WRF-Var Overview

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NCAR/MMM Division Staff
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Taiwanese Central Weather Bureau, Civil Aeronautics Administration

Outline of Talk

1) Introduction to data assimilation.
2) Basics of modern data assimilation.
3) Demonstration with a simple system.
4) WRF-Var.
5) Important issues.

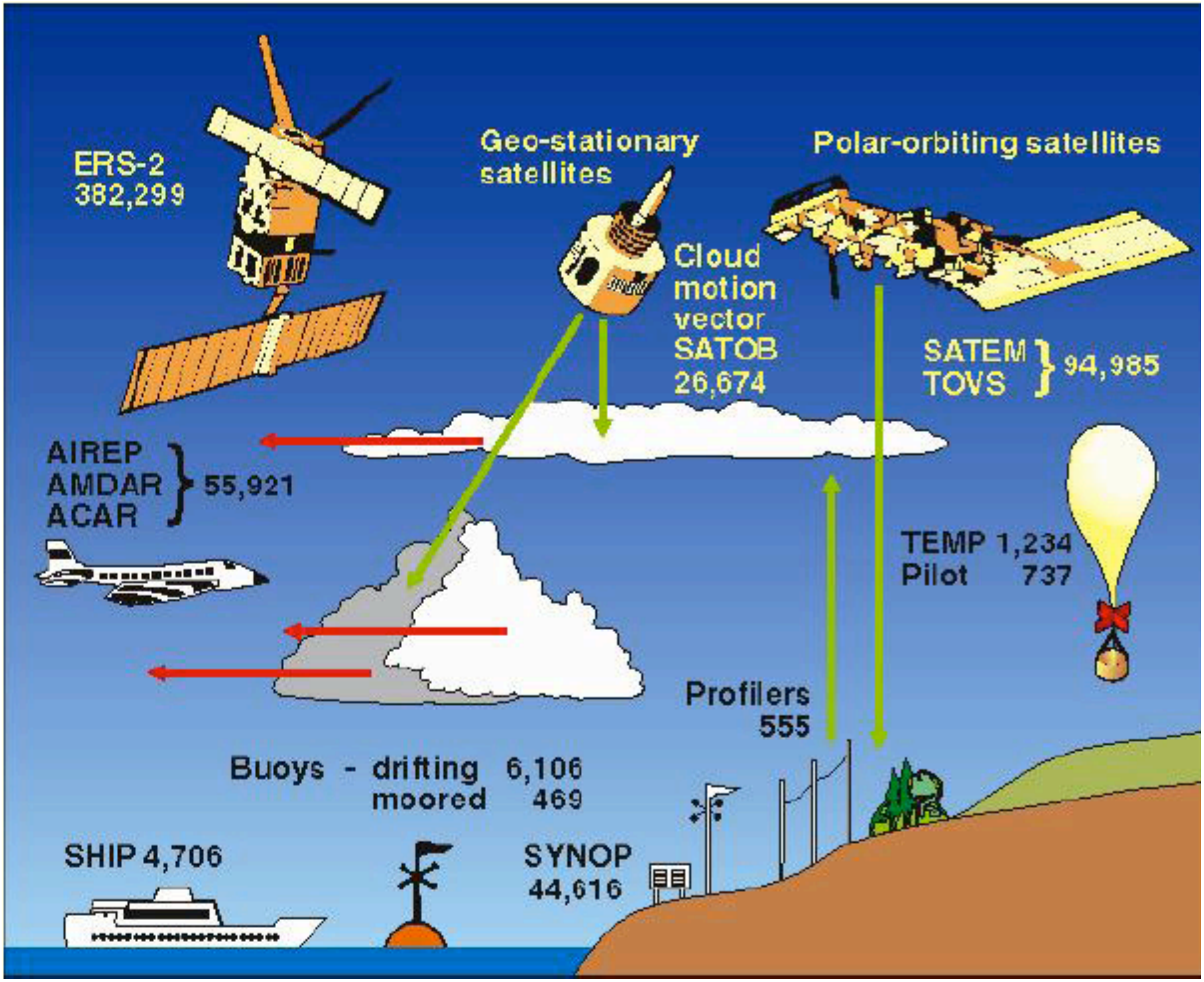
1. Introduction to data assimilation

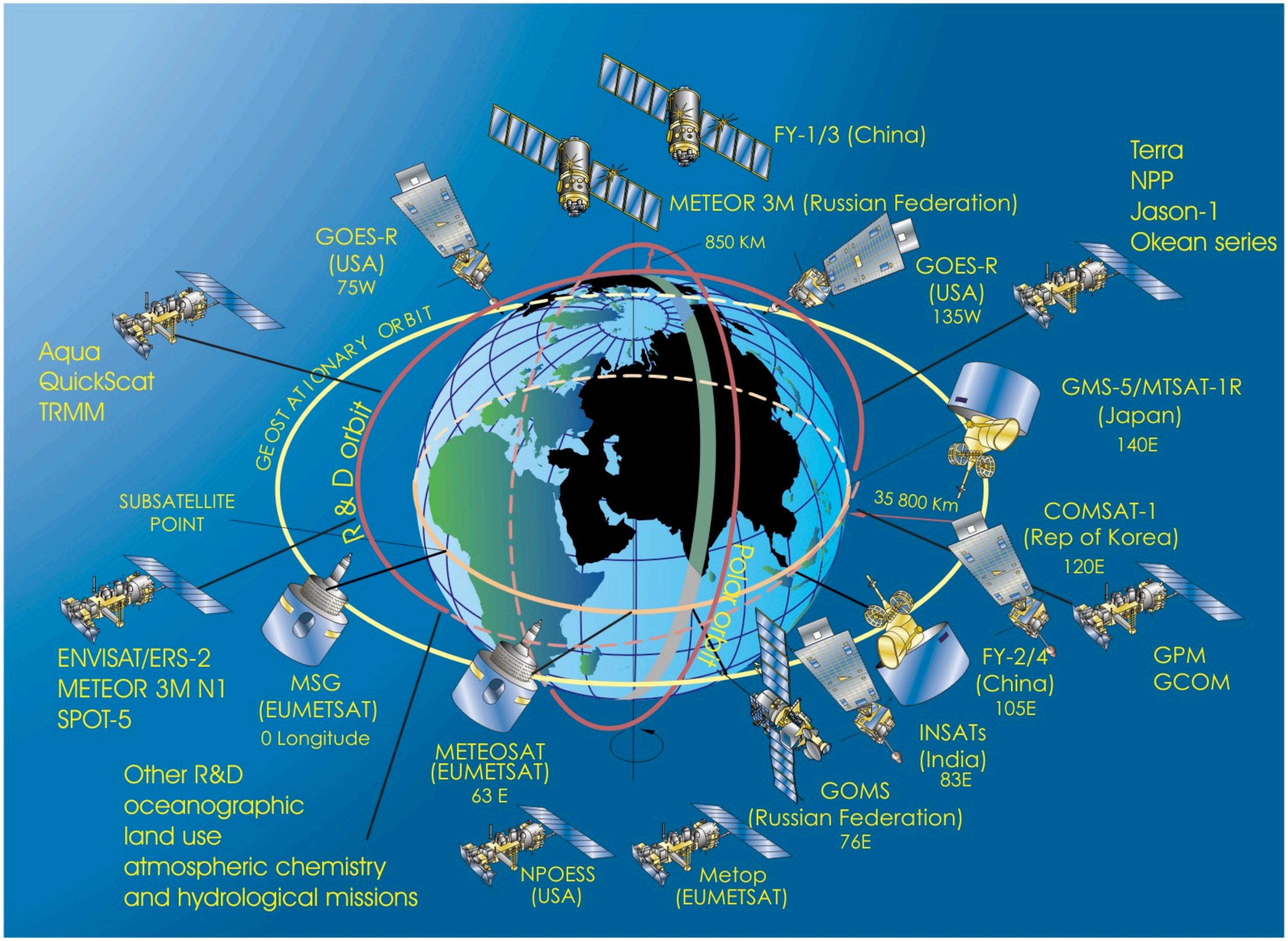
Modern weather forecast (Bjerknes,1904)

- A sufficiently accurate knowledge of the state of the atmosphere at the initial time
- A sufficiently accurate knowledge of the laws according to which one state of the atmosphere develops from another.

