WRFDA Overview

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NCAR/MMM/DAG

NCAR/RAL/JNT/DATC

USWRP, NSF-OPP, NCAR

AFWA, KMA, CWB, CAA, EUMETSAT, AirDat

Outline of Talk

- Introduction to data assimilation.
- Basics of modern data assimilation.
- Demonstration with a simple system.
- WRFDA overview.

Why data assimilation?

- Initial conditions
- Calibration and validation
- Observing system design, monitoring and assessment
- Reanalysis
- Better understanding:
  - Data assimilation methods
  - Model errors
  - Data errors
  - Physical process interactions
  - …

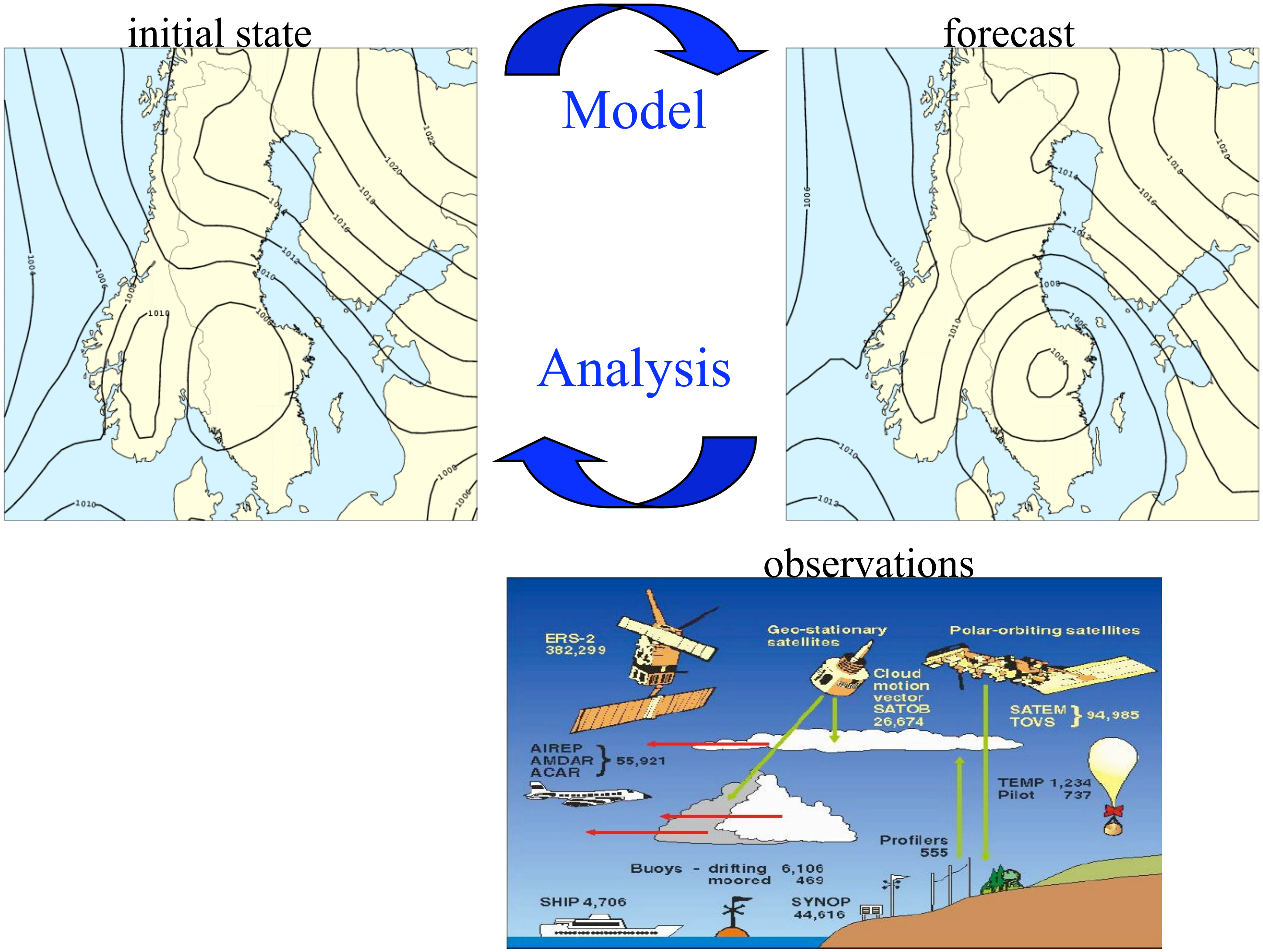
Modern weather forecast
(Bjerknes,1904)

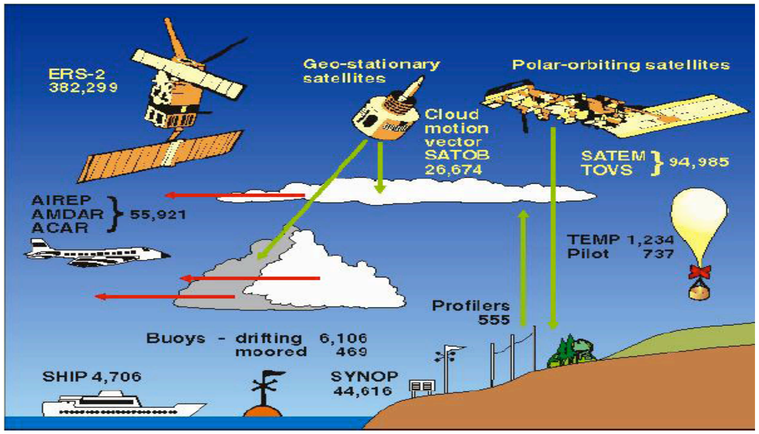
- A sufficiently accurate knowledge of the state of the atmosphere at the initial time
- A sufficiently accurate knowledge of the laws according to which one state of the atmosphere develops from another.

Vilhelm Bjerknes (1862–1951)

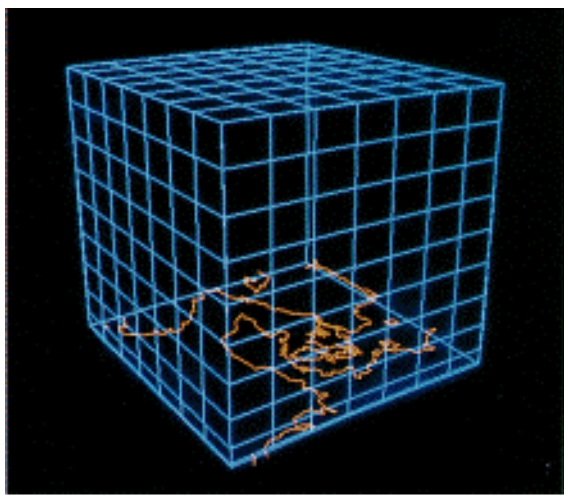
- Analysis: using observations and other information, we can specify the atmospheric state at a given initial time: “Today’s Weather”
- Forecast: using the equations, we can calculate how this state will change over time: “Tomorrow’s Weather”

