, v, T and vertically propagating sound waves

Time Integration

*

Horizontal advection of u, v, T

Adams-Bashforth:

$$\frac{y^{\tau+1} - y^{\tau}}{\Delta t} = \frac{3}{2} f(y^{\tau}) - \frac{1}{2} f(y^{\tau-1})$$

Stability/Amplification:

Method has a weak linear instability (amplification) which can be tolerated in practice or stabilized by a slight off-centering as is done in the WRF-NMM.

$$\frac{y^{\tau+1} - y^{\tau}}{\Delta t} = 1.533 f(y^{\tau}) - 0.533 f(y^{\tau-1})$$

Time Integration

*

Vertical advection of u, v, & T

Crank-Nicolson:

$$\frac{y^{\tau+1} - y^{\tau}}{\Delta t} = \frac{1}{2}[f(y^{\tau+1}) + f(y^{\tau})]$$

Stability:

An implicit method, it is absolutely stable numerically. However, small steps needed for *accuracy*!

Time Integration

Advection of passive substance (Q, Q2, CWM)

- Similar to Janjic (1997) scheme used in Eta model:
  - Starts with an initial upstream advection step
  - anti-diffusion/anti-filtering step to reduce dispersiveness
  - conservation enforced after each step – maintain global sum of advected quantity, and prevent generation of new extrema.

Time Integration – fast adjustment processes

Forward-Backward (Ames, 1968; Janjic and Wiin-Nielsen, 1977; Janjic 1979):

Mass field computed from a forward time difference, while the velocity field comes from a backward time difference.

In a shallow water equation sense:

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x}, \frac{\partial h}{\partial t} = -H\frac{\partial u}{\partial x}$$

$$h^{\tau+1} = h^{\tau} - \Delta t\, H \frac{\partial u^{\tau}}{\partial x}$$

$$u^{\tau+1} = u^{\tau} - \Delta t\, g \frac{\partial h^{\tau+1}}{\partial x}$$

} Mass field forcing to update wind from τ+1 time

Time Integration

Vertically propagating sound waves

Implicit solution (Janjic et al., 2001; Janjic, 2003) of:

$$\frac{\partial^2 p'}{\partial t^2} = \frac{p'^{n+1} - 2p'^{n} + p'^{n-1}}{\Delta t^2} = \frac{c_p}{c_v} R T_0 \frac{\partial^2 p'^{n+1}}{\partial {z_0}^2}$$

Where p' is the perturbation pressure from a hydrostatic basic state, and n represents the time level.

Time Integration

- Sequence of events within a solve_nmm loop (ignoring physics):

(0.6%) ▪ PDTE – integrates mass flux divergence, computes vertical velocity and updated pressure field.

(26.4%) ▪ ADVE – horizontal and vertical advection of T, u, v, Coriolis and curvature terms applied.

(1.2%) ▪ VTOA – updates nonhydrostatic pressure, applies $\omega\alpha$ term to thermodynamic equation

(8.6%) ▪ VADZ/HADZ – vertical/horizontal advection of height. w=dz/dt updated.

(10.6%) ▪ EPS – vertical and horizontal advection of dz/dt, vertical sound wave treatment.

(relative % of dynamics time spent in these subroutines)

Time Integration

- Sequence of events within a solve_nmm loop (cont):

(19.5%) ▪ VAD2/HAD2 (every other step) – vertical/horizontal advection of q, CWM, TKE

(11.8%) ▪ HDIFF – horizontal diffusion

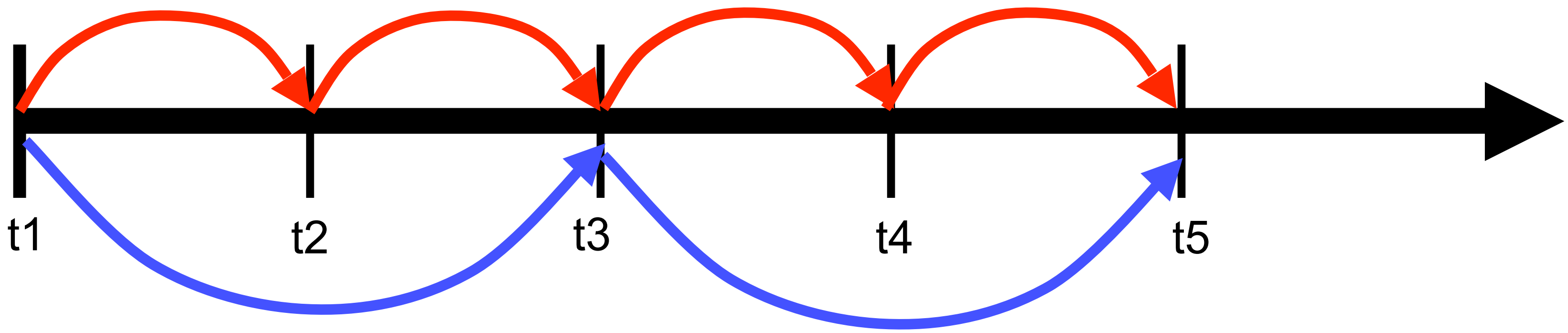
(1.2%) ▪ BOCOH – boundary update at mass points

(17.5%) ▪ PFDHT – calculates PGF, updates winds due to PGF, computes divergence.

(2.3%) ▪ DDAMP – divergence damping

(0.3%) ▪ BOCOV – boundary update at wind points

All dynamical processes every fundamental timestep, except….



…passive substance advection, every other timestep

Model timestep "dt" specified in model namelist.input is for the fundamental timestep.

Generally about 2.25X the horizontal grid spacing (km), or 350X the namelist.input "dy" value (degrees lat).

*

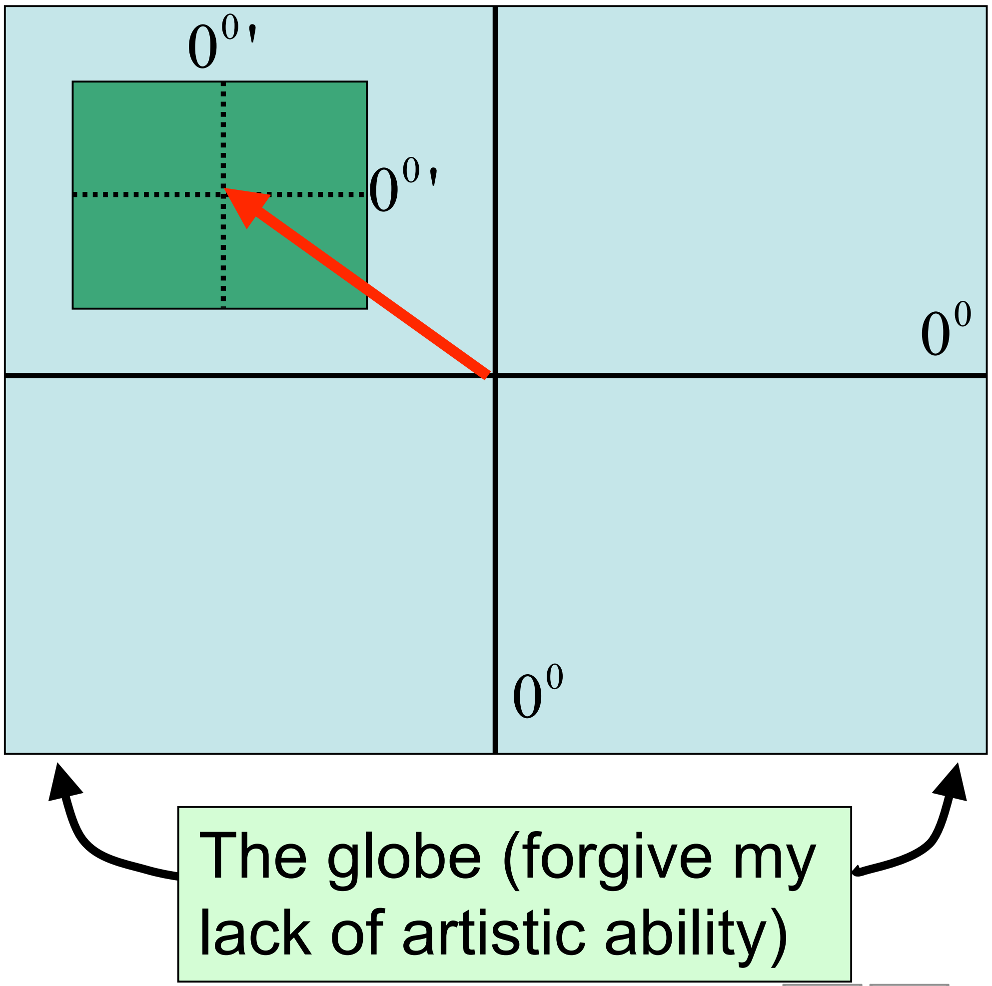
Now we'll take a look at two items specific to the WRF-NMM horizontal grid:

- Rotated latitude-longitude map projection (only one used with WRF-NMM)
- The E-grid stagger

Rotated Latitude-Longitude

- Rotates the earth’s latitude/longitude grid such that the intersection of the equator and prime meridian is at the center of the model domain.
- This rotation minimizes the convergence of meridians over the domain.
- Within the rotated framework the grid spacing is constant, but in an earth-relative sense the scale varies slightly.

