The Advanced Research WRF (ARW) Dynamics Solver

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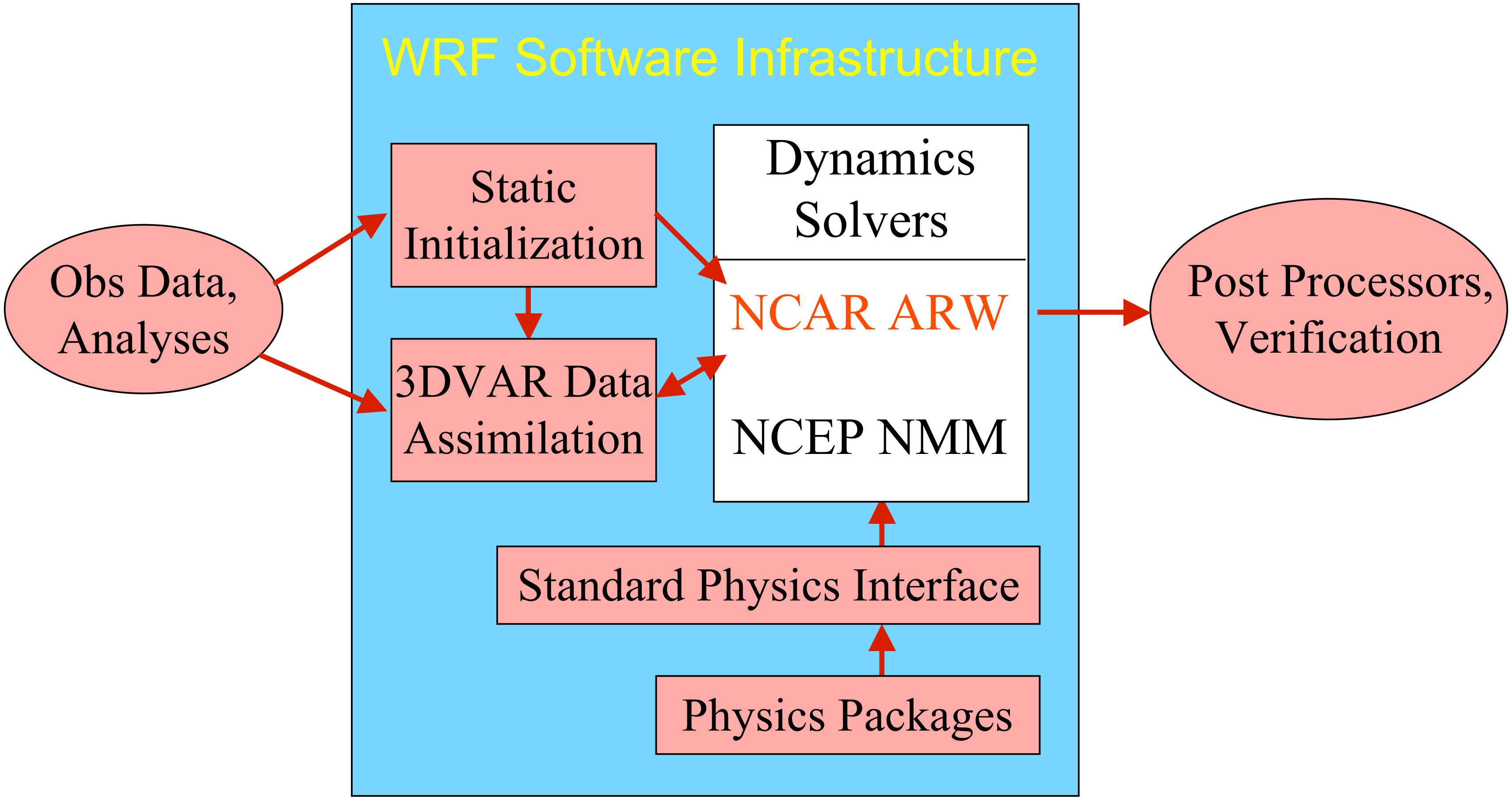

WRF Software Infrastructure
Obs Data,
Analyses
Static
Initialization
3DVAR Data
Assimilation
Dynamics
Solvers
NCAR ARW
NCEP NMM
Standard Physics Interface
Physics Packages
Post Processors,
Verification

ARW Dynamical Solver

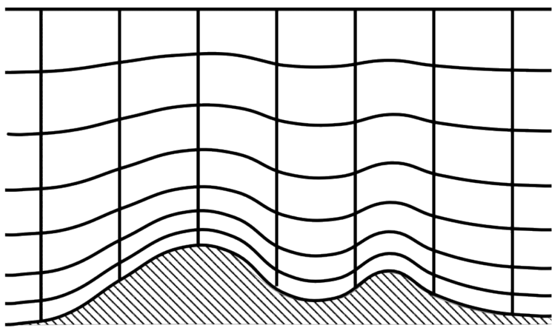
- Terrain representation
- Vertical coordinate
- Equations / variables
- Grid staggering
- Time integration scheme
- Advection scheme
- Time step parameters
- Filters
- Boundary conditions
- Nesting
- Map projections

WRF-ARW	MM5
• Terrain-following hydrostatic pressure vertical coordinate	• Terrain-following height (sigma-z) vertical coordinate
• Arakawa C-grid	• B-grid
• 3rd order Runge-Kutta split-explicit time integration	• 1st order (time-filtered) Leapfrog time integration
• Conserves mass, momentum, entropy, and scalars using flux form prognostic equations	• Advective formulation (no conservation properties)
• 5th order upwind or 6th order centered differencing for advection	• 2nd order centered differencing for advection

ARW, Terrain Representation

Lower boundary condition for the geopotential $(\phi = gz)$ specifies the terrain elevation, and specifying the lowest coordinate surface to be the terrain results in a terrain-following coordinate.

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} + \omega\frac{\partial \phi}{\partial \eta} = gw$$



Vertical coordinate:

hydrostatic pressure π $\qquad \eta = \frac{(\pi - \pi_t)}{\mu}, \qquad \mu = \pi_s - \pi_t$

Flux-Form Equations in ARW

Hydrostatic pressure coordinate:

hydrostatic pressure π

$$\eta = \frac{(\pi - \pi_t)}{\mu}, \qquad \mu = \pi_s - \pi_t$$

Conserved state variables:

$$\mu, \quad U = \mu u, \quad V = \mu v, \quad W = \mu w, \quad \Theta = \mu\theta$$

Non-conserved state variable: $\phi = gz$

Flux-Form Equations in ARW

Inviscid, 2-D equations without rotation:

$$\frac{\partial U}{\partial t} + \mu\alpha\frac{\partial p}{\partial x} + \frac{\partial p}{\partial \eta}\frac{\partial \phi}{\partial x} = -\frac{\partial Uu}{\partial x} - \frac{\partial \Omega u}{\partial \eta}$$

$$\frac{\partial W}{\partial t} + g\left(\mu - \frac{\partial p}{\partial \eta}\right) = -\frac{\partial Uw}{\partial x} - \frac{\partial \Omega w}{\partial \eta}$$

$$\frac{\partial \Theta}{\partial t} + \frac{\partial U\theta}{\partial x} + \frac{\partial \Omega\theta}{\partial \eta} = \mu Q$$

$$\frac{\partial \mu}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial \Omega}{\partial \eta} = 0$$

$$\frac{d\phi}{dt} = gw$$

Diagnostic relations:

$$\frac{\partial \phi}{\partial \eta} = -\mu\alpha, \qquad p = \left(\frac{R\theta}{p_0\alpha}\right)^{\gamma}, \qquad \Omega = \mu\dot{\eta}$$