(Peter Lynch)

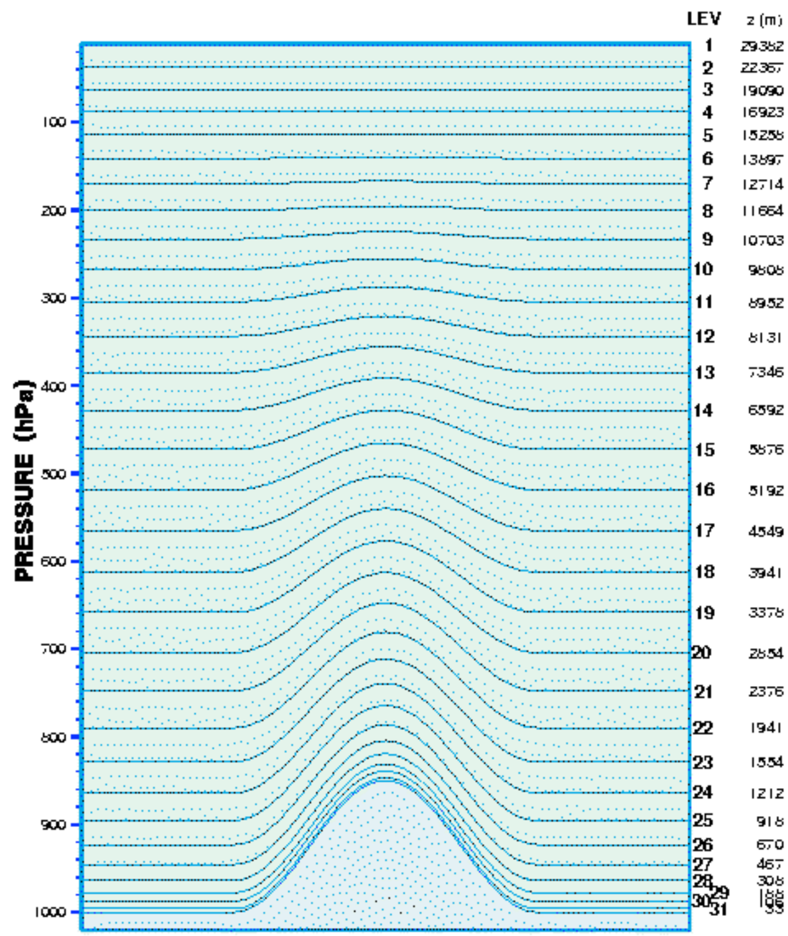

initial state
forecast
Model
Analysis
observations
ERS-2
382,299
Geo-stationary
satellites
Polar-orbiting satellites
Cloud
motion
vector
SATOB
26,674
SATEM
TOVS
94,985
AIREP
AMDAR
ACAR
55,921
TEMP 1,234
Pilot 737
Profilers
555
Buoys - drifting 6,106
moored 469
SHIP 4,706
SYNOP
44,616



Observations
y^o, ~10^5-10^6



Model state
x, ~10^7



Vertical resolution of the DMI-HIRLAM system

Assimilation methods

- Empirical methods
  - Successive Correction Method (SCM)
  - Nudging
  - Physical Initialisation (PI), Latent Heat Nudging (LHN)
- Statistical methods
  - Optimal Interpolation (OI)
  - 3-Dimensional VARiational data assimilation (3DVAR)
  - 4-Dimensional VARiational data assimilation (4DVAR)
- Advanced methods
  - Extended Kalman Filter (EKF)
  - Ensemble Kalman Filter (EnFK)

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The analysis problem for a given time

Consider a scalar x.

The background (normally a short-range forecast):

$$x^b = x^t + b.$$

The observation:

$$x^r = x^t + r.$$

The error statistics are assumed to be known:

$$< b > = 0, \text{ mean error (unbiased),}$$
$$< r > = 0, \text{ mean error (unbiased),}$$
$$< b^2 > = B, \text{ background error variance,}$$
$$< r^2 > = R, \text{ observation error variance,}$$
$$< br > = 0, \text{ no correlation between } b \text{ and } r,$$

where $<\cdot>$ is ensemble average.

BLUE: the Best Liner Unbiased Estimate

The analysis: $x^a = x^t + a$.

Search for the best estimate: $x^a = \alpha x^b + \beta x^r$

Substitute the definitions, we have:

$$\alpha + \beta = 1.$$

$$<a> = 0$$

The variance:

$$A = <a^2> = B - 2\beta B + \beta^2 \left(B + R\right)$$

To determine β: $\dfrac{dA}{d\beta} = -2B + 2\beta + \beta^2 \left(B + R\right) = 0$

we have $\beta = \dfrac{B}{B+R}$

The analysis: $x^a = x^b + \dfrac{B}{B+R}\left(x^r - x^b\right)$

The analysis error variance: $\mathrm{A}^{-1} = B^{-1} + R^{-1}$

The analysis: $x^a = x^b + \frac{B}{B+R}\left(x^r - x^b\right)$

$0 < \frac{B}{B+R} < 1$ — The analysis value should be between background and observation.

$\lim_{B\to 0} x^a = x^b$ — If B is too small, observations are less useful.

$\lim_{R\to 0} x^a = x^r$ — If R can be tuned, analysis can fit observations as close as one wants!

The analysis error variance: $A^{-1} = B^{-1} + R^{-1}$

$A < B$ — Statistically, analyses are better than background.

$A < R$ — Statistically, analyses are better than observations!

3D-Var

The analysis is obtained by minimizing the cost function J, defined as :

$$J=\frac{1}{2}\left(x-x^b\right)^T B^{-1}\left(x-x^b\right)+\frac{1}{2}\left(x-x^r\right)^T R^{-1}\left(x-x^r\right)$$

The gradient of J with respect to x:

$$J'=B^{-1}\left(x-x^b\right)+R^{-1}\left(x-x^r\right)$$

At the minimum, $J'=0$, we have:

$$x^a=x^b+\frac{B}{B+R}\left(x^r-x^b\right)$$

the same as BLUE.

Sequential data assimilation (I)

True states : $\ldots,\ x_{i-1}^{t},\ \ x_{i}^{t},\ x_{i+1}^{t},\ \ldots$

Observations : $\ldots,\ x_{i-1}^{r},\ \ x_{i}^{r},\ x_{i+1}^{r},\ \ldots$

Forecasts : $\ldots,\ x_{i-1}^{f},\ \ x_{i}^{f},\ x_{i+1}^{f},\ \ldots$

Analyses : $\ldots,\ x_{i-1}^{a},\ \ x_{i}^{a},\ x_{i+1}^{a},\ \ldots$

Sequential data assimilation (II)

Forecast model:

$$x_{i+1}^t = M\left(x_i^t\right) + q_i$$

Where q_i is the model error.

