National Weather Service
National Centers for Environmental Prediction

The WRF NMM Core

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Talk modified and presented by
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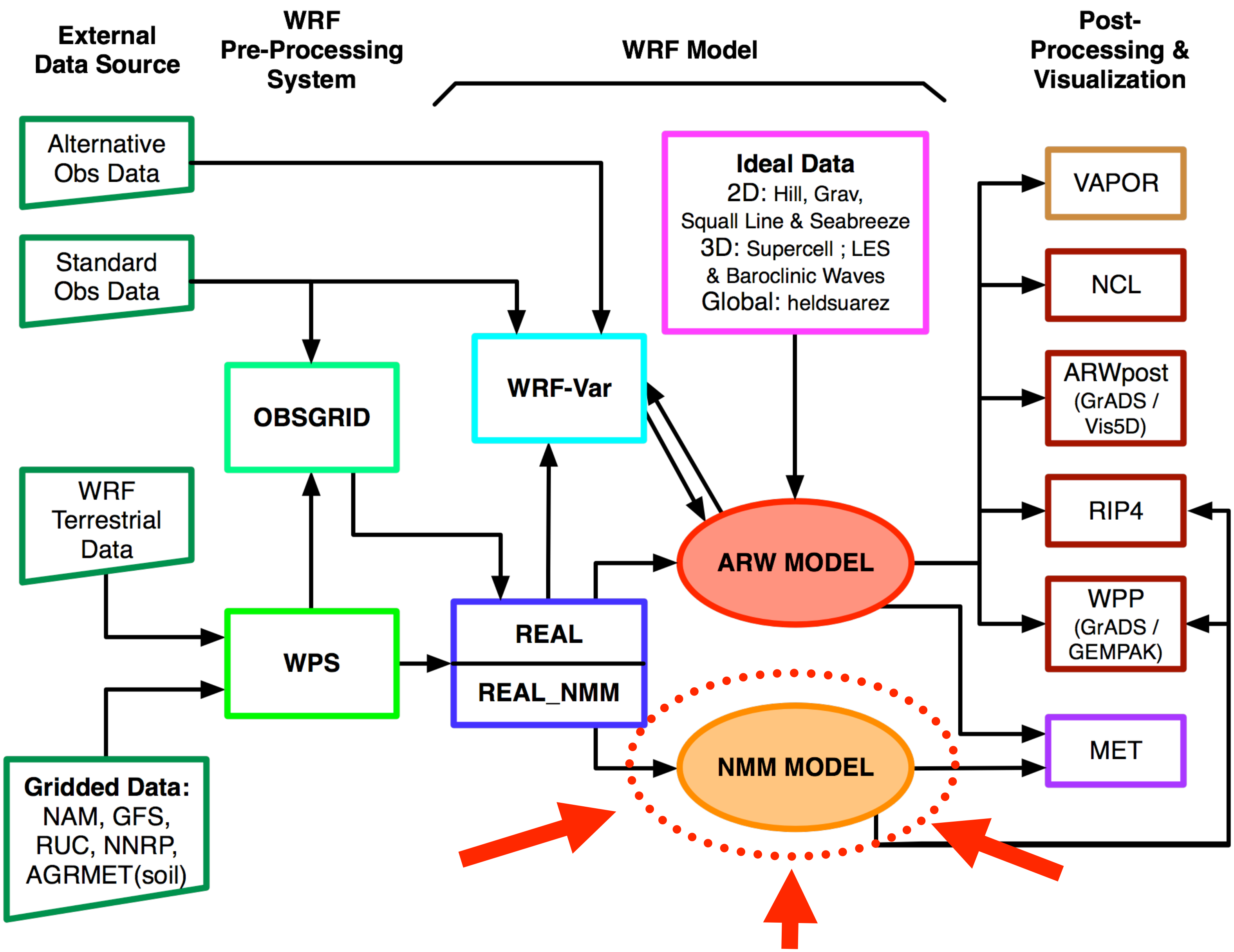
WRF Modeling System Flow Chart



NMM Dynamic Solver

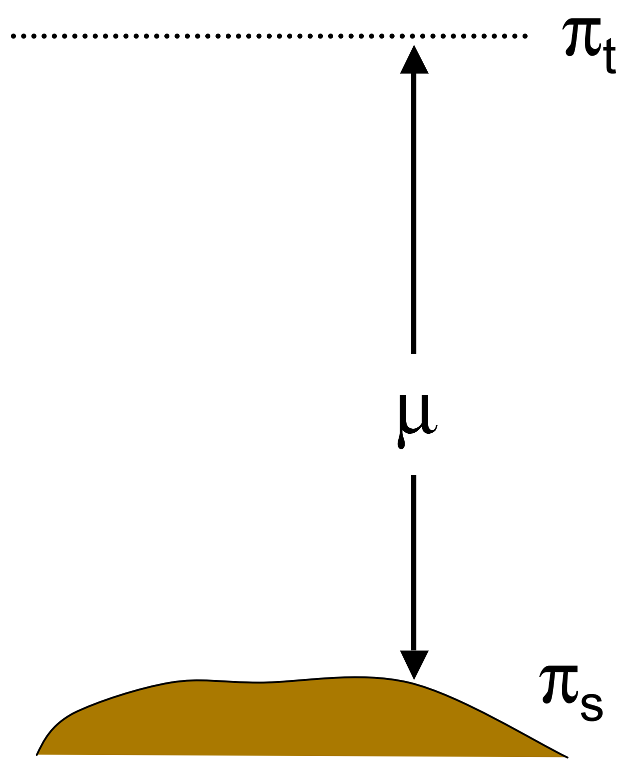
- Basic Principles
- Equations / Variables
- Model Integration
- Horizontal Grid
- Spatial Discretization
- Vertical Grid
- Boundary Conditions
- Dissipative Processes
- Summary

Basic Principles

- Use full compressible equations split into hydrostatic and nonhydrostatic contributions
    - Easy comparison of hydro and nonhydro solutions
    - Reduced computational effort at lower resolutions
- Use modeling principles proven in NWP and regional climate applications
- Use methods that minimize the generation of small-scale noise
- Robust, computationally efficient

Mass Based Vertical Coordinate

For simplicity, consider the sigma coordinate as representative of a vertical coordinate based on hydrostatic pressure (π):



$$\sigma = \frac{\pi - \pi_t}{\mu}$$

$$\mu = \pi_s - \pi_t$$

π_t = model top π
π_s = surface π

Inviscid, adiabatic, sigma (Janjic et al., 2001, MWR)

Analogous to a hydrostatic system, except for p and ε

p is the total (nonhydrostatic) pressure

π is the hydrostatic pressure

Momentum eqn.

$$\frac{\partial \mathbf{v}}{\partial t} = -\mathbf{v}\cdot\nabla_\sigma \mathbf{v} - \dot{\sigma}\frac{\partial \mathbf{v}}{\partial \sigma} - (1+\varepsilon)\nabla_\sigma \Phi - \alpha\nabla_\sigma p + f\mathbf{k}\times\mathbf{v}$$

Thermodynamic eqn.

$$\frac{\partial T}{\partial t} = -\mathbf{v}\cdot\nabla_\sigma T - \dot{\sigma}\frac{\partial T}{\partial \sigma} + \frac{\alpha}{c_p}\left[\frac{\partial p}{\partial t} + \mathbf{v}\cdot\nabla_\sigma p + \dot{\sigma}\frac{\partial p}{\partial \sigma}\right]$$

Continuity eqn.

$$\frac{\partial \mu}{\partial t} + \nabla_\sigma\cdot(\mu\mathbf{v}) + \frac{\partial(\mu\dot{\sigma})}{\partial \sigma} = 0$$

$$\alpha = RT/p$$

$$\varepsilon \equiv \frac{1}{g}\frac{dw}{dt}$$

3rd eqn of motion

$$\frac{\partial p}{\partial \pi} = 1 + \varepsilon$$

Hypsometric eqn.

$$\frac{\partial \Phi}{\partial \sigma} = -\mu \frac{RT}{p}$$

Nonhydrostatic continuity eqn.

$$w = \frac{1}{g}\frac{d\Phi}{dt} = \frac{1}{g}\left(\frac{\partial \Phi}{\partial t} + \mathbf{v} \cdot \nabla_{\sigma}\Phi + \dot{\sigma}\frac{\partial \Phi}{\partial \sigma}\right)$$

Side note: separation of the thermodynamic equation into two parts

$$\frac{\partial T}{\partial t} = -\mathbf{v} \cdot \nabla_\sigma T - \dot{\sigma}\frac{\partial T}{\partial \sigma} + \frac{\alpha}{c_p}[\omega_1 + \omega_2]$$

$$\left(\frac{\partial T}{\partial t}\right)_1 = -\mathbf{v} \cdot \nabla_\sigma T - \dot{\sigma}\frac{\partial T}{\partial \sigma} + \frac{1}{c_p}\alpha\omega_1$$

$$\omega_1 = (1+\varepsilon)\frac{\partial \pi}{\partial t} + \mathbf{v} \cdot \nabla_\sigma p + (1+\varepsilon)\dot{\sigma}\frac{\partial \pi}{\partial \sigma}$$

Reduces to hydrostatic equation for ε=0

$$\left(\frac{\partial T}{\partial t}\right)_2 = \frac{1}{c_p}\alpha\omega_2$$

$$\omega_2 = \frac{\partial p}{\partial t} - (1+\varepsilon)\frac{\partial \pi}{\partial t}$$

Vanishes for purely hydrostatic flow

Properties of system

- Φ, w, ε are not independent, no independent prognostic equation for w!
- $\varepsilon \ll 1$ in meso and large scale atmospheric flows
- Impact of nonhydrostatic dynamics becomes detectable at resolutions <10km, important at 1km.