Moist Equations in ARW

Moist Equations:

$$\frac{\partial U}{\partial t} + \alpha\mu_d \frac{\partial p}{\partial x} + \frac{\alpha}{\alpha_d}\frac{\partial p}{\partial \eta}\frac{\partial \phi}{\partial x} = -\frac{\partial U u}{\partial x} - \frac{\partial \Omega u}{\partial \eta}$$

$$\frac{\partial W}{\partial t} + g\left(\mu_d - \frac{\alpha}{\alpha_d}\frac{\partial p}{\partial \eta}\right) = -\frac{\partial U w}{\partial x} - \frac{\partial \Omega w}{\partial \eta}$$

$$\frac{\partial \mu_d}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial \Omega}{\partial \eta} = 0$$

$$\frac{\partial (\mu_d q_{v,l})}{\partial t} + \frac{\partial (U q_{v,l})}{\partial x} + \frac{\partial (\Omega q_{v,l})}{\partial \eta} = \mu Q_{v,l}$$

Diagnostic relations:

$$\frac{\partial \phi}{\partial \eta} = -\alpha_d \mu_d, \qquad p = \left(\frac{R\Theta}{p_o \mu_d \alpha_v}\right)^{\gamma}$$

Time Integration in ARW

3rd Order Runge-Kutta time integration

advance $\phi^{t} \rightarrow \phi^{t+\Delta t}$

$$\phi^{*} = \phi^{t} + \frac{\Delta t}{3} R\left(\phi^{t}\right)$$

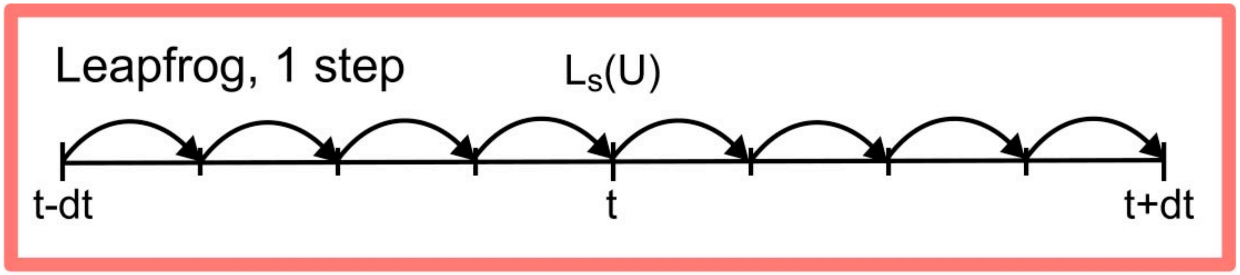
$$\phi^{**} = \phi^{t} + \frac{\Delta t}{2} R\left(\phi^{*}\right)$$

$$\phi^{t+\Delta t} = \phi^{t} + \Delta t\, R\left(\phi^{**}\right)$$

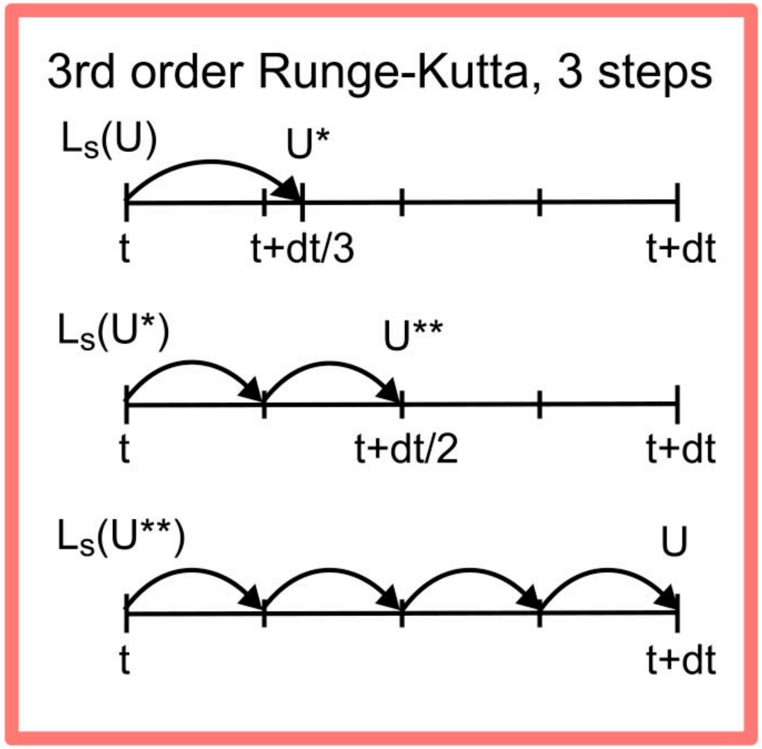
Amplification factor $\phi_{t} = i k \phi; \quad \phi^{n+1} = A \phi^{n}; \quad |A| = 1 - \frac{(k\Delta t)^{4}}{24}$

Time-Split Runge-Kutta Integration Scheme

Integrate
$U_t = L_f(U) + L_s(U)$



- LF is formally 1st order accurate, RK3 is 3rd order accurate
- RK3 stable for centered and upwind advection schemes, LF only stable for centered schemes.
- RK3 is stable for timesteps 2 to 3 times larger than LF.
- LF requires only one advection evaluation per timestep, RK3 requires three per timestep.