As q_i is unknown and x_i^a is the best estimate of x_i^t the forecast model usually takes the form:

$$x_{i+1}^f = M\left(x_i^a\right)$$

OI (and 3DVAR):

$$x_i^a = x_i^f + \frac{B}{B+R}\left(x_i^r - x_i^f\right)$$

Sequential data assimilation (III)

4DVAR analysis is obtained by minimizing the cost function J, defined as:

$$J(x_i) = \frac{1}{2}\left(x_i - x_i^f\right)^T B^{-1}\left(x_i - x_i^f\right)$$

$$+\frac{1}{2}\sum_{k=0}^{K}\left[M_{k-1}(x_i) - x_{i+k}^r\right]^T R^{-1}\left[M_{k-1}(x_i) - x_{i+k}^r\right]$$

where, K is the assimilation window and

$$M_{-1}(x_i) = x_i$$

$$M_0(x_i) = M(x_i)$$

$$M_{k-1}(x_i) = \underbrace{M(M(\ldots M}_{k}(x_i)\ldots))$$

Sequential data assimilation (IV)

4DVAR (continue)

The gradient of J with respect to x:

$$J' = B^{-1}\left(x_i - x_i^f\right) + \sum_{k=0}^{K} \prod_{j=0}^{k-1} M_{i+j}^T R^{-1}\left[M_{k-1}\left(x_i\right) - x_{i+k}^r\right]$$

where, M_{i+j}^T is the adjoint model of M at time step $i+j$.

Sequential data assimilation (V)

Extended Kalman Filters:

True states: $x_{i+1}^{t} = M\left(x_i^t\right) + q_i$

Model states: $x_{i+1}^{f} = M\left(x_i^a\right)$

Forecast error: $x_{i+1}^{f} - x_{i+1}^{t} = M\left(x_i^a\right) - M\left(x_i^t\right) - q_i \approx \mathbf{M}_i\left(x_i^a - x_i^t\right) - q_i$

Forecast error covariance matrix:

$$P_{i+1}^{f} = \left\langle \left(x_{i+1}^{f} - x_{i+1}^{t}\right)\left(x_{i+1}^{f} - x_{i+1}^{t}\right)^{T} \right\rangle$$

$$\approx \mathbf{M}_i \left\langle \left(x_i^a - x_i^t\right)\left(x_i^a - x_i^t\right)^{T} \right\rangle \mathbf{M}_i^T + \left\langle q_i\, q_i^T \right\rangle$$

$$= \mathbf{M}_i\, P_i^a \mathbf{M}_i^T + Q_i$$

Sequential data assimilation (VI)

Extended Kalman Filters (continue):

For the analysis step:

$$K_i = P_i^f \left(P_i^f + R \right)^{-1}$$

$$x_i^a = x_i^f + K_i \left(x_i^r - x_i^f \right)$$

$$P_i^a = \left(I - K_i \right) P_i^f$$

For the forecast step:

$$x_{i+1}^f = M\left(x_i^a \right)$$

$$P_{i+1}^f = \mathrm{M}_i P_i^a \mathrm{M}_i^T + Q_i$$

(Ensemble KF use ensembles to calculate P^f)

Sequential data assimilation (VII)

From scalar to vector:

Number of grid points $\approx 10^7$:

$$x \rightarrow \mathbf{x}$$

$$x^b \rightarrow \mathbf{x}^{\mathrm{b}}$$

Dimension of **B, P** $\approx 10^7 \text{x } 10^7$

Number of observations, 10^6 :

$$x^r \rightarrow \mathbf{y}^o$$

$$x - x^r \rightarrow H(\mathbf{x}) - \mathbf{y}^{\mathrm{b}}$$

Dimension of $\mathbf{R} \approx 10^6 \text{x } 10^6$

Sequential data assimilation (VIII)

OI:

$$\mathbf{x}_i^a = \mathbf{x}_i^f + \mathbf{B}\mathbf{H}^T\left(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R}\right)^{-1}\left[\mathbf{y}^o - H\left(\mathbf{x}_i^f\right)\right]$$

$$\mathbf{x}_{i+1}^f = M\left(\mathbf{x}_i^a\right)$$

4D-Var:

$$J\left(\mathbf{x}_i\right) = \frac{1}{2}\left(\mathbf{x}_i - \mathbf{x}_i^f\right)^T \mathbf{B}^{-1}\left(\mathbf{x}_i - \mathbf{x}_i^f\right)$$

$$+\frac{1}{2}\sum_{k=0}^{k}\left[H\left(M_{k-1}\left(\mathbf{x}_i\right)\right) - \mathbf{y}_{i+k}^o\right]^T \mathbf{R}^{-1}\left[H\left(M_{k-1}\left(\mathbf{x}_i\right)\right) - \mathbf{y}_{i+k}^o\right]$$

$$J' = \mathbf{B}^{-1}\left(\mathbf{x}_i - \mathbf{x}_i^f\right) + \sum_{k=0}^{K}\prod_{j=0}^{k-1}\mathbf{M}_{i+j}^T\mathbf{H}^T\mathbf{R}^{-1}\left[H\left[M_{k-1}\left(\mathbf{x}_i\right)\right] - \mathbf{y}_{i+k}^o\right]$$

Sequential data assimilation (IX)

The Extended Kalman Filter:

For the analysis step i:

$$\mathbf{K}_i = \mathbf{P}_i^f \mathbf{H}_i^T \left(\mathbf{H}_i \mathbf{P}_i^f \mathbf{H}_i^T + \mathbf{R} \right)^{-1}$$

$$\mathbf{x}_i^a = \mathbf{x}_i^f + \mathbf{K}_i \left[\mathbf{y}^o - H\left(\mathbf{x}_i^f \right) \right]$$

$$\mathbf{P}_i^a = \left(\mathbf{I} - \mathbf{K}_i \mathbf{H}_i \right) \mathbf{P}_i^f$$

For the forecast step, from i to i+1:

$$\mathbf{x}_{i+1}^f = M\left(\mathbf{x}_i^a \right)$$

$$\mathbf{P}_{i+1}^f = \mathbf{M}_i \mathbf{P}_i^a \mathbf{M}_i^T + \mathbf{Q}_i$$

(Ensemble KF use ensembles to calculate P^a and P^f)

Sequential data assimilation (X)

EKF

$$\mathbf{K}_i = \mathbf{P}_i^f \mathbf{H}_i^T \left(\mathbf{H}_i \mathbf{P}_i^f \mathbf{H}_i^T + \mathbf{R} \right)^{-1}$$

$$\mathbf{x}_i^a = \mathbf{x}_i^f + \mathbf{K}_i \left[\mathbf{y}^o - H\left(\mathbf{x}_i^f \right) \right]$$

$$\mathbf{P}_i^a = \left(\mathbf{I} - \mathbf{K}_i \mathbf{H}_i \right) \mathbf{P}_i^f$$

$$\mathbf{x}_{i+1}^f = M\left(\mathbf{x}_i^a \right)$$

$$\mathbf{P}_{i+1}^f = \mathbf{M}_i \mathbf{P}_i^a \mathbf{M}_i^T + \mathbf{Q}_i$$

