The Advanced Research WRF (ARW) Dynamics Solver

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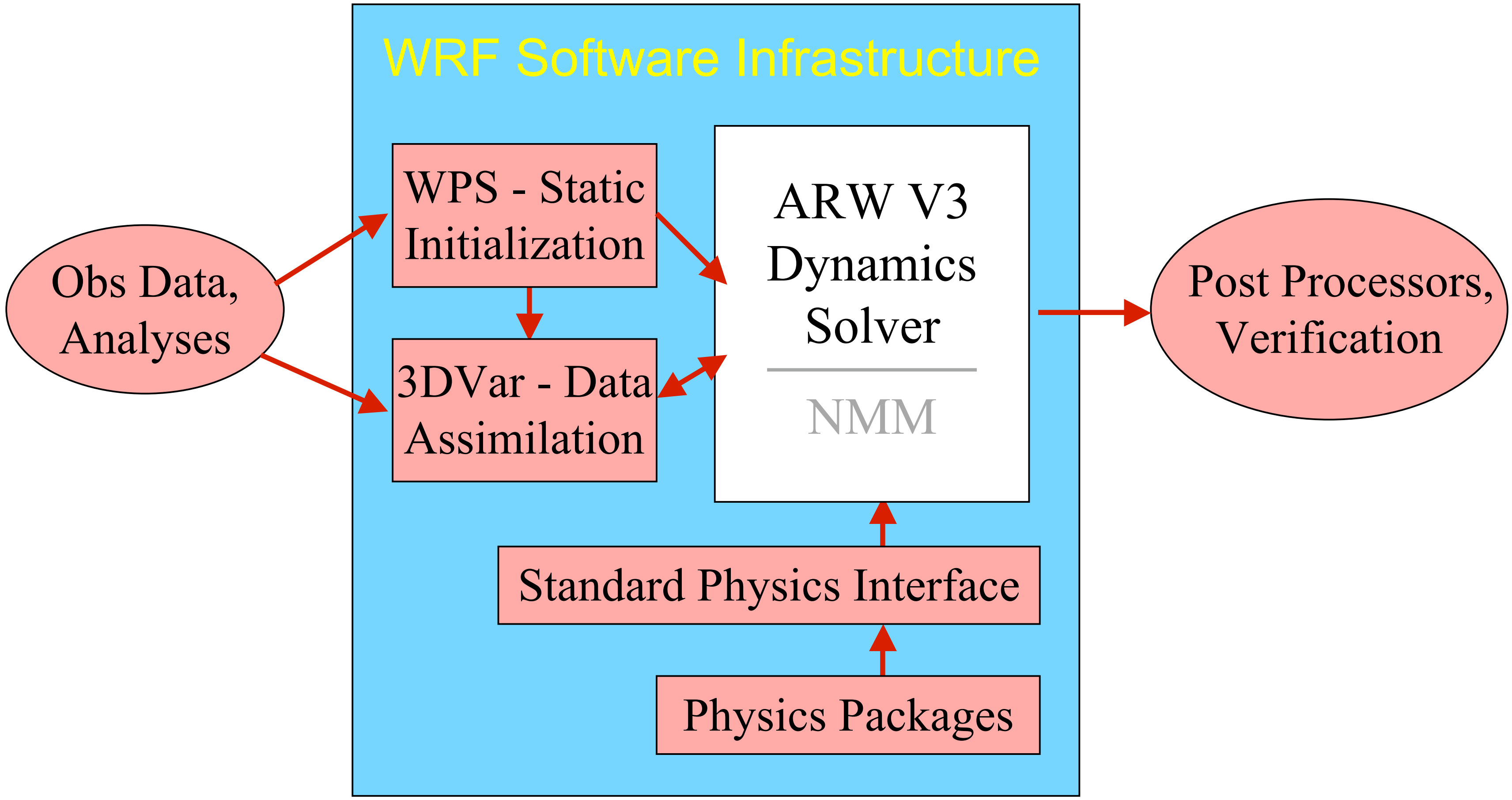

WRF ARW Tech Note

A Description of the Advanced Research WRF Version 3

http://www.mmm.ucar.edu/wrf/users/pub-doc.html

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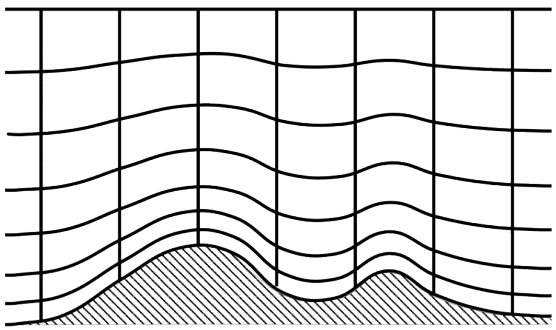
ARW Dynamical Solver

- Terrain representation
- Vertical coordinate
- Equations / variables
- Time integration scheme
- Grid staggering
- Advection scheme
- Time step parameters
- Filters
- Boundary conditions
- Nesting
- Map projections

ARW, Terrain Representation

Lower boundary condition for the geopotential $(\phi = gz)$ specifies the terrain elevation, and specifying the lowest coordinate surface to be the terrain results in a terrain-following coordinate.

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} + \omega\frac{\partial \phi}{\partial \eta} = gw$$



Vertical coordinate:

hydrostatic pressure π $\qquad \eta = \frac{(\pi - \pi_t)}{\mu}, \qquad \mu = \pi_s - \pi_t$

Flux-Form Equations in ARW

Hydrostatic pressure coordinate:

hydrostatic pressure π

$$\eta = \frac{(\pi - \pi_t)}{\mu}, \qquad \mu = \pi_s - \pi_t \qquad \mu(x)\Delta\eta = \Delta\pi = -g\rho\Delta z$$

Conserved state variables:

$$\mu, \quad U = \mu u, \quad V = \mu v, \quad W = \mu w, \quad \Theta = \mu\theta$$

Non-conserved state variable: $\phi = gz$

Flux-Form Equations in ARW

Inviscid, 2-D equations without rotation:

$$\frac{\partial U}{\partial t} + \mu\alpha\frac{\partial p}{\partial x} + \frac{\partial p}{\partial \eta}\frac{\partial \phi}{\partial x} = -\frac{\partial Uu}{\partial x} - \frac{\partial \Omega u}{\partial \eta}$$

$$\frac{\partial W}{\partial t} + g\left(\mu - \frac{\partial p}{\partial \eta}\right) = -\frac{\partial Uw}{\partial x} - \frac{\partial \Omega w}{\partial \eta}$$

$$\frac{\partial \Theta}{\partial t} + \frac{\partial U\theta}{\partial x} + \frac{\partial \Omega\theta}{\partial \eta} = \mu Q$$

$$\frac{\partial \mu}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial \Omega}{\partial \eta} = 0$$

$$\frac{d\phi}{dt} = gw$$

Diagnostic relations:

$$\frac{\partial \phi}{\partial \eta} = -\mu\alpha, \qquad p = \left(\frac{R\theta}{p_0\alpha}\right)^{\gamma}, \qquad \Omega = \mu\dot{\eta}$$

Moist Equations in ARW

Moist Equations:

$$\frac{\partial U}{\partial t} + \alpha\mu_d \frac{\partial p}{\partial x} + \frac{\alpha}{\alpha_d}\frac{\partial p}{\partial \eta}\frac{\partial \phi}{\partial x} = -\frac{\partial Uu}{\partial x} - \frac{\partial \Omega u}{\partial \eta}$$

$$\frac{\partial W}{\partial t} + g\left(\mu_d - \frac{\alpha}{\alpha_d}\frac{\partial p}{\partial \eta}\right) = -\frac{\partial Uw}{\partial x} - \frac{\partial \Omega w}{\partial \eta}$$

$$\frac{\partial \mu_d}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial \Omega}{\partial \eta} = 0$$

$$\frac{\partial (\mu_d q_{v,l})}{\partial t} + \frac{\partial (U q_{v,l})}{\partial x} + \frac{\partial (\Omega q_{v,l})}{\partial \eta} = \mu Q_{v,l}$$

Diagnostic relations:

$$\frac{\partial \phi}{\partial \eta} = -\alpha_d \mu_d, \qquad p = \left(\frac{R\Theta}{p_o \mu_d \alpha_v}\right)^{\gamma}$$