WRF ARW Model Integration Procedure

Begin time step

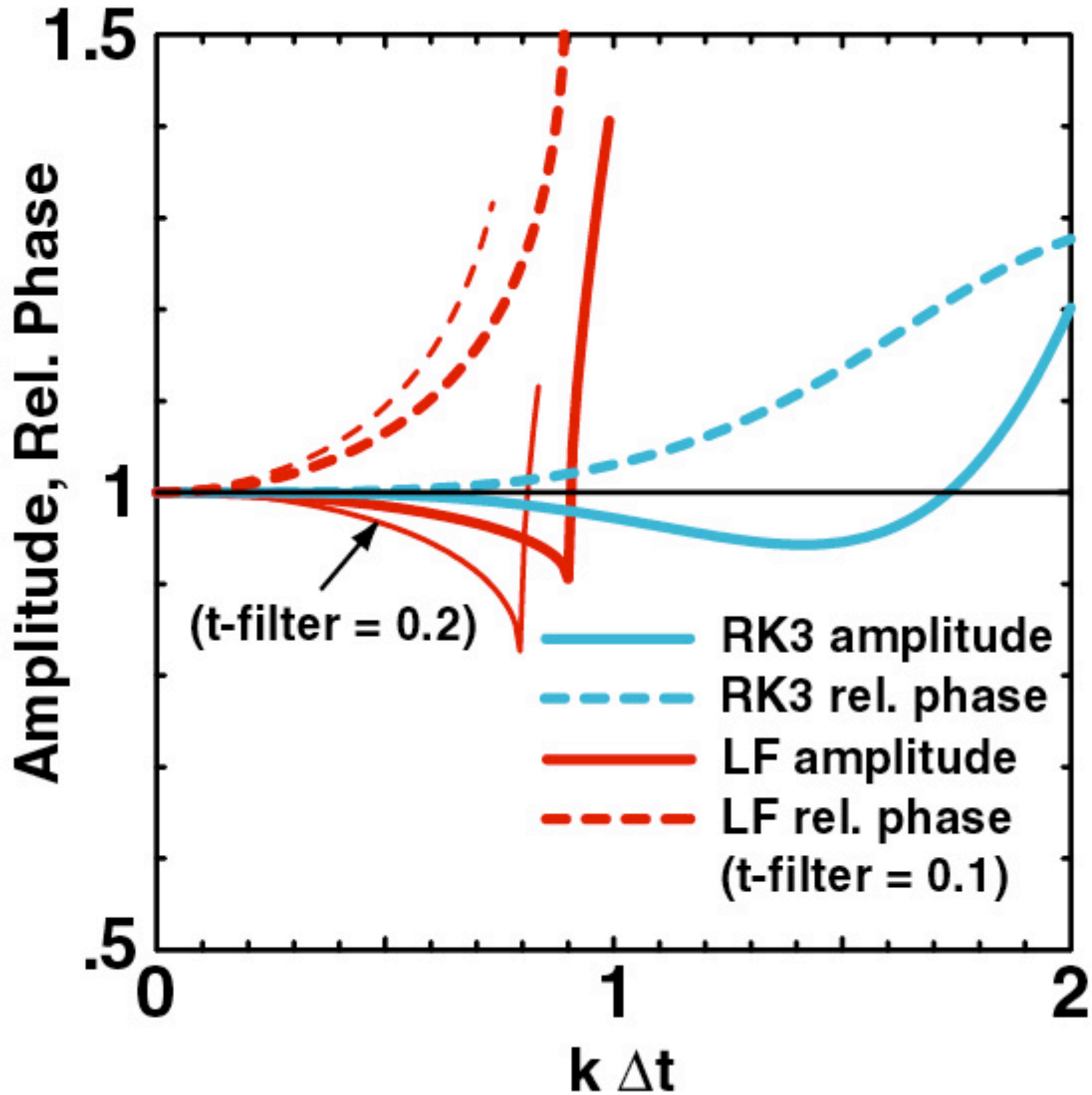
- Runge-Kutta loop (steps 1, 2, and 3)
  - (i) advection, p-grad, buoyancy using $(\phi^{t}, \phi^{*}, \phi^{**})$
  - (ii) physics if step 1, save for steps 2 and 3
  - (iii) mixing, other non-RK dynamics, save…
  - (iv) assemble dynamics tendencies
  - Acoustic step loop
    - (i) advance U,V, then μ, Θ, then w, ϕ
    - (ii) time-average U,V, Ω
  - End acoustic loop
  - Advance scalars using time-averaged U,V, Ω
- End Runge-Kutta loop
- Adjustment physics (currently microphysics)

End time step

Phase and amplitude errors for LF, RK3

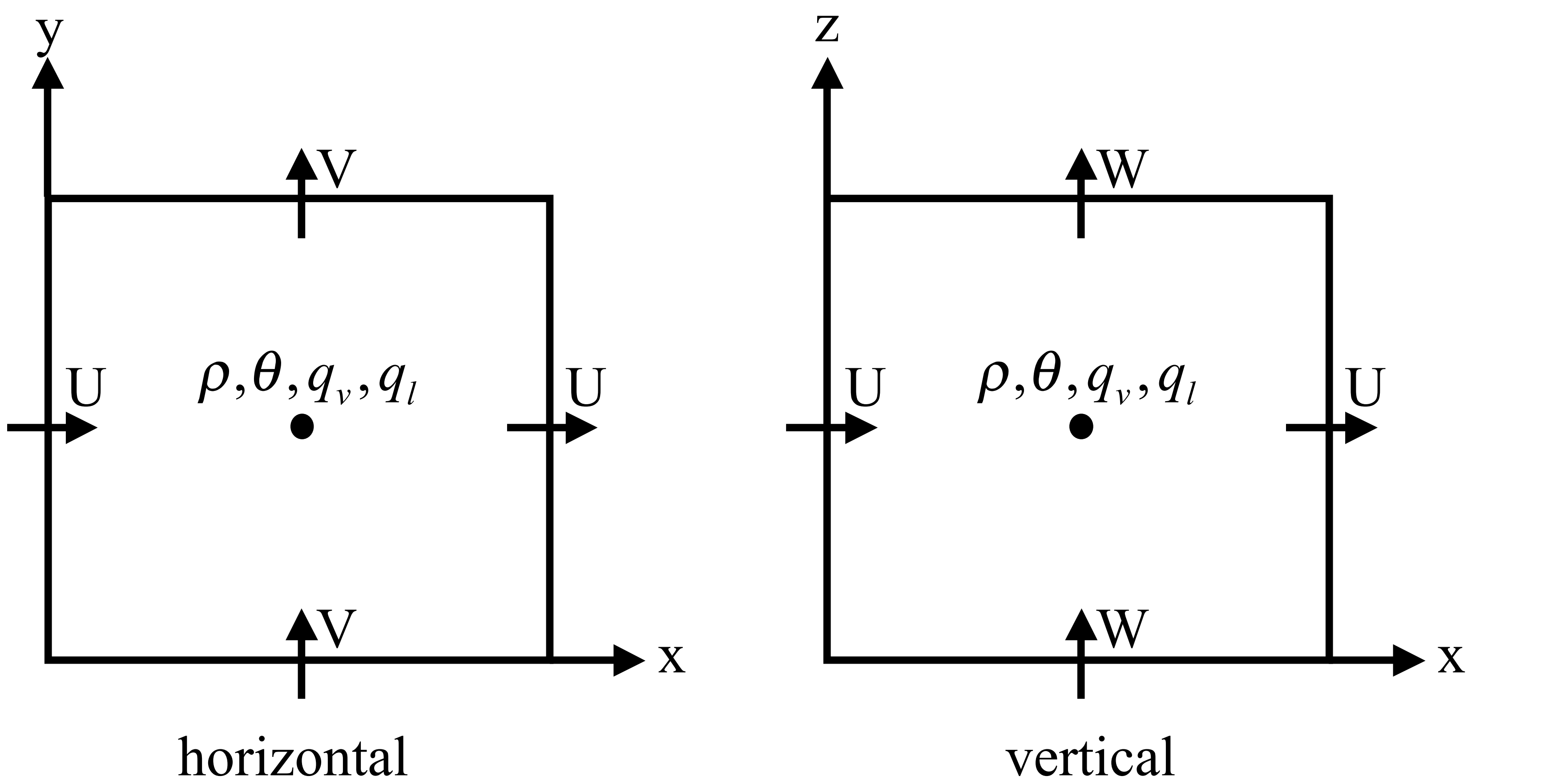
Oscillation equation analysis

$$\phi_t = ik\phi$$



ARW model, grid staggering

C-grid staggering



Advection in the ARW Model

2nd, 3rd, 4th, 5th and 6th order centered and upwind-biased schemes are available in the ARW model.