Boundary Conditions for model equations

Vertical boundaries:

Top: $\dot{\sigma} = 0$, $p - \pi = 0$

Surface: $\dot{\sigma} = 0$, $\dfrac{\partial (p - \pi)}{\partial \sigma} = 0$

WRF-NMM predictive variables

- Mass variables:
  - PD – hydrostatic pressure depth (time and space varying component) (Pa)
  - PINT – nonhydrostatic pressure (Pa)
  - T – sensible temperature (K)
  - Q – specific humidity (kg/kg)
  - CWM – total cloud water condensate (kg/kg)
  - Q2 – 2 * turbulent kinetic energy (m^2/s^2)
- Wind variables:
  - U, V – wind components (m/s)

Model Integration

General Philosophy

- Explicit time differencing preferred where possible, as allows for better phase speeds and more transparent coding:
  - horizontal advection of u, v, T
  - passive substance advection of q, cloud water, TKE
- Implicit time differencing for very fast processes that would require a restrictively short time step for numerical stability:
  - vertical advection of u, v, T and vertically propagating sound waves

Model Integration

Horizontal advection of u, v, T

2nd order Adams-Bashforth:

$$\frac{y^{\tau+1} - y^{\tau}}{\Delta t} = \frac{3}{2} f(y^{\tau}) - \frac{1}{2} f(y^{\tau-1})$$

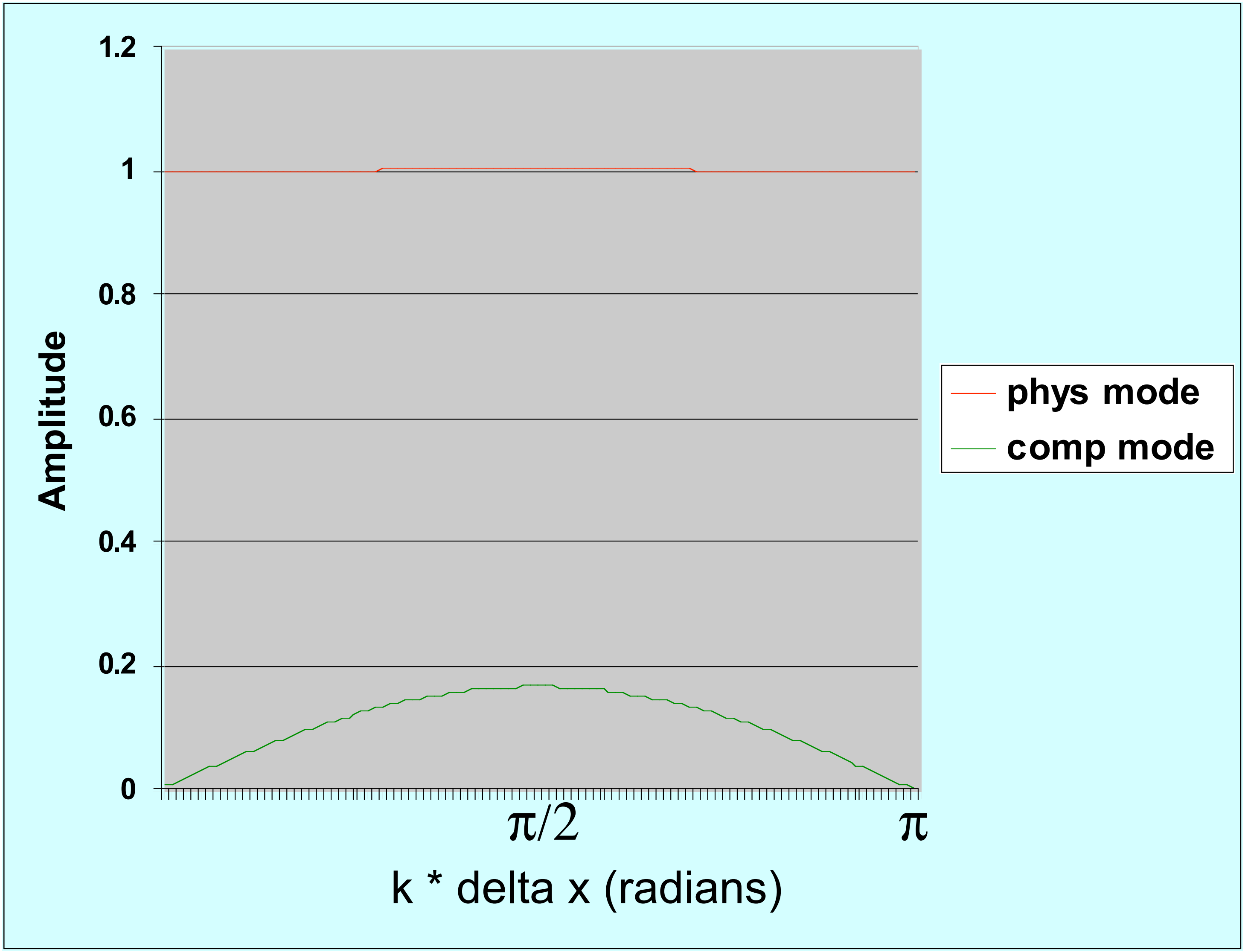
Stability/Amplification:

A-B has a weak linear instability (amplification) which can be tolerated in practice or stabilized by a slight off-centering as is done in the WRF-NMM.

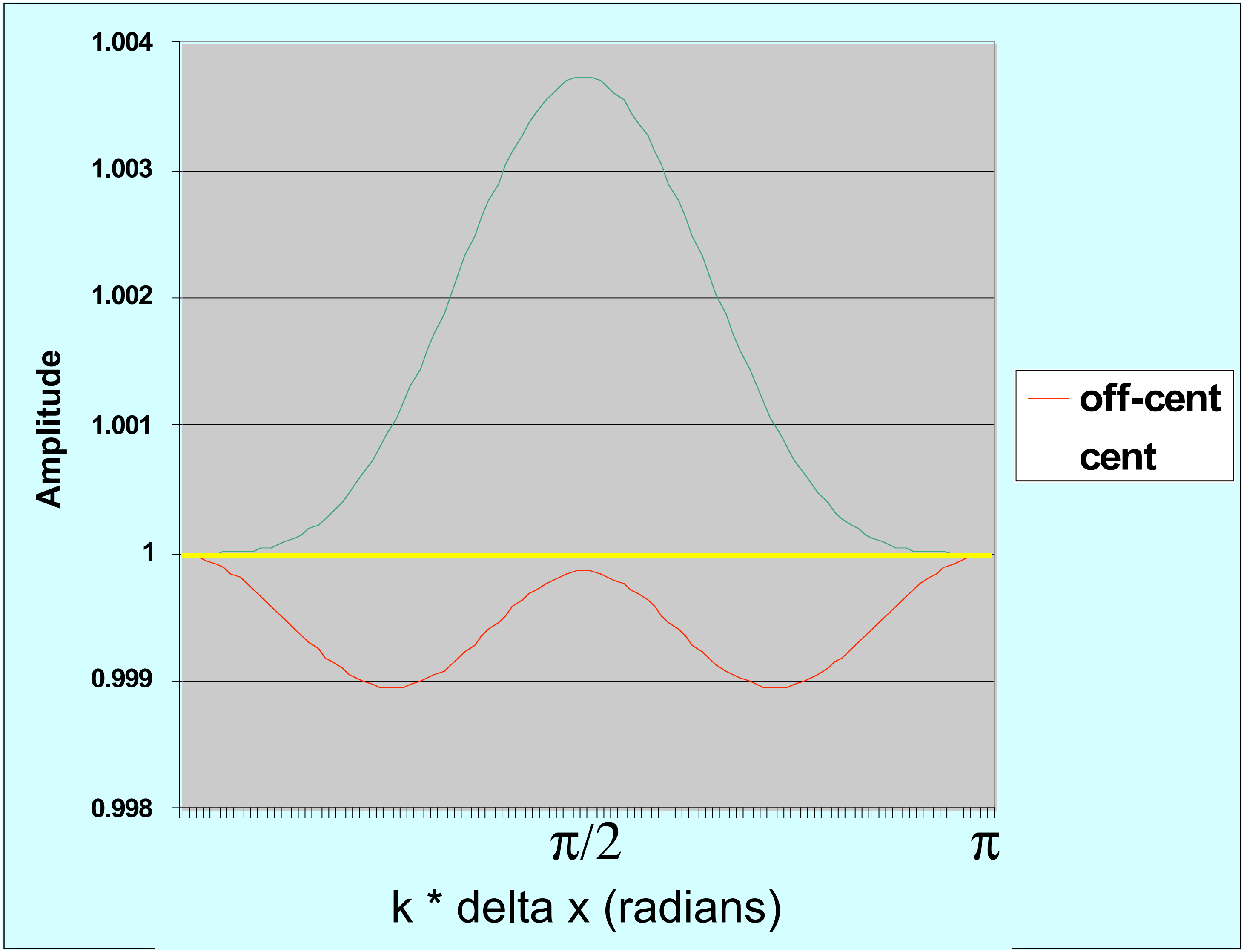
$$\frac{y^{\tau+1} - y^{\tau}}{\Delta t} = 1.533 f(y^{\tau}) - 0.533 f(y^{\tau-1})$$

Adams-Bashforth amplification factor derived from

$$\frac{\Delta u}{\Delta t} + c\frac{\Delta u}{\Delta x} = 0$$ with $c\Delta t/\Delta x = 0.33$



Adams-Bashforth amplification factor, Impact of off-centering



Model Integration

Vertical advection of u, v, & T

Crank-Nicolson:

$$\frac{y^{\tau+1} - y^{\tau}}{\Delta t} = \frac{1}{2}[f(y^{\tau+1}) + f(y^{\tau})]$$

Stability:

An implicit method, it is absolutely stable numerically.

Model Integration

Advection of TKE (Q2) and moisture (Q, CWM)

- Similar to Janjic (1997) scheme used in Eta model:
  - Starts with an initial upstream advection step
  - anti-diffusion/anti-filtering step to reduce dispersiveness
  - conservation enforced after each anti-filtering step – maintain global sum of advected quantity, and prevent generation of new extrema.

Model Integration

Fast adjustment processes

Forward-Backward (Ames, 1968; Janjic and Wiin-Nielsen, 1977; Janjic 1979): Mass field computed from a forward time difference, while the velocity field comes from a backward time difference.

In a shallow water equation sense:

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x}, \frac{\partial h}{\partial t} = -H\frac{\partial u}{\partial x}$$

$$h^{\tau+1} = h^{\tau} - \Delta t\, H\frac{\partial u^{\tau}}{\partial x}$$

$$u^{\tau+1} = u^{\tau} - \Delta t\, g\frac{\partial h^{\tau+1}}{\partial x}$$

Mass field forcing to update wind from τ+1 time

Model Integration

Vertically propagating sound waves

In case of linearized equations, equivalent to implicit solution of:

$$\frac{\partial^2 p'}{\partial t^2} \rightarrow \frac{p'^{\tau+1} - 2p'^{\tau} + p'^{\tau-1}}{\Delta t^2} = \frac{c_p}{c_v} R T_0 \frac{\partial^2 p'^{\tau+1}}{\partial {z_0}^2}$$

Where p' is the perturbation pressure from a hydrostatic basic state, and τ represents the time level. (Janjic et al., 2001; Janjic, 2003). Embedded into full equations, not actually used in the model in this form.

