

# **WRF-Var Background Error Generation**

Dr. Dale M. Barker

MMM Division, NCAR,  
PO Box 3000, Boulder, Colorado,  
80307-3000, USA

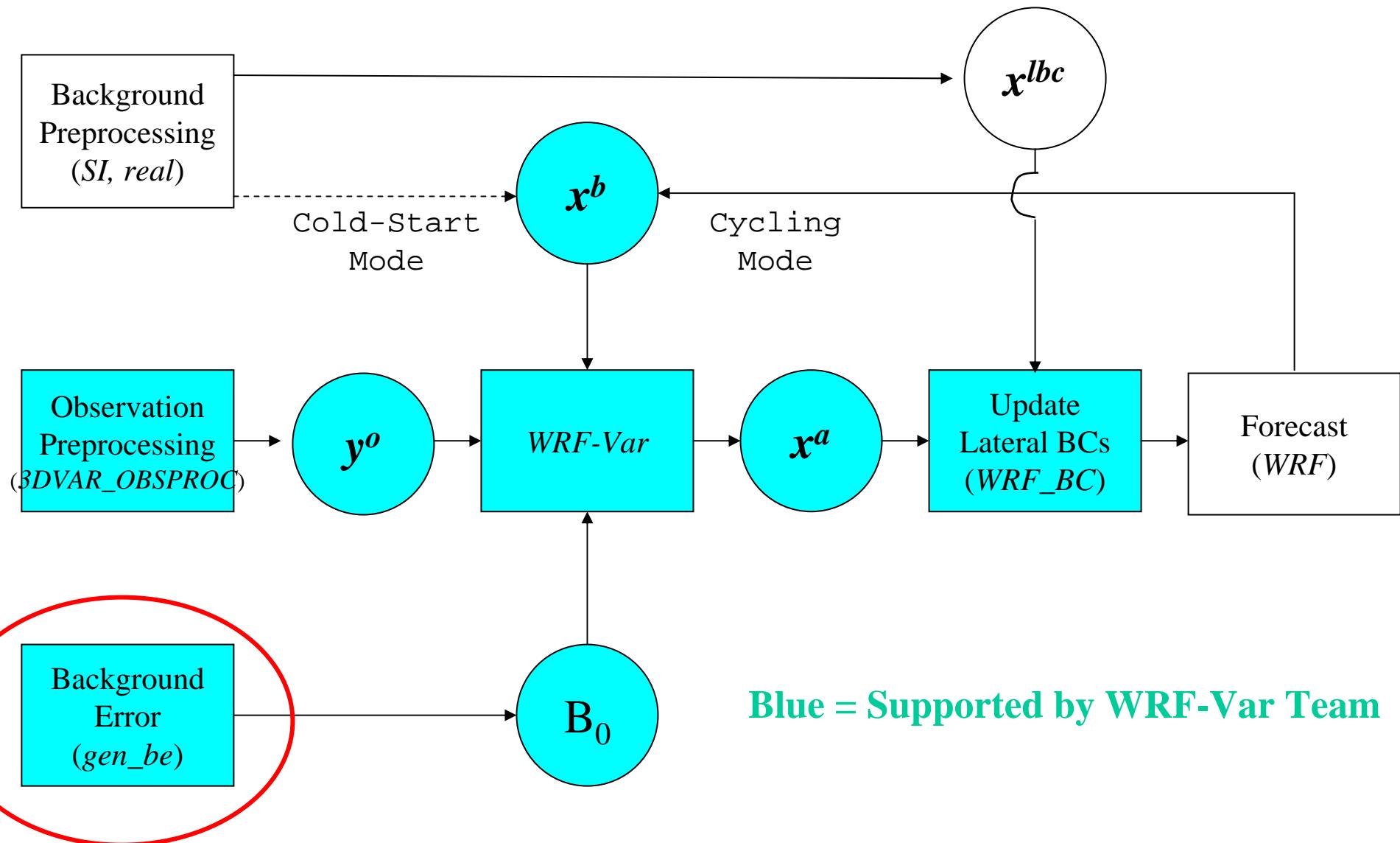
<http://www.mmm.ucar.edu/people/barker/>

# Outline of Talk

- Use of Background Errors in WRF-Var.
- WRF-Var Background Error Covariance Generation (gen\_be).

# **1. Use of Background Errors in WRF-Var**

# WRF-Var in the WRF Modeling System



# Background Error (BE) Estimation in WRF-Var

The number 1 question from WRF-Var users is

**“What background error covariances are best for my application?”.**

Procedure:

1. Use default statistics files supplied with code (MM5, GFS-based).
2. Create your own, once you have run your system for ~a few weeks.
3. Implement, tune, and iterate.

A new utility *gen\_be* has been developed at NCAR to calculate BEs.

# Model-Based Estimation of Climatological Background Errors

- Assume background error covariance estimated by model perturbations  $\mathbf{x}'$  :

$$\mathbf{B}_0 = \overline{(\mathbf{x}^b - \mathbf{x}^t)(\mathbf{x}^b - \mathbf{x}^t)^T} \approx \overline{\mathbf{x}' \mathbf{x}'^T}$$

Two ways of defining  $\mathbf{x}'$ :

- The NMC-method (Parrish and Derber 1992):

$$\mathbf{B}_0 = \overline{\mathbf{x}' \mathbf{x}'^T} \approx A \overline{(\mathbf{x}^{t2} - \mathbf{x}^{t1})(\mathbf{x}^{t2} - \mathbf{x}^{t1})^T}$$