Time Integration in ARW

3rd Order Runge-Kutta time integration

advance $\phi^t \rightarrow \phi^{t+\Delta t}$

$$\phi^* = \phi^t + \frac{\Delta t}{3} R\left(\phi^t\right)$$

$$\phi^{**} = \phi^t + \frac{\Delta t}{2} R\left(\phi^*\right)$$

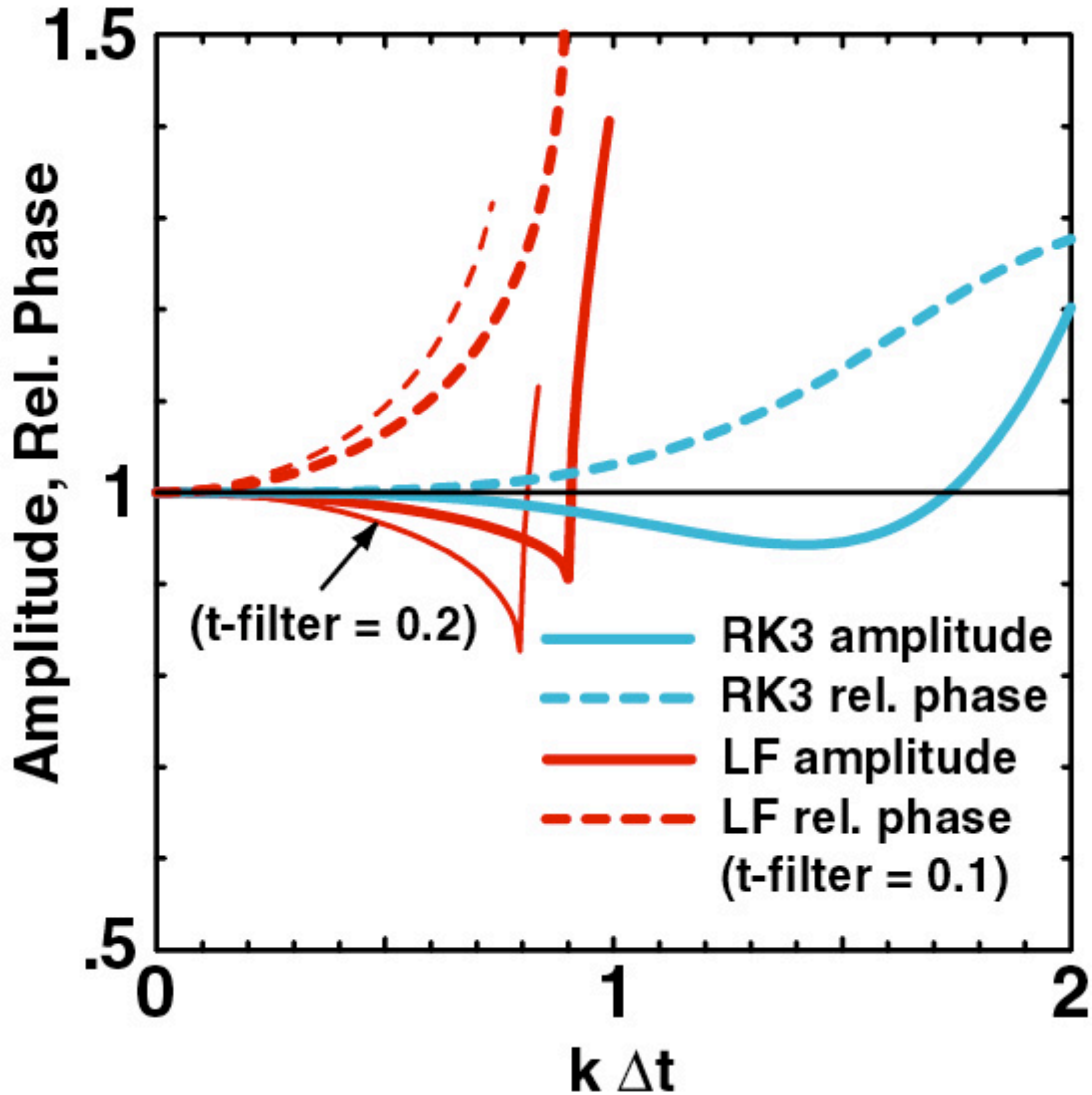
$$\phi^{t+\Delta t} = \phi^t + \Delta t\, R\left(\phi^{**}\right)$$

Amplification factor $\phi_t = i\,k\phi;\quad \phi^{n+1} = A\,\phi^n;\quad |A| = 1 - \frac{(k\Delta t)^4}{24}$

Phase and amplitude errors for LF, RK3

Oscillation equation analysis

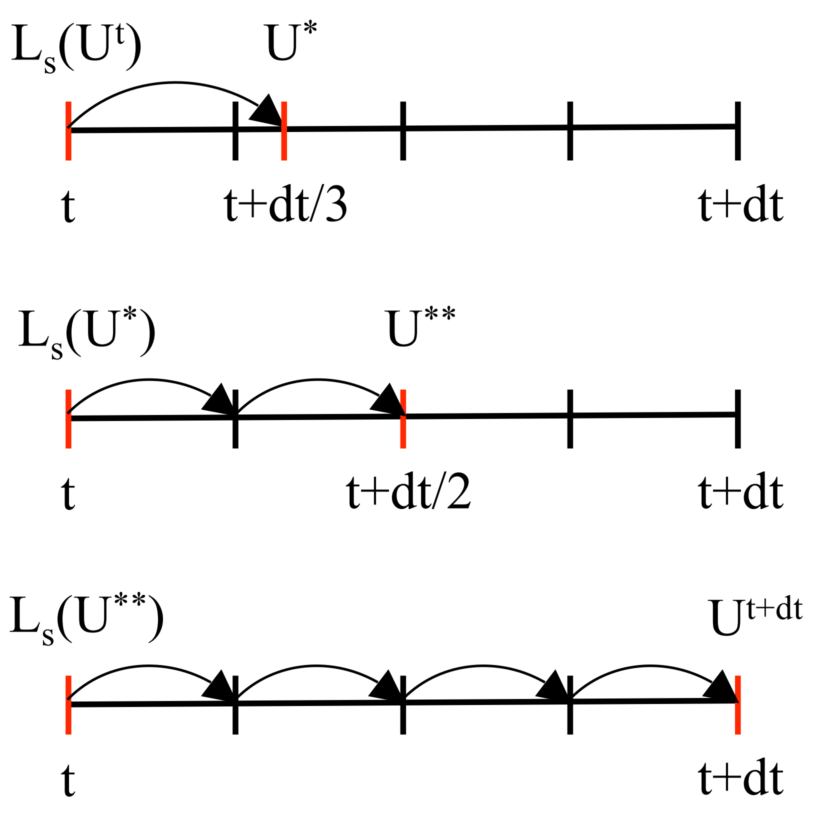
$$\phi_t = ik\phi$$



Time-Split Runge-Kutta Integration Scheme

$$U_t = L_{fast}(U) + L_{slow}(U)$$

3rd order Runge-Kutta, 3 steps



- RK3 is 3rd order accurate for linear eqns, 2nd order accurate for nonlinear eqns.
- Stable for centered and upwind advection schemes.
- Stable for Courant number $Udt/dx < 1.73$
- Three $L_{slow}(U)$ evaluations per timestep.

WRF ARW Model Integration Procedure

Begin time step

- Runge-Kutta loop (steps 1, 2, and 3)
    - (i) advection, p-grad, buoyancy using $\left(\phi^{t}, \phi^{*}, \phi^{**}\right)$
    - (ii) physics if step 1, save for steps 2 and 3
    - (iii) mixing, other non-RK dynamics, save…
    - (iv) assemble dynamics tendencies
    - Acoustic step loop
        - (i) advance U,V, then μ, Θ, then w, ϕ
        - (ii) time-average U,V, Ω
    - End acoustic loop
    - Advance scalars using time-averaged U,V, Ω
- End Runge-Kutta loop
- Adjustment physics (currently microphysics)

End time step

Flux-Form Perturbation Equations

Introduce the perturbation variables:

$$\phi = \overline{\phi}(z) + \phi', \mu = \overline{\mu} \quad + \mu';$$
$$p = \overline{p}(z) + p', \alpha = \overline{\alpha}(z) + \alpha'$$

Note – $\quad \phi = \overline{\phi}(z) = \overline{\phi}(x, y, \eta),$

likewise $\quad \overline{p}(x, y, \eta), \overline{\alpha}(x, y, \eta)$

Momentum and hydrostatic equations become:

$$\frac{\partial U}{\partial t} + \mu\alpha \frac{\partial p'}{\partial x} + \eta\mu\alpha' \frac{\partial \overline{\mu}}{\partial x} + \mu \frac{\partial \phi'}{\partial x} + \frac{\partial \phi'}{\partial x}\left(\frac{\partial p'}{\partial \eta} - \mu'\right) = -\frac{\partial Uu}{\partial x} - \frac{\partial \Omega u}{\partial \eta}$$

$$\frac{\partial W}{\partial t} + g\left(\mu' - \frac{\partial p'}{\partial \eta}\right) = -\frac{\partial Uw}{\partial x} - \frac{\partial \Omega w}{\partial \eta}$$

$$\frac{\partial \phi'}{\partial \eta} = -\overline{\mu}\alpha' - \overline{\alpha}\mu'$$