(Ensemble KF use ensembles to calculate P^a and P^f)

$\left(\mathbf{B} = \mathbf{P}_{i+1}^f \right)$ EKF $\rightarrow$ OI or VAR

$$\left(\mathbf{K} = \mathbf{B}\mathbf{H}^T \left(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R} \right)^{-1} \right)$$

OI or VAR

$$\mathbf{x}_i^a = \mathbf{x}_i^f + \mathbf{K} \left[\mathbf{y}^o - H\left(\mathbf{x}_i^f \right) \right]$$

$$\mathbf{x}_{i+1}^f = M\left(\mathbf{x}_i^a \right)$$

WRF DA

Issues on data assimilation

- Observations $\mathbf{y}^o$
- Observation operator H
- Observation errors $\mathbf{R}$
- Backgound $\mathbf{x}^b$
- Size of $\mathbf{B}$: statistical model and tuning
- Minimization algorithm $\left(\text{Quasi}-\text{Newton; Conjugate Gradient; } \ldots\right)$
- $\mathbf{M}$ and $\mathbf{M}^T$: development and validity
- Size of $\mathbf{P}^f$ and $\mathbf{P}^a$: simplifications
- Model errors $\mathbf{Q}$

H - Observation operator

H maps variables from "model space" to "observation space"

$$\mathbf{x} \Longrightarrow \mathbf{y}$$

– Interpolations from model grids to observation locations
– Extrapolations using PBL schemes
– Time integration using full NWP models
– Transformations of model variables (u, v, T, q, p_s, etc.) to "indirect" observations (e.g. satellite radiance, radar radial winds, etc.)
  - Simple relations like PW, radial wind, refractivity, …
  - Radar reflectivity $Z = Z(T, IWC, LWC, RWC, SWC)$
  - Radiative transfer models $L(\nu) \approx \int_0^\infty B(\nu, T(z)) \left[\frac{dTR(\nu)}{dz} \right] dz$
  - Precipitation using simple or complex models
  - …

!!! Need *H*, **H** and $\mathbf{H}^T$, not $\mathbf{H}^{-1}$!!!

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The Lorenz 1964 equation

Nonlinear equation (NL):

$$x_{i+1} = ax_i - x_i^2 = M\left(x_i\right)$$

Depending on a, three types of solution are found; steady state; limited cycle; chaotic.

Tangent Linear equations (TL):

$$x_{i+1}^{tl} = \left(a - 2x_i^{bs}\right)x_i^{tl} = \mathbf{M}_i x_i^{tl}$$

(linearized around basic state x_i^{bs})

Adjoint equation (AD):

$$x_i^{ad} = \left(a - 2x_i^{bs}\right)x_{i+1}^{ad} = \mathbf{M}_i^T x_{i+1}^{ad}$$

Note here for this simple case we have

$$\mathbf{M}_i = \mathbf{M}_i^T = \left(a - 2x_i^{bs}\right)$$

Issues on data assimilation for the system based on the Lorenz 64 equation

- Observation operator $H = \mathrm{H} = \mathrm{H}^T = 1$
- Estimate of the true states - generated (with model error!)
- Observation errors $x^o - x^t = \sigma_o G$
- Size of B is 1, but $B = \sigma_b^2$ still needs attention
- Model errors Q: $q_i = \sigma_m G$ When "true" states are generated; but we assume $Q = B/4$
- Size of P^f or P^a is 1.
- M and M^T: no eort in development but their validity is still a major problem

  (model, assimilation window length, ...)
- Gaussian statistics

Too simple?

Try: Lorenz63 model

Try: 1-D advection equation

Try: 2-D shallow water equation

…

Try: ARW!!!

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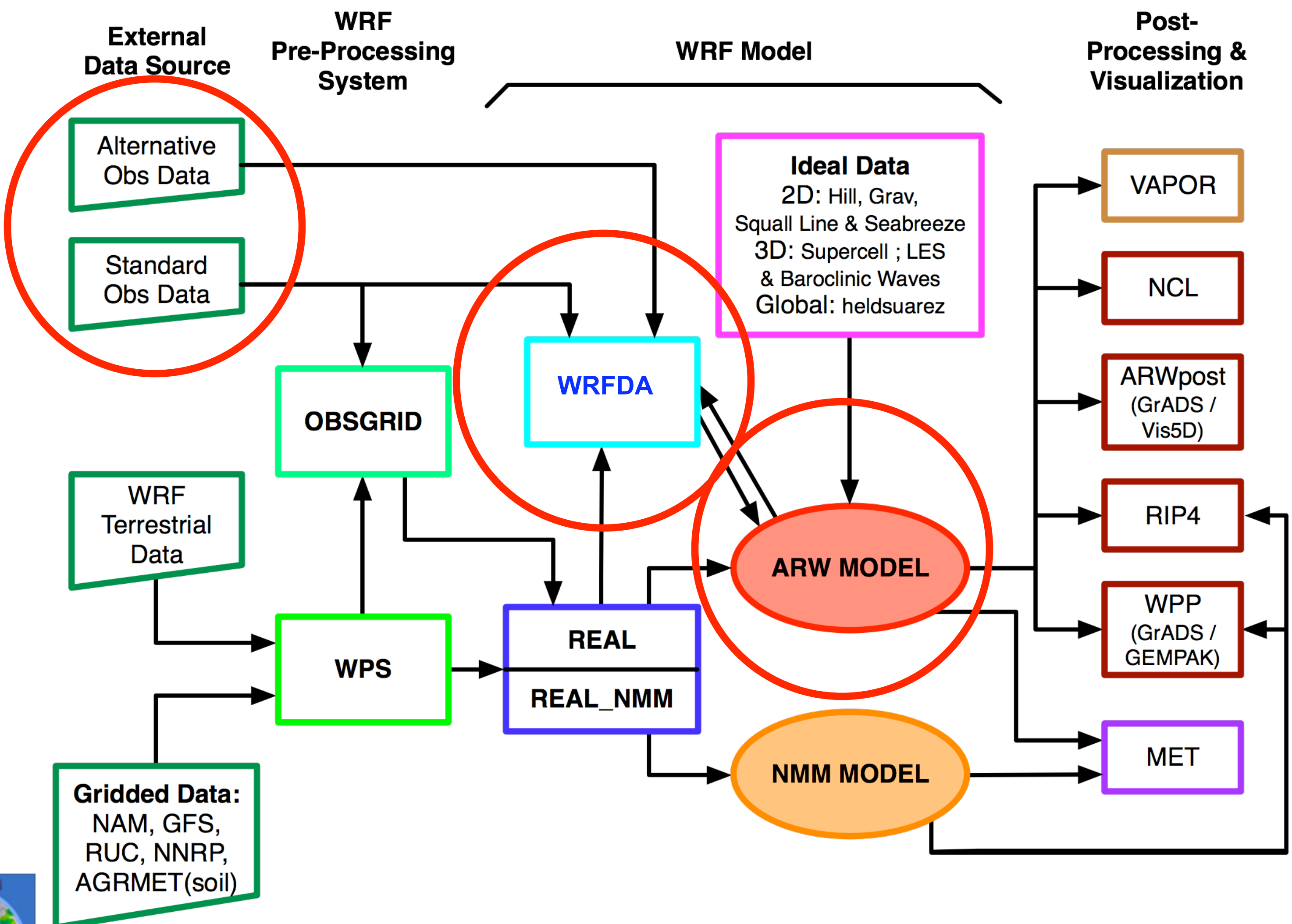
- WRFDA overview…