Rotated Latitude-Longitude



For a domain spanning 10N to 70N:

$$\Delta x \propto \cos(lat)$$

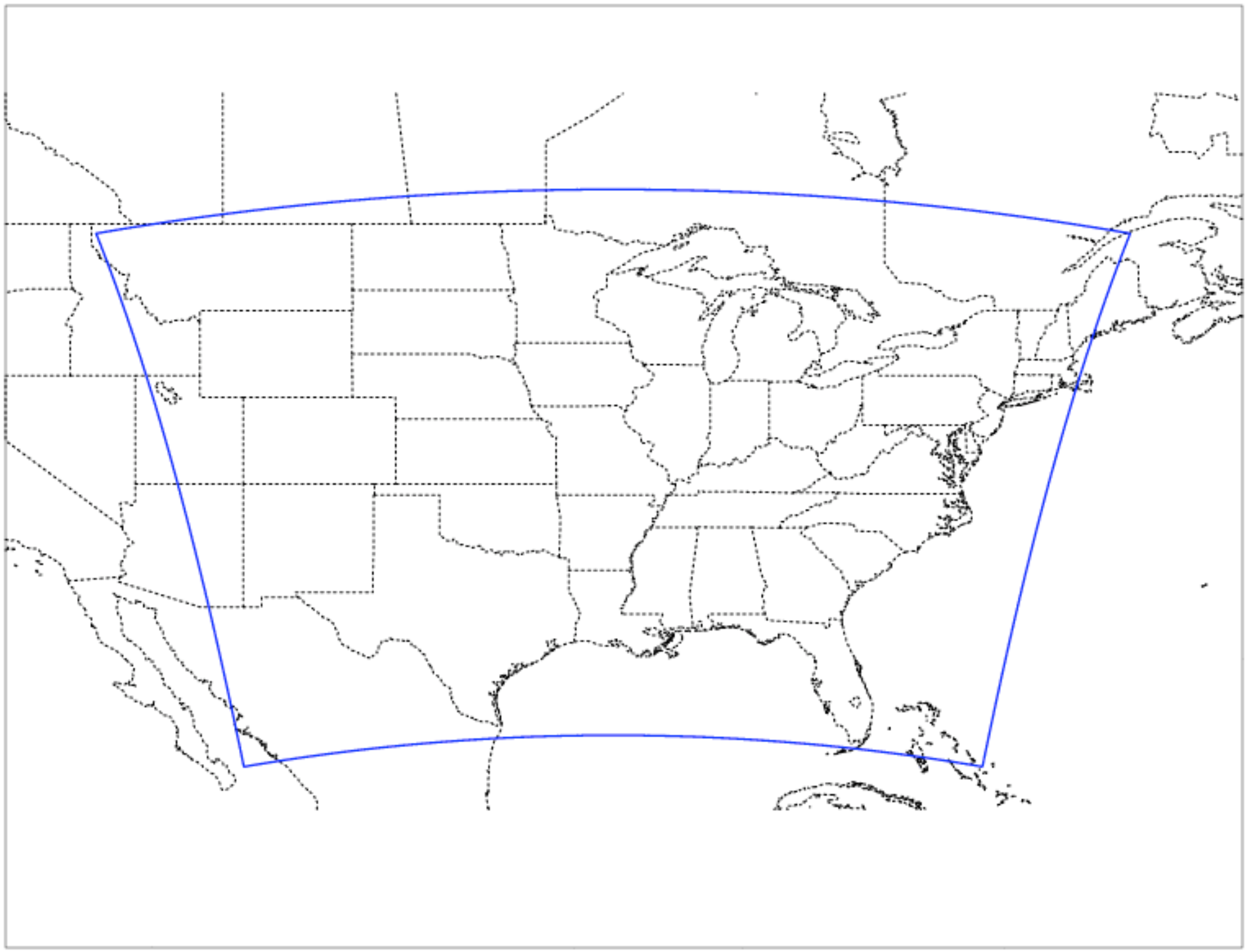
Regular lat-lon grid

$$\cos(70^0)/\cos(10^0) = 0.347$$

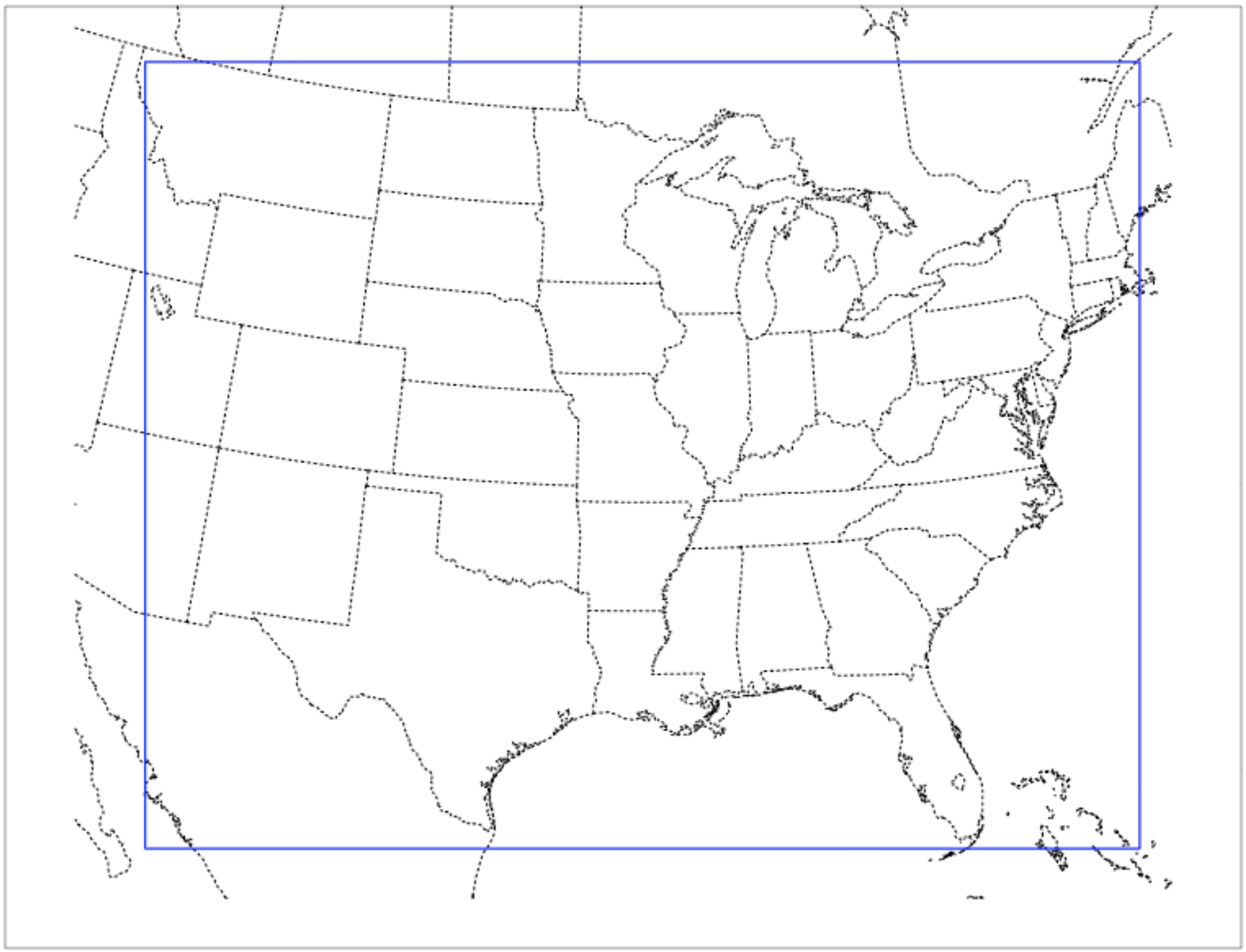
Rotated lat-lon grid

$$\cos(30^0)/\cos(0^0) = 0.866$$

Rotated latitude-longitude domain (center 38N, 92W) projected on regular lat-lon map background