Vilhelm Bjerknes (1862–1951)

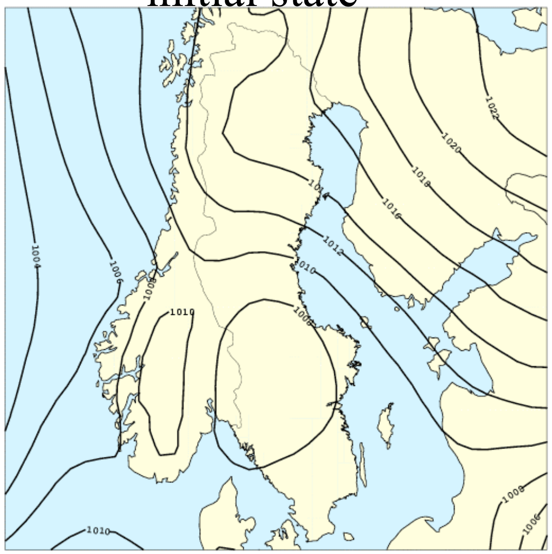
- Analysis: using observations and other information, we can specify the atmospheric state at a given initial time: “Today’s Weather”
- Forecast: using the equations, we can calculate how this state will change over time: “Tomorrow’s Weather”


ERS-2
382,299
Geo-stationary
satellites
Polar-orbiting satellites
Cloud
motion
vector
SATOB
26,674
SATEM
TOVS
94,985
AIREP
AMDAR
ACAR
55,921
TEMP 1,234
Pilot 737
Profilers
555
Buoys - drifting 6,106
moored 469
SHIP 4,706
SYNOP
44,616



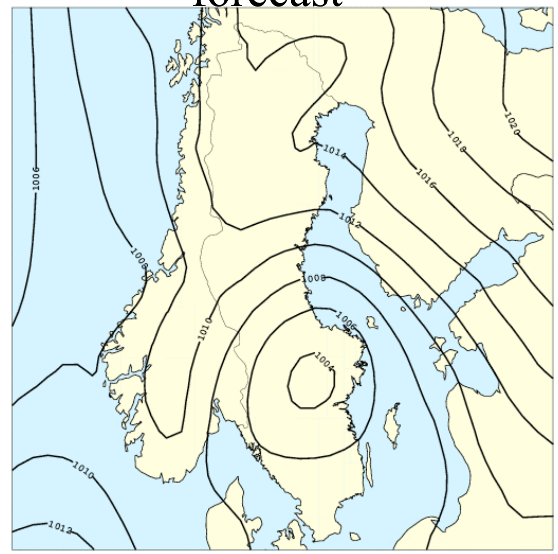
FY-1/3 (China)
METEOR 3M (Russian Federation)
Terra
NPP
Jason-1
Okean series
GOES-R
(USA)
75W
850 KM
GOES-R
(USA)
135W
Aqua
QuickScat
TRMM
GEOSTATIONARY ORBIT
R & D orbit
GMS-5/MTSAT-1R
(Japan)
140E
SUBSATELLITE
POINT
35 800 Km
COMSAT-1
(Rep of Korea)
120E
Polar orbit
ENVISAT/ERS-2
METEOR 3M N1
SPOT-5
MSG
(EUMETSAT)
0 Longitude
FY-2/4
(China)
105E
GPM
GCOM
INSATs
(India)
83E
METEOSAT
(EUMETSAT)
63 E
Other R&D
oceanographic
land use
atmospheric chemistry
and hydrological missions
GOMS
(Russian Federation)
76E
NPOESS
(USA)
Metop
(EUMETSAT)