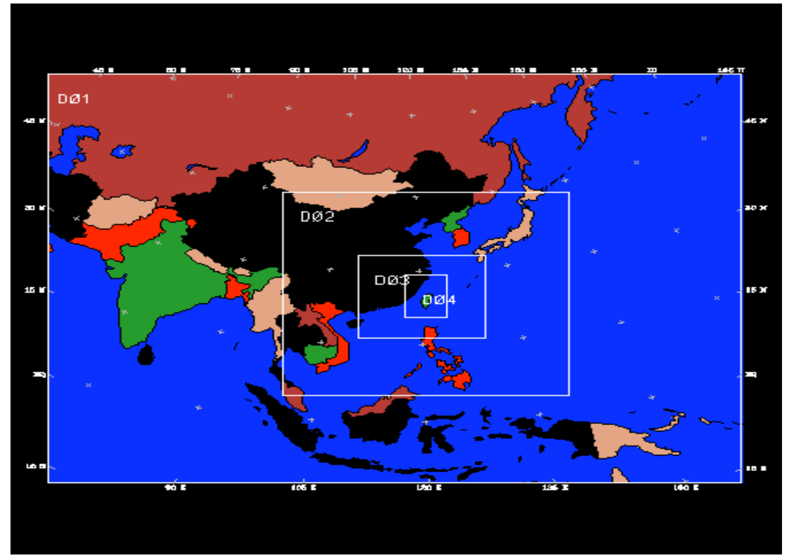
WRFDA is a **D**ata **A**ssimilation system built within the **WRF** software framework, used for application in both research and operational environments….

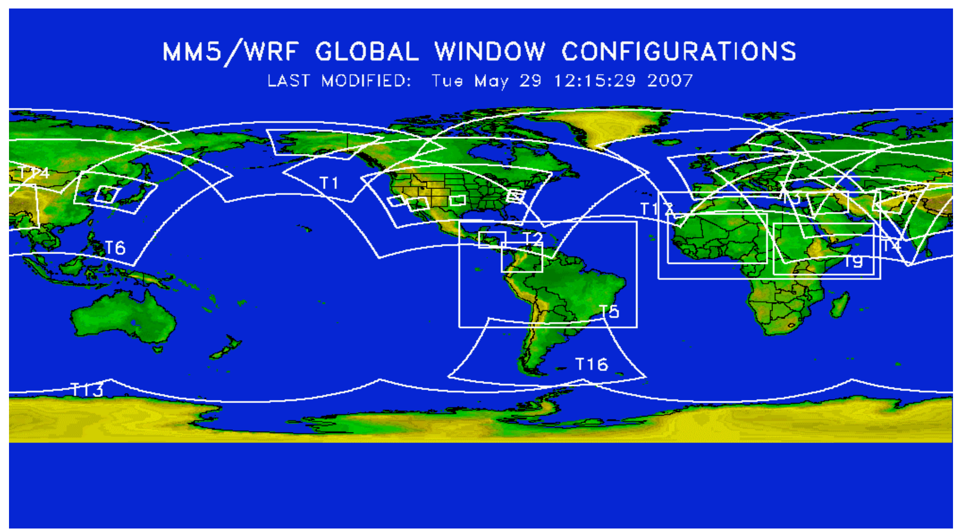
WRFDA in WRF Modeling System



WRFDA

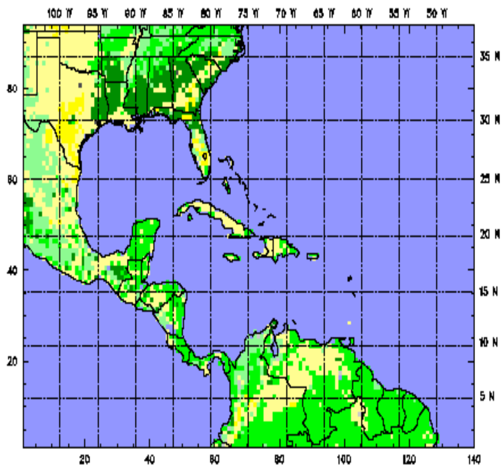
- **Goal:** Community WRF DA system for
  - regional/global,
  - research/operations, and
  - deterministic/probabilistic applications.
- **Techniques:**
  - 3D-Var
  - 4D-Var (regional)
  - Ensemble DA,
  - Hybrid Variational/Ensemble DA.
- **Model:** WRF (ARW, NMM, Global)
- **Support:**
  - NCAR/ESSL/MMM/DAG
  - NCAR/RAL/JNT/DATC
- **Observations:** Conv.+Sat.+Radar (+bogus)