Flux-Form Perturbation Equations: Acoustic Step

Acoustic mode separation:

Recast Equations in terms of perturbation about time t

$$U' = U'^{t} + U'', \; V' = V'^{t} + V'', \; W' = W'^{t} + W'',$$

$$\Theta' = \Theta'^{t} + \Theta'', \quad \mu' = \mu'^{t} + \mu'', \; \phi' = \phi'^{t} + \phi'';$$

$$p' = p'^{t} + p'', \; \alpha' = \alpha'^{t} + \alpha''$$

Linearize ideal gas law about time t

$$p'' = \frac{c_s^2}{\alpha^t}\left(\frac{\Theta''}{\Theta^t} - \frac{\alpha''}{\alpha^t} - \frac{\mu''}{\mu^t}\right)$$

$$\alpha'' = \frac{1}{\mu^t}\left(\frac{\partial \phi''}{\partial \eta} + \alpha^t \mu''\right)$$

Vertical pressure gradient becomes

$$\frac{\partial p''}{\partial \eta} = \frac{\partial}{\partial \eta}\left(\frac{c_s^2}{\mu^t {\alpha^t}^2}\frac{\partial \phi''}{\partial \eta} + \frac{c_s^2}{\mu^t}\frac{\Theta''}{\Theta^t}\right)$$

Acoustic Integration in ARW

Forward-backward scheme, first advance the horizontal momentum

$$\delta_\tau U'' + \mu^t \alpha^t \frac{\partial p''}{\partial x} + \eta \mu^t \frac{\partial \bar{\mu}}{\partial x} \alpha'' + \mu^t \frac{\partial \phi''}{\partial x} + \frac{\partial \phi^t}{\partial x} \left(\frac{\partial p''}{\partial \eta} - \mu'' \right) = R_u{}^t$$

Second, advance continuity equation,
diagnose omega,
and advance thermodynamic equation

$$\delta_\tau \mu'' + \left(\nabla \cdot \mathrm{V}'' \right)_\eta^{\tau+\Delta\tau} = R_\mu{}^t$$

$$\delta_\tau \Theta'' + \left(\nabla \cdot \mathrm{V}'' \theta^t \right)_\eta^{\tau+\Delta\tau} = R_\Theta{}^t$$

Finally, vertically-implicit integration of the acoustic and gravity wave terms

$$\delta_\tau W'' + g \overline{\left[\mu'' - \frac{\partial}{\partial \eta} \left(\frac{c_s^2}{\mu^t \alpha^{t2}} \frac{\partial \phi''}{\partial \eta} + \frac{c_s^2}{\alpha^t} \frac{\Theta''}{\Theta^t} \right) \right]}^\tau = R_w{}^t$$

$$\delta_\tau \phi'' + \frac{1}{\mu^t} \left[\left(\mathrm{V}'' \cdot \nabla \phi^t \right)_\eta^{\tau+\Delta\tau} - g \overline{W''}^\tau \right] = R_\varphi{}^t$$

Hydrostatic Option

Instead of solving vertically implicit equations for W'' and ϕ''

Integrate the hydrostatic equation to obtain $p''^{\tau+\Delta\tau}$:

$$\frac{\partial p''}{\partial \eta} = \left(\frac{\alpha_d}{\alpha}\right)^t \mu_d''$$

Solve the linearized ideal gas law for $\alpha_d''^{\tau+\Delta\tau}$:

$$p'' = \frac{c_s^2}{\alpha^t}\left(\frac{\Theta''}{\Theta^t} - \frac{\alpha_d''}{\alpha_d^t} - \frac{\mu_d''}{\mu_d^t}\right)$$

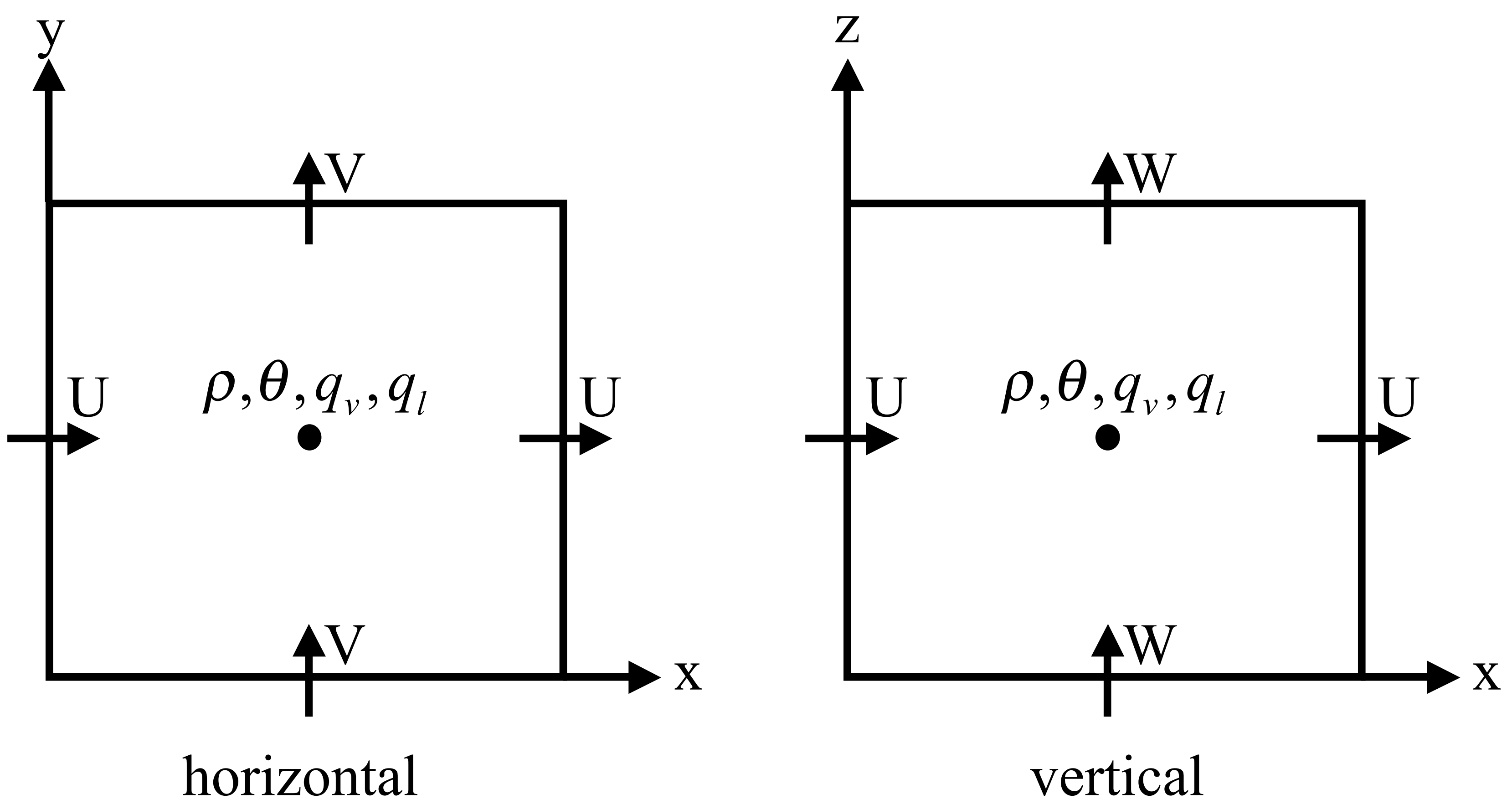
and recover $\phi''^{\tau+\Delta\tau}$ from:

$$\alpha_d'' = \frac{1}{\mu_d^t}\left(\frac{\partial \phi''}{\partial \eta} + \alpha_d^t \mu_d''\right)$$

W'' is no longer required during the integration.

ARW model, grid staggering

C-grid staggering



Advection in the ARW Model

2nd, 3rd, 4th, 5th and 6th order centered and upwind-biased schemes are available in the ARW model.