Same domain projected on a similarly rotated lat/lon map background



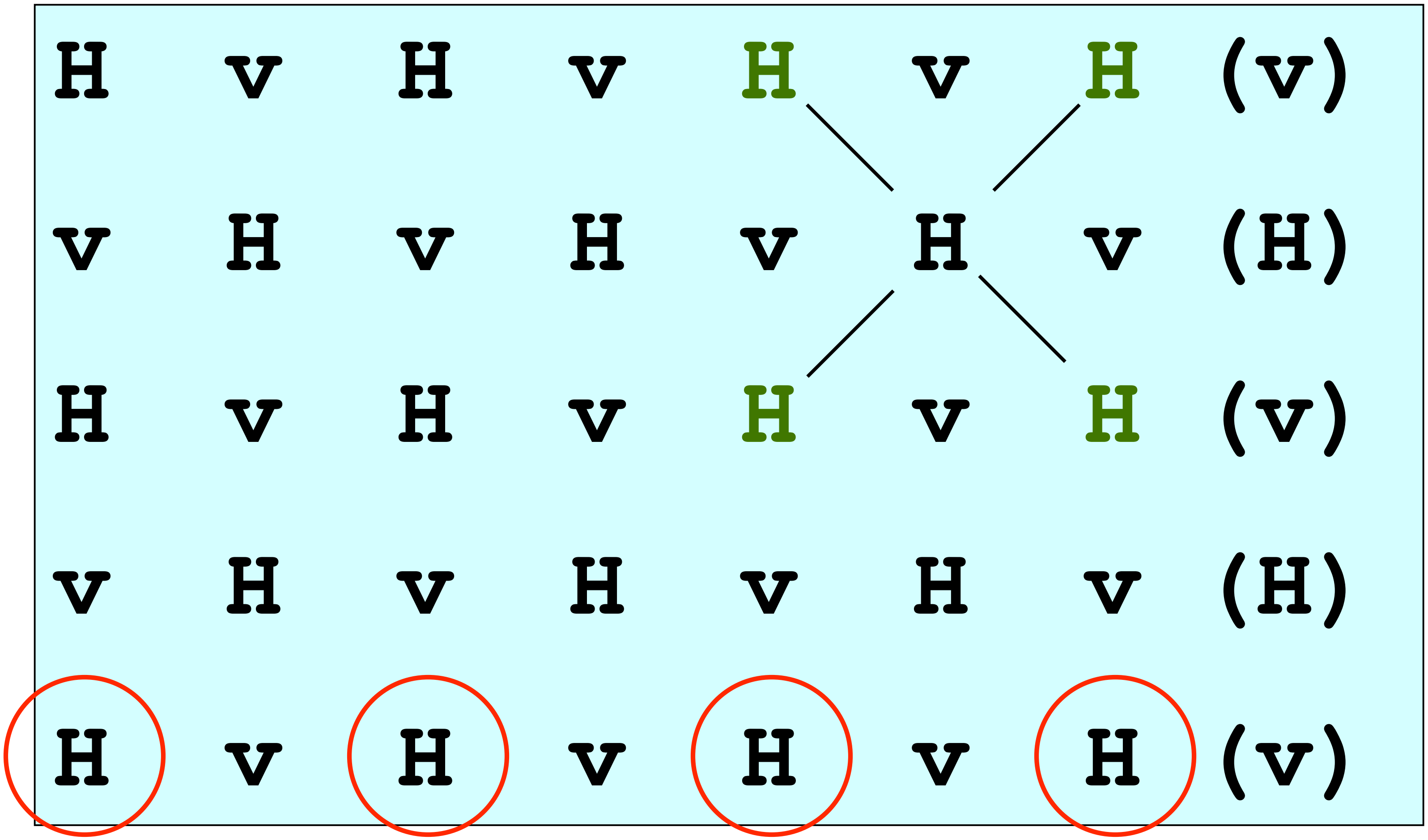
The E-grid Stagger

H	v	H	v	H	v	H	(v)
v	H	v	H	v	H	v	(H)
H	v	H	v	H	v	H	(v)
v	H	v	H	v	H	v	(H)
H	v	H	v	H	v	H	(v)

H=mass point, v=wind point

red=(1,1) ; blue=(1,2)

The E-grid Stagger



XDIM=4 (# of mass points on odd numbered row)

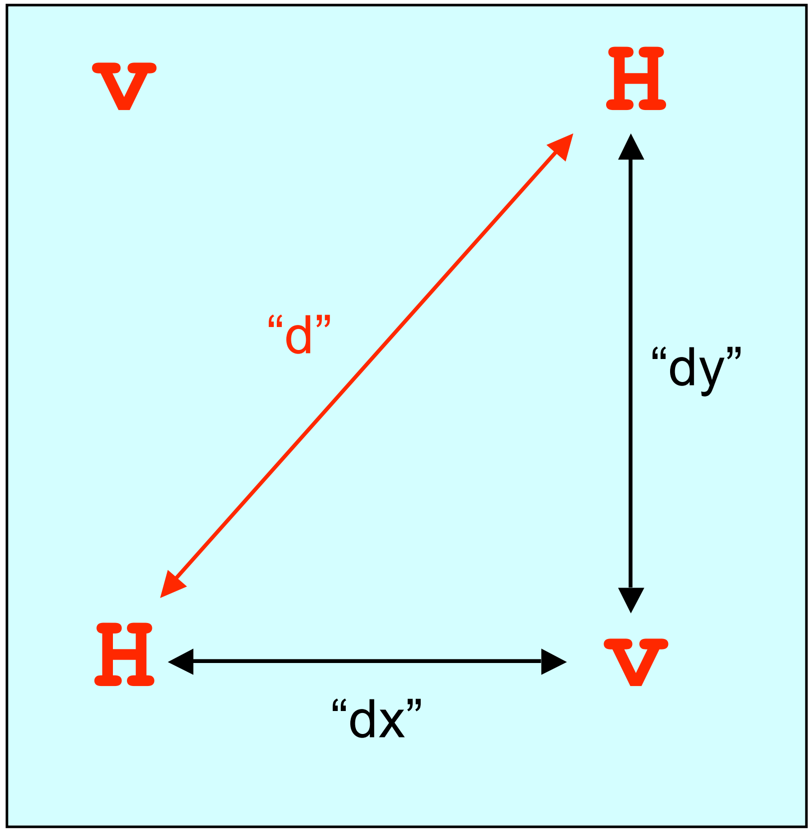
YDIM=5 (number of rows)

The E-grid Stagger - properties

- Due to the indexing convention, the X-dimension is half as large as would be expected from a C-grid domain (typically XDIM < YDIM for the E-grid).
- "Think diagonally" –the shortest distance between adjacent like points is along the diagonals of the grid.

- Nonlinear energy cascade toward small scales minimized, and eliminated for 2 delta-x wavelength [discretization methodologies developed for E-grid that can't be replicated for the C-grid – Janjic (1984) MWR].