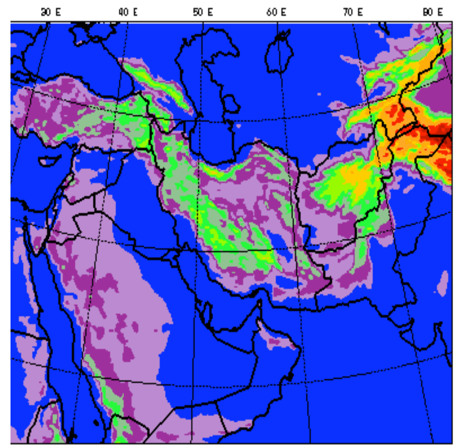


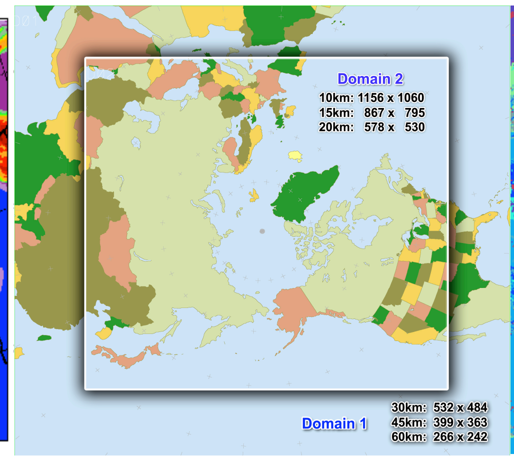


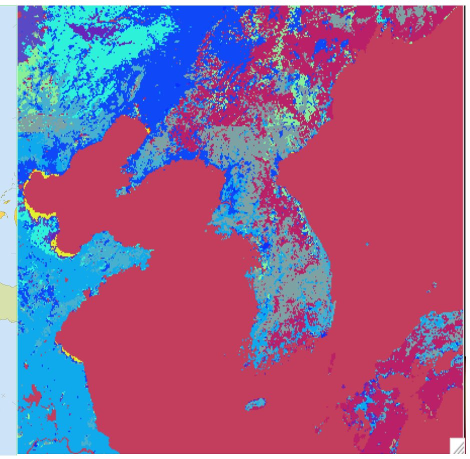
The WRFDA Program

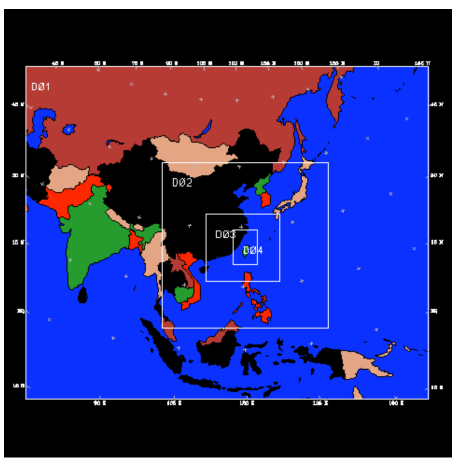
- NCAR staff (DAG,DATC): 20FTE, ~10 projects.
- Non-NCAR collaborators (AFWA, KMA, CWB, BMB, etc): ~10FTE.
- Community users: ~30 (more in 10,000 general WRF downloads?).

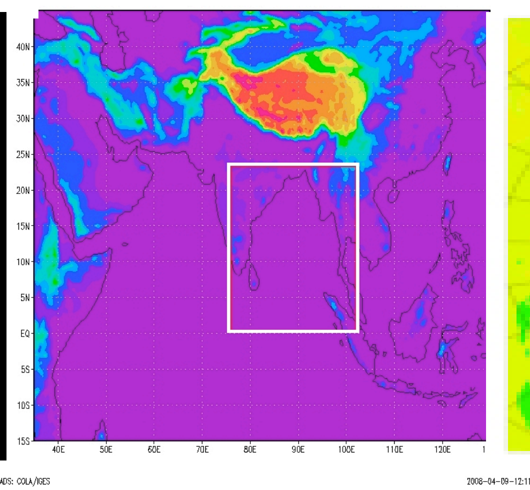


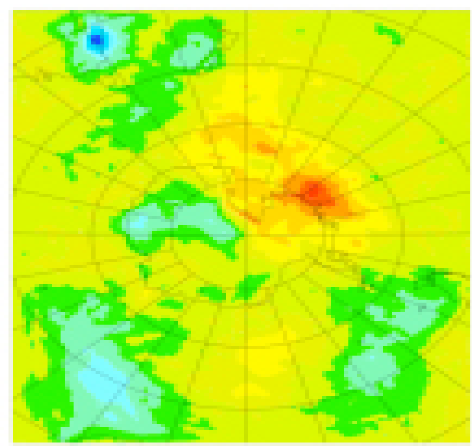


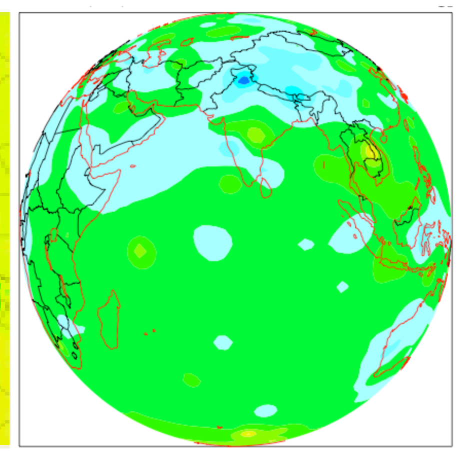












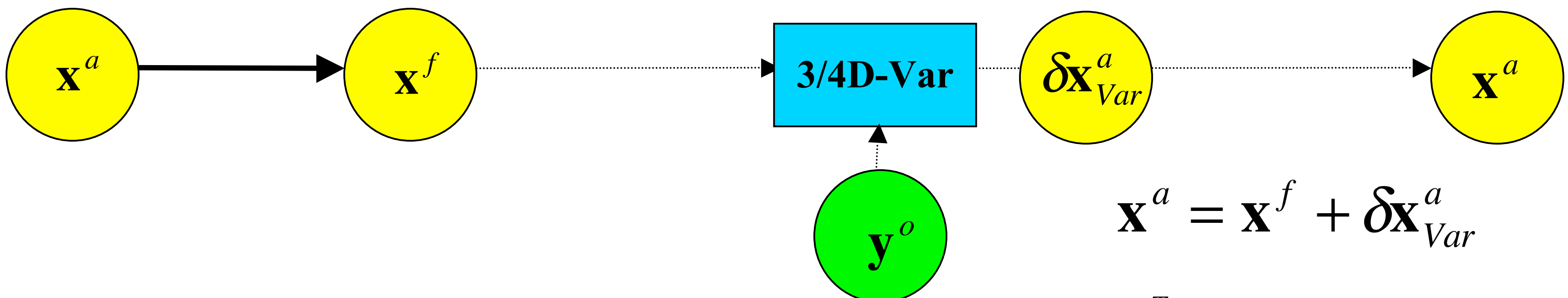
WRFDA Observations

- **In-Situ:**
  - Surface (SYNOP, METAR, SHIP, BUOY).
  - Upper air (TEMP, PIBAL, AIREP, ACARS, TAMDAR).
- **Remotely sensed retrievals:**
  - Atmospheric Motion Vectors (geo/polar).
  - SATEM thickness.
  - Ground-based GPS Total Precipitable Water/Zenith Total Delay.
  - SSM/I oceanic surface wind speed and TPW.
  - Scatterometer oceanic surface winds.
  - Wind Profiler.
  - Radar radial velocities and reflectivities.
  - Satellite temperature/humidity/thickness profiles.
  - GPS refractivity (e.g. COSMIC).
- **Radiative Transfer (RTTOV or CRTM):**

– HIRS	from NOAA-16, NOAA-17, NOAA-18, METOP-2
– AMSU-A	from NOAA-15, NOAA-16, NOAA-18, EOS-Aqua, METOP-2
– AMSU-B	from NOAA-15, NOAA-16, NOAA-17
– MHS	from NOAA-18, METOP-2
– AIRS	from EOS-Aqua
– SSMIS	from DMSP-16