initial state



Model

forecast



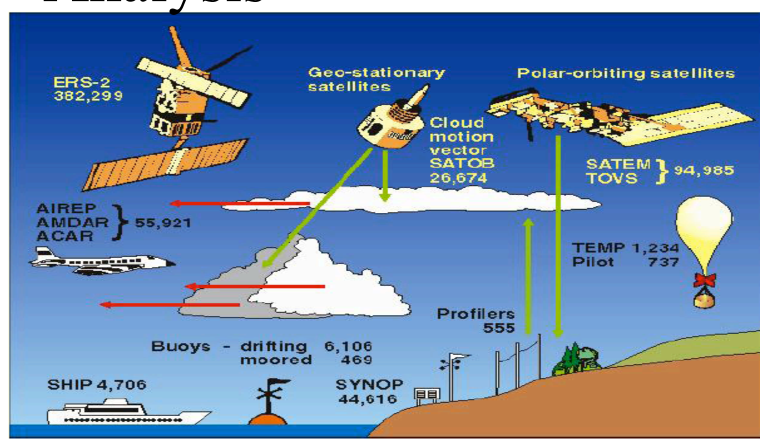
Analysis

Model state

x, $\sim 10^7$

Observations

y^o, $\sim 10^5\text{-}10^6$



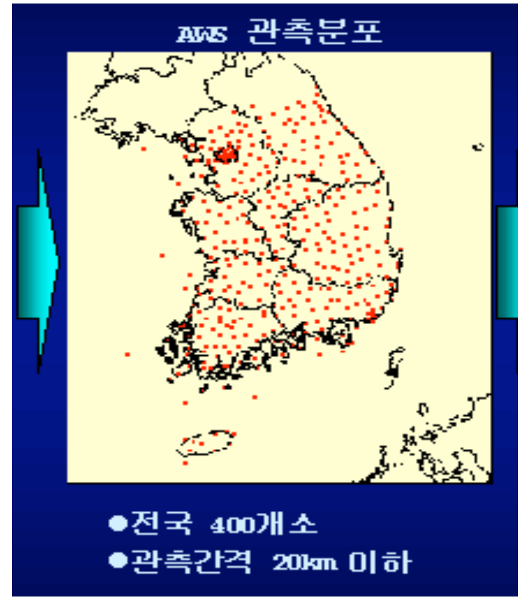
Need For Data Assimilation in NWP

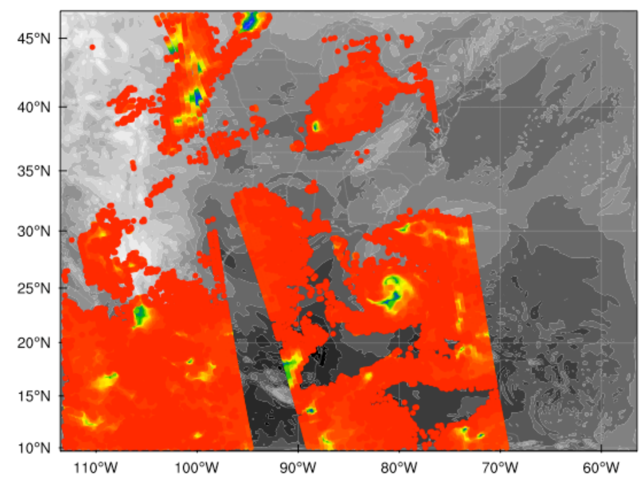
Fact: There are never enough good observations!!

- Consider NWP model:
  - Typical global model – 425 * 325 * 50 = 7 million gridpts.
  - Minimum number of variables = 6 *(u, v, w, T, p, q)*.
  - Number of degrees of freedom = 41.4 million.

- Typical number of observations = few x 10^6 but:
  - Inhomogeneous distribution of data.
  - Observations not always in sensitive areas.
  - Observations have errors.