Example: 5th order scheme

$$\frac{\partial(U\phi)}{\partial x} = \frac{1}{\Delta x}\left(F_{i+\frac{1}{2}}(U\phi) - F_{i-\frac{1}{2}}(U\phi) \right)$$

where

$$F_{i-\frac{1}{2}}(U\phi) = U_{i-\frac{1}{2}}\left\{ \frac{37}{60}(\phi_i + \phi_{i-1}) - \frac{2}{15}(\phi_{i+1} + \phi_{i-2}) + \frac{1}{60}(\phi_{i+2} + \phi_{i-3}) \right\}$$
$$- sign(1,U)\frac{1}{60}\left\{ (\phi_{i+2} - \phi_{i-3}) - 5(\phi_{i+1} - \phi_{i-2}) + 10(\phi_i - \phi_{i-1}) \right\}$$

Advection in the ARW Model

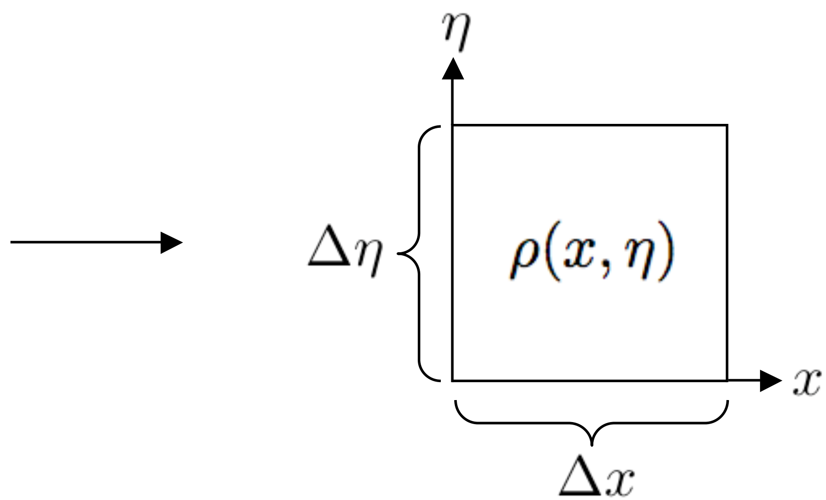
For constant U, the 5th order flux divergence tendency becomes

$$\Delta t \frac{\delta(U\phi)}{\Delta x}\bigg|_{5th} = \Delta t \frac{\delta(U\phi)}{\Delta x}\bigg|_{6th} - \underbrace{\left|\frac{U\Delta t}{\Delta x}\right| \frac{1}{60}\left(-\phi_{i-3} + 6\phi_{i-2} - 15\phi_{i-1} + 20\phi_i - 15\phi_{i+1} + 6\phi_{i+2} - \phi_{i+3}\right)}_{\frac{Cr}{60}\frac{\partial^6 \phi}{\partial x^6} + H.O.T}$$

The odd-ordered flux divergence schemes are equivalent to the next higher ordered (even) flux-divergence scheme plus a dissipation term of the higher even order with a coefficient proportional to the Courant number.

Mass Conservation in the ARW Model

control volume
(2D example)



Mass in a control volume is proportional to

$$(\Delta x \Delta \eta)(\mu)^t$$

since $\mu(x)\Delta\eta = \Delta\pi = -g\rho\Delta z$

Mass Conservation in the ARW Model

Mass in a control volume $(\Delta x \Delta \eta)(\mu)^t$
2D example

Mass conservation equation

$$\Delta t^{-1}(\Delta x \Delta \eta) \cdot \left[(\mu)^{t+\Delta t} - (\mu)^t)\right] = \left[(\mu u \Delta \eta)_{x-\Delta x/2,\eta} - (\mu u \Delta \eta)_{x+\Delta x/2,\eta})\right] + \left[(\mu \omega \Delta x)_{x,\eta-\Delta \eta/2} - (\mu \omega \Delta x)_{x,\eta+\Delta \eta/2})\right]$$

Change in mass over a time step

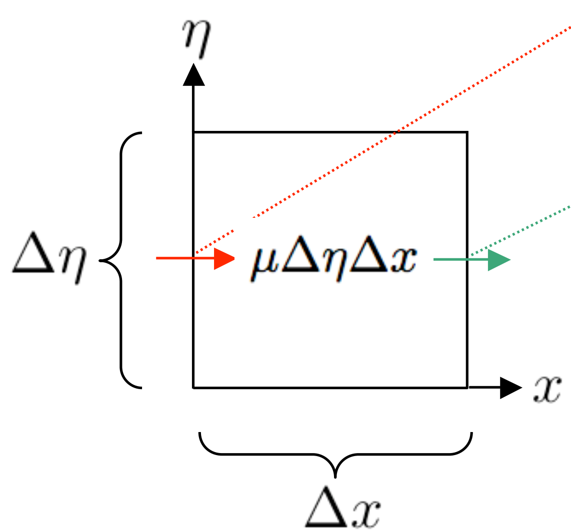
mass fluxes through control volume faces

Mass Conservation in the ARW Model

Mass in a control volume $(\Delta x \Delta \eta)(\mu)^t$

Mass conservation equation

$$\Delta t^{-1}(\Delta x \Delta \eta) \cdot [(\mu)^{t+\Delta t} - (\mu)^t)] = [(\mu u \Delta \eta)_{x-\Delta x/2,\eta} - (\mu u \Delta \eta)_{x+\Delta x/2,\eta})] + [(\mu \omega \Delta x)_{x,\eta-\Delta \eta/2} - (\mu \omega \Delta x)_{x,\eta+\Delta \eta/2})]$$



Horizontal fluxes through the vertical control-volume faces

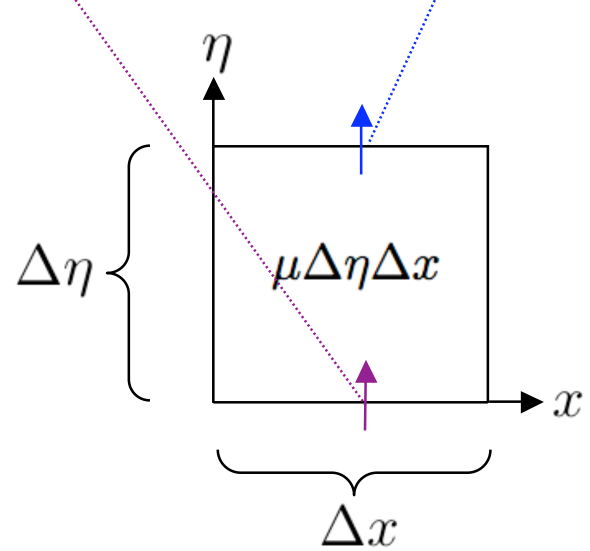
Mass Conservation in the ARW Model

Mass in a control volume $(\Delta x \Delta \eta)(\mu)^t$

Mass conservation equation

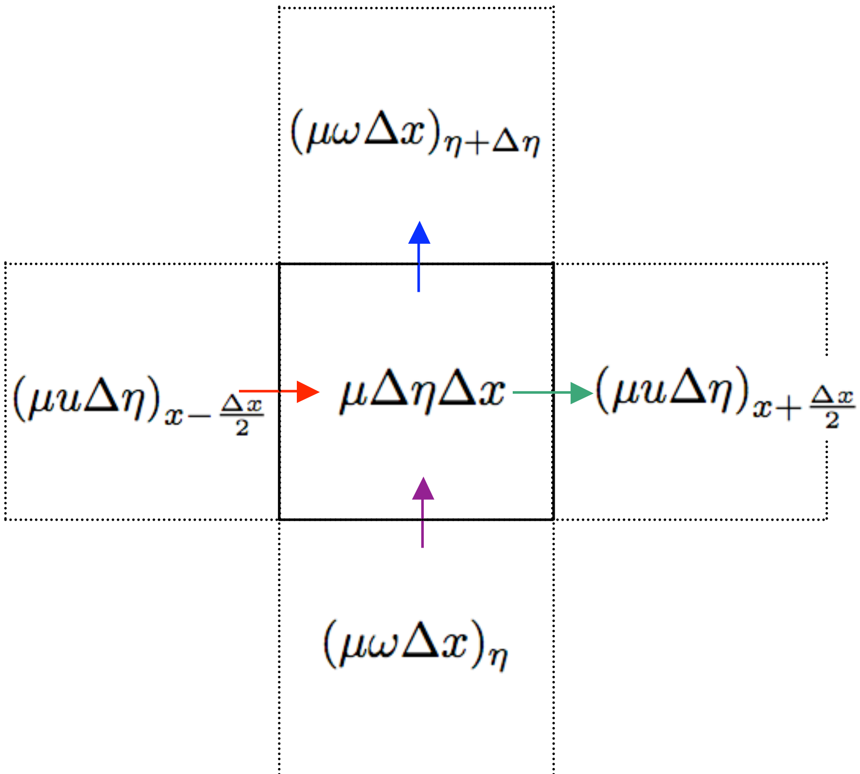
$$\Delta t^{-1}(\Delta x \Delta \eta) \cdot \left[(\mu)^{t+\Delta t} - (\mu)^t)\right] = \left[(\mu u \Delta \eta)_{x-\Delta x/2,\eta} - (\mu u \Delta \eta)_{x+\Delta x/2,\eta})\right] + \left[(\mu \omega \Delta x)_{x,\eta-\Delta \eta/2} - (\mu \omega \Delta x)_{x,\eta+\Delta \eta/2})\right]$$

Vertical fluxes through the horizontal control-volume faces



Mass Conservation in the ARW Model

The same mass fluxes are used for neighboring grid cells - hence mass is conserved locally and globally.