Model Integration

- Sequence of events within a solve_nmm loop (ignoring physics):

(0.6%) ▪ PDTE – integrates mass flux divergence, computes vertical velocity and updates hydrostatic pressure.

(26.4%) ▪ ADVE – horizontal and vertical advection of T, u, v, Coriolis and curvature terms applied.

(1.2%) ▪ VTOA – updates nonhydrostatic pressure, applies $\omega\alpha$ term to thermodynamic equation

(8.6%) ▪ VADZ/HADZ – vertical/horizontal advection of height. w=dz/dt updated.

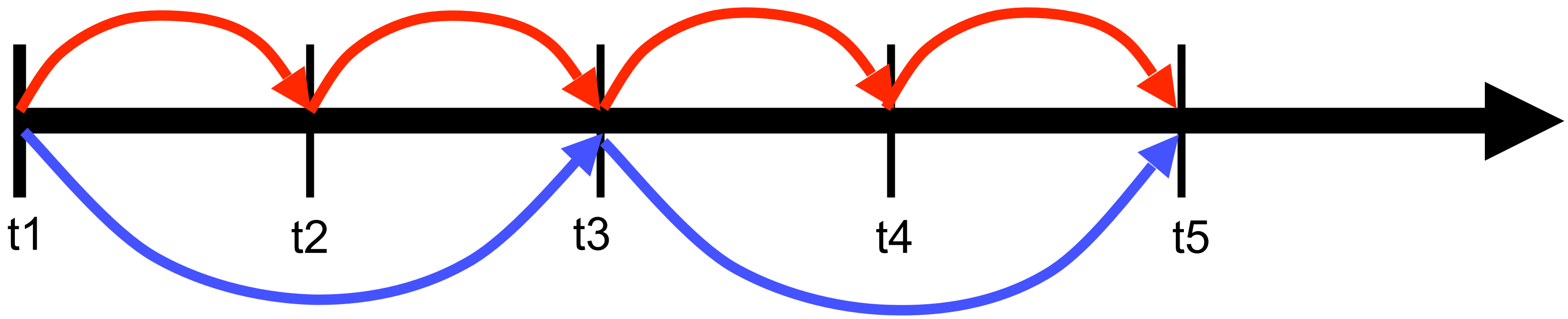
(10.6%) ▪ EPS – vertical and horizontal advection of dz/dt, vertical sound wave treatment.

(relative % of dynamics time spent in these subroutines)

Model Integration

- Sequence of events within a solve_nmm loop (cont):
  - (19.5%) VAD2/HAD2 (every other step) – vertical/horizontal advection of q, CWM, TKE
  - (11.8%) HDIFF – horizontal diffusion
  - (1.2%) BOCOH – boundary update at mass points
  - (17.5%) PFDHT – calculates PGF, updates winds due to PGF, computes divergence.
  - (2.3%) DDAMP – divergence damping
  - (0.3%) BOCOV – boundary update at wind points

All dynamical processes every fundamental time step, except….



…passive substance advection, every other time step

Model time step "dt" specified in model namelist.input is for the fundamental time step.

Generally about 2.25X the horizontal grid spacing (km), or 350X the namelist.input "dy" value (degrees lat).

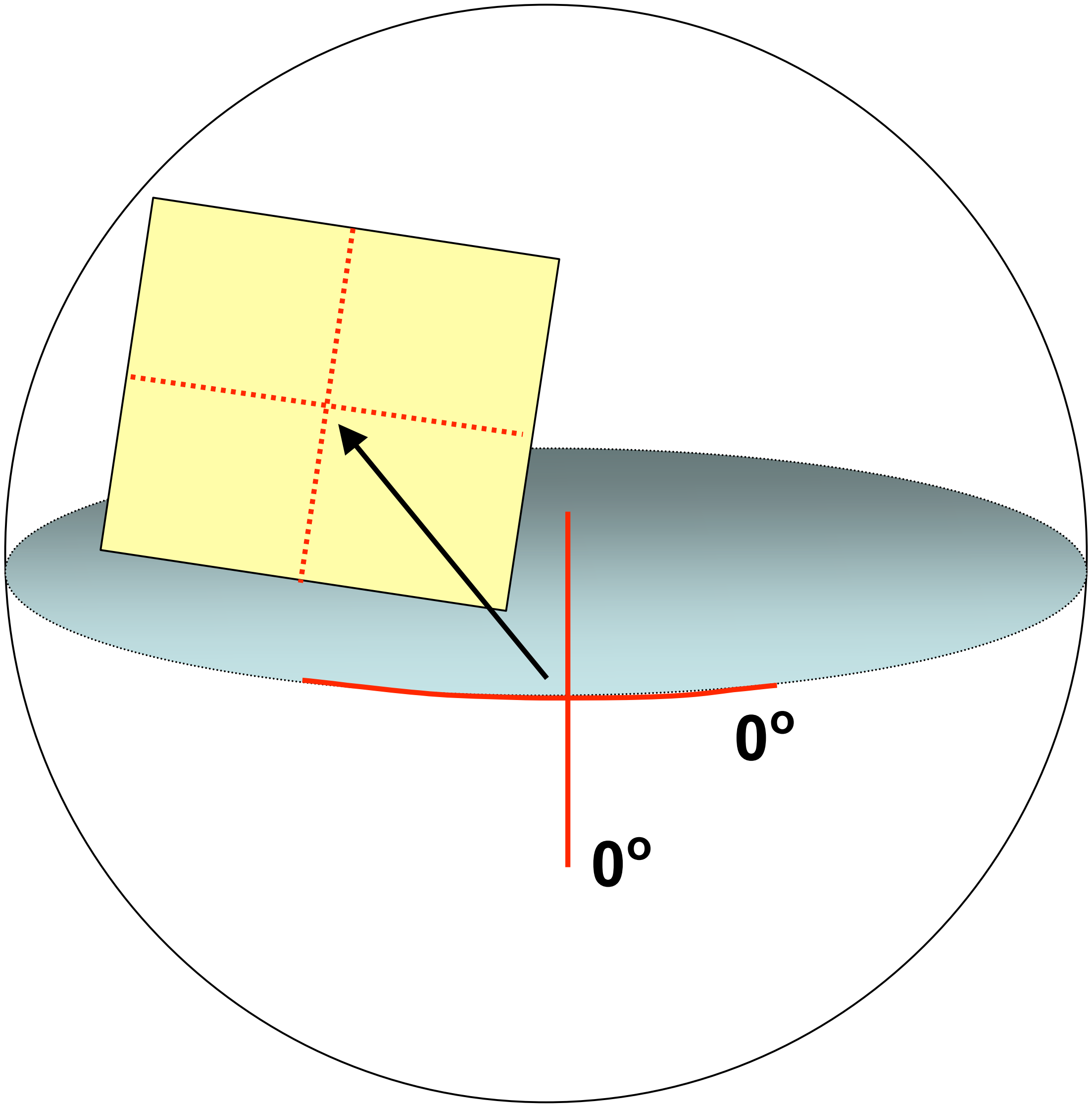
Now we'll take a look at two items specific to the WRF-NMM horizontal grid:

- Rotated latitude-longitude map projection (only projection used with the WRF-NMM)
- The Arakawa E-grid stagger

Rotated Latitude-Longitude

- Rotates the earth's latitude/longitude grid such that the intersection of the equator and prime meridian is at the center of the model domain.

- The rotation minimizes the convergence of meridians over the domain, and maintains a more uniform earth-relative grid spacing than exists for a regular lat-lon grid.



For a domain spanning 10N to 70N:

$$\Delta x \propto \cos(lat)$$

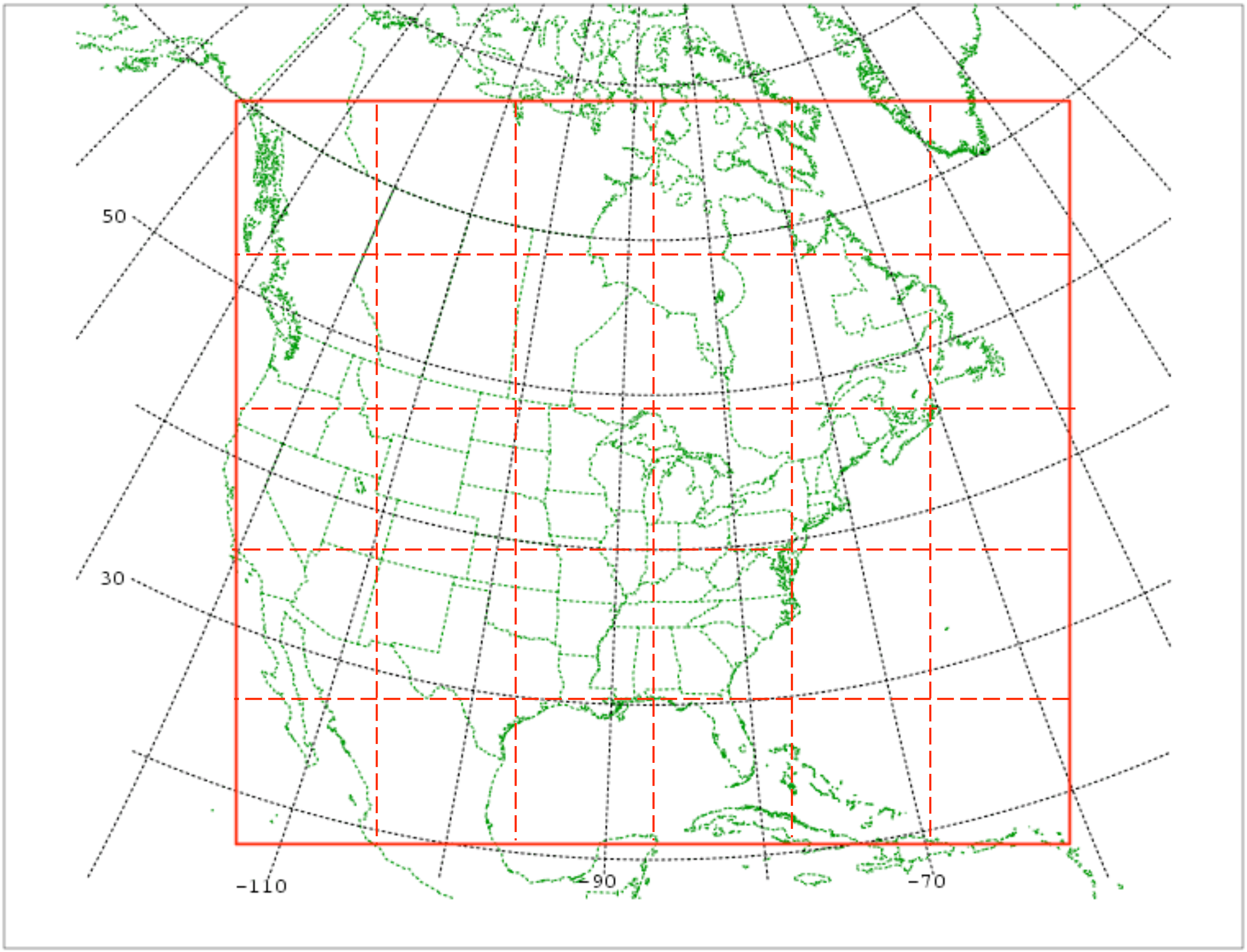
Regular lat-lon grid

$$\cos(70^0)/\cos(10^0) = 0.347$$

Rotated lat-lon grid

$$\cos(30^0)/\cos(0^0) = 0.866$$

Sample rotated lat-lon domain (red), with earth lat-lon lines (black)