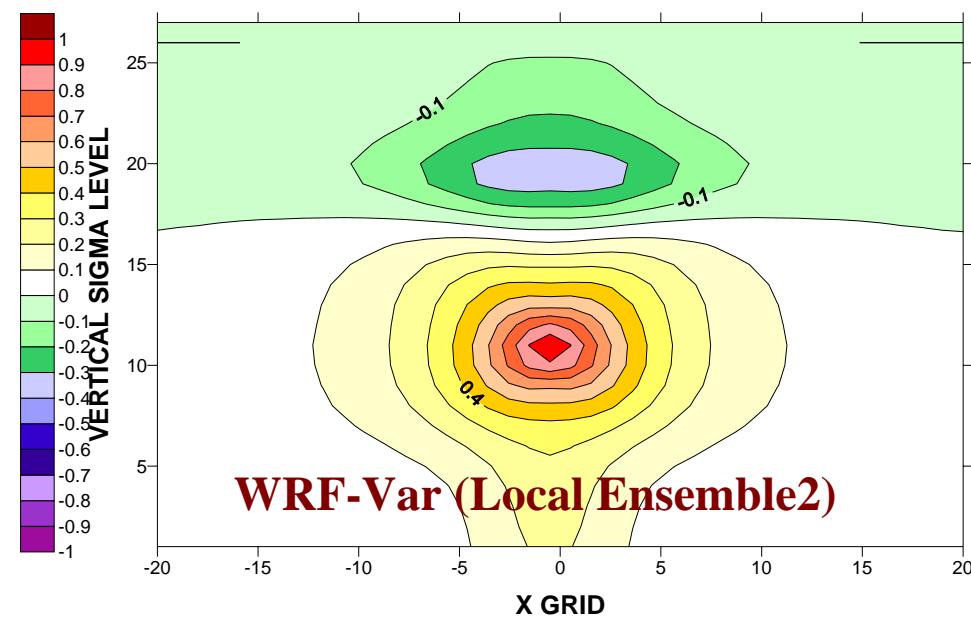
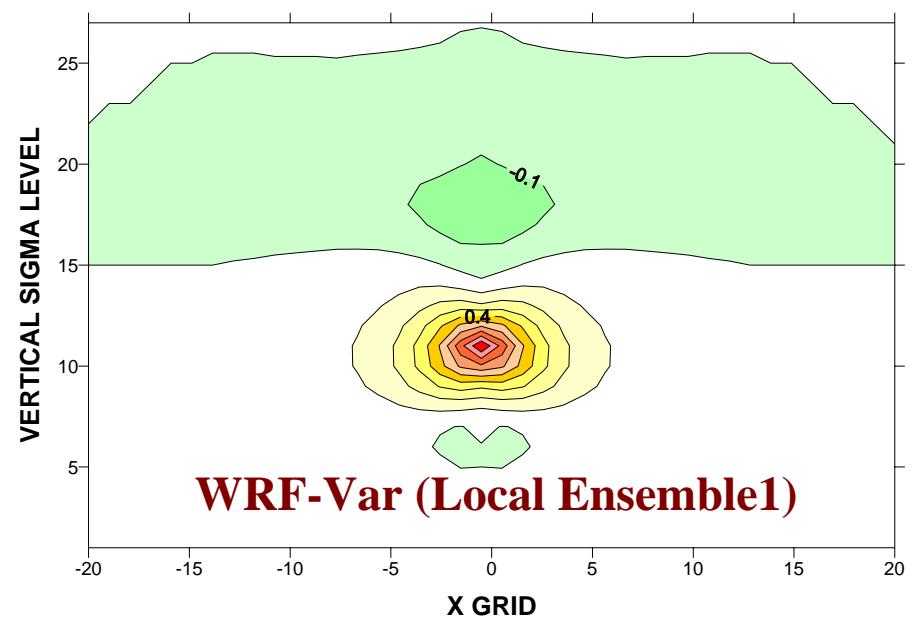
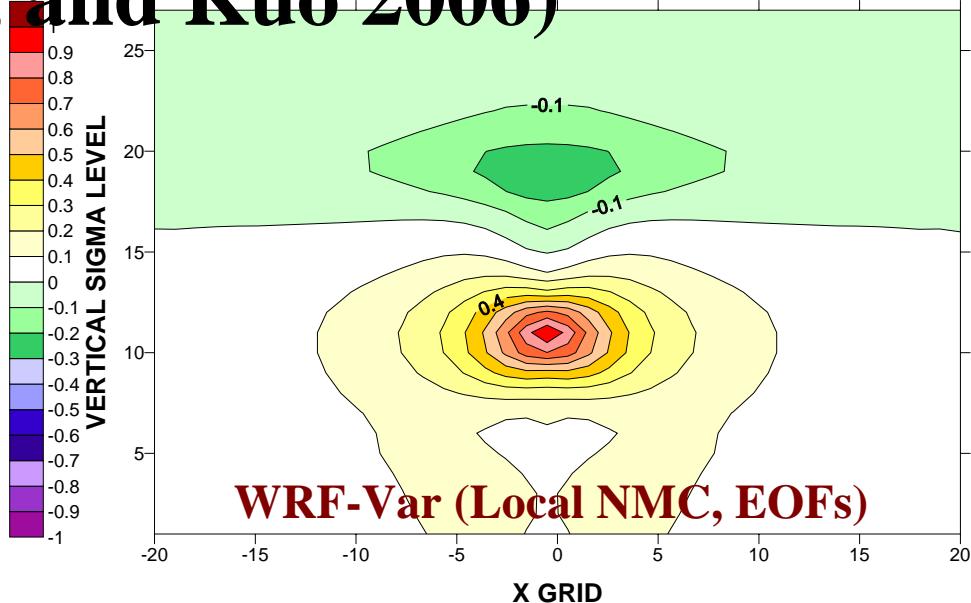
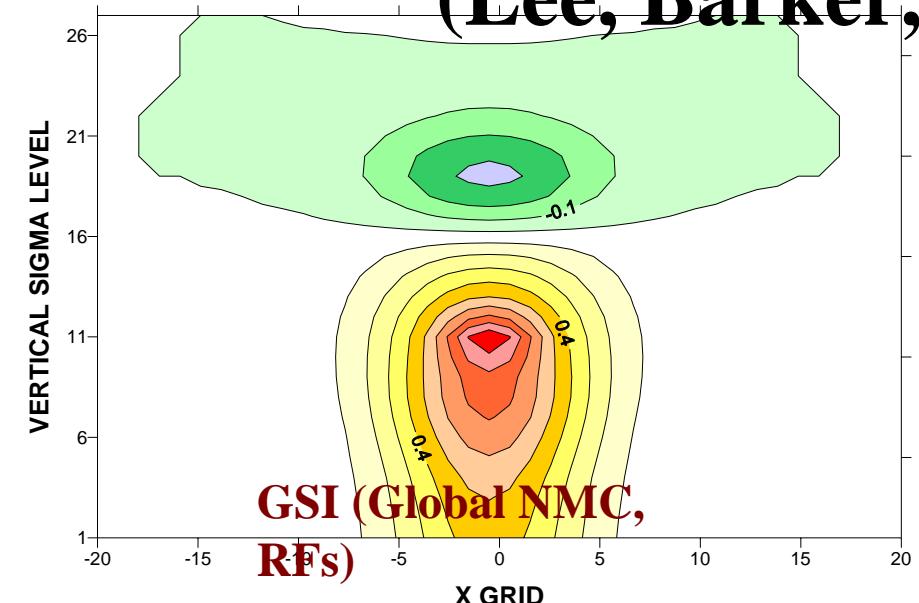
where e.g. t2=24hr, t1=12hr forecasts...

- ...or ensemble perturbations (Fisher 2003):

$$\mathbf{B}_0 = \overline{\mathbf{x}' \mathbf{x}'^T} \approx C \overline{(\mathbf{x}^k - \langle \mathbf{x} \rangle)(\mathbf{x}^k - \langle \mathbf{x} \rangle)^T}$$