Scalar Mass Conservation in the ARW Model

Mass in a control volume	$(\Delta x \Delta \eta)(\mu)^t$
Scalar mass	$(\Delta x \Delta \eta)(\mu \phi)^t$

Mass conservation equation:

$$\Delta t^{-1}(\Delta x \Delta \eta) \cdot \left[(\mu)^{t+\Delta t} - (\mu)^t)\right] = \left[(\mu u \Delta \eta)_{x-\Delta x/2,\eta} - (\mu u \Delta \eta)_{x+\Delta x/2,\eta})\right] + \left[(\mu \omega \Delta x)_{x,\eta-\Delta \eta/2} - (\mu \omega \Delta x)_{x,\eta+\Delta \eta/2})\right]$$

change in mass over a time step — mass fluxes through control volume faces

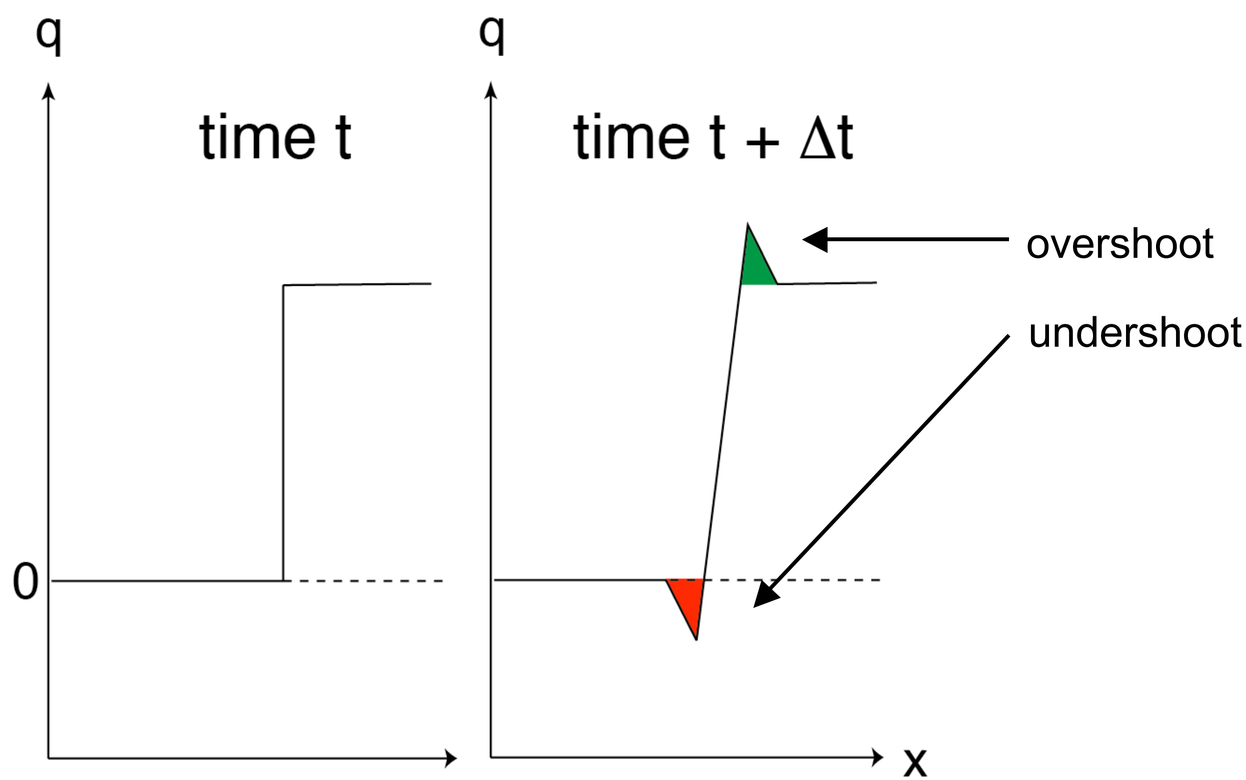
Scalar mass conservation equation:

$$\Delta t^{-1}(\Delta x \Delta \eta) \cdot \left[(\mu \phi)^{t+\Delta t} - (\mu \phi)^t)\right] = \left[(\mu u \phi \Delta \eta)_{x-\Delta x/2,\eta} - (\mu u \phi \Delta \eta)_{x+\Delta x/2,\eta})\right] + \left[(\mu \omega \phi \Delta x)_{x,\eta-\Delta \eta/2} - (\mu \omega \phi \Delta x)_{x,\eta+\Delta \eta/2})\right]$$

change in tracer mass over a time step — tracer mass fluxes through control volume faces

Moisture Transport in ARW

1D advection



ARW scheme is conservative,
but not positive definite nor monotonic.
Removal of negative q
results in spurious source of q .

Positive-Definite Flux Renormalization

Scalar update, last RK3 step

$$(\mu\phi)^{t+\Delta t} = (\mu\phi)^t - \Delta t \sum_{i=1}^{n} \delta_{x_i}[f_i] \qquad (1)$$

(1) Decompose flux: $f_i = f_i^{upwind} + f_i^c$

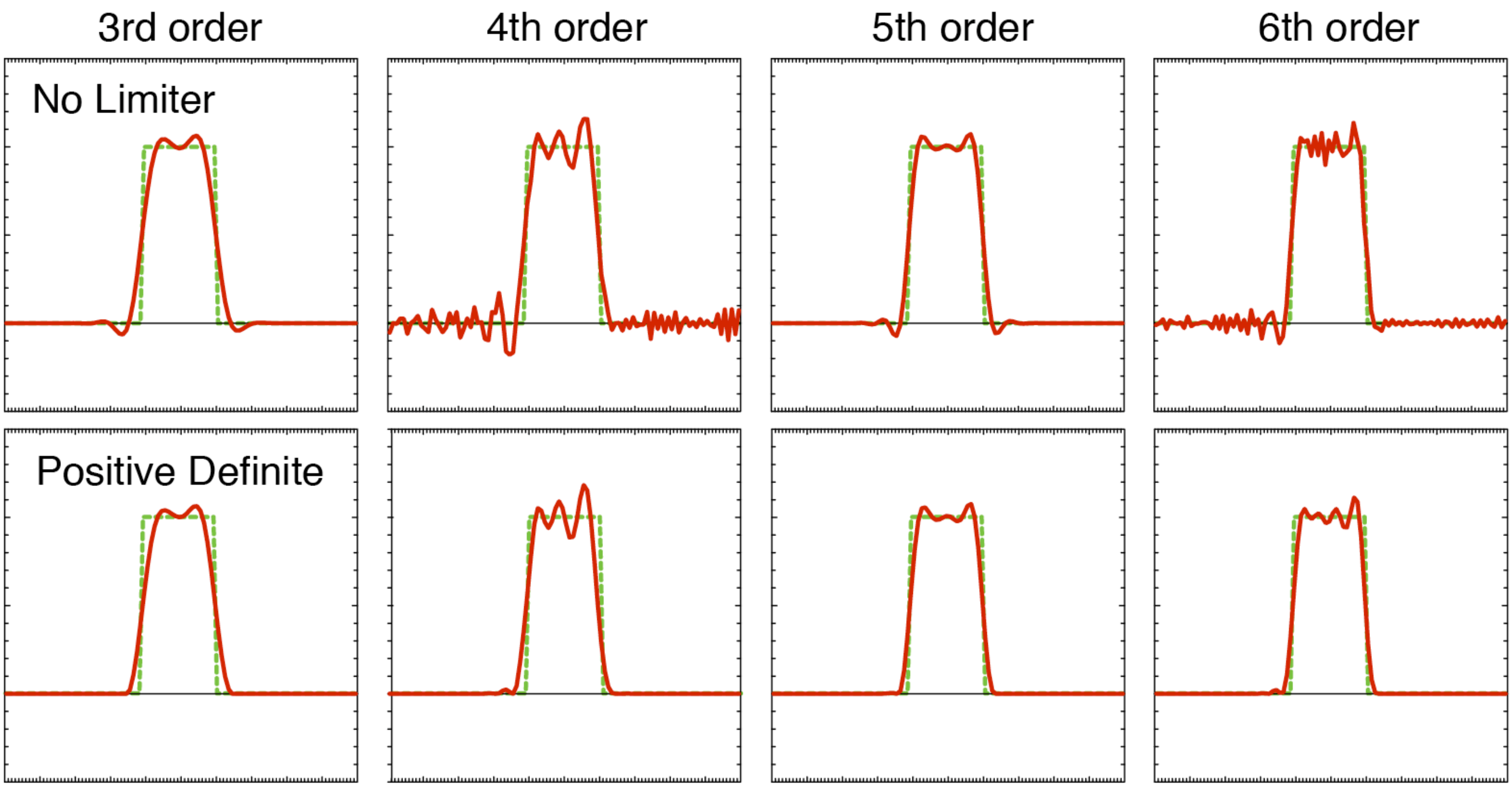
(2) Renormalize high-order correction fluxes f_i^c such that solution is positive definite: $f_i^c = R(f_i^c)$