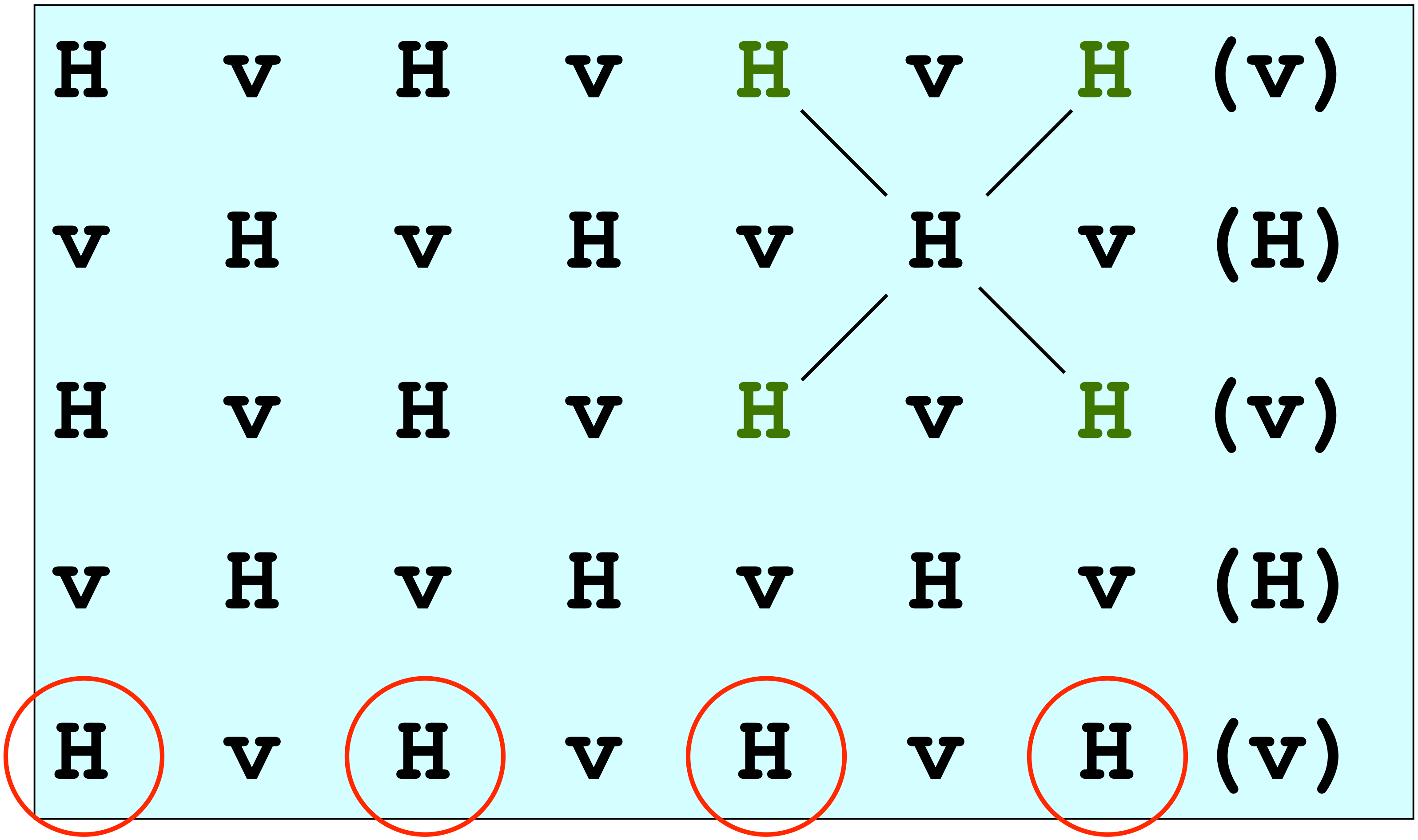
The E-grid Stagger

H	v	H	v	H	v	H	(v)
v	H	v	H	v	H	v	(H)
H	v	H	v	H	v	H	(v)
v	H	v	H	v	H	v	(H)
H	v	H	v	H	v	H	(v)

H=mass point, v=wind point

red=(1,1) ; blue=(1,2)

The E-grid Stagger



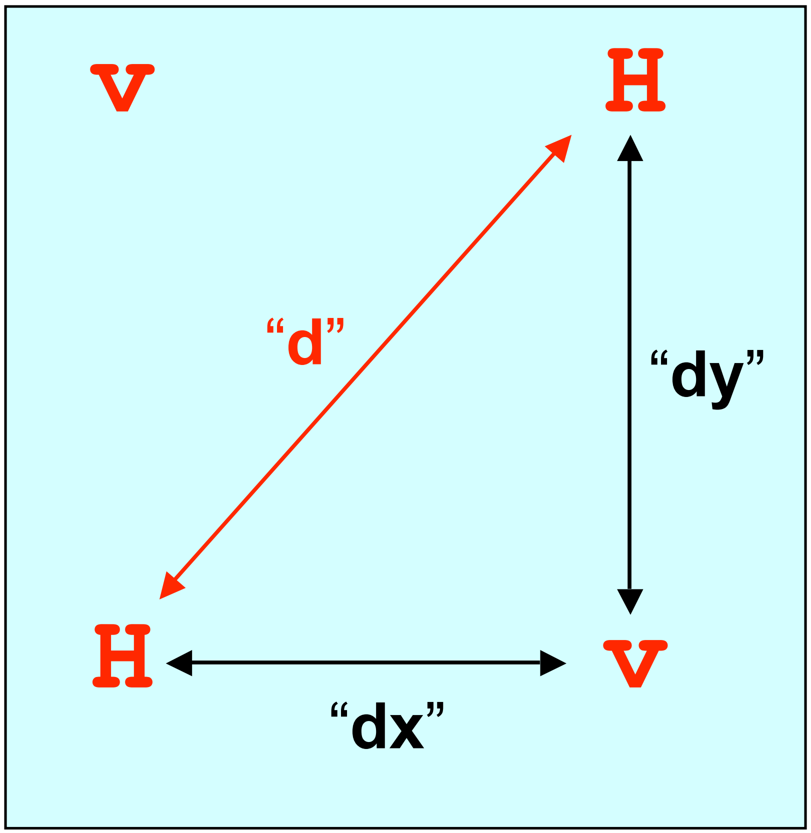
XDIM=4 (# of mass points on odd numbered row)
YDIM=5 (number of rows)

The E-grid Stagger - properties

- Due to the indexing convention, the X-dimension is half as large as would be expected from a C-grid domain (typically XDIM < YDIM for the E-grid).
- “Think diagonally” –the shortest distance between adjacent like points is along the diagonals of the grid.

- E-grid energy and enstrophy conserving momentum advection scheme (Janjic, 1984, MWR) more effectively controls the spurious nonlinear energy cascade (accumulation of small scale computational noise due to nonlinearity) than schemes on the C grid – an argument in favor of the E grid.

The E-grid Stagger



- Conventional grid spacing is the diagonal distance "**d**".

- Grid spacings in the WPS and WRF namelists are the "**dx**" and "**dy**" values, specified in fractions of a degree *for the WRF-NMM*.

- "WRF domain wizard" takes input grid spacing "**d**" in km and computes the angular distances "**dx**" and "**dy**" for the namelist.

Spatial Discretization

- Basic discretization principle is conservation of important properties of the continuous system.
  - "Mimetic" approach