- Solutions:
  - Use sophisticated (variational/ensemble) techniques.
  - Use previous forecast to propagate past observations.
  - Use approximate physical balance relationships.
  - More/better observations!





Assimilation methods

- Empirical methods
  - Successive Correction Method (SCM)
  - Nudging
  - Physical Initialisation (PI), Latent Heat Nudging (LHN)
- Statistical methods
  - Optimal Interpolation (OI)
  - 3-Dimensional VARiational data assimilation (3DVAR)
  - 4-Dimensional VARiational data assimilation (4DVAR)
- Advanced methods
  - Extended Kalman Filter (EKF)
  - Ensemble Kalman Filter (EnFK)

2. Basics of modern data assimilation

The analysis problem for a given time

Consider a scalar x.

The background (normally a short-range forecast):

$$x^b = x^t + b.$$

The observation:

$$x^r = x^t + r.$$

The error statistics are assumed to be known:

$$\begin{aligned}
< b > &= 0, \quad mean\ error\ (unbiased), \\
< r > &= 0, \quad mean\ error\ (unbiased), \\
< b^2 > &= B, \quad background\ error\ variance, \\
< r^2 > &= R, \quad observation\ error\ variance, \\
< br > &= 0, \quad nocorrelation\ between\ b\ and\ r,
\end{aligned}$$

where $< . >$ is ensemble average.

BLUE: the Best Linear Unbiased Estimate

The analysis: $x^a = x^t + a$.
Search for the best estimate: $x^a = \alpha x^b + \beta x^r$

Substitute the definitions, we have:

$$\alpha + \beta = 1.$$

$$< a >= 0$$

The variance:

$$A =< a^2 >= B - 2\beta B + \beta^2(B + R)$$

To determine β: $\frac{dA}{d\beta} = -2B + 2\beta(B + R) = 0$
we have

$$\beta = \frac{B}{B + R}.$$

$$A^{-1} = B^{-1} + R^{-1}$$

The analysis: $x^a = x^b + \frac{B}{B+R}\left(x^r - x^b\right).$

“3DVAR”

The analysis is obtained by minimizing the cost function J, defined as:

$$J = \frac{1}{2}\left(x - x^b\right)^T B^{-1}\left(x - x^b\right) + \frac{1}{2}\left(x - x^r\right)^T R^{-1}\left(x - x^r\right).$$

The gradient of J with respect to x:

$$J' = B^{-1}\left(x - x^b\right) + R^{-1}\left(x - x^r\right).$$

At the minimum, $J' = 0$, we have:

$$x^a = x^b + \frac{B}{B + R}\left(x^r - x^b\right),$$

the same as BLUE.

Sequential data assimilation (I)

$$
\begin{array}{lll}
True\ states & : & \ldots,\ x^t_{i-1},\ x^t_i,\ x^t_{i+1},\ \ldots \\
 & & \\
Observations & : & \ldots,\ x^r_{i-1},\ x^r_i,\ x^r_{i+1},\ \ldots \\
Forecasts & : & \ldots,\ x^f_{i-1},\ x^f_i,\ x^f_{i+1},\ \ldots \\
 & & \\
Analyses & : & \ldots,\ x^a_{i-1},\ x^a_i,\ x^a_{i+1},\ \ldots
\end{array}
$$

Sequential data assimilation (II)

Forecast model:

$$x_{i+1}^t = M(x_i^t) + q_i,$$

where q_i is the model error.