(3) Update scalar eqn. (1) using $f_i = f_i^{upwind} + R(f_i^c)$

Skamarock, MWR 2006, 2241-2250

PD Limiter in ARW - 1D Example
Top-Hat Advection

Cr = 0.5, 1 revolution (200 steps)



ARW Model: Dynamics Parameters

3rd order Runge-Kutta time step

Courant number limited, 1D: $C_r = \frac{U \Delta t}{\Delta x} < 1.73$

Generally stable using a timestep approximately twice as large as used in a leapfrog model.

Acoustic time step

2D horizontal Courant number limited: $C_r = \frac{C_s \Delta \tau}{\Delta h} < \frac{1}{\sqrt{2}}$

$$\Delta \tau_{sound} = \Delta t_{RK} / (\text{number of acoustic steps})$$

Guidelines for time step

Δt in seconds should be about $6 * \Delta x$ (grid size in kilometers). Larger Δt can be used in smaller-scale dry situations, but *time_step_sound* (default = 4) should increase proportionately if larger Δt is used.

Maximum Courant Number for Advection

$$C_a = U\Delta t / \Delta x$$

Time Integration Scheme	*Advection Scheme*				
	2^{nd}	*3^{rd}*	*4^{th}*	*5^{th}*	*6^{th}*
Leapfrog (α=0.1)	0.91	U	0.66	U	0.57
RK2	U	0.90	U	0.39	U
RK3	1.73	1.63	1.26	1.43	1.09

U = unstable

(Wicker & Skamarock, 2002)

ARW Filters: Divergence Damping

Purpose: filter acoustic modes

$$p^{*\tau} = p^{\tau} + \gamma_d \left(p^{\tau} - p^{\tau - \Delta\tau} \right)$$ since $p_t \sim c^2 \nabla \cdot \rho \mathbf{V}$

$$\delta_\tau U'' + \mu^{t^*} \alpha^{t^*} \partial_x p''^{\tau} + (\mu^{t^*} \partial_x \bar{p}) \alpha''^{\tau} + (\alpha/\alpha_d)[\mu^{t^*} \partial_x \phi''^{\tau} + (\partial_x \phi^{t^*})(\partial_\eta p'' - \mu'')^{\tau}] = R_U{}^{t^*}$$

$$\delta_\tau V'' + \mu^{t^*} \alpha^{t^*} \partial_y p''^{\tau} + (\mu^{t^*} \partial_y \bar{p}) \alpha''^{\tau} + + (\alpha/\alpha_d)[\mu^{t^*} \partial_y \phi''^{\tau} + (\partial_y \phi^{t^*})(\partial_\eta p'' - \mu'')^{\tau}] = R_V{}^{t^*}$$

$\gamma_d = 0.1$ recommended (default)

ARW Filters: External Mode Filter

Purpose: filter the external mode
(primarily for real-data applications)

Additional terms:

$$\delta_\tau U'' = \ldots \quad \underline{- \gamma_e \left(\Delta x^2 / \Delta\tau\right) \delta_x \left(\delta_{\tau - \Delta\tau} \mu''_d\right)}$$

$$\delta_\tau V'' = \ldots \quad \underline{- \gamma_e \left(\Delta y^2 / \Delta\tau\right) \delta_y \left(\delta_{\tau - \Delta\tau} \mu''_d\right)}$$

$$\delta_\tau \mu_d = m^2 \int_1^0 \left[\partial_x U'' + \partial_y V''\right]^{\tau + \Delta\tau} d\eta$$

$\gamma_e = 0.01$ recommended (default)

ARW Filters: Vertically Implicit Off-Centered Acoustic Step

Purpose: damp vertically-propagating acoustic modes

$$\delta_\tau W'' - m^{-1} g \overline{\left[(\alpha/\alpha_d)^{t^*} \partial_\eta (C \partial_\eta \phi'') + \partial_\eta \left(\frac{c_s^2}{\alpha^{t^*}} \frac{\Theta''}{\Theta^{t^*}} \right) - \mu_d'' \right]}^{\tau} = {R_W}^{t^*}$$

$$\delta_\tau \phi'' + \frac{1}{\mu_d^{t^*}} [m \Omega^{\tau+\Delta\tau} \phi_\eta - \overline{g W''}^{\tau}] = {R_\phi}^{t^*}.$$

$$\overline{a}^{\tau} = \frac{1+\beta}{2} a^{\tau+\Delta\tau} + \frac{1-\beta}{2} a^{\tau}$$

β= 0.1 recommended (default)

ARW Filters: Vertical Velocity Damping

Purpose: damp anomalously-large vertical velocities
(usually associated with anomalous physics tendencies)

Additional term:

$$\partial_t W = \ldots \underline{- \mu_d \, \mathrm{sign}(W) \gamma_w (Cr - Cr_\beta)}$$

$$Cr = \left| \frac{\Omega dt}{\mu d\eta} \right|$$

$Cr_\beta = 1.0$ typical value (default)
$\gamma_w = 0.3$ m/s^2 recommended (default)

ARW Filters: 2nd-Order Horizontal Mixing, Horizontal-Deformation-Based K_h

Purpose: mixing on horizontal coordinate surfaces (real-data applications)

$$K_h = C_s^2\, l^2 \left[0.25(D_{11} - D_{22})^2 + \overline{D_{12}^2}^{xy} \right]^{\frac{1}{2}}$$

where

$$l = (\Delta x \Delta y)^{1/2}$$

$$D_{11} = 2\, m^2 \left[\partial_x (m^{-1} u) - z_x \partial_z (m^{-1} u) \right]$$
$$D_{22} = 2\, m^2 \left[\partial_y (m^{-1} v) - z_y \partial_z (m^{-1} v) \right]$$
$$D_{12} = m^2 \left[\partial_y (m^{-1} u) - z_y \partial_z (m^{-1} u) + \partial_x (m^{-1} v) - z_x \partial_z (m^{-1} v) \right]$$

$C_s = 0.25$ (Smagorinsky coefficient, default value)

ARW Filters: Upper Level Gravity-Wave Absorbers

(1) Absorbing layer using spatial filtering

Horizontal and vertical 2nd order diffusion operators with eddy viscosities that increase with height.

$$K_{dh} = \frac{\Delta x^2}{\Delta t}\gamma_g \cos\left(\frac{\pi}{2}\frac{z_{top} - z}{z_d}\right)$$

$$K_{dv} = \frac{\Delta z^2}{\Delta t}\gamma_g \cos\left(\frac{\pi}{2}\frac{z_{top} - z}{z_d}\right)$$

z_d - depth of the damping layer
K_{dh}, K_{dv} - horizontal and vertical eddy viscosities
γ_g - dimensionless damping coefficient, typical value 0.003

Not recommended !