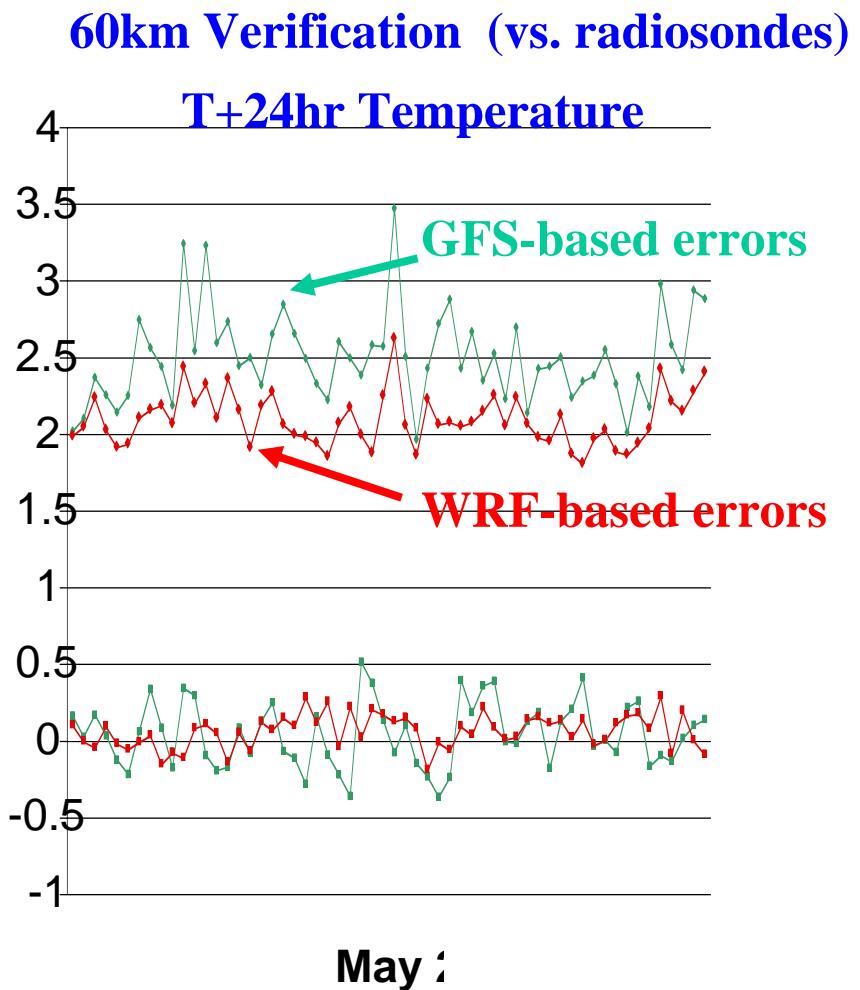
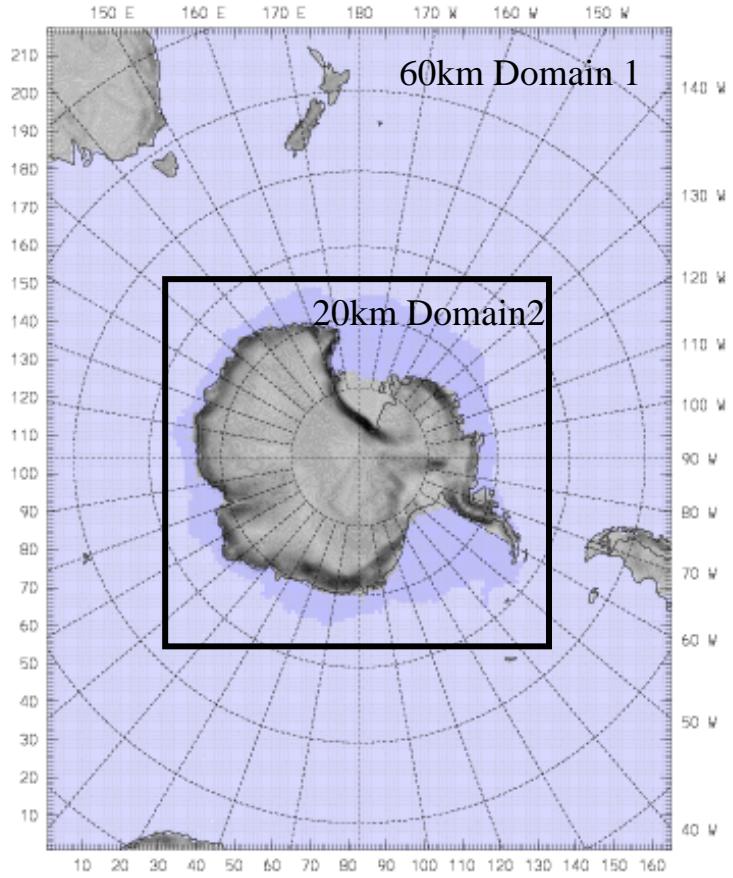
- Tuning via innovation vector statistics and/or variational methods.

# Analysis Sensitivity to Background Error Model (Lee, Barker, and Kuo 2006)



T increments : T Observation (1 Deg , 0.001 error around 850 hPa)

# Sensitivity to Forecast Error Covariances in Antarctica (Rizvi et al 2006)



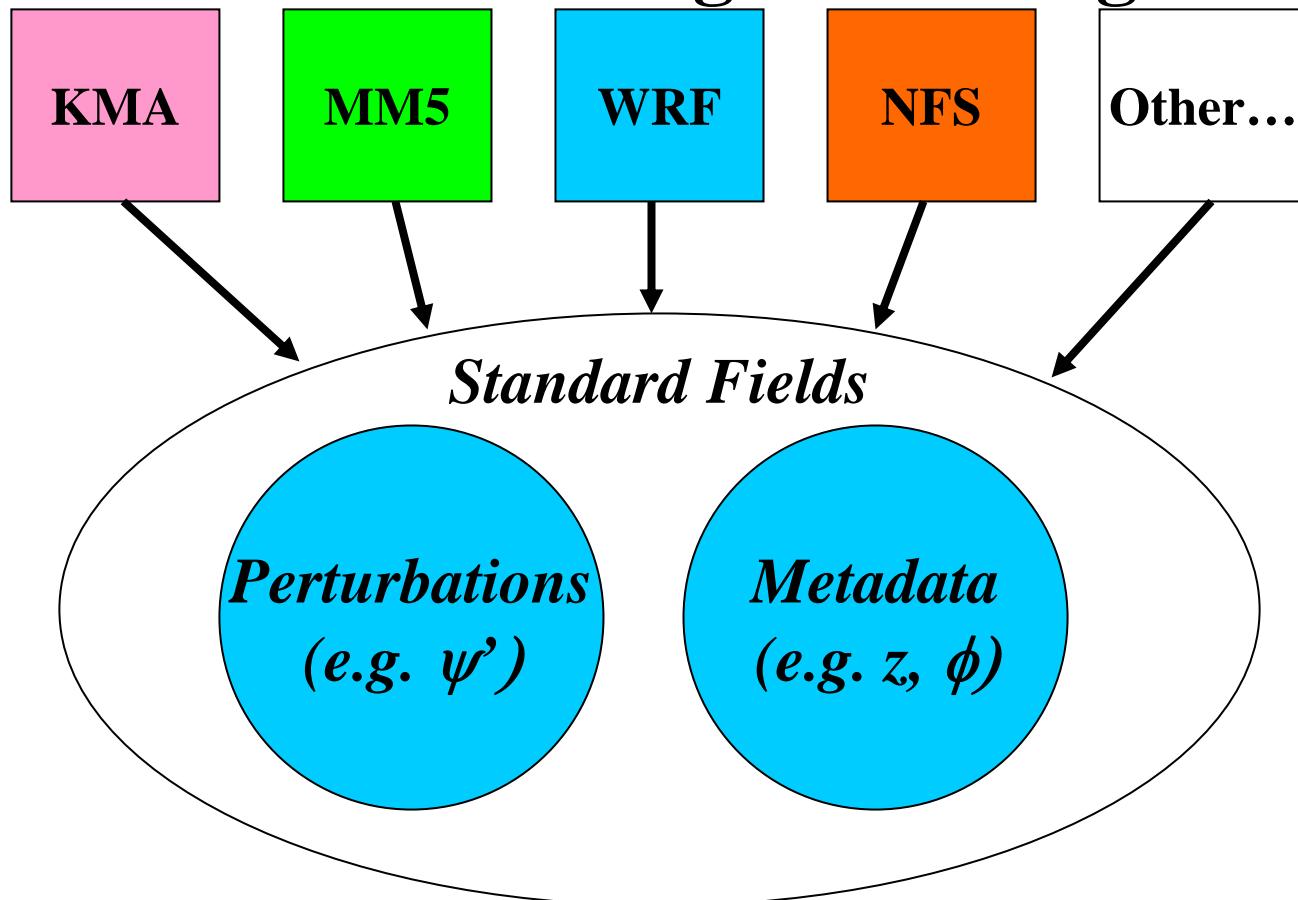
## **2. WRF-Var Background Error Covariance Generation (gen\_be)**

# Background Error (BE) Estimation in WRF-Var

A new utility *gen\_be* has been developed at NCAR to calculate BEs. The new *gen\_be* code is split into a number of stages:

- **Stage0: Convert model-specific data to “standard fields”.**
- Stage1: Remove time-domain mean from fields.
- Stage2: Calculate regression coefficients, and use them to define “unbalanced” control variables.
- Stage3: Calculate vertical error covariances (eigenvectors and eigenvalues) for control variables.
- Stage 4: Calculate horizontal error correlations: lengthscales (in regional domains), and “power spectra” in global domains.