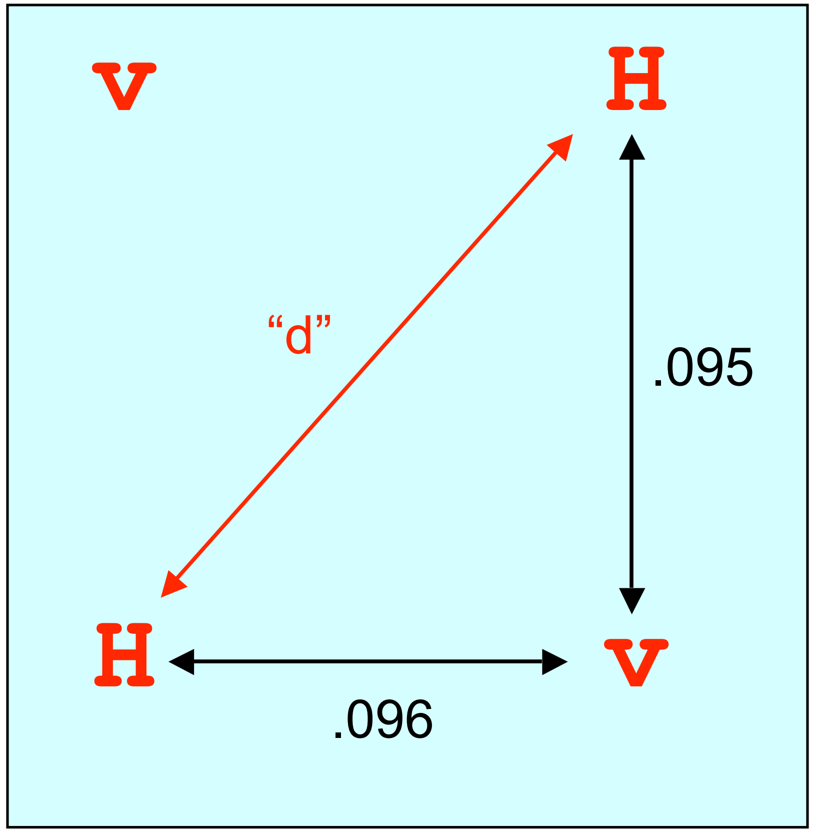
The E-grid Stagger



- Conventional grid spacing is the diagonal distance "d".

- Grid spacings in the WPS and WRF namelists are the "dx" and "dy" values, specified in fractions of a degree.

The E-grid Stagger



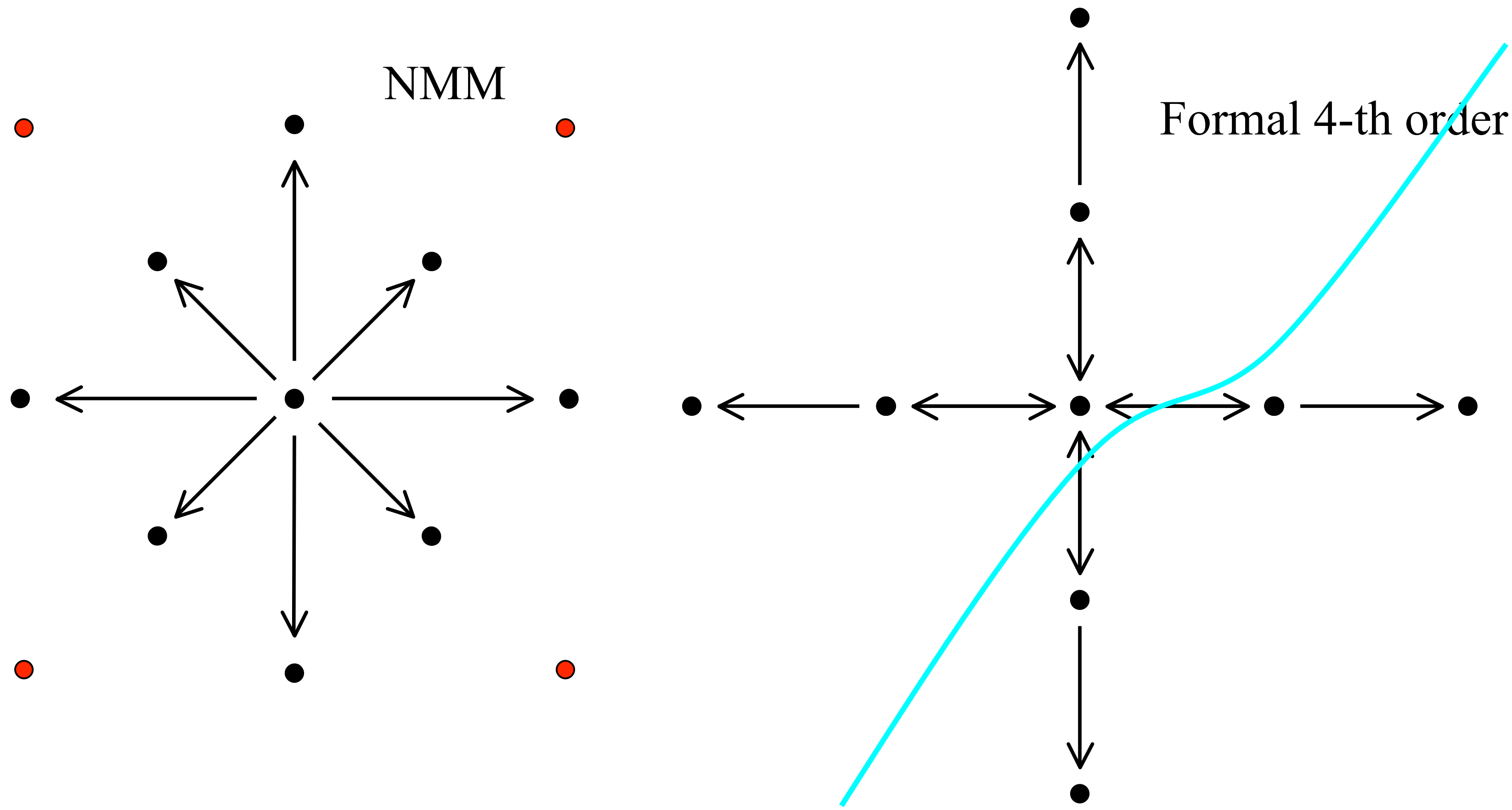
- d ~= sqrt $(0.096^2+0.095^2)$ * (111.2 km/deg) ~= 15 km

- "WRF domain wizard" takes input grid spacing in km and computes the angular distances for the namelist.