Example: 5th order scheme

$$\frac{\partial (U\phi)}{\partial x} = \frac{1}{\Delta x}\left(F_{i+\frac{1}{2}}(U\phi) - F_{i-\frac{1}{2}}(U\phi) \right)$$

where

$$F_{i-\frac{1}{2}}(U\phi) = U_{i-\frac{1}{2}} \left\{ \frac{37}{60}(\phi_i + \phi_{i-1}) - \frac{2}{15}(\phi_{i+1} + \phi_{i-2}) + \frac{1}{60}(\phi_{i+2} + \phi_{i-3}) \right\}$$

$$- sign(1, U)\frac{1}{60}\left\{ (\phi_{i+2} - \phi_{i-3}) - 5(\phi_{i+1} - \phi_{i-2}) + 10(\phi_i - \phi_{i-1}) \right\}$$

Advection in the ARW Model

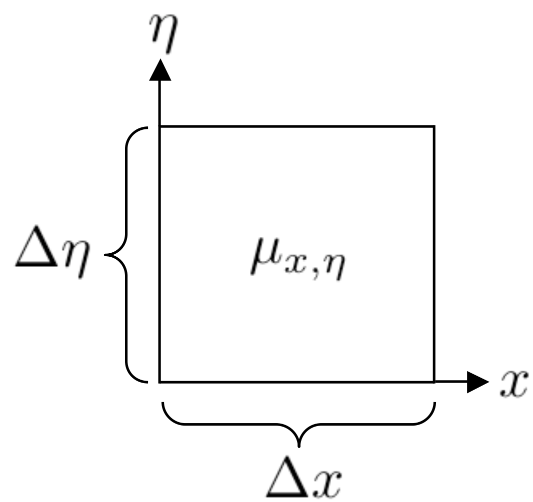
For constant U, the 5[th] order flux divergence tendency becomes

$$\Delta t \frac{\delta(U\phi)}{\Delta x}\bigg|_{5th} = \Delta t \frac{\delta(U\phi)}{\Delta x}\bigg|_{6th} - \underbrace{\left|\frac{U\Delta t}{\Delta x}\right| \frac{1}{60}\left(-\phi_{i-3} + 6\phi_{i-2} - 15\phi_{i-1} + 20\phi_i - 15\phi_{i+1} + 6\phi_{i+2} - \phi_{i+3}\right)}_{\frac{Cr}{60}\frac{\partial^6 \phi}{\partial x^6} + H.O.T}$$

The odd-ordered flux divergence schemes are equivalent to the next higher ordered (even) flux-divergence scheme plus a dissipation term of the higher even order with a coefficient proportional to the Courant number.

Mass Conservation in the ARW Model

control volume
(2D example)



Mass in a control volume

$$(\Delta x \Delta \eta)(\mu)^t$$

Mass Conservation in the ARW Model

Mass in a control volume $(\Delta x \Delta \eta)(\mu)^t$

2D example

Mass conservation equation

$$\Delta t^{-1}(\Delta x \Delta \eta) \cdot \left[(\mu)^{t+\Delta t} - (\mu)^t)\right] = \left[(\mu u \Delta \eta)_{x-\Delta x/2,\eta} - (\mu u \Delta \eta)_{x+\Delta x/2,\eta})\right] + \left[(\mu \omega \Delta x)_{x,\eta-\Delta \eta/2} - (\mu \omega \Delta x)_{x,\eta+\Delta \eta/2})\right]$$

Change in mass over a time step

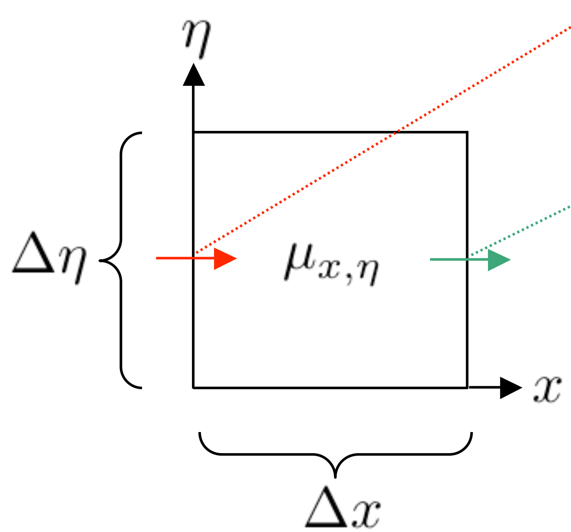
mass fluxes through control volume faces

Mass Conservation in the ARW Model

Mass in a control volume $(\Delta x \Delta \eta)(\mu)^t$

Mass conservation equation

$$\Delta t^{-1}(\Delta x \Delta \eta) \cdot [(\mu)^{t+\Delta t} - (\mu)^t)] = [(\mu u \Delta \eta)_{x-\Delta x/2,\eta} - (\mu u \Delta \eta)_{x+\Delta x/2,\eta})] + [(\mu \omega \Delta x)_{x,\eta-\Delta\eta/2} - (\mu \omega \Delta x)_{x,\eta+\Delta\eta/2})]$$