# BE Generation: gen\_be\_stage0

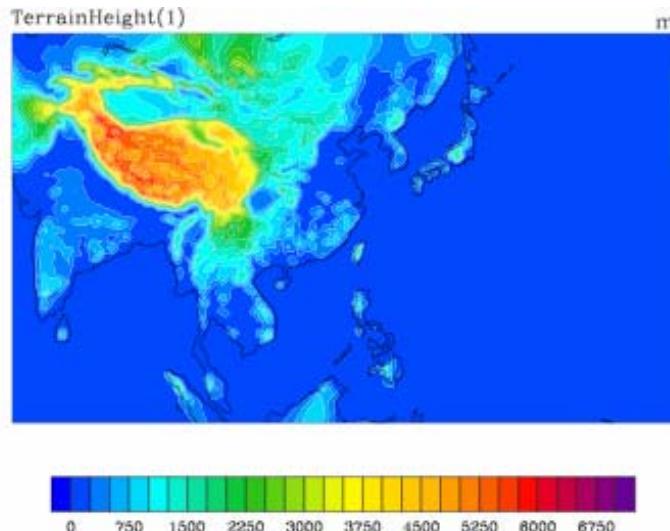


- Stage0 is the only stage that is model specific (e.g. gen\_be\_stage0\_nfs).
- Perturbed fields are either forecast differences (NMC-Method) or ensemble perturbations (ensemble method).
- Standard fields are perturbations of streamfunction, velocity potential, temperature, relative humidity, and surface pressure.

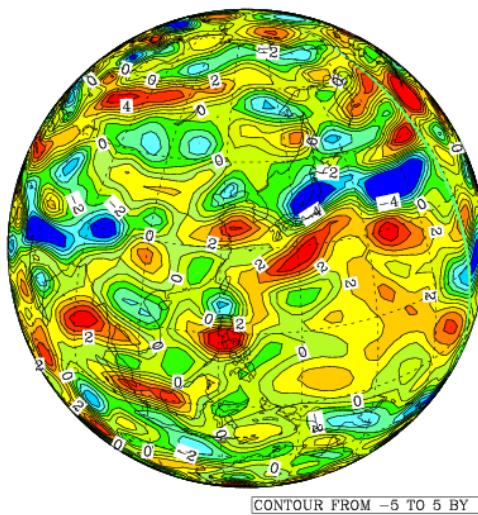
# Background Error (BE) Estimation for different domains/models/times

In the following results, we apply gen\_be stages 1-4 to a variety of models to see robustness of results:

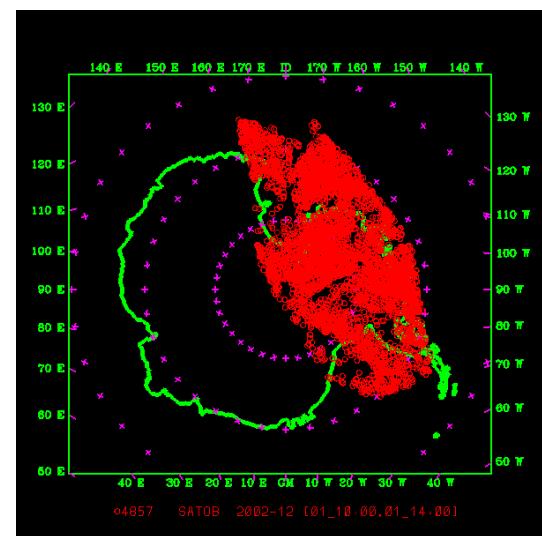
CWB 45km WRF



Korean T213 Global:



AMPS 30km WRF



# Background Error (BE) Estimation in WRF-Var

A new utility *gen\_be* has been developed at NCAR to calculate BEs. The new *gen\_be* code is split into a number of stages:

- Stage0: Convert model-specific data to “standard fields”.
- Stage1: Remove time-domain mean from fields.
- **Stage2: Calculate regression coefficients, and use them to define “unbalanced” control variables.**
- Stage3: Calculate vertical error covariances (eigenvectors and eigenvalues) for control variables.
- Stage 4: Calculate horizontal error correlations: lengthscales (in regional domains), and “power spectra” in global domains.

# WRF-Var Background Error Modeling

cv_options		2 (original MM5)	3(GSI)	4 (Global)	5(regional)
Analysis increments	$\mathbf{x}'$	$u', v', T', q', p_s'(i, j, k)$			
Change of Variable	$U_p$	$\psi', \chi', p_u', q'(i, j, k)$	$\psi', \chi_u', T_u', \tilde{r}', p_{su}'(i, j, k)$		
Vertical Covariances	$U_v$	$\mathbf{B} = \mathbf{E} \Lambda \mathbf{E}^T$	RF	$\mathbf{B} = \mathbf{E} \Lambda \mathbf{E}^T$	
Horizontal Correlations	$U_h$	RF	Spectral	RF	
Control Variables	$\mathbf{v}$	$\mathbf{v}(i, j, m)$	$\mathbf{v}(l, n, m)$	$\mathbf{v}(i, j, m)$	

$$\delta\mathbf{x}(t_0) = \mathbf{U}\mathbf{v} = \mathbf{U}_p \mathbf{U}_v \mathbf{U}_h \mathbf{v}$$

Define control variables:

$$\psi'$$

$$r' = q'/q_s(T_b, q_b, p_b)$$

$$\chi' = \chi_u' + \chi_b'(\psi')$$

$$T' = T_u' + T_b'(\psi')$$

$$p_s' = p_{su}' + p_{sb}'(\psi')$$

# 3D-Var Statistical Balance Constraints

- Define statistical balance after Wu et al (2002):

$$\dot{\chi}_b = c \psi' \quad T_b'(k) = \sum_{k1} G(k, k1) \psi'(k1) \quad p_{sb}' = \sum_k W(k) \psi'(k)$$

# 3D-Var Statistical Balance Constraints

- Define statistical balance after Wu et al (2002):

Regression Coefficients

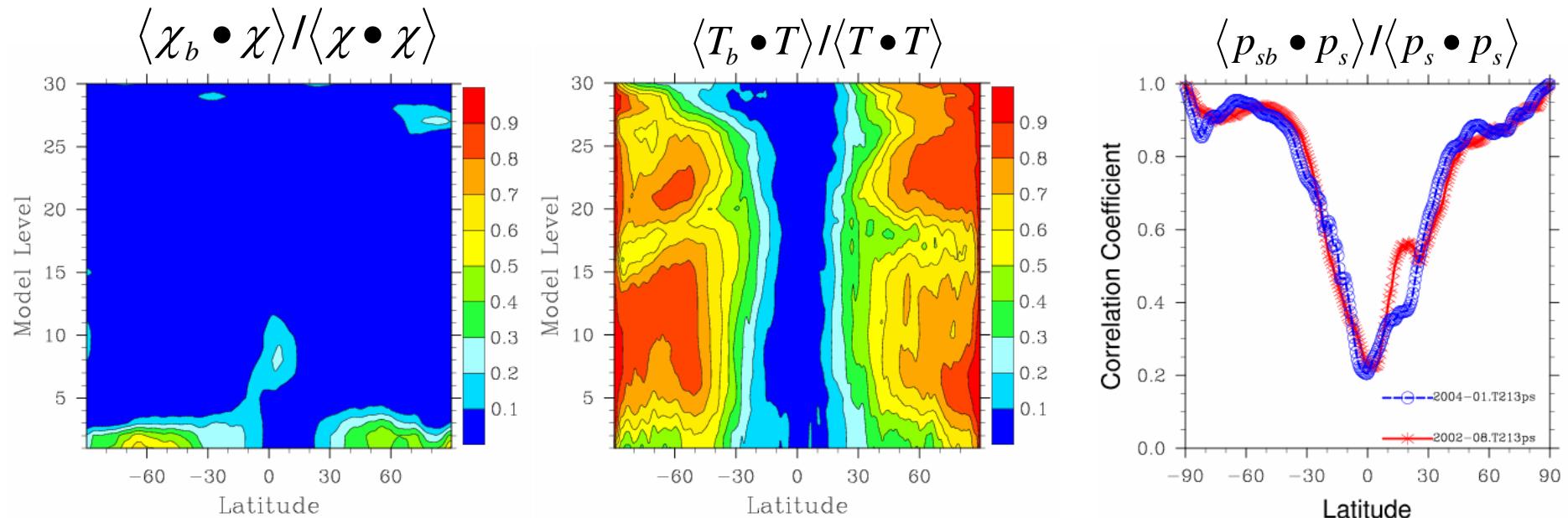
$$\chi_b' = c \psi' \quad T_b'(k) = \sum_{k1} G(k, k1) \psi'(k1) \quad p_{sb}' = \sum_k W(k) \psi'(k)$$