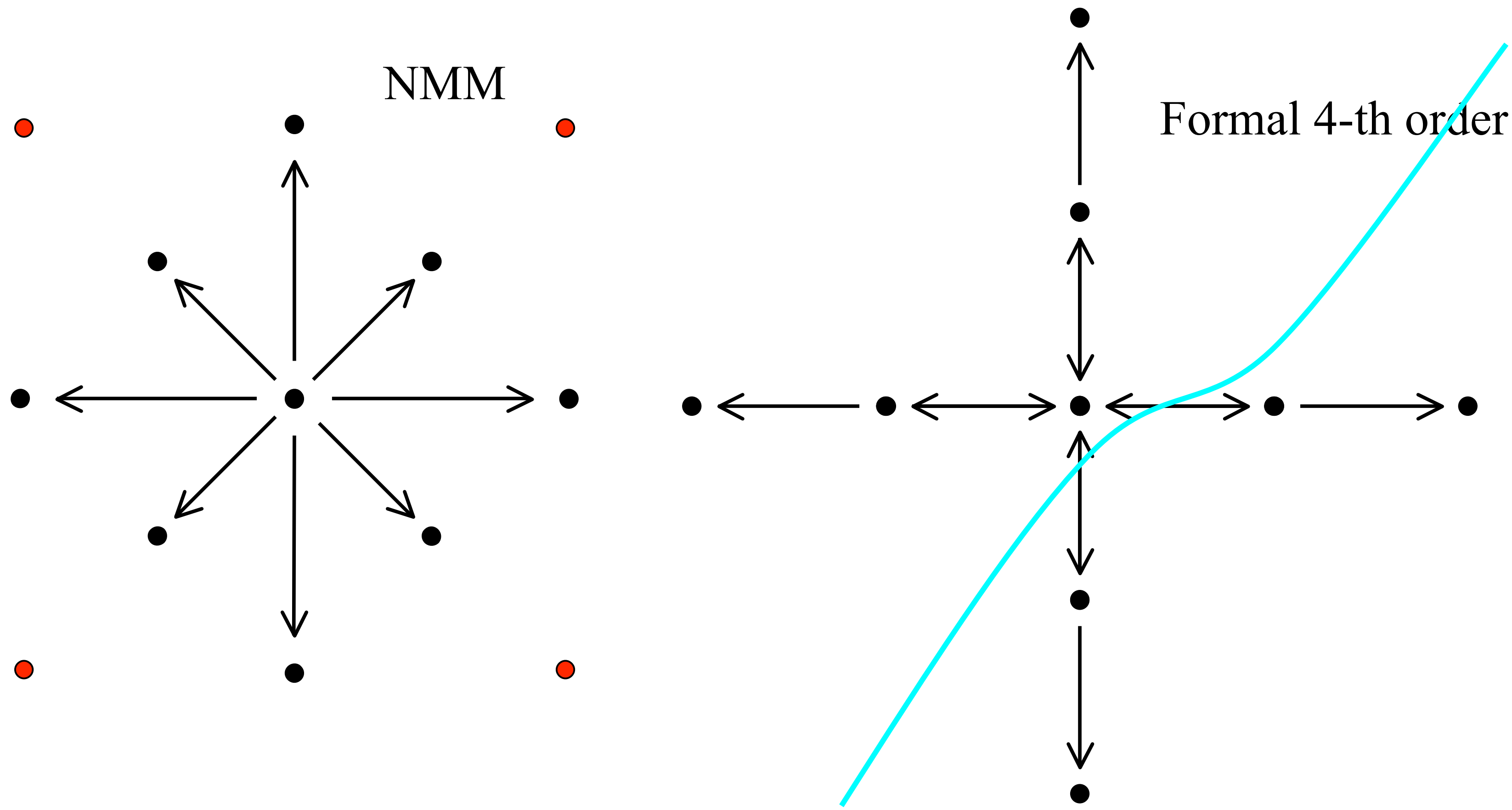
    http://www.math.unm.edu/~stanly/mimetic/mimetic.html

  - Major novelty in applied mathematics, …
  - … but well established in atmospheric modeling (Arakawa, 1966, 1972, 1977 …; Sadourny, 1975 …; Janjic, 1977, 1984 …; Tripoli, 1992, …)

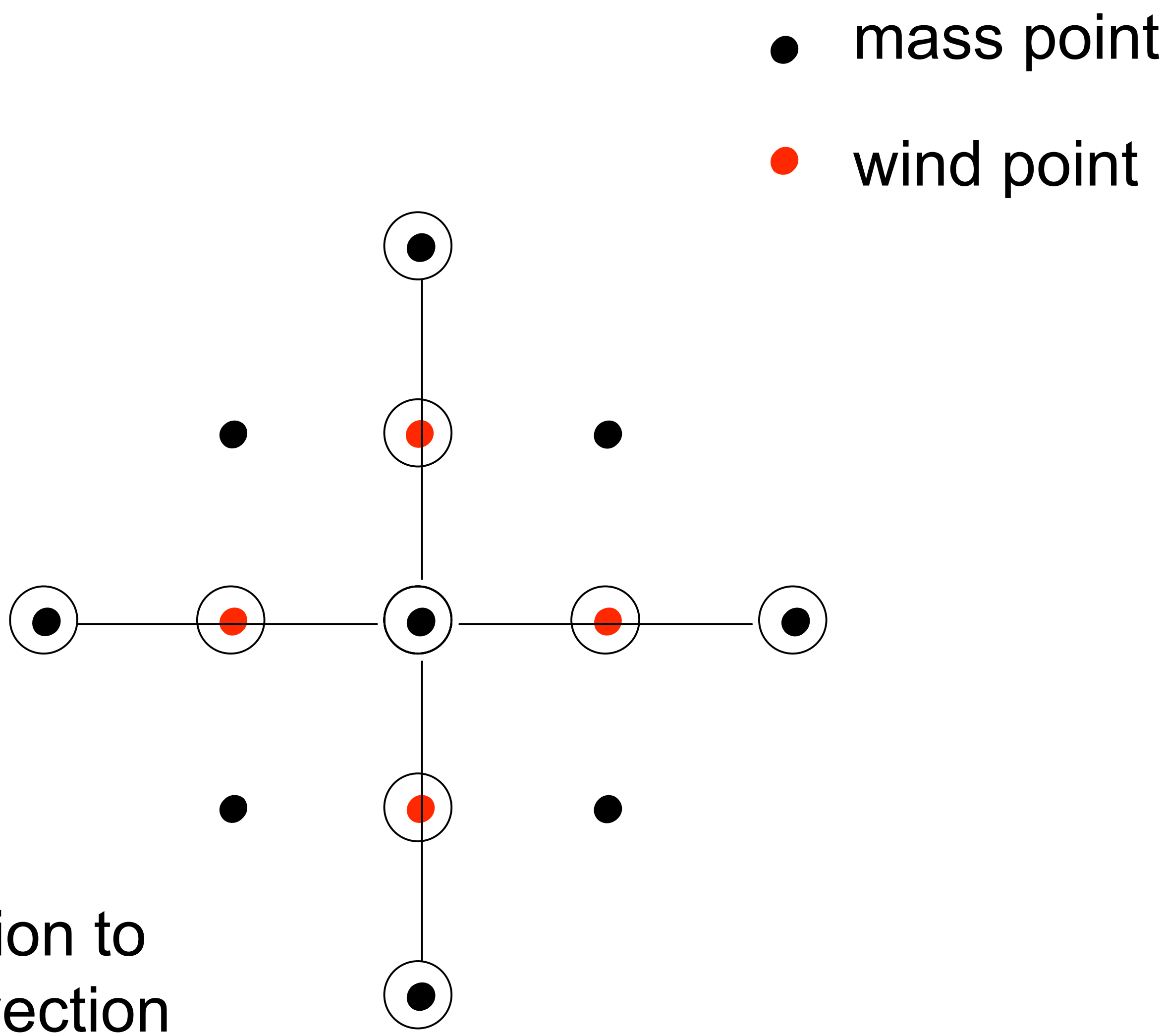
Spatial Discretization

General Philosophy

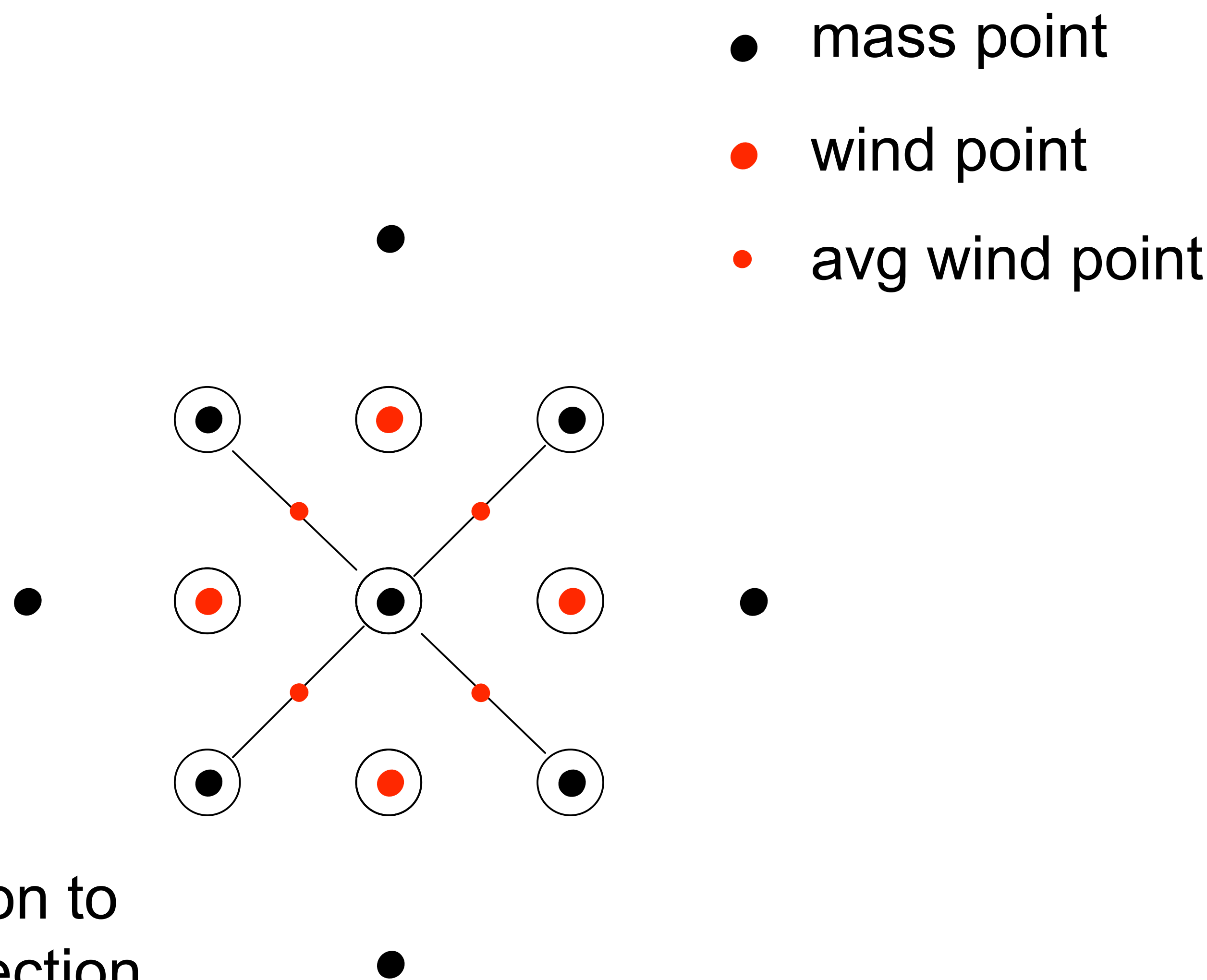
- Conserve energy and enstrophy in order to control nonlinear energy cascade; eliminate the need for numerical filtering to the extent possible.
- Conserve a number of first order and quadratic quantities (mass, momentum, energy, …).
- Use consistent order of accuracy for advection and divergence operators and the omega-alpha term; consistent transformations between KE and PE.
- Preserve properties of differential operators.



Advection and divergence operators – each point talks to all eight neighboring points (isotropic)



1/3 of contribution to
divergence/advection
comes from these N/S
and E/W fluxes.