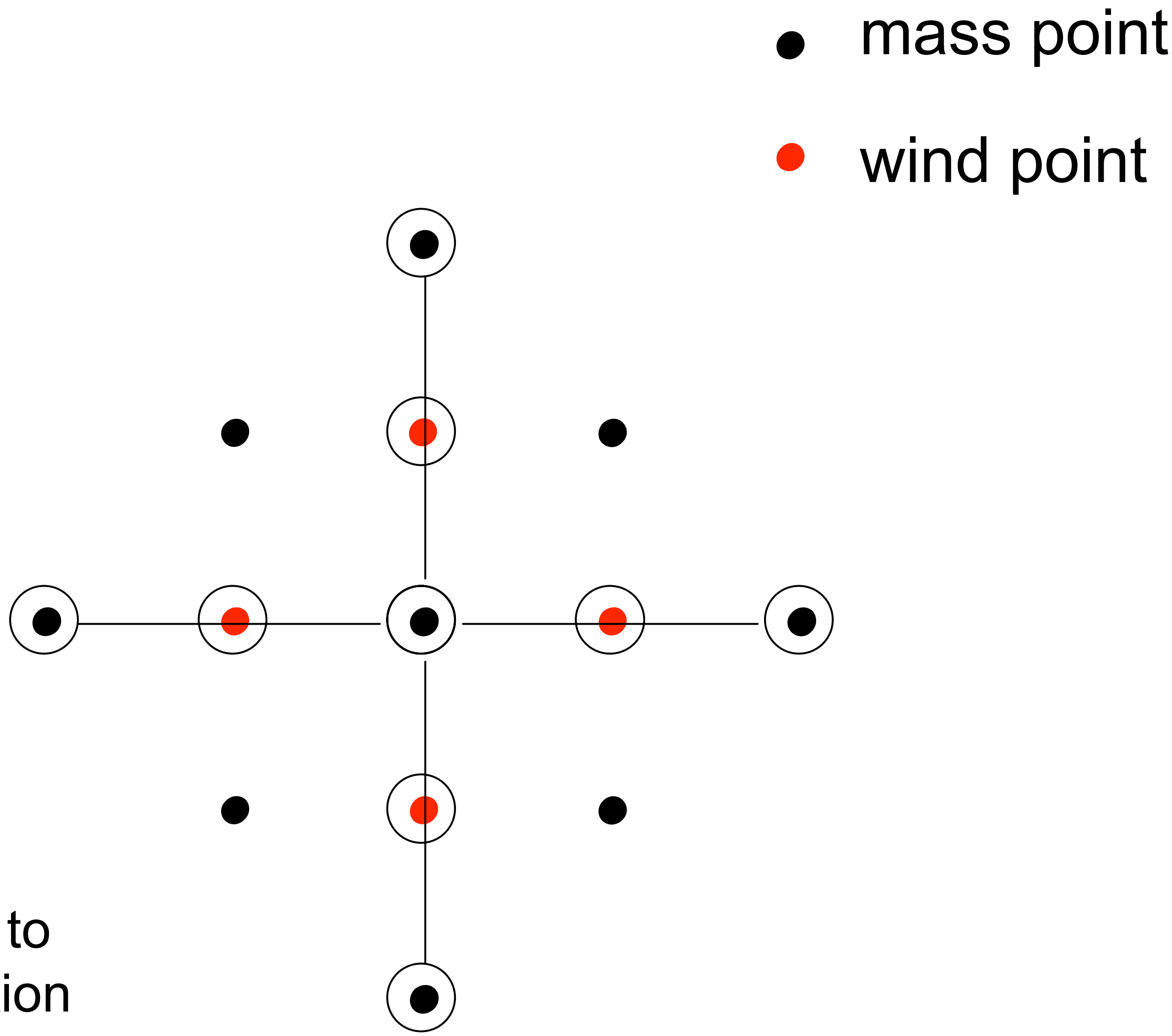
Spatial Discretization

General Philosophy

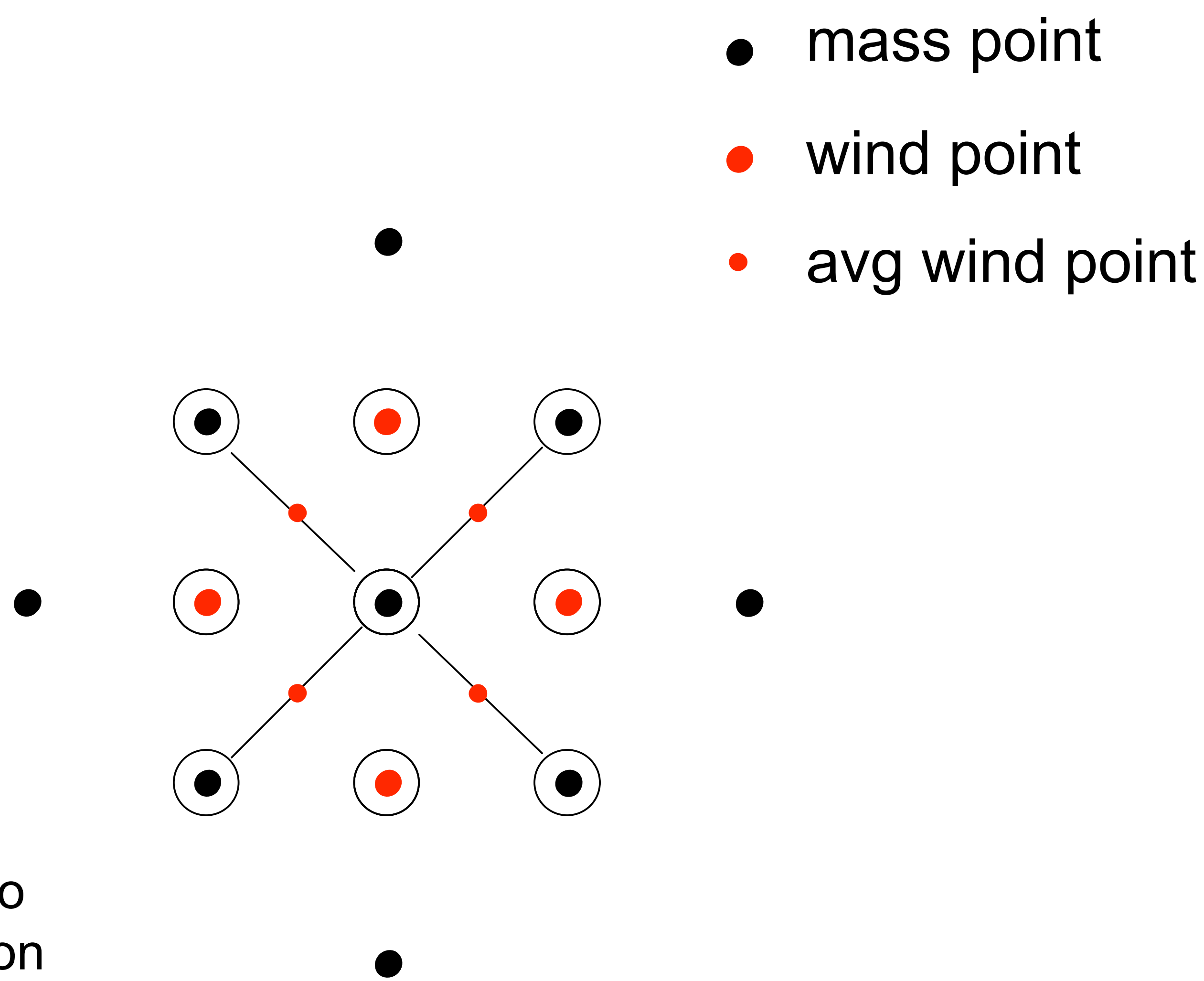
- Conserve energy and enstrophy in order to control nonlinear energy cascade; eliminate the need for numerical filtering to the extent possible.
- Use consistent order of accuracy for advection and divergence operators and the omega-alpha term; consistent transformations between KE and PE .
- Conserve a number of first order and quadratic quantities, such as momentum and advected quantities.



Advection and divergence operators – each point talks to all neighboring points (isotropic)



1/3 of contribution to divergence/advection comes from these N/S and E/W fluxes.



2/3 of contribution to divergence/advection comes from these diagonal fluxes.