# 3D-Var Statistical Balance Constraints

- Define statistical balance after Wu et al (2002):

$$\chi'_b = c \psi' \quad T'_b(k) = \sum_{k1} G(k, k1) \psi'(k1) \quad p'_{sb} = \sum_k W(k) \psi'(k)$$

- How good are these balance constraints? Test on KMA global model data. Plot correlation between “Full” and balanced components of field:



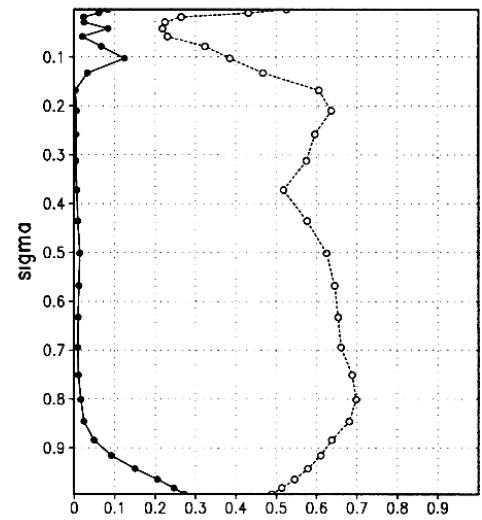
# Stage2: CWB/KMA/AMPS Comparison

Calculate domain averaged correlations:

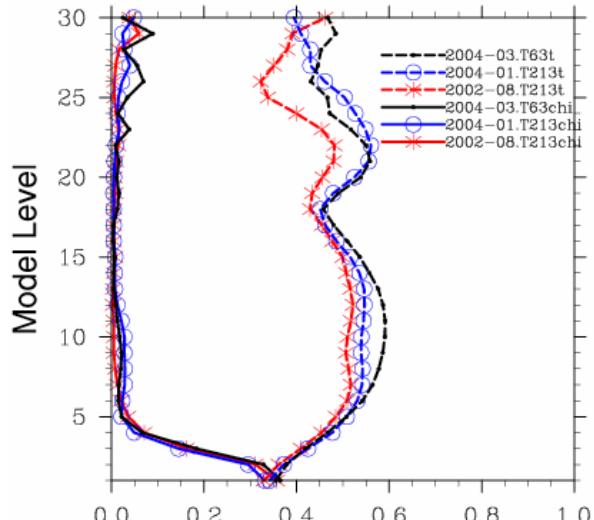
$$\langle T_b \bullet T \rangle / \langle T \bullet T \rangle \quad \langle \chi_b \bullet \chi \rangle / \langle \chi \bullet \chi \rangle$$

For a variety of models using NMC-method.

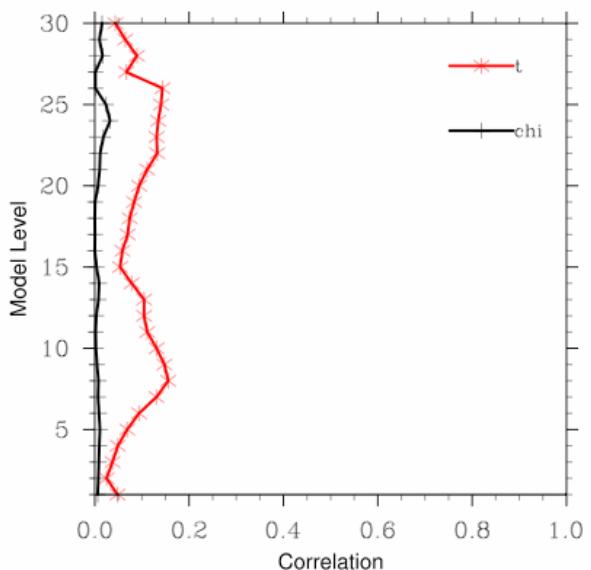
NCEP (Wu et al 2002):



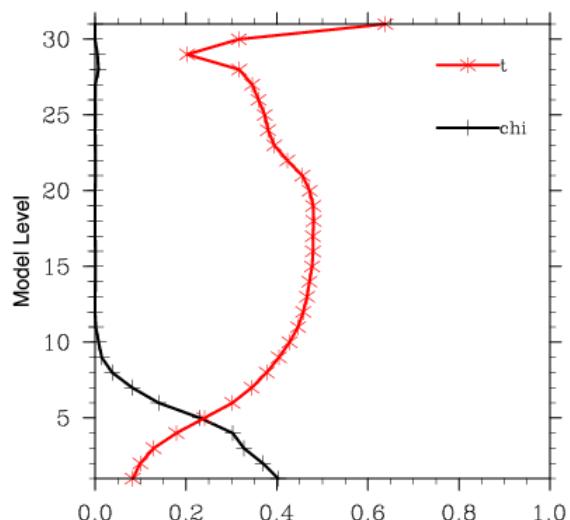
Korean T213 Global:



CWB 45km WRF:



AMPS 30km WRF:



# Background Error (BE) Estimation in WRF-Var

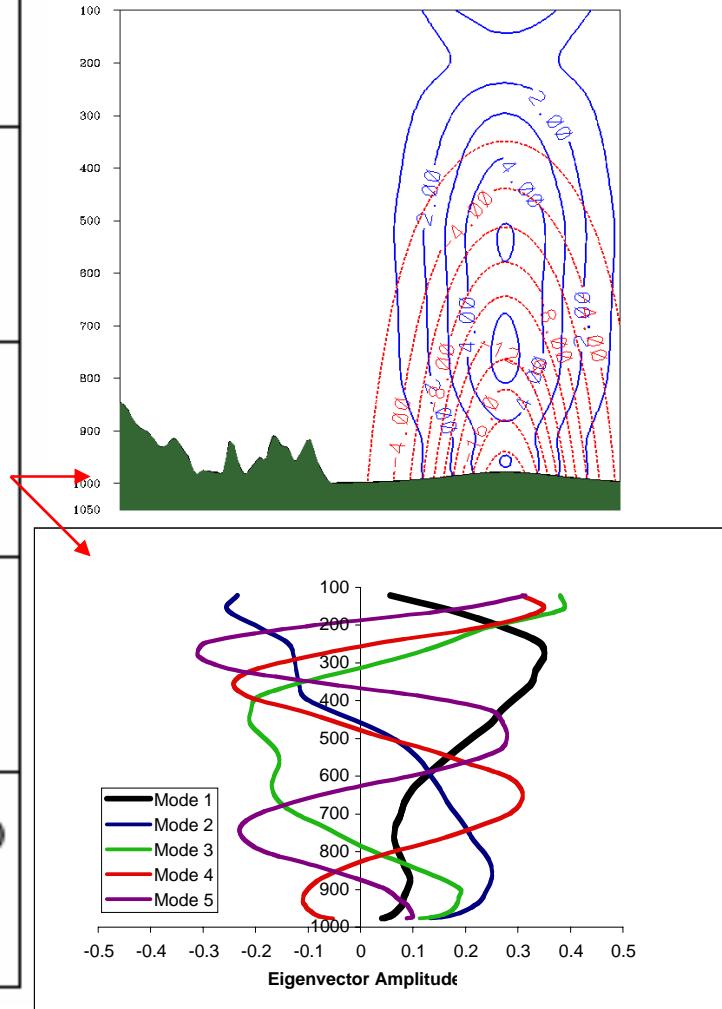
A new utility *gen\_be* has been developed at NCAR to calculate BEs. The new *gen\_be* code is split into a number of stages:

- Stage0: Convert model-specific data to “standard fields”.
- Stage1: Remove time-domain mean from fields.
- Stage2: Calculate regression coefficients, and use them to define “unbalanced” control variables.
- **Stage3: Calculate vertical error covariances (eigenvectors and eigenvalues) for control variables.**
- Stage 4: Calculate horizontal error correlations: lengthscales (in regional domains), and “power spectra” in global domains.