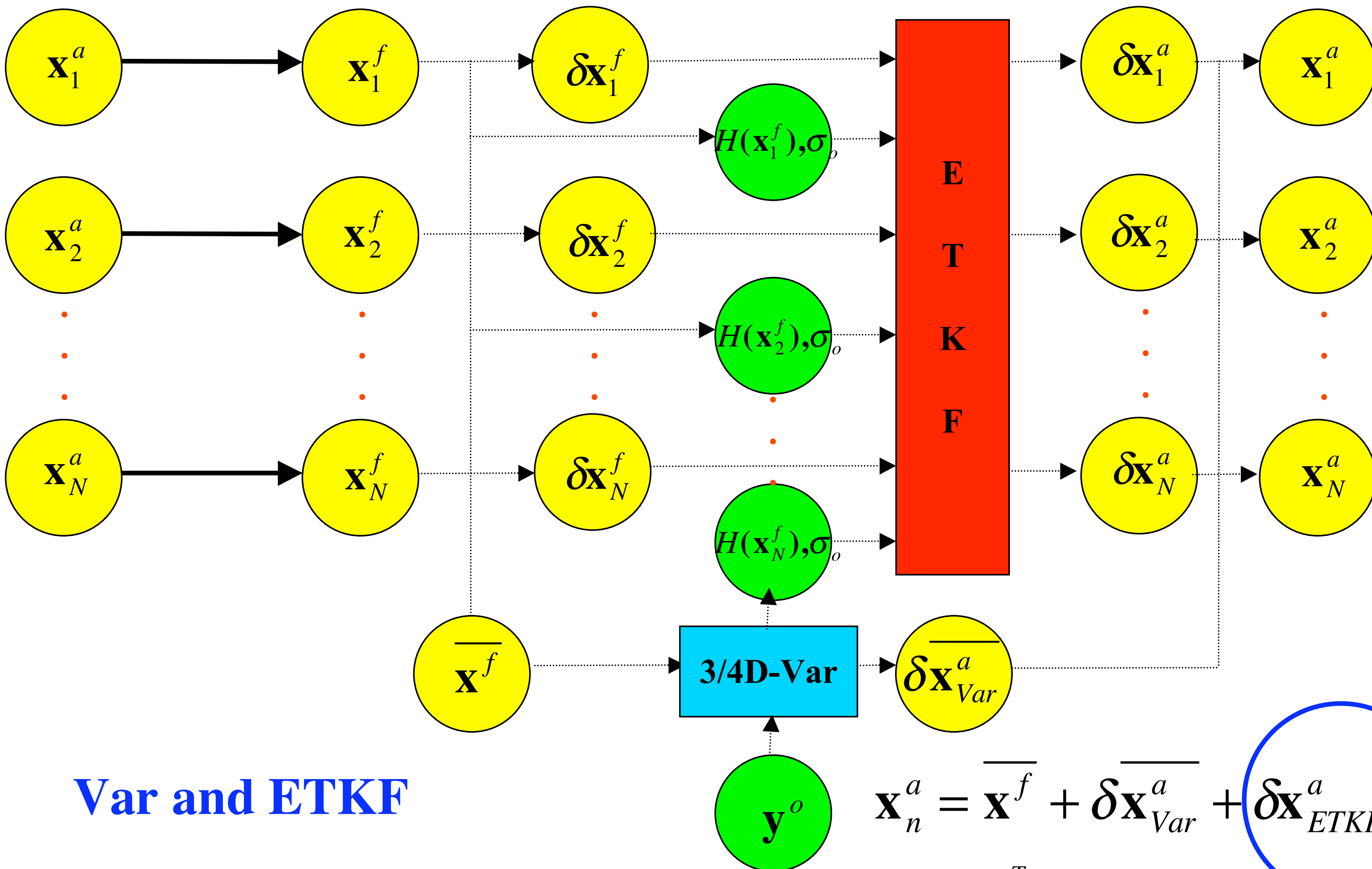
- **Bogus:**
  - TC bogus.
  - Global bogus.

WRFDA
3/4D-Var



$$\mathbf{x}^a = \mathbf{x}^f + \delta\mathbf{x}^a_{Var}$$

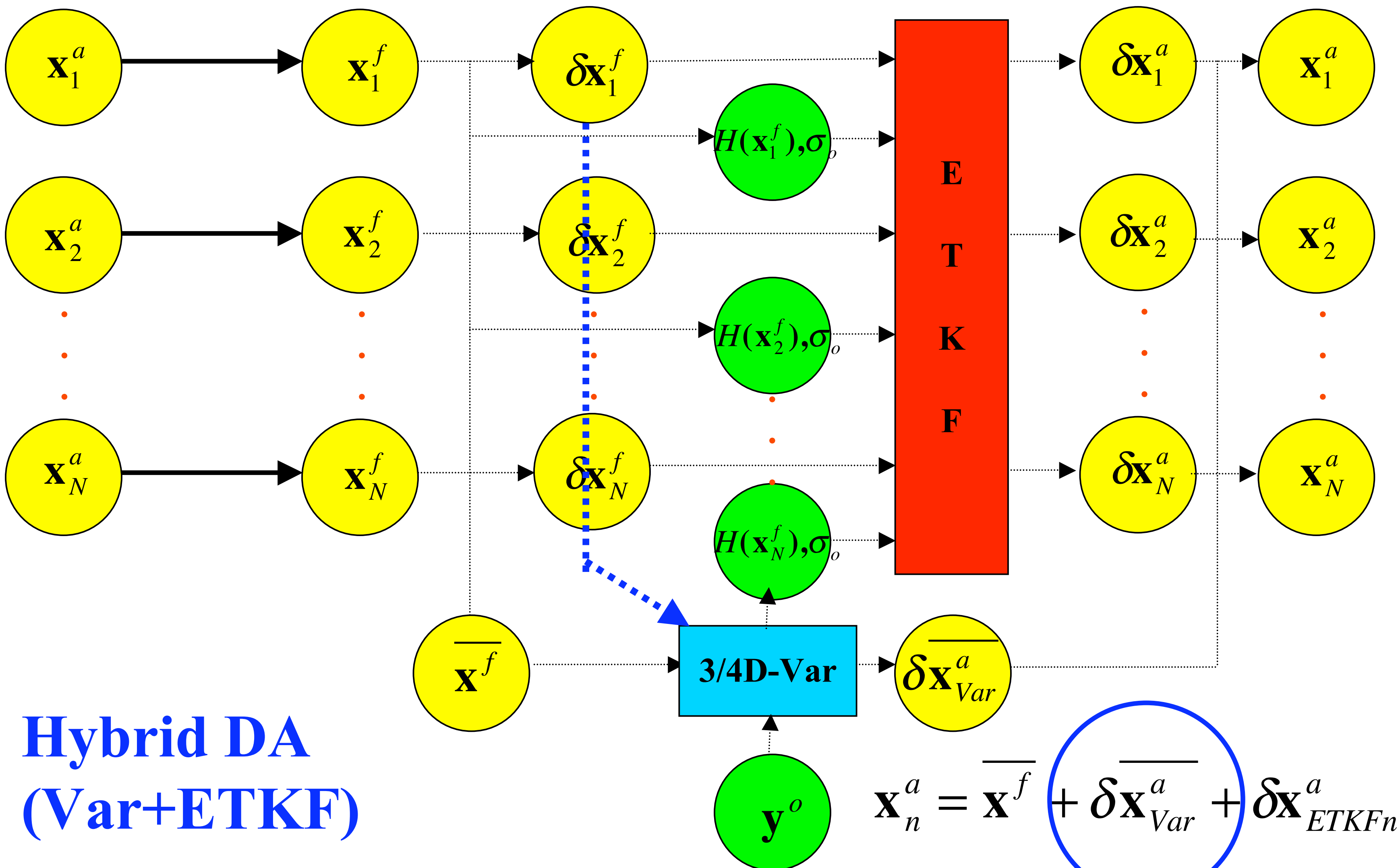
$$J = \frac{1}{2}\delta\mathbf{x}_0{}^T \mathbf{B}_o^{-1}\delta\mathbf{x}_0 + \frac{1}{2}\sum_{i=0}^{n}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]^T \mathbf{R}_i^{-1}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]$$