ARW Filters: Upper Level Gravity-Wave Absorbers

(2) Traditional Rayleigh Damping - idealized cases only!

$$\frac{\partial u}{\partial t} = -\tau(z)\left(u - \overline{u}\right)$$
$$\frac{\partial v}{\partial t} = -\tau(z)\left(v - \overline{v}\right)$$
$$\frac{\partial w}{\partial t} = -\tau(z) w,$$
$$\frac{\partial \theta}{\partial t} = -\tau(z)\left(\theta - \overline{\theta}\right)$$

$$\tau(z) = \begin{cases} \gamma_r \sin^2\left[\frac{\pi}{2}\left(1 - \frac{z_{top}-z}{z_d}\right)\right] & \text{for } z \geq (z_{top} - z_d); \\ 0 & \text{otherwise,} \end{cases}$$

$\tau(z)$ - damping rate (t^{-1})
z_d - depth of the damping layer
γ_r - dimensionless damping coefficient

ARW Filters: Upper Level Gravity-Wave Absorbers

(3) Implicit Rayleigh W - damping (nonhydrostatic equations only!)

$$\tilde{W}''^{\tau+\Delta\tau} - W''^{\tau} - g\Delta\tau \frac{\alpha}{\alpha_d} \frac{\partial}{\partial_\eta} \left(C \frac{\partial \overline{\phi''}^{\tau}}{\partial \eta} \right) = \Delta\tau R_W^* \quad (1)$$

$$W''^{\tau+\Delta\tau} - \tilde{W}''^{\tau+\Delta\tau} = -\tau(z)\Delta\tau W''^{\tau+\Delta\tau} \quad (2)$$

$$\phi''^{\tau+\Delta\tau} - \phi''^{\tau} - g\Delta\tau \frac{1}{\mu} \overline{W''}^{\tau} = \Delta\tau R_\phi^* \quad (3)$$

Vertically implicit solution procedure:
Eliminate $\phi''^{\tau+\Delta\tau}$ from (1) using (3), solve for $\tilde{W}''^{\tau+\Delta\tau}$.
Apply implicit Rayleigh damping - solve for $W''^{\tau+\Delta\tau}$ using (2).
Recover the geopotential using $W''^{\tau+\Delta\tau}$ in (3).

$$\tau(z) = \begin{cases} \gamma_r \sin^2\left[\frac{\pi}{2}\left(1 - \frac{z_{top}-z}{z_d}\right)\right] & \text{for } z \geq (z_{top} - z_d); \\ 0 & \text{otherwise,} \end{cases}$$

$\tau(z)$ - damping rate (t^{-1})
z_d - depth of the damping layer
γ_r - dimensionless damping coefficient

Global WRF - Latitude-Longitude Grid

WRF Version 3 Release

Additions to WRF Version 2

- Map factors are generalized - m_x and m_y
  - Computational grid poles need not be geographic poles.
  - Limited area and nesting capable.
- Polar boundary conditions
- Polar filtering

ARW Map Projections

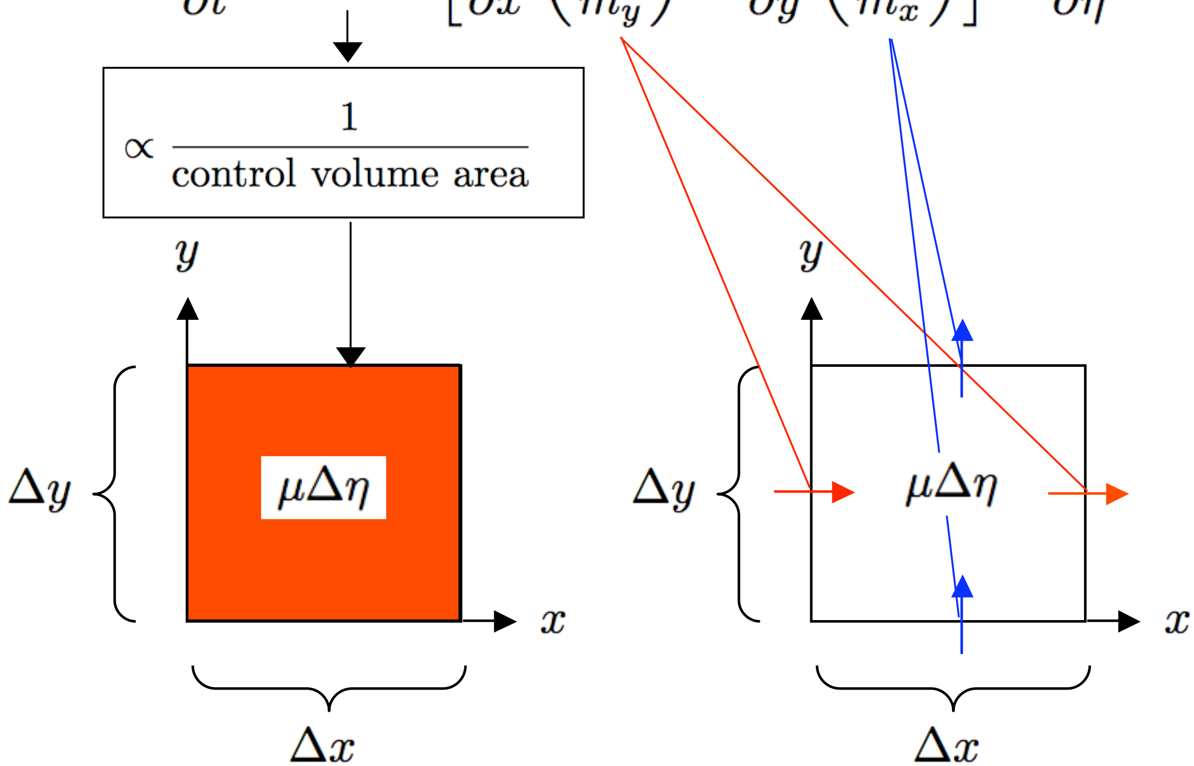
ARW map factors

Map-scale factor: $m_x = \frac{\Delta x}{\text{distance on the earth}}, \quad m_y = \frac{\Delta y}{\text{distance on the earth}}$

Continuity equation:

$$\frac{\partial \mu}{\partial t} + m_x m_y \left[\frac{\partial}{\partial x}\left(\frac{\mu u}{m_y}\right) + \frac{\partial}{\partial y}\left(\frac{\mu v}{m_x}\right) \right] + \frac{\partial}{\partial \eta}(\mu \omega) = 0$$

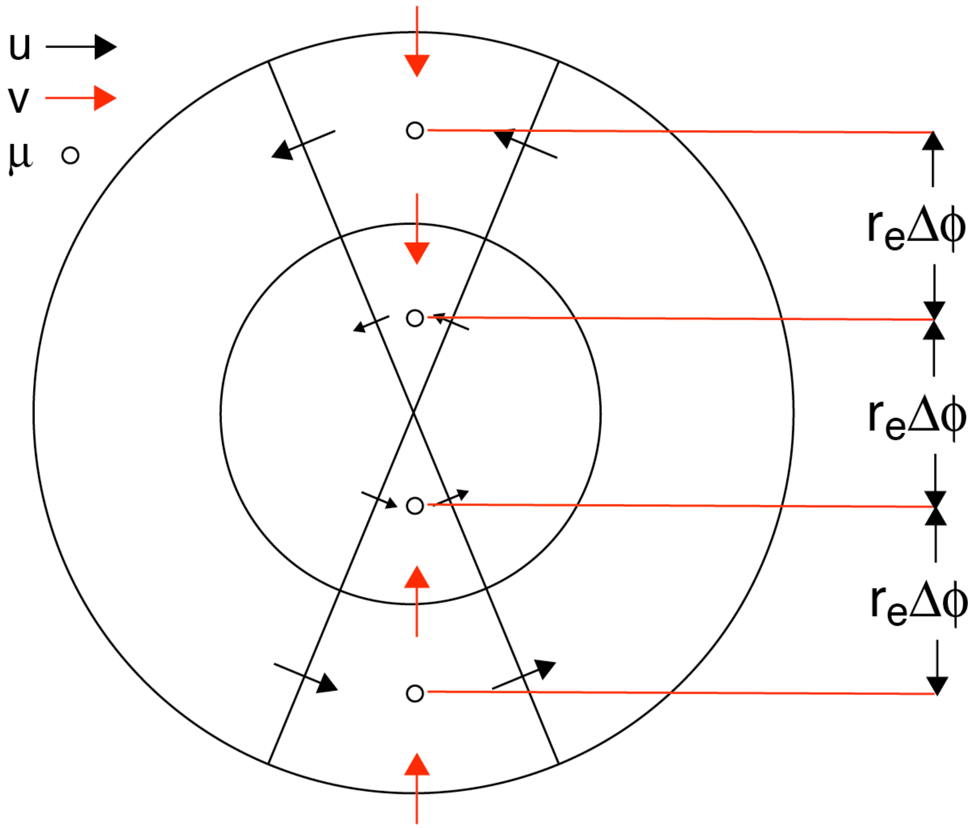
Control volume:



Lat-Long Grid Global WRF

Lat-Long WRFV3

Polar boundary condition (pole point).



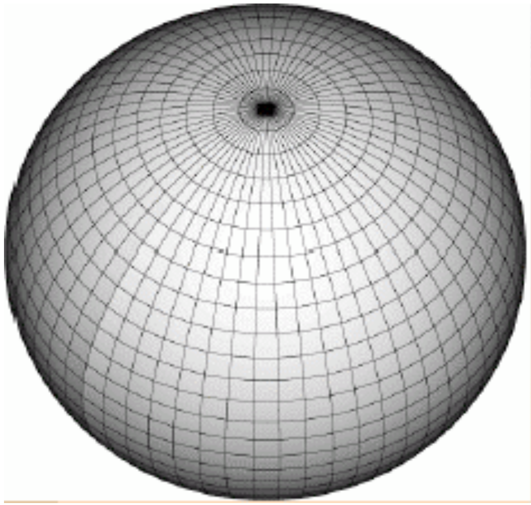
Meridional velocity (v) is undefined at the poles.