2/3 of contribution to divergence/advection comes from these diagonal fluxes.

NMM Vertical Coordinate

Pressure-sigma hybrid (Arakawa and Lamb, 1977)

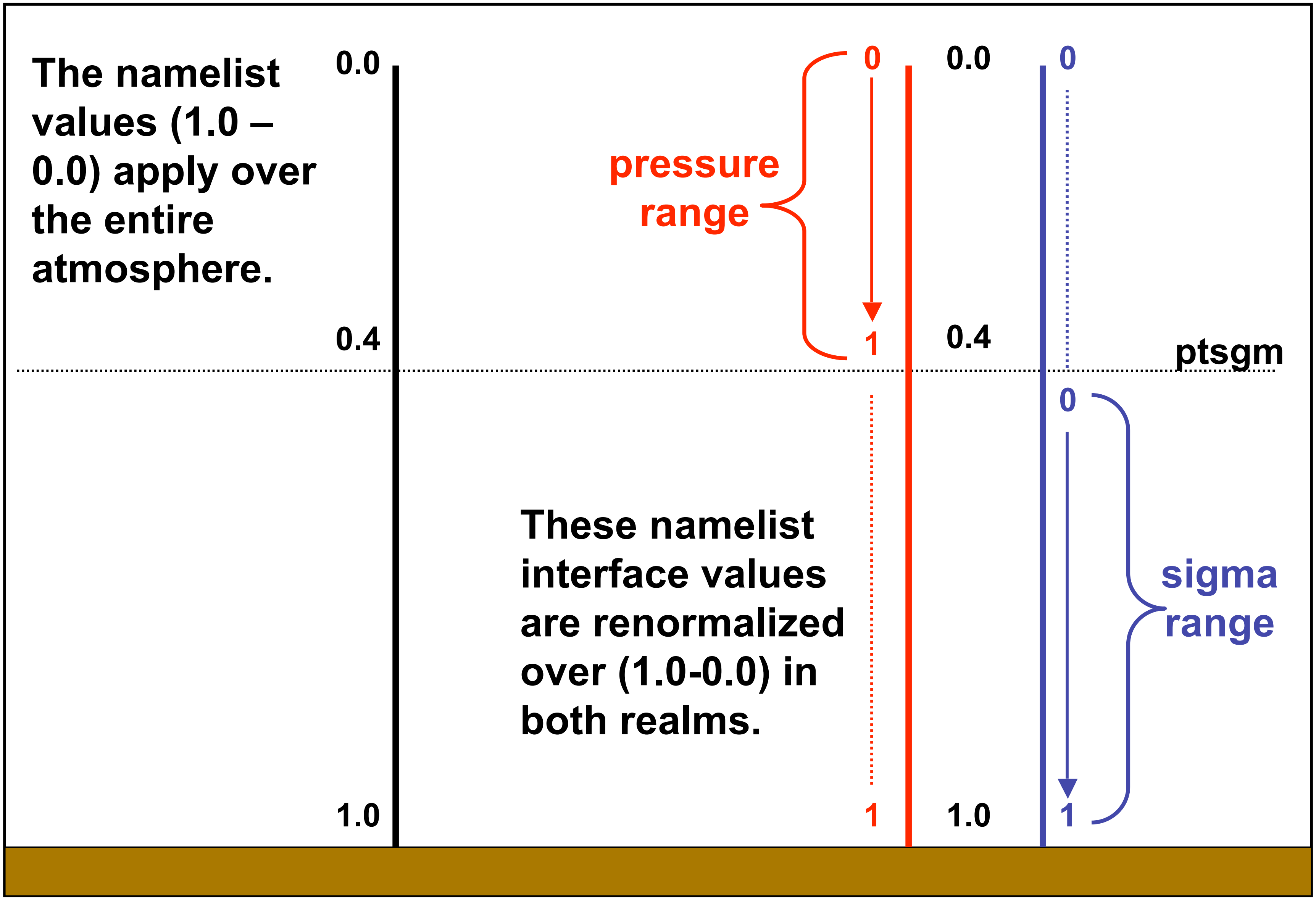
Has the desirable properties of a terrain-following, pressure coordinate:

- Exact mass (etc.) conservation
- Nondivergent flow on pressure surfaces
- No problems with weak static stability
- No discontinuities or internal boundary conditions

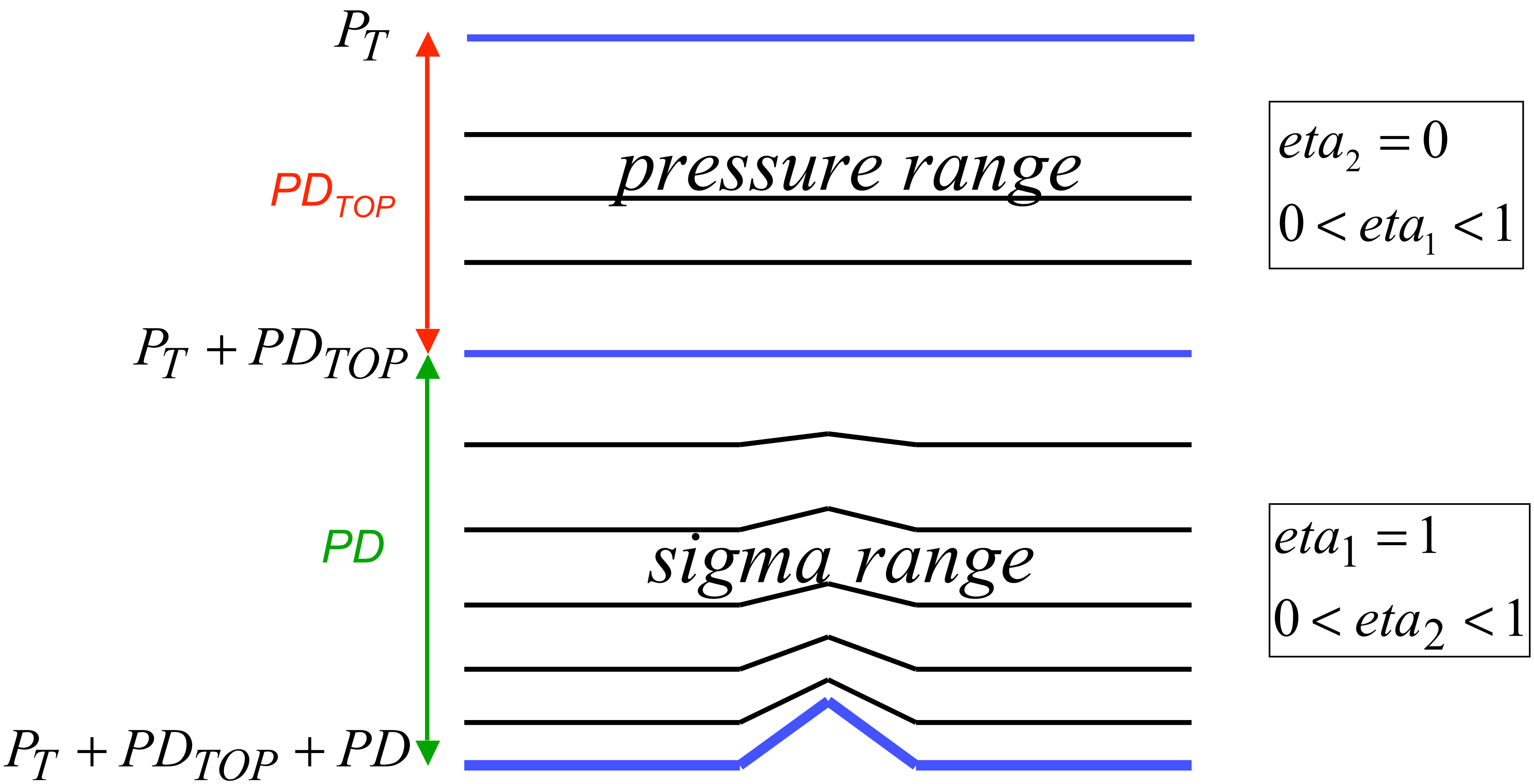
And an additional benefit from the hybrid:

- Flat coordinate surfaces at high altitudes where sigma problems worst (e.g., Simmons and Burridge, 1981)