Horizontal fluxes through the vertical control-volume faces

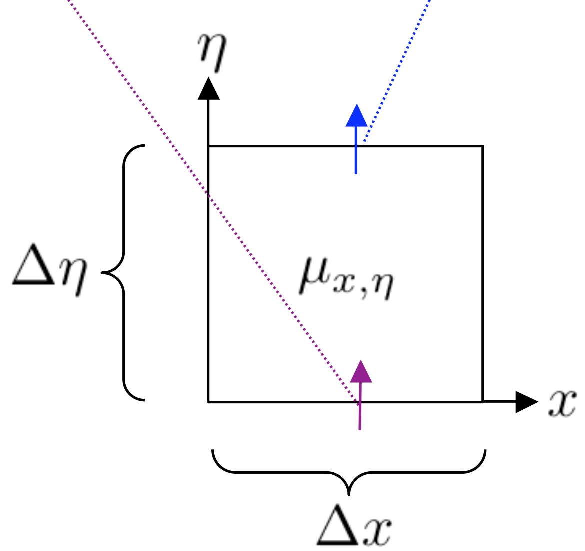
Mass Conservation in the ARW Model

Mass in a control volume $(\Delta x \Delta \eta)(\mu)^t$

Mass conservation equation

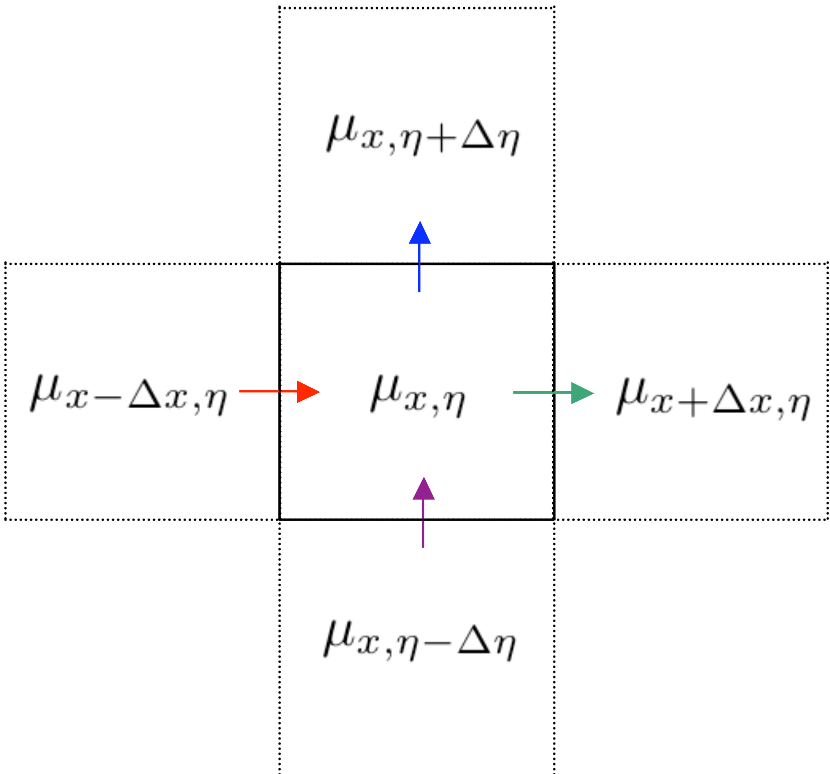
$$\Delta t^{-1}(\Delta x \Delta \eta) \cdot \left[(\mu)^{t+\Delta t} - (\mu)^t)\right] = \left[(\mu u \Delta \eta)_{x-\Delta x/2,\eta} - (\mu u \Delta \eta)_{x+\Delta x/2,\eta})\right] + \left[(\mu \omega \Delta x)_{x,\eta-\Delta \eta/2} - (\mu \omega \Delta x)_{x,\eta+\Delta \eta/2})\right]$$

Vertical fluxes through the horizontal control-volume faces



Mass Conservation in the ARW Model

The same mass fluxes are used for neighboring grid cells - hence mass is conserved locally and globally.



Scalar Mass Conservation in the ARW Model

Mass in a control volume	$(\Delta x \Delta \eta)(\mu)^t$
Scalar mass	$(\Delta x \Delta \eta)(\mu\phi)^t$

Mass conservation equation:

$$\Delta t^{-1}(\Delta x \Delta \eta) \cdot \left[(\mu)^{t+\Delta t} - (\mu)^t\right] = \left[(\mu u \Delta \eta)_{x-\Delta x/2,\eta} - (\mu u \Delta \eta)_{x+\Delta x/2,\eta}\right] + \left[(\mu \omega \Delta x)_{x,\eta-\Delta \eta/2} - (\mu \omega \Delta x)_{x,\eta+\Delta \eta/2}\right]$$

change in mass over a time step

mass fluxes through control volume faces

Scalar mass conservation equation:

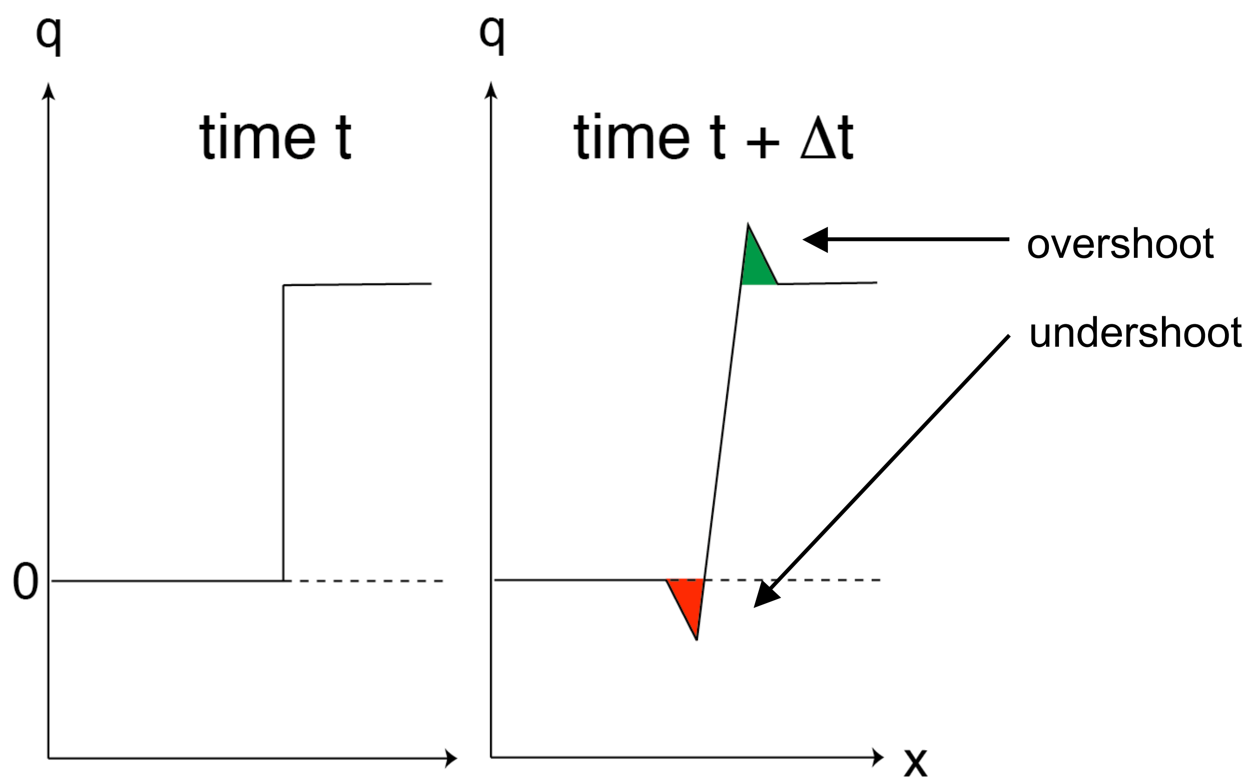
$$\Delta t^{-1}(\Delta x \Delta \eta) \cdot \left[(\mu\phi)^{t+\Delta t} - (\mu\phi)^t\right] = \left[(\mu u \phi \Delta \eta)_{x-\Delta x/2,\eta} - (\mu u \phi \Delta \eta)_{x+\Delta x/2,\eta}\right] + \left[(\mu \omega \phi \Delta x)_{x,\eta-\Delta \eta/2} - (\mu \omega \phi \Delta x)_{x,\eta+\Delta \eta/2}\right]$$

change in tracer mass over a time step

tracer mass fluxes through control volume faces

Moisture Transport in ARW

1D advection



ARW scheme is conservative,
but not positive definite nor monotonic.
Removal of negative q
results in spurious source of q .