# WRF-Var Background Error Modeling

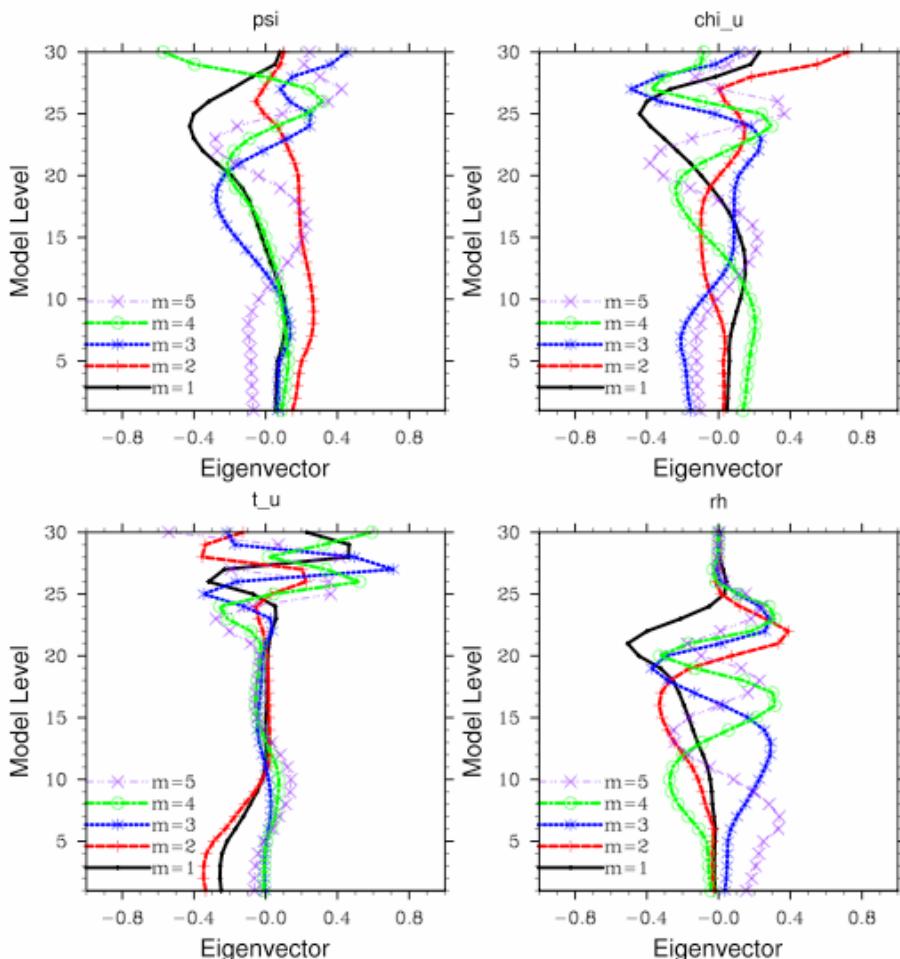
cv_options		2 (original MM5)	3(GSI)	4 (Global)	5(regional)
Analysis increments	$\mathbf{x}^t$	$u^t, v^t, T^t, q^t, p_s^t(i, j, k)$			
Change of Variable	$U_p$	$\psi^t, \chi^t, p_u^t, q^t(i, j, k)$	$\psi^t, \chi_u^t, T_u^t, \tilde{r}^t, p_{su}^t(i, j, k)$		
Vertical Covariances	$U_v$	$\mathbf{B} = \mathbf{E} \Lambda \mathbf{E}^T$	RF	$\mathbf{B} = \mathbf{E} \Lambda \mathbf{E}^T$	
Horizontal Correlations	$U_h$	RF	Spectral	RF	
Control Variables	$\mathbf{v}$	$\mathbf{v}(i, j, m)$	$\mathbf{v}(l, n, m)$	$\mathbf{v}(i, j, m)$	

$$\delta\mathbf{x}(t_0) = \mathbf{U}\mathbf{v} = \mathbf{U}_p \mathbf{U}_v \mathbf{U}_h \mathbf{v}$$

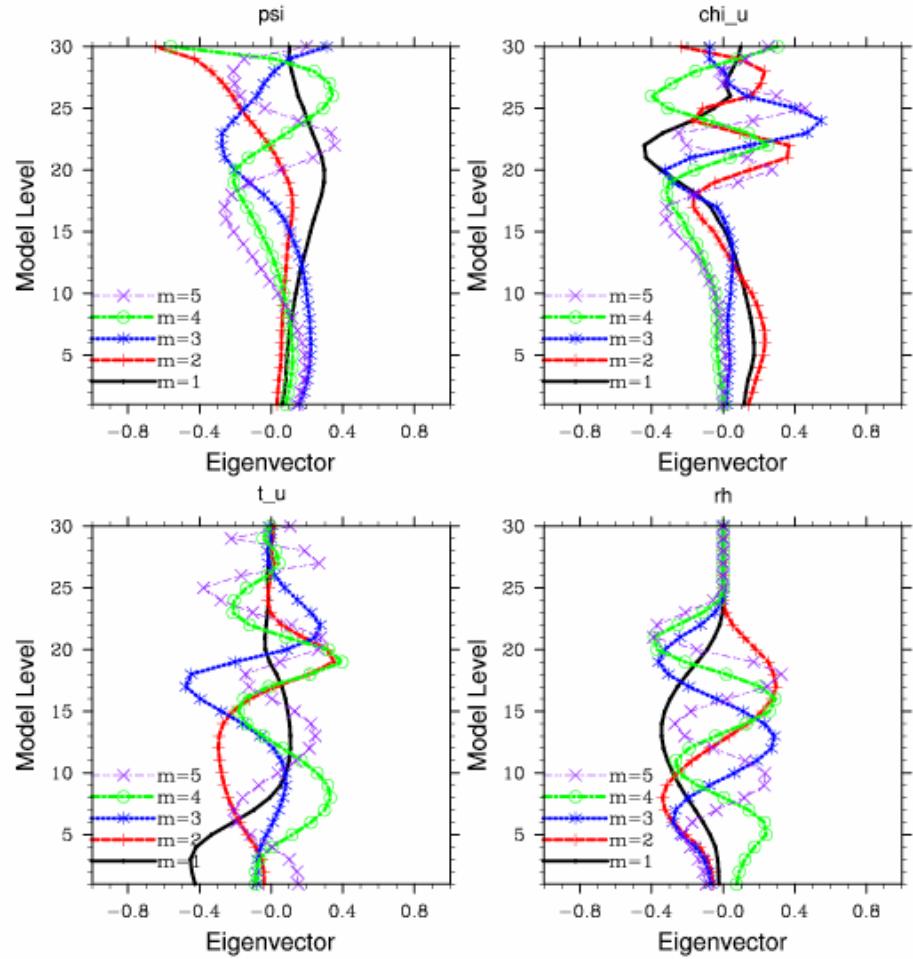


# Comparison of Domain-Averaged Eigenvectors $E_v^g$

CWB 45km WRF:



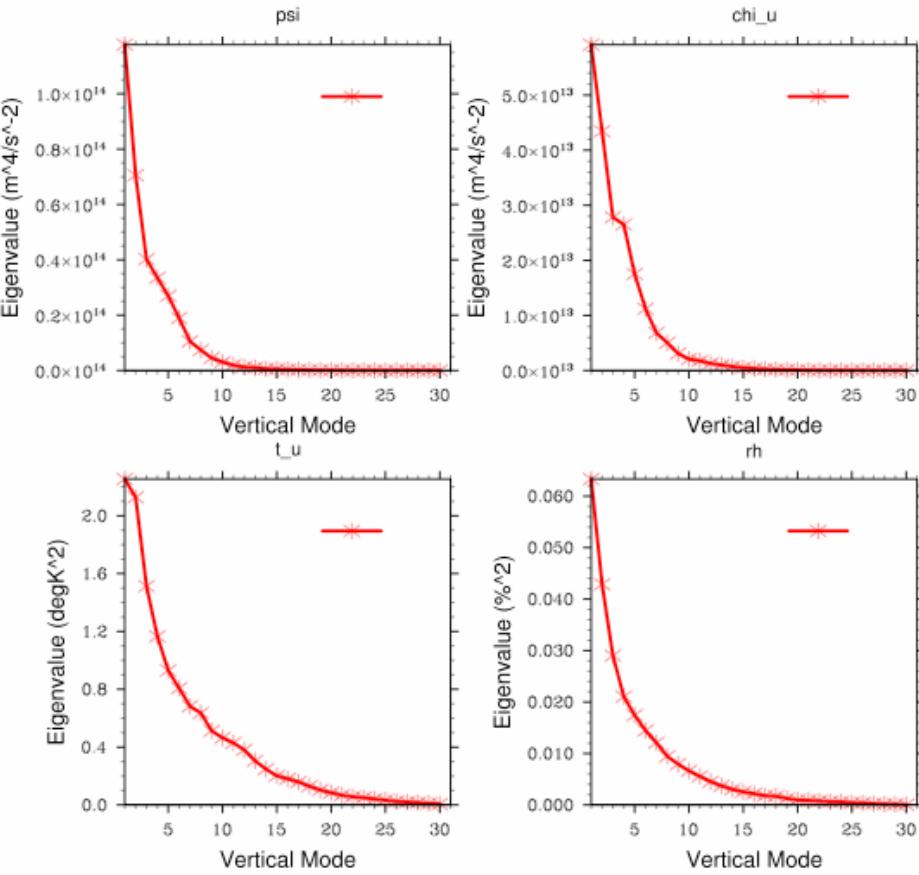
Korean T213 Global:



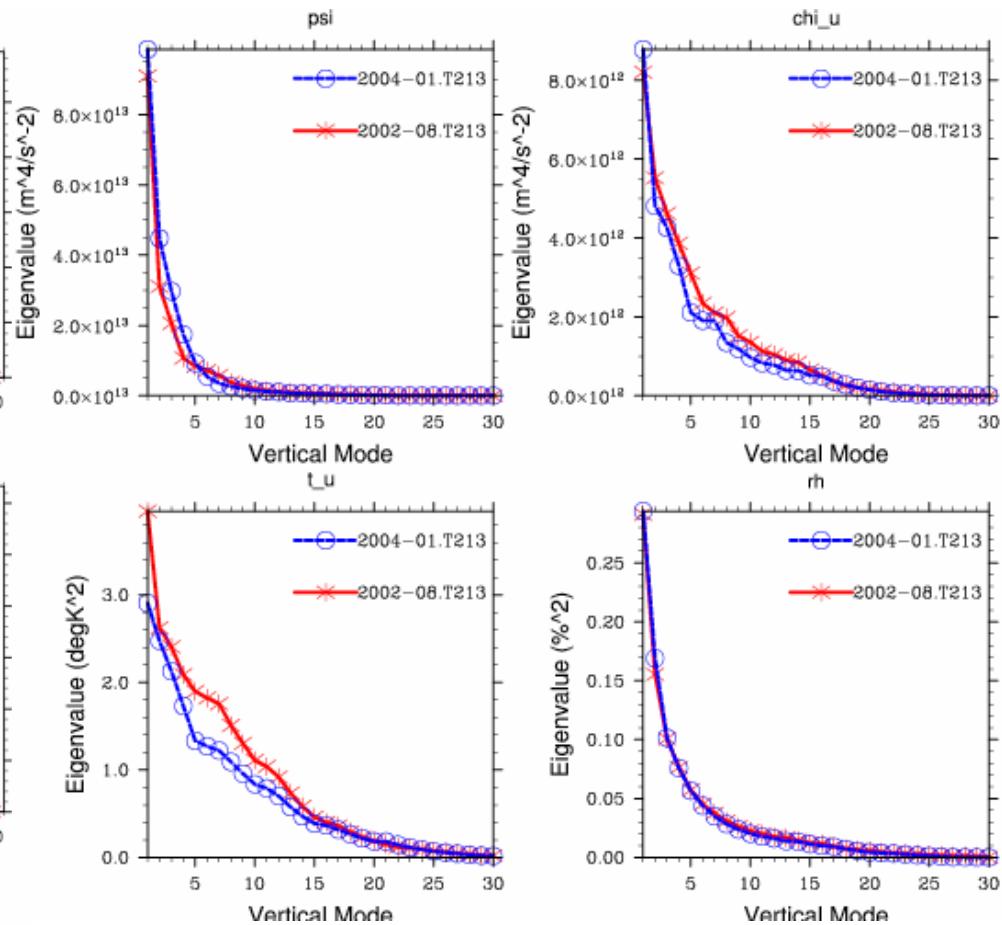
Conclusion: Significant differences in regional/global models (as one might expect), so it is important to calculate domain-specific background error correlations.

# Comparison of Domain-Averaged Eigenvalues $\Lambda_v^g$

CWB 45km WRF:



Korean T213 Global:



Conclusion: Significant differences in regional/global models (as one might expect), so it is important to calculate domain-specific background error variances.

# Background Error (BE) Estimation in WRF-Var

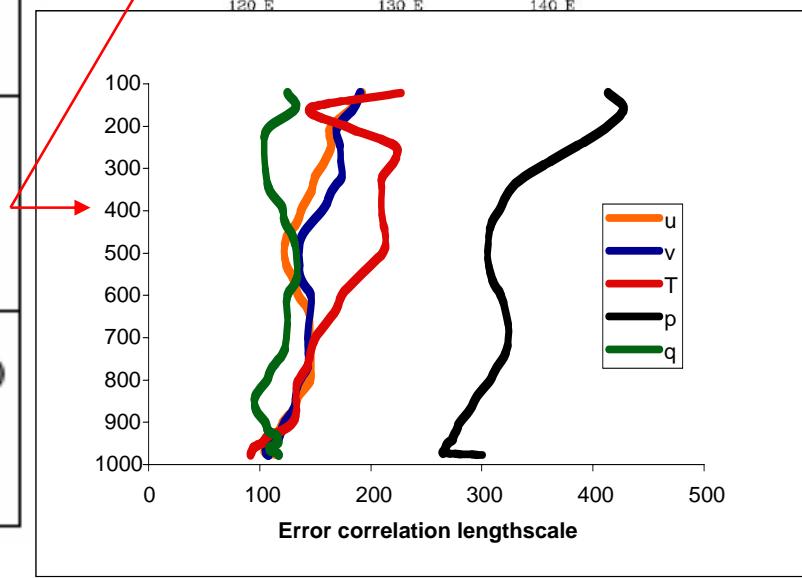
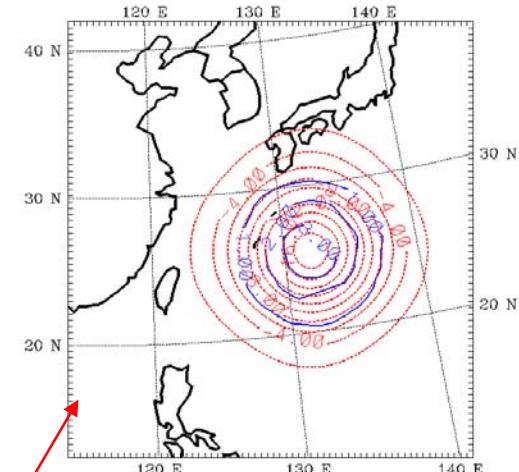
A new utility *gen\_be* has been developed at NCAR to calculate BEs. The new *gen\_be* code is split into a number of stages:

- Stage0: Convert model-specific data to “standard fields”.
- Stage1: Remove time-domain mean from fields.
- Stage2: Calculate regression coefficients, and use them to define “unbalanced” control variables.
- Stage3: Calculate vertical error covariances (eigenvectors and eigenvalues) for control variables.
- **Stage 4: Calculate horizontal error correlations: lengthscales (in regional domains), and “power spectra” in global domains.**

# WRF-Var Background Error Modeling

cv_options		2 (original MM5)	3(GSI)	4 (Global)	5(regional)
Analysis increments	$\mathbf{x}^*$	$u^*, v^*, T^*, q^*, p_s^*(i, j, k)$			
Change of Variable	$U_p$	$\psi^*, \chi^*, p_u^*, q^*(i, j, k)$	$\psi^*, \chi_u^*, T_u^*, \tilde{r}^*, p_{su}^*(i, j, k)$		
Vertical Covariances	$U_v$	$\mathbf{B} = \mathbf{E} \Lambda \mathbf{E}^T$	RF	$\mathbf{B} = \mathbf{E} \Lambda \mathbf{E}^T$	
Horizontal Correlations	$U_h$	RF	Spectral	RF	
Control Variables	$\mathbf{x}$	$\mathbf{v}(i, j, m)$	$\mathbf{v}(l, n, m)$	$\mathbf{v}(i, j, m)$	

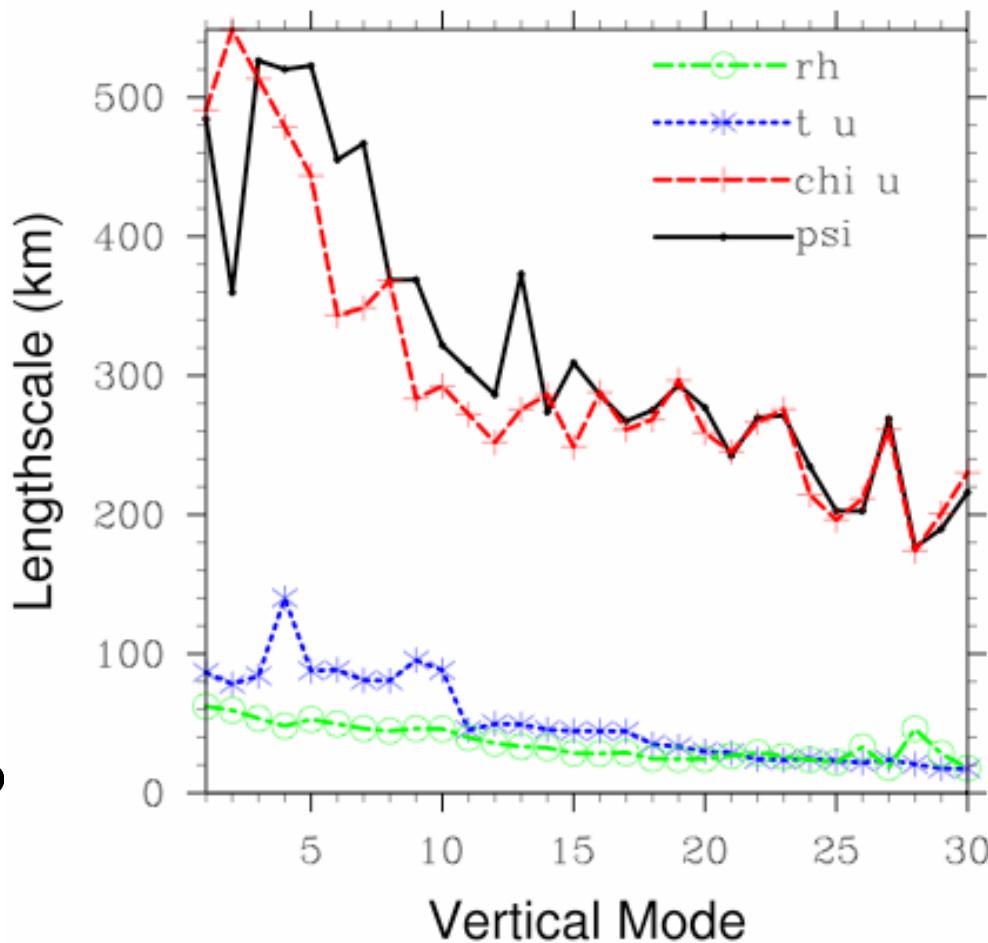
$$\delta\mathbf{x}(t_0) = \mathbf{U}\mathbf{v} = \mathbf{U}_p \mathbf{U}_v \mathbf{U}_h \mathbf{v}$$



## Stage4: Regional model (CWB Sept. 2003) data

Horizontal error correlation lengthscales are produced as follows:

- a. Bin each 2D perturbation field as a function of gridpoint separation.
- b. Calculate correlation matrix for all data.
- c. Fit Gaussian curve to data to estimate horizontal error correlation lengthscales.



# Conclusions

- Background Error Statistics are a crucial input to 3/4D-Var systems.
- BE estimation methods are somewhat ad-hoc, and require significant efforts to accumulate the necessary forecast input data (e.g. 1 month 24hr forecasts)
- A new utility *gen\_be* has been written to calculate BEs for WRF-Var applications (for a variety of models, and techniques).
- The output of *gen\_be* must be further tuned to optimize WRF-Var performance. A number of additional utilities are available to perform this work.

# Components of Global Background Error Statistics

- a) **Regression Coefficients** (used to derive “balanced” components of fields) after Wu et al (2002) (output from stage2):

$$\chi_b' = c \psi' \quad T_b'(k) = \sum_{k1} G(k, k1) \psi'(k1) \quad p_{sb}' = \sum_k W(k) \psi'(k)$$

- b) **Eigenvectors/eigenvalues** of vertical component of background error (output from stage3):

Domain/time averaged “global” values -  $\mathbf{P}_{fv}^g = \mathbf{E}_v^g \Lambda_v^g \mathbf{E}_v^{gT}$   
 Longitude/time averaged “local” values -  $\mathbf{P}_{fv}^l(\phi) = \mathbf{E}_{fv}^l(\phi) \Lambda_{fv}^l(\phi) \mathbf{E}_{fv}^{lT}(\phi)$

- a) **Power spectra** (model horizontal error correlations for each vertical mode - output from stage4\_global):

$$F^m(\phi) = \frac{1}{I} \sum_{i=1}^I F(\lambda_i, \phi) e^{-im\lambda_i} \quad \longrightarrow \quad F_n^m = \sum_{j=1}^J W_j F^m(\mu_j) P_n^m(\mu_j) \quad \longrightarrow \quad D_n = \sum_{m=-n}^n (F_n^m)^2$$

# Components of Regional Background Error Statistics

- a) **Regression Coefficients** (used to derive “balanced” components of fields) after Wu et al (2002) (output from stage2):

$$\chi_b' = c \psi' \quad T_b'(k) = \sum_{k1} G(k, k1) \psi'(k1) \quad p_{sb}' = \sum_k W(k) \psi'(k)$$

- b) **Eigenvectors/eigenvalues** of vertical component of background error (output from stage3):

Domain/time averaged “global” values -  $\mathbf{P}_{fv}^g = \mathbf{E}_v^g \Lambda_v^g \mathbf{E}_v^{gT}$   
Longitude/time averaged “local” values -  $\mathbf{P}_{fv}^l(\phi) = \mathbf{E}_{fv}^l(\phi) \Lambda_{fv}^l(\phi) \mathbf{E}_{fv}^{lT}(\phi)$

- a) **Lengthscales** (model horizontal error correlations  $s$  for each vertical mode - output from stage4Regional):

$$B(r) = B(0) \exp[-r^2 / 8s^2]$$