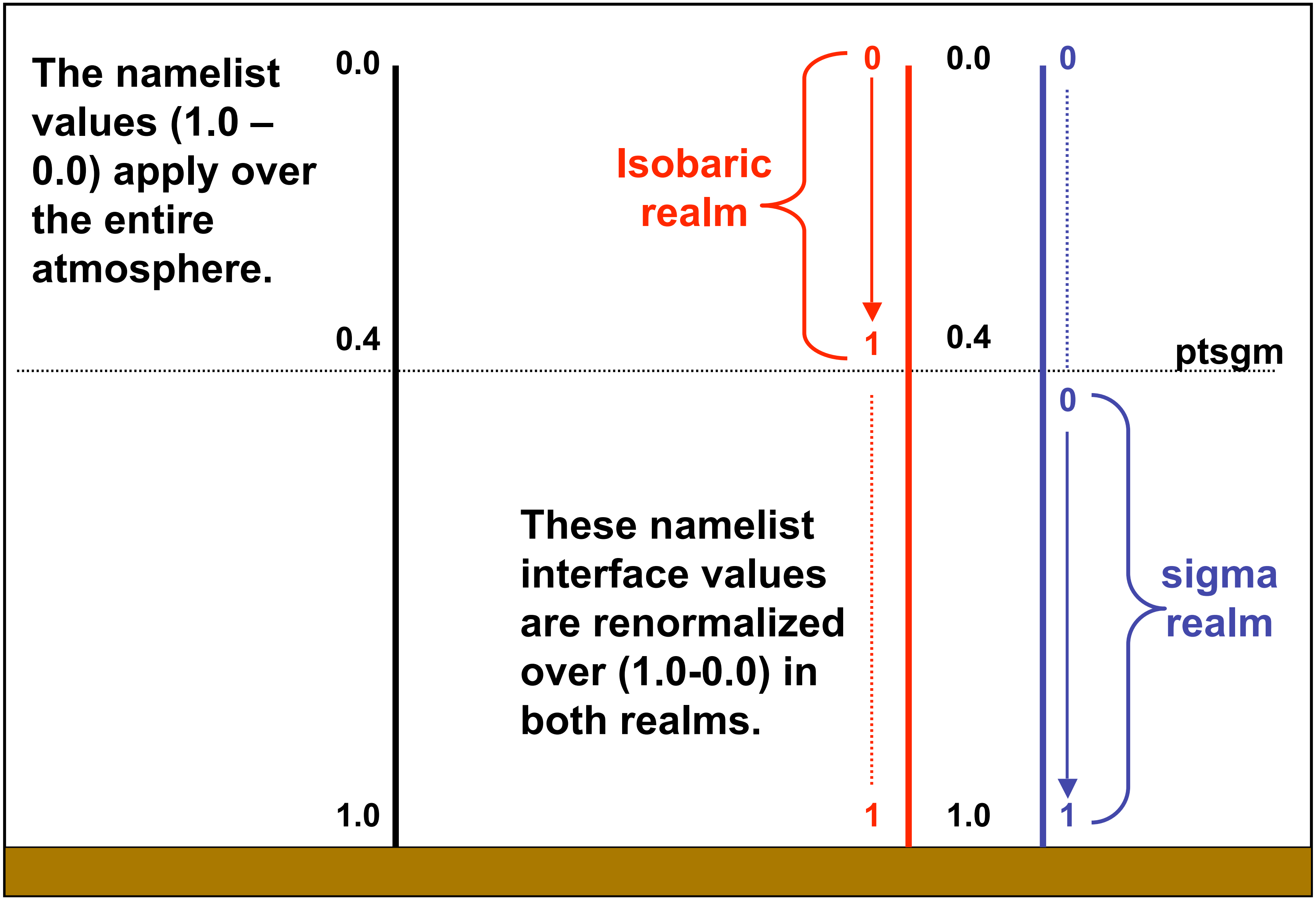
NMM Vertical Coordinate

Pressure-sigma hybrid (Arakawa and Lamb, 1977)

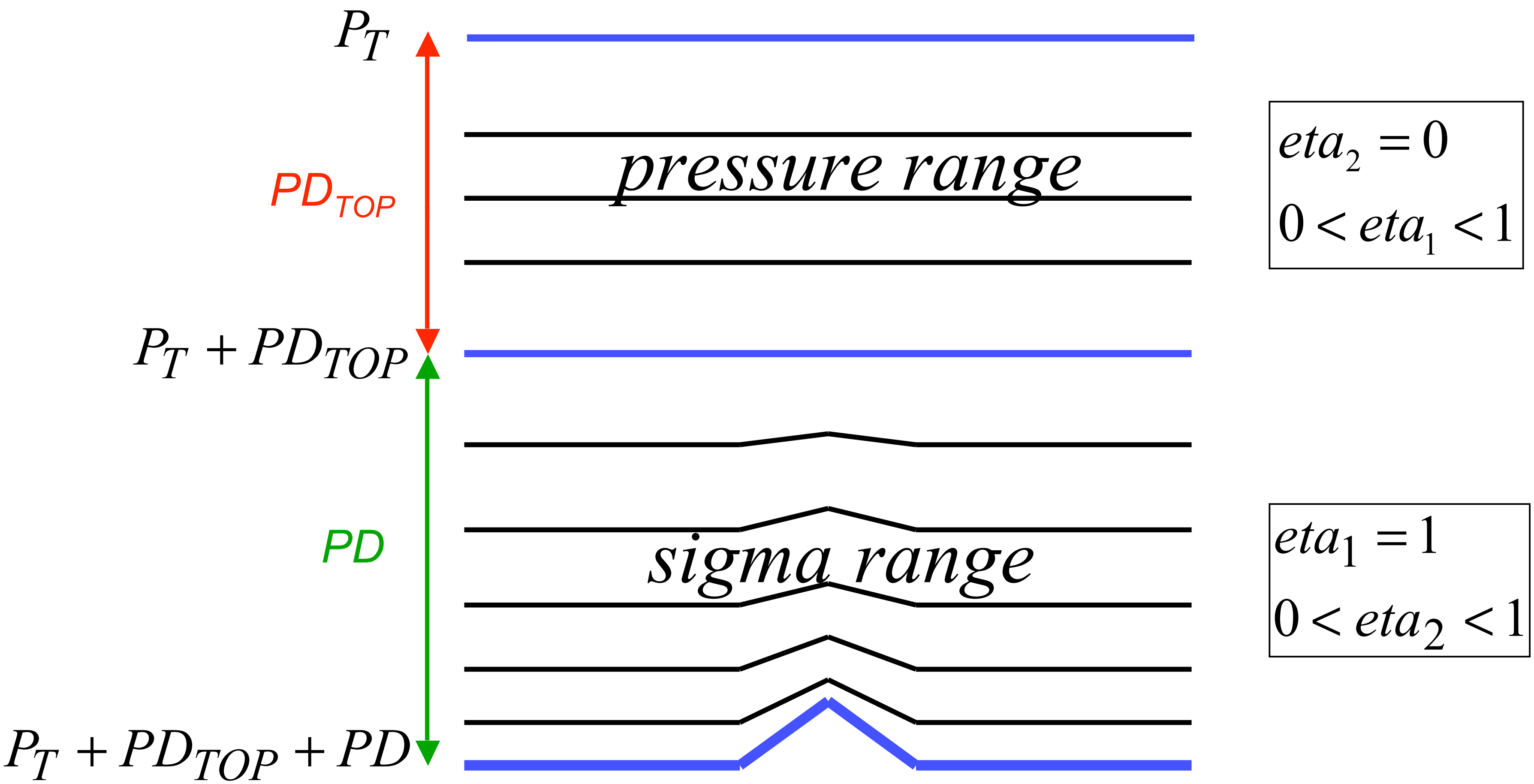
- Exact mass (etc.) conservation
- Nondivergent flow on pressure surfaces
- No problems with weak static stability
- No discontinuities or internal boundary conditions

- Flat coordinate surfaces at high altitudes where sigma problems worst (e.g., Simmons and Burridge, 1981)