Positive-Definite Flux Renormalization

Scalar update, last RK3 step

$$(\mu\phi)^{t+\Delta t} = (\mu\phi)^t - \Delta t \sum_{i=1}^{n} \delta_{x_i}[f_i]$$

(1) Decompose flux into upwind (1st order) flux
and a higher order corrective flux.

$$f_i = f_i^{upwind} + f_i^c$$

(2) Update solution with the upwind fluxes.
This update is monotonic.

$$(\mu\phi)^* = (\mu\phi)^t - \Delta t \sum_{i=1}^{n} \delta_{x_i}[f_i^{upwind}]$$

(3) Compute partial update using only outgoing higher order corrective fluxes (only outgoing fluxes can reduce the scalar mass in a volume).

$$(\mu\phi)^{**} = (\mu\phi)^{*} - \Delta t \sum_{i=1}^{n} \delta_{x_i} [f_i^c]^+$$

(4) If the partial update is negative, renormalize the higher order corrective fluxes such that the update will be zero.

$$[f_j^{c*}]^+ = [f_j^c]^+ \cdot (\mu\phi)^{*} \cdot \left(\Delta t \sum_{i=1}^{n} \delta_{x_i} [f_i^c]^+ \right)^{-1}$$

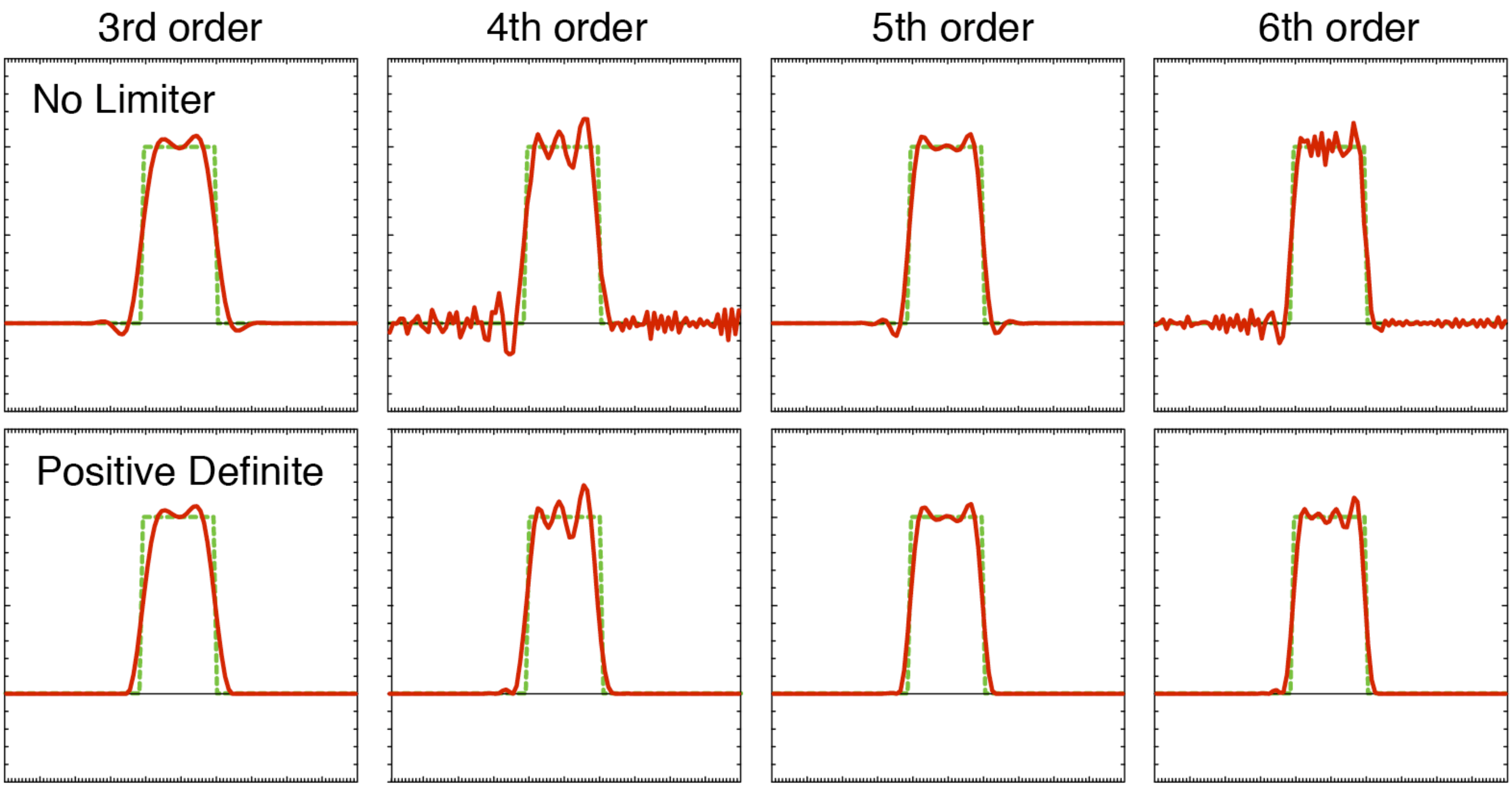
(5) After all fluxes have been renormalized, compute the full update.

$$(\mu\phi)^{t+\Delta t} = (\mu\phi)^{*} - \Delta t \sum_{j=1}^{n} \delta_{x_j} [f_j^{c*}]$$

Skamarock, MWR 2006, 2241-2250

PD Limiter in ARW - 1D Example
Top-Hat Advection

Cr = 0.5, 1 revolution (200 steps)



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- Runge-Kutta loop (steps 1, 2, and 3)
    - (i) advection, p-grad, buoyancy using $\left(\phi^{t}, \phi^{*}, \phi^{**}\right)$
    - (ii) physics if step 1, save for steps 2 and 3
    - (iii) mixing, other non-RK dynamics, save…
    - (iv) assemble dynamics tendencies
    - Acoustic step loop
        - (i) advance U,V, then μ, Θ, then w, ϕ
        - (ii) time-average U,V, Ω
    - End acoustic loop
    - Advance scalars using time-averaged U,V, Ω
- End Runge-Kutta loop
- Adjustment physics (currently microphysics)

End time step

Flux-Form Perturbation Equations

Introduce the perturbation variables:

$$\phi = \overline{\phi}(z) + \phi', \mu = \overline{\mu}(z) + \mu';$$
$$p = \overline{p}(z) + p', \alpha = \overline{\alpha}(z) + \alpha'$$

Note – $\phi = \overline{\phi}(z) = \overline{\phi}(x, y, \eta),$

likewise $\overline{p}(x, y, \eta), \overline{\alpha}(x, y, \eta)$

Momentum and hydrostatic equations become:

$$\frac{\partial U}{\partial t} + \mu\alpha \frac{\partial p'}{\partial x} + \eta\mu\alpha' \frac{\partial \overline{\mu}}{\partial x} + \mu \frac{\partial \phi'}{\partial x} + \frac{\partial \phi'}{\partial x}\left(\frac{\partial p'}{\partial \eta} - \mu'\right) = -\frac{\partial Uu}{\partial x} - \frac{\partial \Omega u}{\partial \eta}$$

$$\frac{\partial W}{\partial t} + g\left(\mu' - \frac{\partial p'}{\partial \eta}\right) = -\frac{\partial Uw}{\partial x} - \frac{\partial \Omega w}{\partial \eta}$$

$$\frac{\partial \phi'}{\partial \eta} = -\overline{\mu}\alpha' - \overline{\alpha}\mu'$$

Flux-Form Perturbation Equations: Acoustic Step

Acoustic mode separation:

Recast Equations in terms of perturbation about time t

$$U' = U'^{t} + U'', \; V' = V'^{t} + V'', \; W' = W'^{t} + W'',$$

$$\Theta' = \Theta'^{t} + \Theta'', \quad \mu' = \mu'^{t} + \mu'', \; \phi' = \phi'^{t} + \phi'';$$

$$p' = p'^{t} + p'', \; \alpha' = \alpha'^{t} + \alpha''$$

Linearize ideal gas law about time t

$$p'' = \frac{c_s^2}{\alpha^t}\left(\frac{\Theta''}{\Theta^t} - \frac{\alpha''}{\alpha^t} - \frac{\mu''}{\mu^t}\right)$$

$$\alpha'' = \frac{1}{\mu^t}\left(\frac{\partial \phi''}{\partial \eta} + \alpha^t \mu''\right)$$

Vertical pressure gradient becomes

$$\frac{\partial p''}{\partial \eta} = \frac{\partial}{\partial \eta}\left(\frac{c_s^2}{\mu^t {\alpha^t}^2}\frac{\partial \phi''}{\partial \eta} + \frac{c_s^2}{\mu^t}\frac{\Theta''}{\Theta^t}\right)$$

Flux-Form Perturbation Equations: Acoustic Step

Small (acoustic) timestep equations:

$$\delta_\tau U'' + \mu^t \alpha^t \frac{\partial p''}{\partial x} + \eta \mu^t \frac{\partial \overline{\mu}}{\partial x} \alpha'' + \mu^t \frac{\partial \phi''}{\partial x} + \frac{\partial \phi^t}{\partial x} \left(\frac{\partial p''}{\partial \eta} - \mu'' \right) = R_u{}^t$$

$$\delta_\tau \mu'' + \left(\nabla \cdot \mathrm{V}'' \right)_\eta^{\tau + \Delta\tau} = R_\mu{}^t$$

$$\delta_\tau \Theta'' + \left(\nabla \cdot \mathrm{V}'' \theta^t \right)_\eta^{\tau + \Delta\tau} = R_\Theta{}^t$$

$$\delta_\tau W'' + g \overline{\left[\mu'' - \frac{\partial}{\partial \eta} \left(\frac{c_s^2}{\mu^t {\alpha^t}^2} \frac{\partial \phi''}{\partial \eta} + \frac{c_s^2}{\alpha^t} \frac{\Theta''}{\Theta^t} \right) \right]}^{\tau} = R_w{}^t$$

$$\delta_\tau \phi'' + \frac{1}{\mu^t} \left[\left(\mathrm{V}'' \cdot \nabla \phi^t \right)_\eta^{\tau + \Delta\tau} - g \overline{W''}^{\tau} \right] = R_\varphi{}^t$$

Acoustic Integration in ARW

Forward-backward scheme, first advance the horizontal momentum

$$\delta_\tau U'' + \mu^t \alpha^t \frac{\partial p''}{\partial x} + \eta \mu^t \frac{\partial \overline{\mu}}{\partial x} \alpha'' + \mu^t \frac{\partial \phi''}{\partial x} + \frac{\partial \phi^t}{\partial x}\left(\frac{\partial p''}{\partial \eta} - \mu''\right) = R_u{}^t$$

Second, advance continuity equation,
diagnose omega,
and advance thermodynamic equation

$$\delta_\tau \mu'' + \left(\nabla \cdot \mathrm{V}''\right)_\eta^{\tau+\Delta\tau} = R_\mu{}^t$$

$$\delta_\tau \Theta'' + \left(\nabla \cdot \mathrm{V}''\theta^t\right)_\eta^{\tau+\Delta\tau} = R_\Theta{}^t$$

Finally, vertically-implicit integration of the acoustic and gravity wave terms

$$\delta_\tau W'' + g\,\overline{\left[\mu'' - \frac{\partial}{\partial \eta}\left(\frac{c_s^2}{\mu^t \alpha^{t\,2}} \frac{\partial \phi''}{\partial \eta} + \frac{c_s^2}{\alpha^t} \frac{\Theta''}{\Theta^t}\right)\right]}^{\tau} = R_w{}^t$$

$$\delta_\tau \phi'' + \frac{1}{\mu^t}\left[\left(\mathrm{V}'' \cdot \nabla \phi^t\right)_\eta^{\tau+\Delta\tau} - g\overline{W''}^{\tau}\right] = R_\varphi{}^t$$

ARW Model: Dynamics Parameters

3rd order Runge-Kutta time step

Courant number limited, 1D: $C_r = \frac{U\Delta t}{\Delta x} < 1.73$

Generally stable using a timestep approximately twice as large as used in a leapfrog model.

Acoustic time step

2D horizontal Courant number limited: $C_r = \frac{C_s \Delta \tau}{\Delta x} < \frac{1}{\sqrt{2}}$

$$\Delta \tau_{sound} = \Delta t_{RK} / (\text{number of acoustic steps})$$

Guidelines for time step

Δt in seconds should be about 6**Δx* (grid size in kilometers). Larger *Δt* can be used in smaller-scale dry situations, but *time_step_sound* (default = 4) should increase proportionately if larger *Δt* is used.

ARW Filters: Divergence Damping

Purpose: filter acoustic modes

$$p^{*\tau} = p^{\tau} + \gamma_d \left(p^{\tau} - p^{\tau - \Delta\tau} \right)$$

$$\delta_\tau U'' + \mu^{t^*} \alpha^{t^*} \partial_x p''^{\tau} + (\mu^{t^*} \partial_x \bar{p}) \alpha''^{\tau} + (\alpha/\alpha_d)[\mu^{t^*} \partial_x \phi''^{\tau} + (\partial_x \phi^{t^*})(\partial_\eta p'' - \mu'')^{\tau}] = R_U{}^{t^*}$$

$$\delta_\tau V'' + \mu^{t^*} \alpha^{t^*} \partial_y p''^{\tau} + (\mu^{t^*} \partial_y \bar{p}) \alpha''^{\tau} + + (\alpha/\alpha_d)[\mu^{t^*} \partial_y \phi''^{\tau} + (\partial_y \phi^{t^*})(\partial_\eta p'' - \mu'')^{\tau}] = R_V{}^{t^*}$$

$\gamma_d = 0.1$ recommended (default)

ARW Filters: External Mode Filter

Purpose: filter the external mode

(primarily for real-data applications)

Additional terms:

$$\delta_\tau U'' = \ldots \quad \underline{- \gamma_e \left(\Delta x^2 / \Delta\tau\right) \delta_x (\delta_{\tau-\Delta\tau} \mu''_d)}$$

$$\delta_\tau V'' = \ldots \quad \underline{- \gamma_e \left(\Delta y^2 / \Delta\tau\right) \delta_y (\delta_{\tau-\Delta\tau} \mu''_d)}$$

$$\delta_\tau \mu_d = m^2 \int_1^0 [\partial_x U'' + \partial_y V'']^{\tau+\Delta\tau} d\eta$$

$\gamma_e = 0.01$ recommended (default)

ARW Filters: Vertically Implicit Off-Centered Acoustic Step

Purpose: damp vertically-propagating acoustic modes

$$\delta_\tau W'' - m^{-1} g \overline{\left[(\alpha/\alpha_d)^{t^*} \partial_\eta (C \partial_\eta \phi'') + \partial_\eta \left(\frac{c_s^2}{\alpha^{t^*}} \frac{\Theta''}{\Theta^{t^*}} \right) - \mu_d'' \right]}^\tau = {R_W}^{t^*}$$

$$\delta_\tau \phi'' + \frac{1}{\mu_d^{t^*}} [m \Omega^{\tau+\Delta\tau} \phi_\eta - \overline{g W''}^\tau] = {R_\phi}^{t^*}.$$

$$\overline{a}^\tau = \frac{1+\beta}{2} a^{\tau+\Delta\tau} + \frac{1-\beta}{2} a^\tau$$