Pressure-Sigma Hybrid Vertical Coordinate

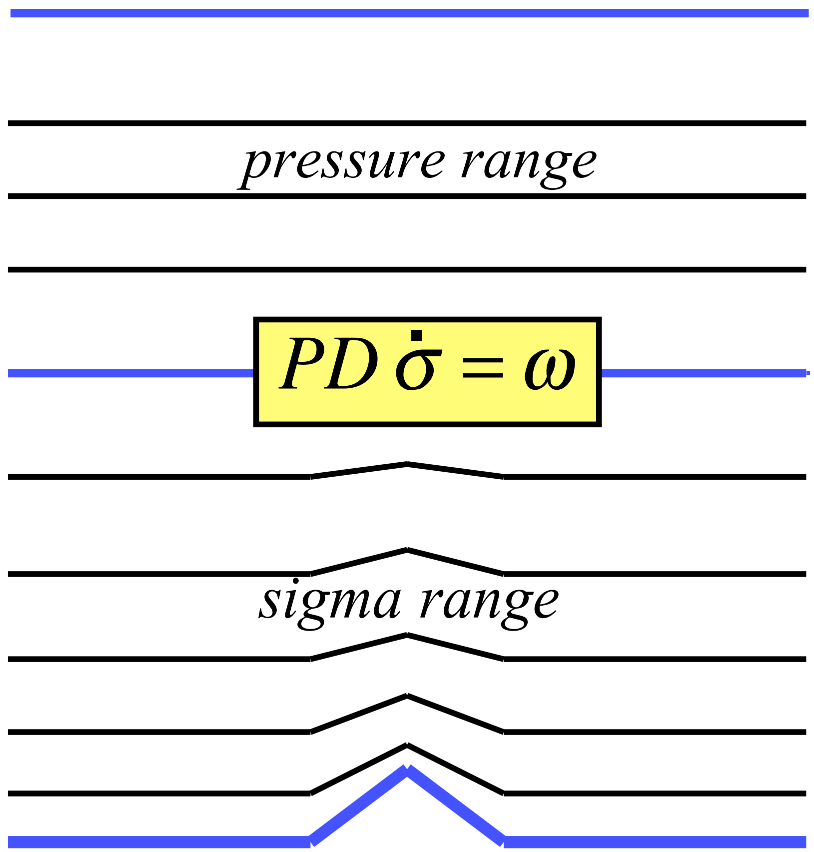


Pressure-Sigma Hybrid Vertical Coordinate



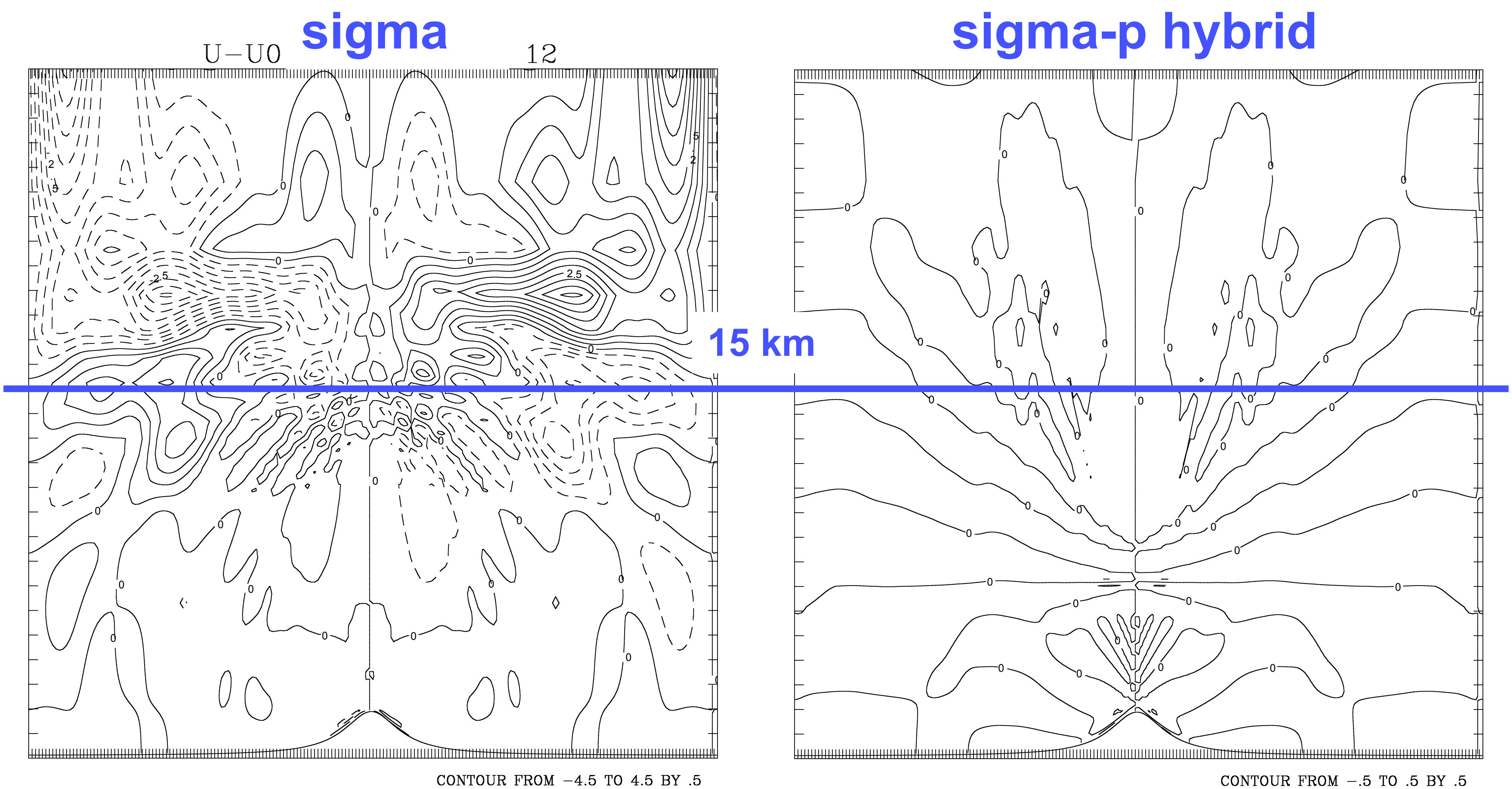
$$p = eta_1 * PD_{TOP} + eta_2 * PD + P_T$$

Equations in Hybrid Coordinate



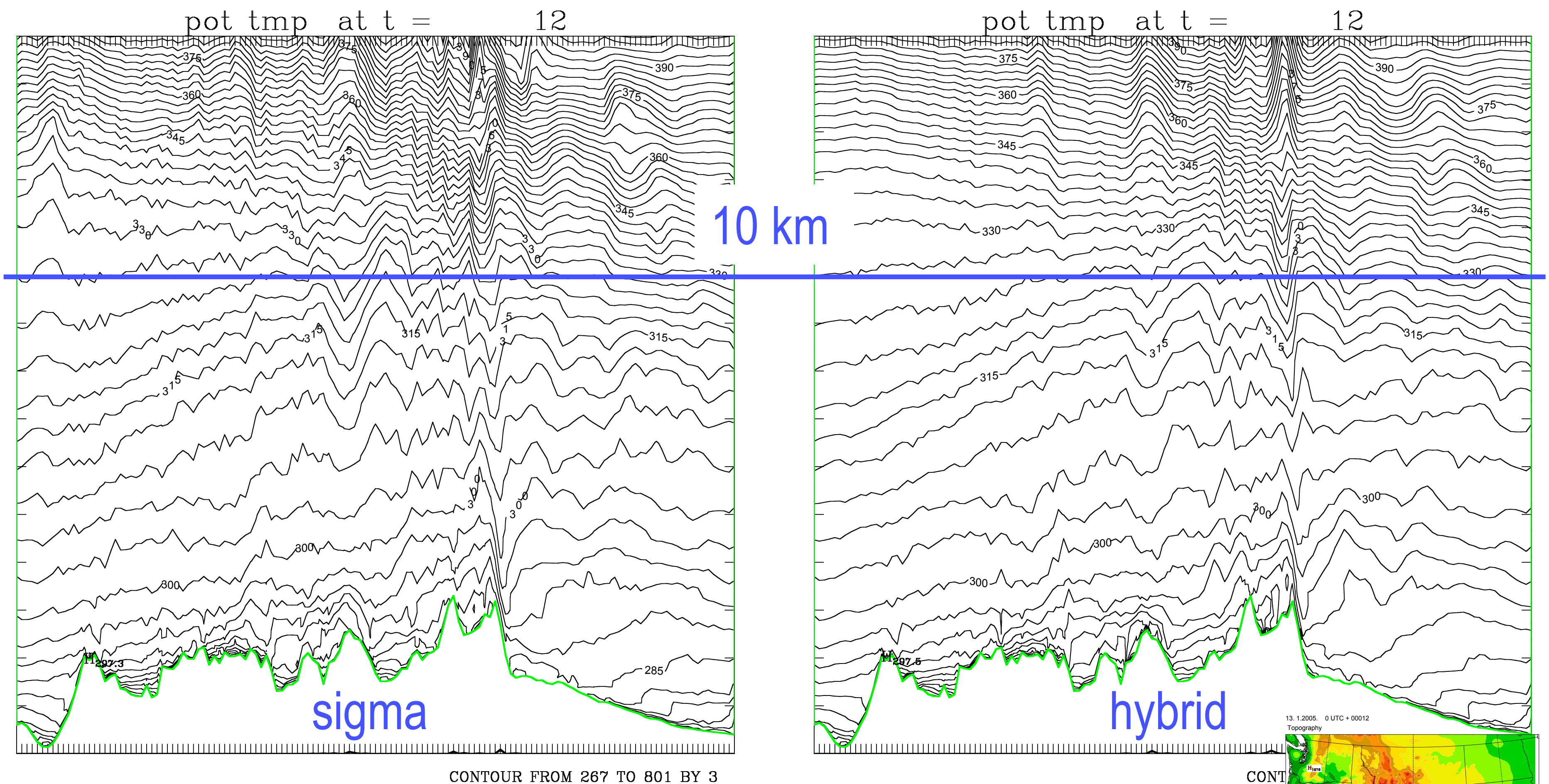
$$\nabla_p \bullet (\mathbf{v}) + \frac{\partial \omega}{\partial p} = 0$$

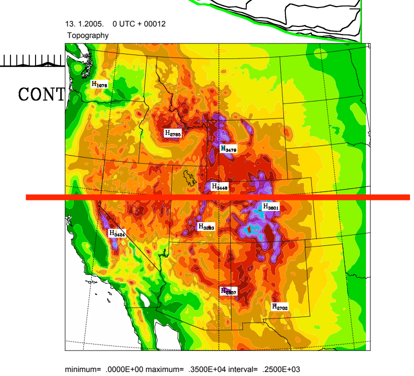
$$\frac{\partial PD}{\partial t} + \nabla_\sigma \cdot (PD\ \mathbf{v}) + \frac{\partial (PD\dot{\sigma})}{\partial \sigma} = 0$$



Wind component developing due to the spurious pressure gradient force in an idealized integration. The hybrid coordinate boundary between the pressure and sigma domains is at at about 400 hPa.

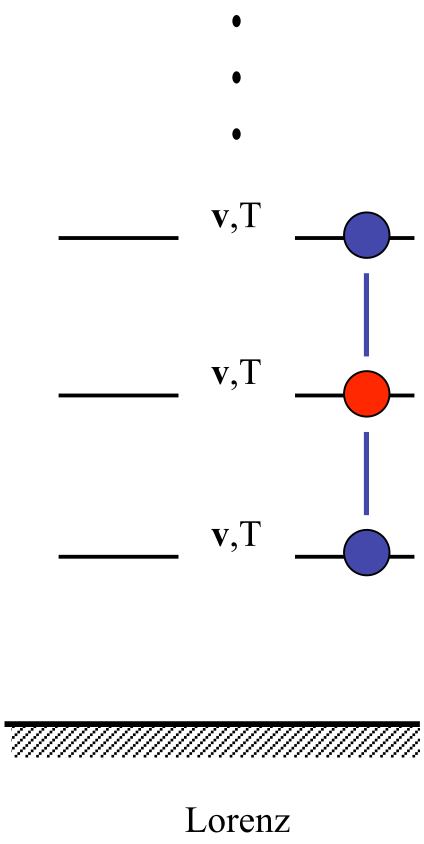
Example of nonphysical small scale energy source





12 h potential temperature forecasts (CINT=3 C) from 00Z January 13, 2005.