Pressure-Sigma Hybrid Vertical Coordinate



Pressure-Sigma Hybrid Vertical Coordinate

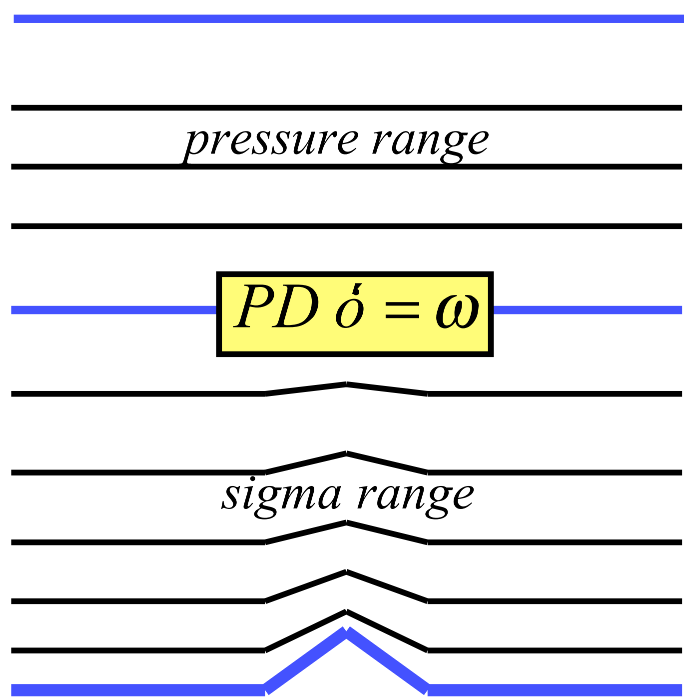


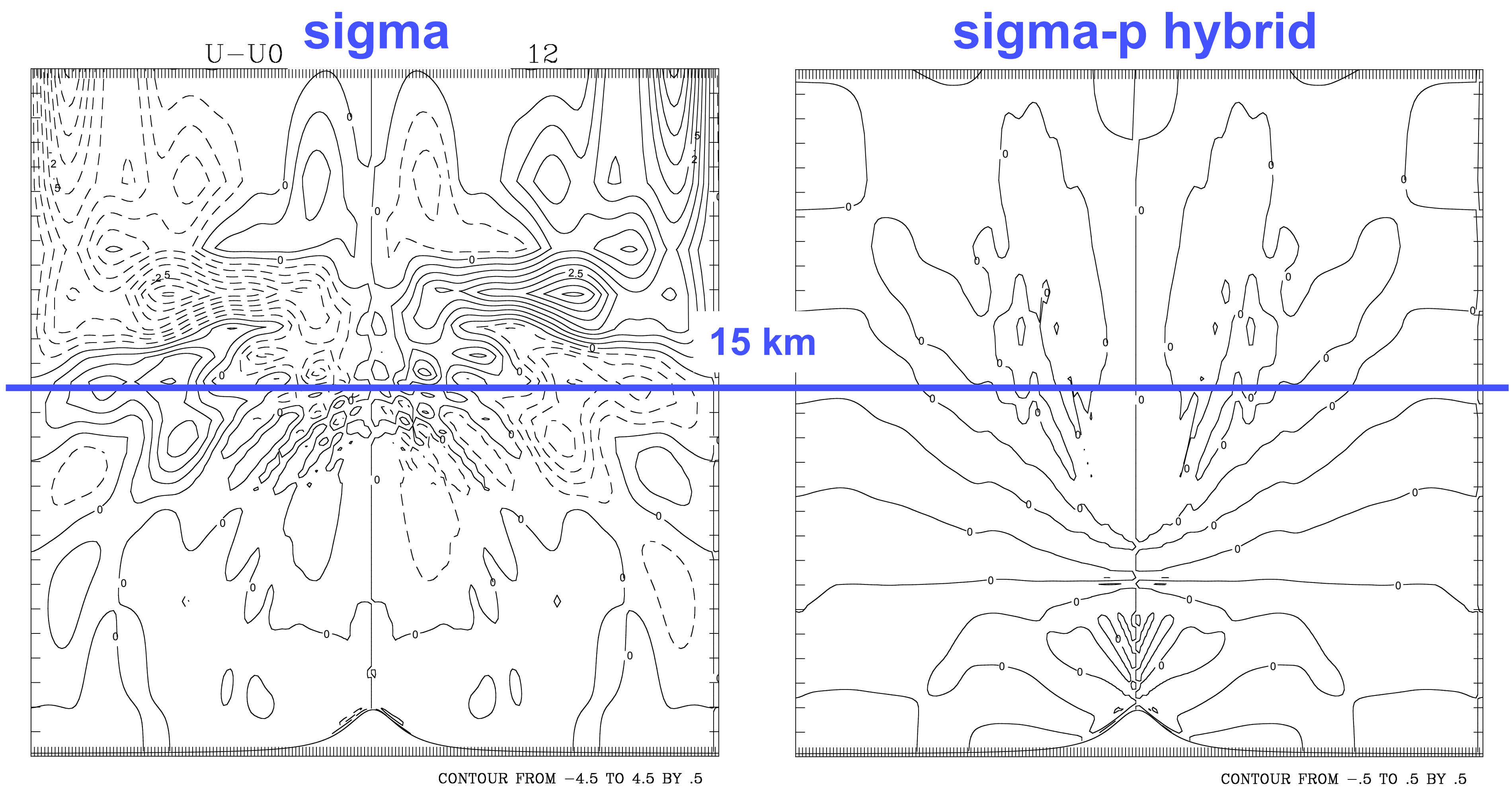
$$p = eta_1 * PD_{TOP} + eta_2 * PD + P_T$$

Equations in Hybrid Coordinate

$$\frac{\partial PD}{\partial t} + \nabla_{\sigma} \cdot (PD\ \mathbf{v}) + \frac{\partial (PD\,\dot{\sigma})}{\partial \sigma} = 0$$

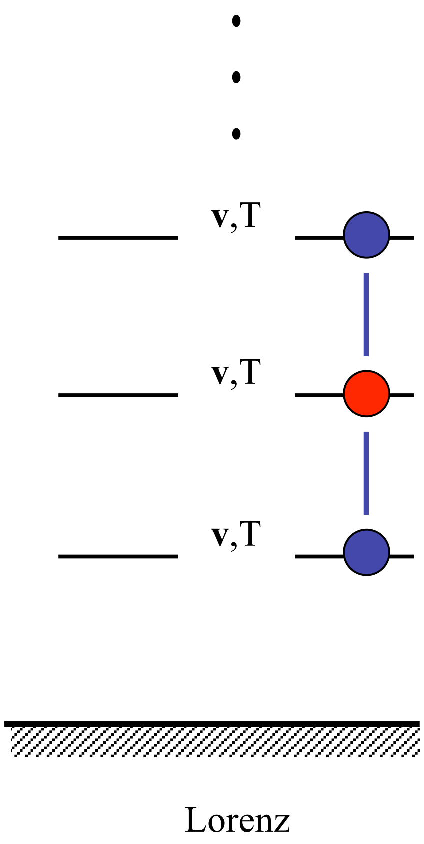
$$\nabla_{p} \bullet (\mathbf{v}) + \frac{\partial \omega}{\partial p} = 0$$





Wind component developing due to the spurious pressure gradient force in an idealized integration. The hybrid coordinate boundary between the pressure and sigma domains is at at about 400 hPa.

Vertical discretization



Vertical advection combines the advective fluxes computed above and below the layer of interest.

Boundary Conditions

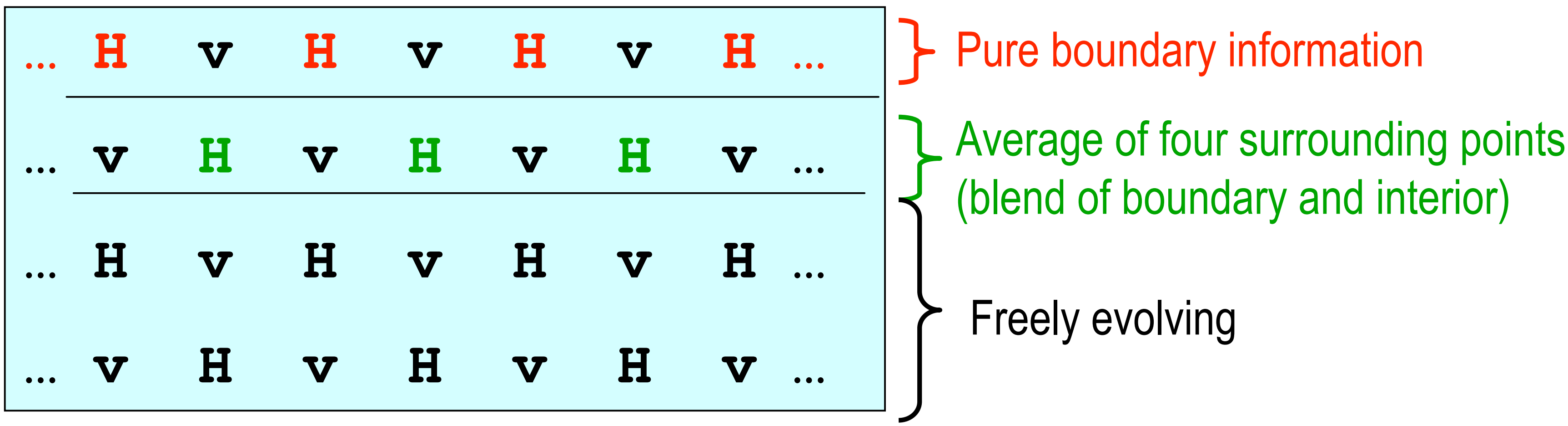
Vertical boundaries:

Top: $\dot{\sigma} = 0$, $p - \pi = 0$

Surface: $\dot{\sigma} = 0$, $\frac{\partial (p - \pi)}{\partial \sigma} = 0$

Boundary Conditions

- Lateral boundary information prescribed only on outermost row:



- Upstream advection in three rows next to the boundary
    - No computational outflow boundary condition for advection

- Enhanced divergence damping close to the boundaries.