β= 0.1 recommended (default)

ARW Filters: Vertical Velocity Damping

Purpose: damp anomalously-large vertical velocities

(usually associated with anomalous physics tendencies)

Additional term:

$$\partial_t W = \ldots \underline{- \mu_d \ \mathrm{sign}(W) \gamma_w (Cr - Cr_\beta)}$$

$$Cr = \left| \frac{\Omega dt}{\mu d\eta} \right|$$

$Cr_\beta = 1.0$ typical value (default)

$\gamma_w = 0.3 \ \mathrm{m/s^2}$ recommended (default)

ARW Filters: 2nd-Order Horizontal Mixing, Horizontal-Deformation-Based K_h

Purpose: mixing on horizontal coordinate surfaces (real-data applications, 2 km < Δx <= 10 km)

$$K_h = C_s^2\, l^2 \left[0.25(D_{11} - D_{22})^2 + \overline{D_{12}^2}^{xy} \right]^{\frac{1}{2}}$$

where $$l = (\Delta x \Delta y)^{1/2}$$

$$D_{11} = 2\, m^2 \left[\partial_x(m^{-1}u) - z_x \partial_z(m^{-1}u) \right]$$
$$D_{22} = 2\, m^2 \left[\partial_y(m^{-1}v) - z_y \partial_z(m^{-1}v) \right]$$
$$D_{12} = m^2 \left[\partial_y(m^{-1}u) - z_y \partial_z(m^{-1}u) + \partial_x(m^{-1}v) - z_x \partial_z(m^{-1}v) \right]$$

$C_s = 0.25$ (Smagorinsky coefficient, default value)

ARW Model: Boundary Condition Options

Lateral boundary conditions

1. Specified (Coarse grid, real-data applications).
2. Open lateral boundaries (gravity-wave radiative).
3. Symmetric lateral boundary condition (free-slip wall).
4. Periodic lateral boundary conditions.
5. Nested boundary conditions (specified).

Top boundary conditions

1. Constant pressure.
2. Rayleigh damping upper layer.
3. Absorbing upper layer (increased horizontal diffusion).
4. Gravity-wave radiative condition (not yet implemented).

Bottom boundary conditions

1. Free slip.
2. Various B.L. implementations of surface drag, fluxes.

ARW Model: Nesting

2-way nesting

1. Multiple domains run concurrently
2. Multiple levels, multiple nests per level
3. Any integer ratio grid size and time step
4. Parent domain provides nest boundaries
5. Nest feeds back interior values to parent

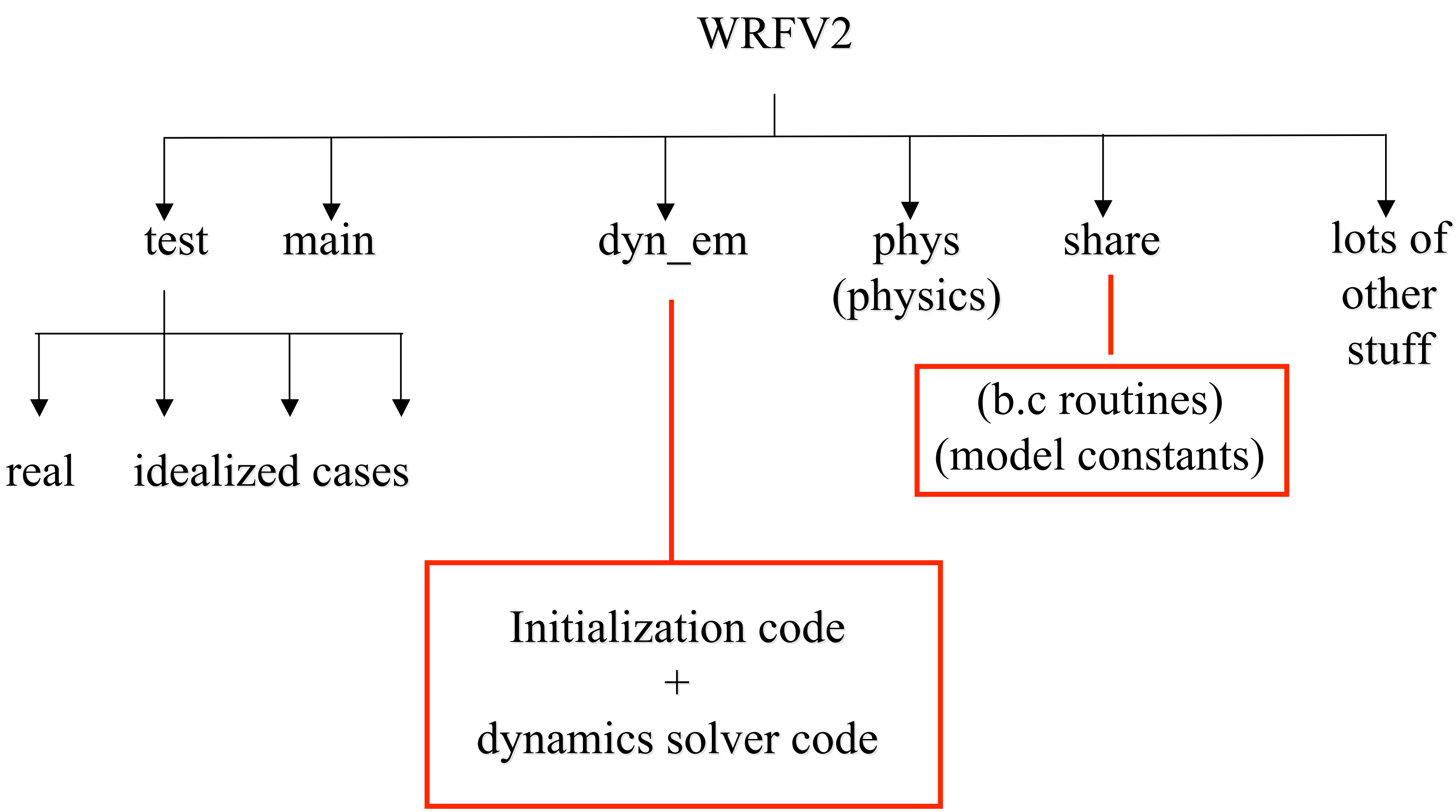
1-way nesting

1. Parent domain is run first
2. *ndown* uses coarse output to generate nest boundary conditions
3. Nest initial conditions from fine-grid input file
4. Nest is run after *ndown*

ARW Model: Coordinate Options

1. Cartesian geometry:
   idealized cases
2. Lambert Conformal:
   mid-latitude applications
3. Polar Stereographic:
   high-latitude applications
4. Mercator:
   low-latitude applications

WRF ARW code



WRFV2
test
main
dyn_em
phys
(physics)
share
lots of
other
stuff
real
idealized cases
(b.c routines)
(model constants)
Initialization code
+
dynamics solver code