Var and ETKF

$$\mathbf{x}_n^a = \overline{\mathbf{x}^f} + \overline{\delta\mathbf{x}_{Var}^a} + \delta\mathbf{x}_{ETKFn}^a$$

$$J = \frac{1}{2}\delta\mathbf{x}_0^T\mathbf{B}_o^{-1}\delta\mathbf{x}_0 + \frac{1}{2}\sum_{i=0}^{n}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]^T\mathbf{R}_i^{-1}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]$$

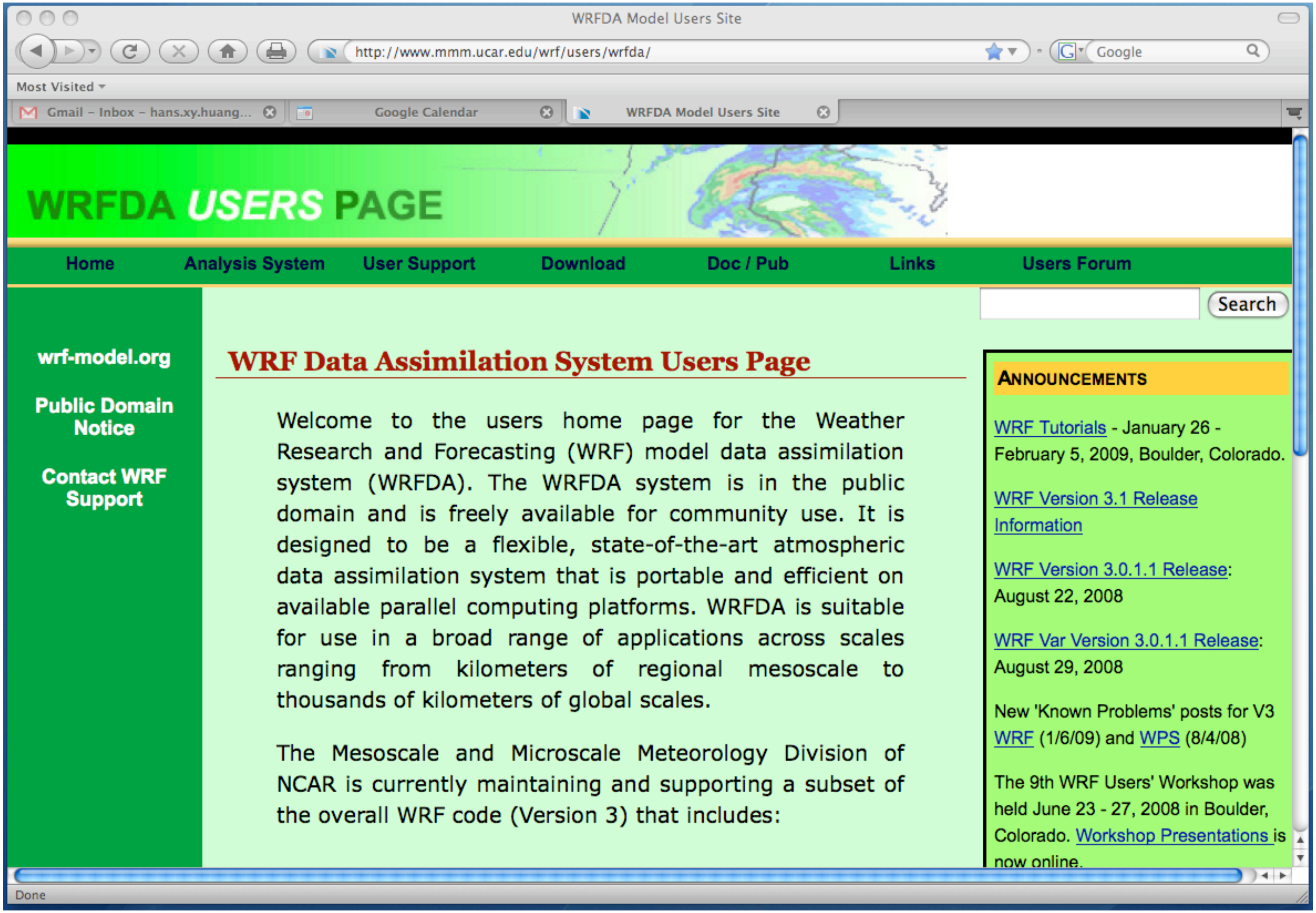


Hybrid DA (Var+ETKF)

$$\mathbf{x}_n^a = \overline{\mathbf{x}^f} + \delta\overline{\mathbf{x}^a_{Var}} + \delta\mathbf{x}^a_{ETKFn}$$

$$J = \frac{W_b}{2}\delta\mathbf{x}_0^T\mathbf{B}_o^{-1}\delta\mathbf{x}_0 + \frac{W_\alpha}{2}\mathbf{a}^T\mathbf{A}^{-1}\mathbf{a} + \frac{1}{2}\sum_{i=0}^{n}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]^T\mathbf{R}_i^{-1}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]$$

www.mmm.ucar.edu/wrf/users/wrfda



Issues on data assimilation
(covered by this tutorial)

- Observations $\mathbf{y}^o$
- Observation errors $\mathbf{R}$
- Observation operator H
- Backgound $\mathbf{x}^b$
- Size of $\mathbf{B}$: statistical model and tuning
- Minimization algorithm (Quasi–Newton; Conjugate Gradient; …)
- $\mathbf{M}$ and $\mathbf{M}^T$: development and validity
- Size of $\mathbf{P}^f$ and $\mathbf{P}^a$: simplifications
- Model errors $\mathbf{Q}$