Dissipative Processes – lateral diffusion

A 2nd order, nonlinear Smagorinsky-type horizontal diffusion is utilized:

- Diffusion strength a function of the local TKE and 3D wind field deformation, gradients of the field being diffused, and a code-specified constant (COAC*).
- Lateral diffusion is zeroed for sloping model surfaces (> ~ 6 m / 12 km grid point), although this detail will be changed in a future model release.

* COAC has a default value of 1.6 and is specified in ./dyn_nmm/module_initialize_real.F. Larger values generate more diffusive smoothing.

Dissipative Processes - divergence damping

- Horizontal divergence damping with enhanced damping of the external mode (in future WRFV3 code).

- Internal mode damping (on each vertical layer)

$$v_j = v_j + \frac{(\nabla \cdot dp_{j+1}\mathbf{v}_{j+1} - \nabla \cdot dp_{j-1}\mathbf{v}_{j-1})}{(dp_{j+1} + dp_{j-1})} \cdot DDMPV$$

- External mode damping (vertical integrated)

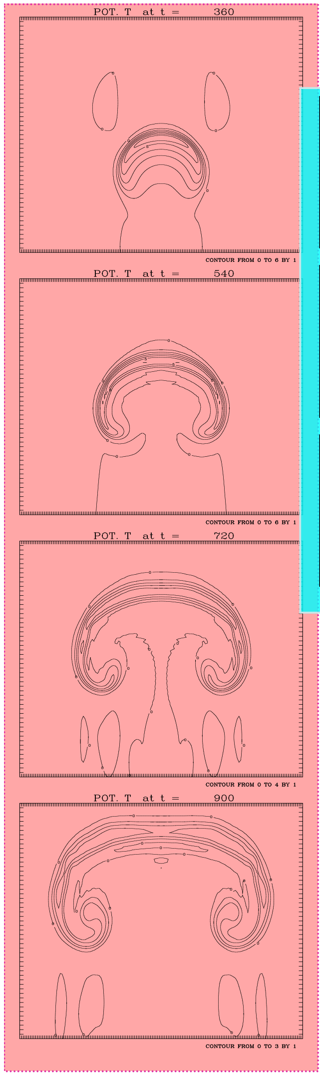
$$v_j = v_j + \frac{(\int \nabla \cdot dp_{j+1}\mathbf{v}_{j+1} - \int \nabla \cdot dp_{j-1}\mathbf{v}_{j-1})}{(\int dp_{j+1} + \int dp_{j-1})} \cdot DDMPV$$

$$DDMPV \approx \sqrt{2} \cdot dt \cdot CODAMP$$

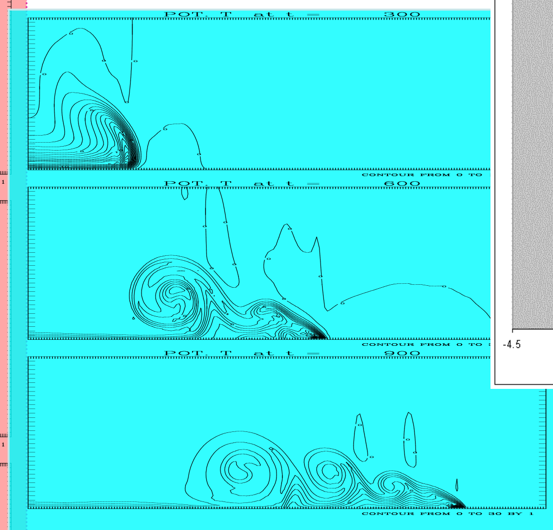
CODAMP is a code-specified parameter = 6.4 by default.

Dynamics formulation tested on various scales

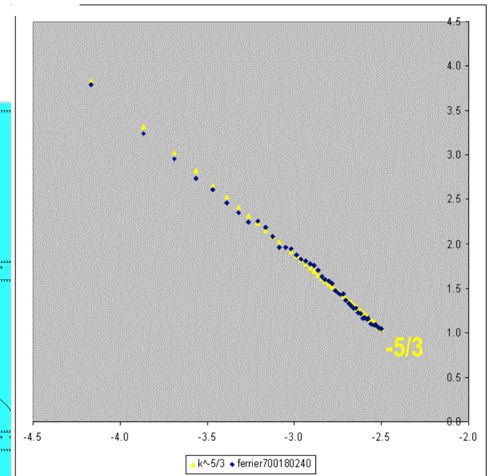
Warm bubble



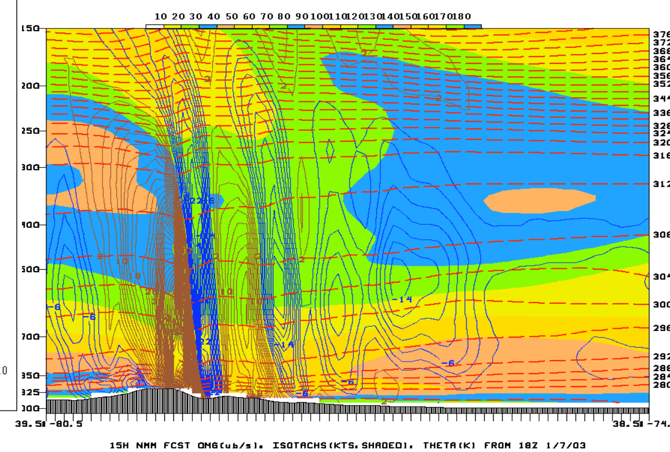
Cold bubble



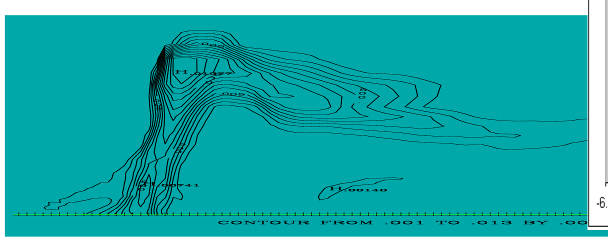
Decaying 3D turbulence



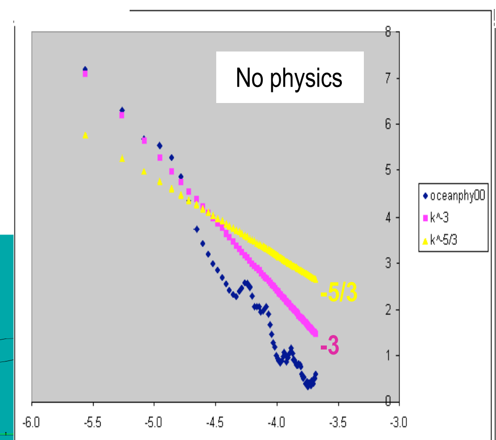
Mountain waves

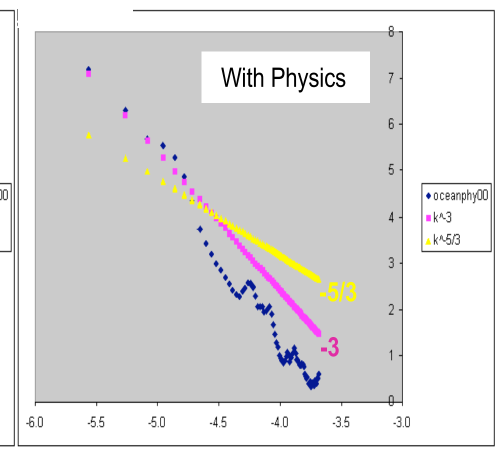


Convection



Atmospheric spectra





Summary

- Robust, reliable, fast
- NWP on near-cloud scales successful more frequently and with stronger signal than if only by chance
- Replaced the Eta as NAM at NCEP on June 20, 2006
- Near-cloud-scale runs (~4 km grid spacing) operational at NCEP for severe weather forecasting
- Operational as Hurricane WRF in 2007
- Operational and quasi-operational elsewhere (Europe, Canada)