Zero meriodional flux at the poles (cell-face area is zero).

v (poles) only needed for meridional derivative of v near the poles (some approximation needed).

All other meriodional derivatives are well-defined near/at poles.

ARW Filters: Polar Filter

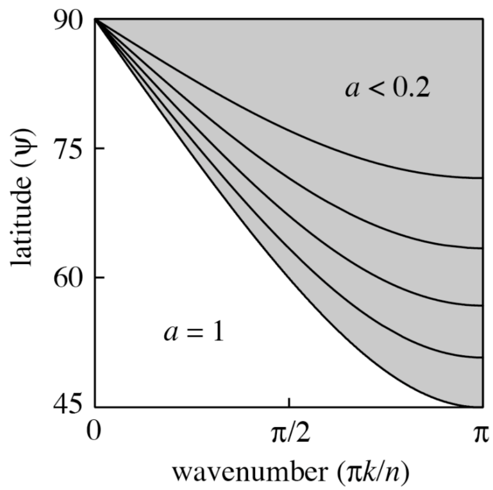


Converging gridlines severely limit timestep.
The polar filter removes this limitation.

Filter procedure - Along a grid latitude circle:

1. Fourier transform variable.
2. Filter Fourier coefficients.
3. Transform back to physical space.

Filter Coefficient $a(k)$, $\psi_o = 45^o$



$$\hat{\phi}(k)_{filtered} = a(k)\,\hat{\phi}(k), \quad \text{for all } k$$

$$a(k) = \min\left[1., \max\left(0., \left(\frac{\cos\psi}{\cos\psi_o}\right)^2 \frac{1}{\sin^2(\pi k/n)}\right)\right]$$

$k =$ dimensionless wavenumber
$\hat{\phi}(k) =$ Fourier coefficients from forward transform
$a(k) =$ filter coefficients
$\psi =$ latitude $\psi_o =$ polar filter latitude, filter when $|\psi| > \psi_o$

WRF ARW Model Integration Procedure

Begin time step

Runge-Kutta loop (steps 1, 2, and 3)

(i) advection, p-grad, buoyancy, physics

Acoustic step loop

(i) advance U,V

(ii) advance μ, Θ

(iii) advance w, ϕ

End acoustic loop

Advance scalars

End Runge-Kutta loop

Adjustment physics (currently microphysics)

End time step

WRF ARW Model Integration Procedure

Begin time step

Runge-Kutta loop (steps 1, 2, and 3)

(i) advection, p-grad, buoyancy, physics

Acoustic step loop

(i) advance U,V (Fourier Filter U,V)

(ii) advance μ, Θ (Fourier Filter μ, Θ)

(iii) advance w, ϕ (Fourier Filter w, ϕ)

End acoustic loop

Advance scalars (Fourier Filter Sc)

End Runge-Kutta loop

Adjustment physics (currently microphysics) (Fourier Filter Sc)

End time step

Timestep limited by minimum Δx outside of polar-filter region.

ARW Model: Coordinate Options

1. Cartesian geometry:
   idealized cases
2. Lambert Conformal:
   mid-latitude applications
3. Polar Stereographic:
   high-latitude applications
4. Mercator:
   low-latitude applications
5. Latitude-Longitude (new in ARW V3)
   global
   regional

Projections 1-4 are isotropic ($m_x = m_y$)
Latitude-Longitude projection is anistropic ($m_x \neq m_y$)

ARW Model: Boundary Condition Options

Lateral boundary conditions

1. Specified (Coarse grid, real-data applications).
2. Open lateral boundaries (gravity-wave radiative).
3. Symmetric lateral boundary condition (free-slip wall).
4. Periodic lateral boundary conditions.
5. Nested boundary conditions (specified).

Top boundary conditions

1. Constant pressure.

Bottom boundary conditions

1. Free slip.
2. Various B.L. implementations of surface drag, fluxes.

ARW Model: Nesting

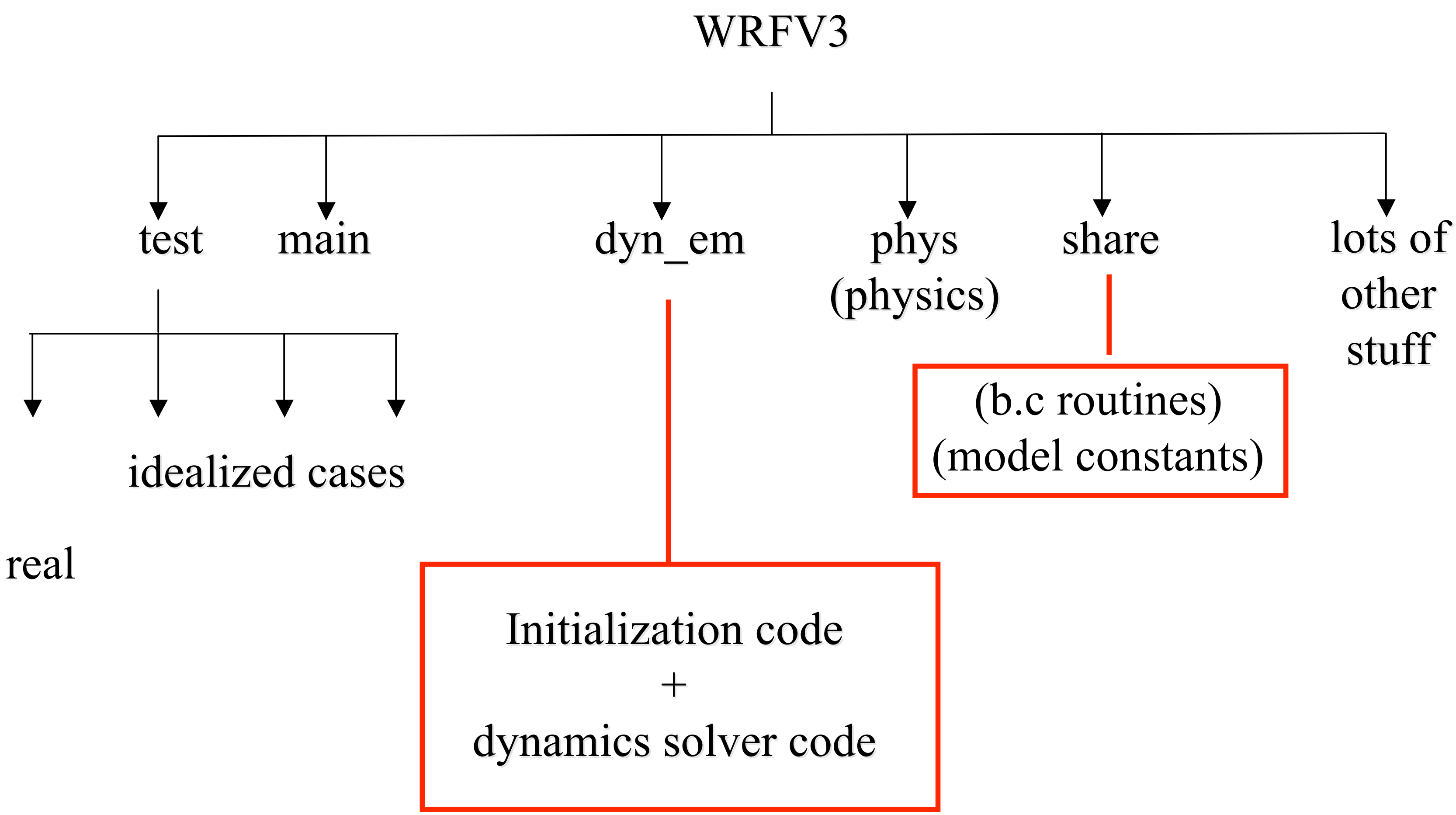
2-way nesting

1. Multiple domains run concurrently
2. Multiple levels, multiple nests per level
3. Any integer ratio grid size and time step
4. Parent domain provides nest boundaries
5. Nest feeds back interior values to parent

1-way nesting

1. Parent domain is run first
2. *ndown* uses coarse output to generate nest boundary conditions
3. Nest initial conditions from fine-grid input file
4. Nest is run after *ndown*

WRF ARW code



WRF ARW Tech Note
A Description of the Advanced Research WRF Version 3 (June 2008)
http://www.mmm.ucar.edu/wrf/users/pub-doc.html