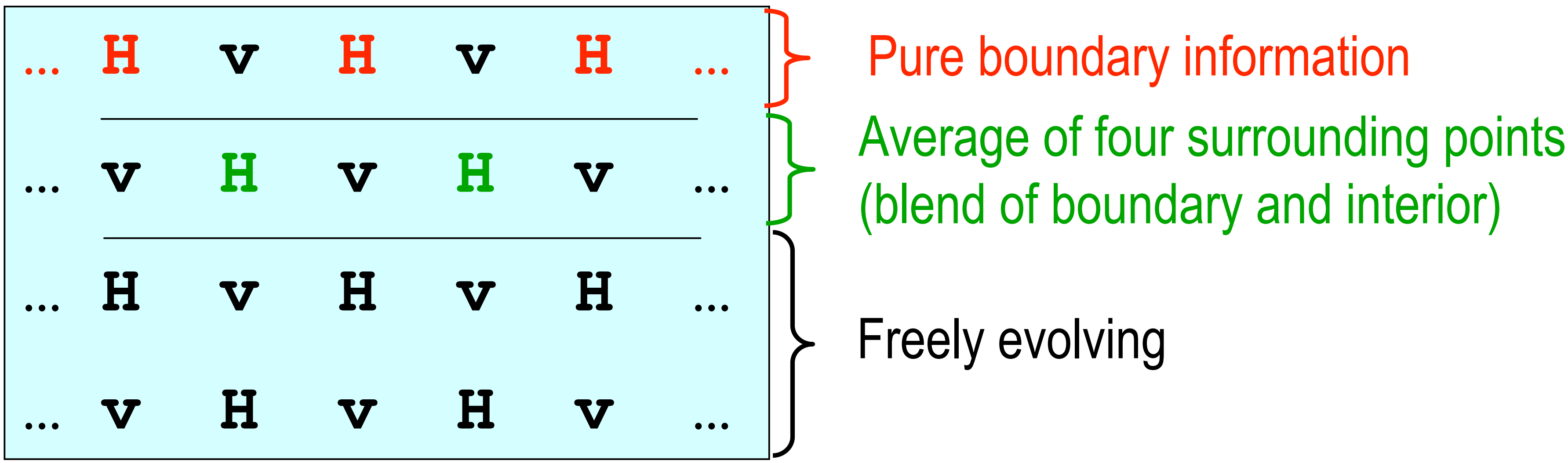
Vertical discretization



Vertical advection combines the advective fluxes computed above and below the layer of interest.

Boundary Conditions

- Lateral boundary information prescribed only on outermost row:



- Upstream advection in three rows next to the boundary
  - No computational outflow boundary condition for advection

- Enhanced divergence damping close to the boundaries.

Dissipative Processes – lateral diffusion

A 2^{nd} order, nonlinear Smagorinsky-type horizontal diffusion is utilized:

- Diffusion strength a function of the local TKE and 3D wind field deformation, gradients of the field being diffused, and a code-specified constant (COAC*).
- Lateral diffusion is zeroed for sloping model surfaces (> ~ 54 m / 12 km grid point), although this detail may be changed in a future model release.

* COAC has a default value of 1.6 and is specified in ./dyn_nmm/module_initialize_real.F. Larger values generate more diffusive smoothing.

Dissipative Processes - divergence damping

- Horizontal divergence damping with enhanced damping of the external mode.

- Internal mode damping (on each vertical layer)

$$v_j = v_j + \frac{(\nabla \cdot dp_{j+1}\mathbf{v}_{j+1} - \nabla \cdot dp_{j-1}\mathbf{v}_{j-1})}{(dp_{j+1} + dp_{j-1})} \cdot DDMPV$$

- External mode damping (vertically integrated)

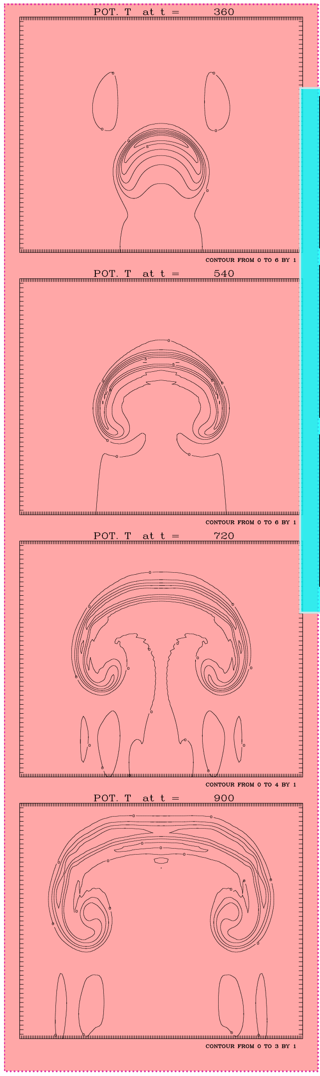
$$v_j = v_j + \frac{(\int \nabla \cdot dp_{j+1}\mathbf{v}_{j+1} - \int \nabla \cdot dp_{j-1}\mathbf{v}_{j-1})}{(\int dp_{j+1} + \int dp_{j-1})} \cdot DDMPV$$

$$DDMPV \approx \sqrt{2} \cdot dt \cdot CODAMP$$

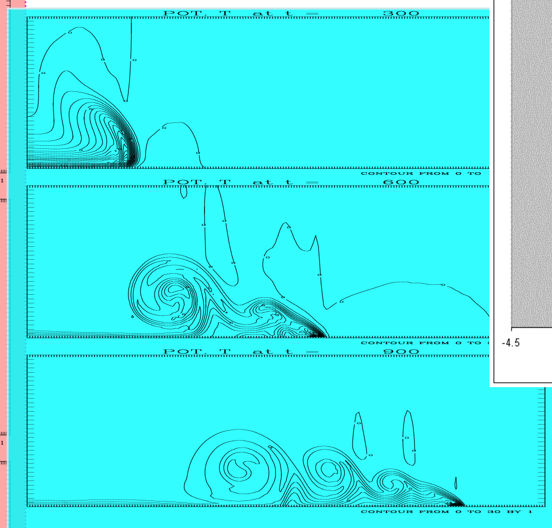
CODAMP is a code-specified parameter = 6.4 by default.

Dynamics formulation tested on various scales

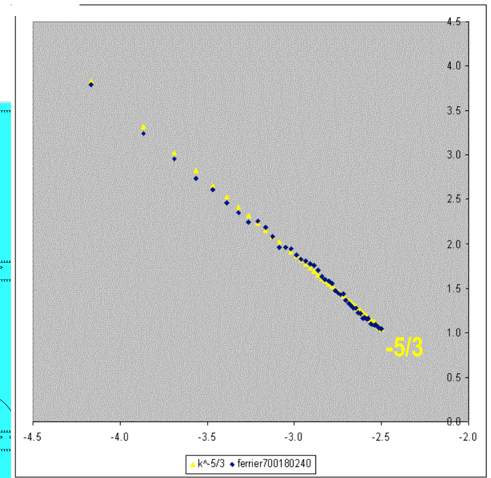
Warm bubble



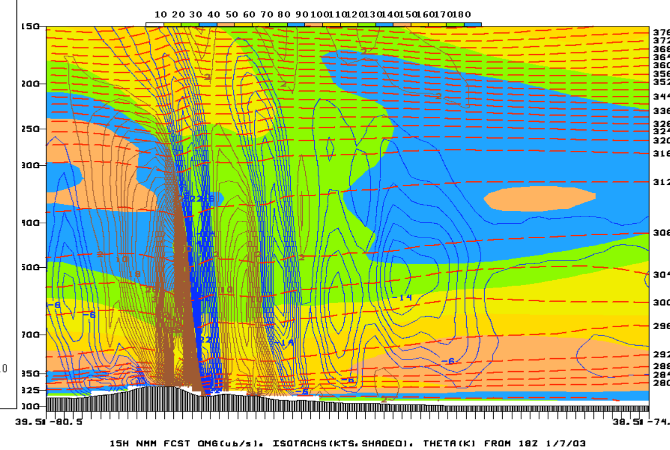
Cold bubble



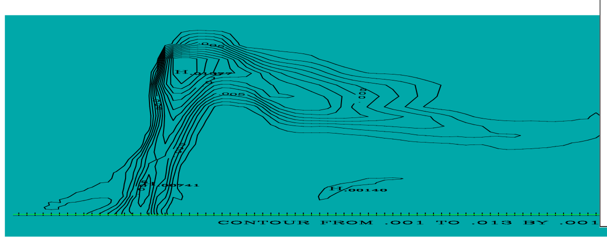
Decaying 3D turbulence



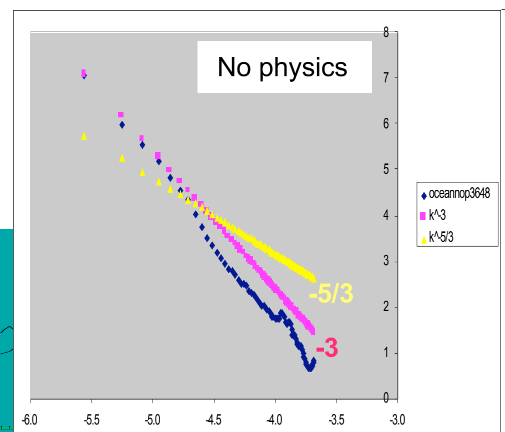
Mountain waves

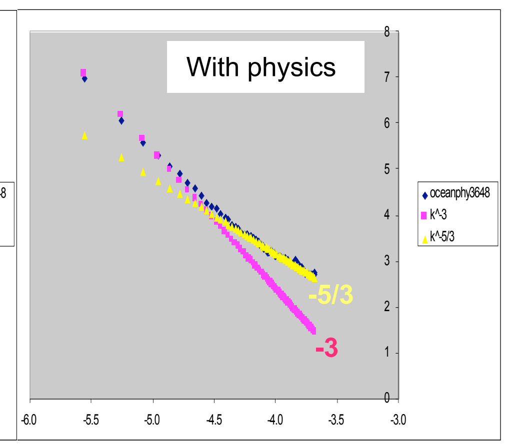


Convection



Atmospheric spectra





Summary

- Robust, reliable, fast
- NWP on near-cloud scales successful more frequently and with stronger signal than if only by chance
- Replaced the Eta as NAM at NCEP on June 20, 2006
- Near-cloud-scale runs (~4 km grid spacing) operational at NCEP for severe weather forecasting
- Operational as Hurricane WRF in 2007
- Operational and quasi-operational elsewhere.