As q_i is unknown and x_i^a is the best estimate of x_i^t, the forecast model usually takes the form:

$$x_{i+1}^f = M(x_i^a).$$

OI (and 3DVAR):

$$x_i^a = x_i^f + \frac{B}{B+R}\left(x_i^r - x_i^f\right).$$

Sequential data assimilation (III)

4DVAR

4DVAR analysis is obtained by minimizing the cost function J, defined as:

$$J(x_i) = \frac{1}{2}\left(x_i - x_i^f\right)^T B^{-1}\left(x_i - x_i^f\right)$$
$$+\frac{1}{2}\sum_{k=0}^{K}\left[M_{k-1}(x_i) - x_{i+k}^r\right]^T R^{-1}\left[M_{k-1}(x_i) - x_{i+k}^r\right]$$

where, K is the assimilation window and

$$\begin{aligned} M_{-1}(x_i) &= x_i \\ M_0(x_i) &= M(x_i) \\ M_{k-1}(x_i) &= \underbrace{M(M(\ldots M}_{k}(x_i)\ldots)) \end{aligned}$$

Sequential data assimilation (IV)

4DVAR (continue)

The gradient of J with respect to x:

$$J = B^{-1}\left(x_i - x_i^f\right) + \sum_{k=0}^{K} \prod_{j=0}^{k-1} \mathbf{M}_{i+j}^T R^{-1}\left[M_{k-1}(x_i) - x_{i+k}^r\right]$$

where, $\mathbf{M}_{i+j}^T$ is the adjoint model of M at time step $i + j$.

Sequential data assimilation (V)

Extended Kalman Filters:

True states: $x_{i+1}^t = M(x_i^t) + q_i$

Model states: $x_{i+1}^f = M(x_i^a)$

Forecast error: $x_{i+1}^f - x_{i+1}^t = M(x_i^a) - M(x_i^t) - q_i$

A major assumption in KF:

$$M(x_i^a) - M(x_i^t) \approx \mathbf{M}_i(x_i^a - x_i^t)$$

Sequential data assimilation (VI)

Extended Kalman Filters (continue):

Forecast error covariance matrix:

$$
\begin{aligned}
P^f_{i+1} &= \left\langle \left(x^f_{i+1} - x^t_{i+1}\right)\left(x^f_{i+1} - x^t_{i+1}\right)^T \right\rangle \\
&\approx \mathbf{M}_i \left\langle \left(x^a_i - x^t_i\right)\left(x^a_i - x^t_i\right)^T \right\rangle \mathbf{M}_i^T + \left\langle q_i q_i^T \right\rangle \\
&= \mathbf{M}_i P^a_i \mathbf{M}_i^T + Q_i
\end{aligned}
$$

Sequential data assimilation (VII)

Extended Kalman Filters (continue):

For the analysis step:

$$K_i = P_i^f \left(P_i^f + R\right)^{-1}$$

$$x_i^a = x_i^f + K_i(x_i^r - x_i^f)$$

$$P_i^a = (I - K_i)P_i^f$$

For the forecast step:

$$x_{i+1}^f = M(x_i^a)$$

$$P_{i+1}^f = \mathbf{M}_i P_i^a \mathbf{M}_i^T + Q_i$$

Sequential data assimilation (VIII)

From scalar to vector:

Number of grid points $\approx 10^7$:

$$x \rightarrow \mathbf{x}$$

$$x^b \rightarrow \mathbf{x^b}$$

Dimension of $\mathbf{B}$, $\mathbf{P} \approx 10^7 \times 10^7$.

Number of observations, 10^6:

$$x^r \rightarrow \mathbf{y^o}$$

$$x - x^r \rightarrow H(\mathbf{x}) - \mathbf{y^o}$$

Dimension of $\mathbf{R} \approx 10^6 \times 10^6$.

Sequential data assimilation (IX)

OI:

$$\mathbf{x}_i^a = \mathbf{x}_i^f + \mathbf{B}\mathbf{H}^T \left(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R}\right)^{-1} \left[\mathbf{y^o} - H(\mathbf{x}_i^f)\right]$$

$$\mathbf{x}_{i+1}^f = M(\mathbf{x}_i^a)$$

4DVAR:

$$J(\mathbf{x}_i) = \frac{1}{2}\left(\mathbf{x}_i - \mathbf{x}_i^f\right)^T \mathbf{B}^{-1} \left(\mathbf{x}_i - \mathbf{x}_i^f\right)$$
$$+\frac{1}{2}\sum_{k=0}^{K} \left[H\left(M_{k-1}\left(\mathbf{x}_i\right)\right) - \mathbf{y^o}_{i+k}\right]^T \mathbf{R}^{-1} \left[H\left(M_{k-1}\left(\mathbf{x}_i\right)\right) - \mathbf{y^o}_{i+k}\right]$$

$$J' = \mathbf{B}^{-1}\left(\mathbf{x}_i - \mathbf{x}_i^f\right) + \sum_{k=0}^{K}\prod_{j=0}^{k-1} \mathbf{M}_{i+j}^T \mathbf{H}^T \mathbf{R}^{-1} \left[H\left(M_{k-1}\left(\mathbf{x}_i\right)\right) - \mathbf{y^o}_{i+k}\right]$$

Sequential data assimilation (X)

The Extended Kalman Filter:

For the analysis step i:

$$\mathbf{K}_i = \mathbf{P}_i^f \mathbf{H}_i^T \left(\mathbf{H}_i \mathbf{P}_i^f \mathbf{H}_i^T + \mathbf{R} \right)^{-1}$$

$$\mathbf{x}_i^a = \mathbf{x}_i^f + \mathbf{K}_i \left[\mathbf{y}^{\mathbf{o}} - H(\mathbf{x}_i^f) \right]$$

$$\mathbf{P}_i^a = (\mathbf{I} - \mathbf{K}_i \mathbf{H}_i) \mathbf{P}_i^f$$

For the forecast step, from i to $i+1$:

$$\mathbf{x}_{i+1}^f = M(\mathbf{x}_i^a)$$

$$\mathbf{P}_{i+1}^f = \mathbf{M}_i \mathbf{P}_i^a \mathbf{M}_i^T + \mathbf{Q}_i$$

Issues on data assimilation

- Observations $\mathbf{y}^o$
- Observation operator H
- Observation errors $\mathbf{R}$
- Background $\mathbf{x}^b$
- Size of $\mathbf{B}$: statistical model and tuning
- $\mathbf{M}$ and $\mathbf{M}^T$: development and validity
- Minimization algorithm (Quasi-Newton; Conjugate Gradient; ...)

- Model errors $\mathbf{Q}$
- Size of $\mathbf{P}^f$ and $\mathbf{P}^a$: simplifications

3. Demonstration with a simple system

The Lorenz 1964 equation

Nonlinear equation (NL):

$$x_{i+1} = ax_i - x_i^2 = M(x_i).$$

Depending on a, three types of solution are found: steady state; limited cycle; chaotic.

Tangent Linear equations (TL):

$$x_{i+1}^{tl} = \left(a - 2x_i^{bs}\right) x_i^{tl} = \mathbf{M}_i x_i^{tl}.$$

(linearized around basic state x_i^{bs})
Adjoint equation (AD):

$$x_i^{ad} = \left(a - 2x_i^{bs}\right) x_{i+1}^{ad} = \mathbf{M}_i^T x_{i+1}^{ad}.$$

Note here for this simple case we have

$$\mathbf{M}_i = \mathbf{M}_i^T = \left(a - 2x_i^{bs}\right).$$

Issues on data assimilation

for the system based on the Lorenz 64 eqation

- Observation operator $H = \mathbf{H} = \mathbf{H}^T = 1$
- Estimate of the true states - generated (with model error!)
- Observation error $x^o - x^t = \sigma_o G$
- Size of $\mathbf{B}$ is 1, but $B = \sigma_b^2$ still needs attention
- Model errors $\mathbf{Q}$: $q_i = \sigma_m G$ when "true" states are generated; but we assume $Q = B/4$
- Size of $\mathbf{P}^f$ or $\mathbf{P}^a$ is 1.
- $\mathbf{M}$ and $\mathbf{M}^T$: no e ort in development but their validity is still a major problem
  (model, assimilation window length, ...)
- Gaussian statistics

Too simple?

Try: Lorenz63 model

Try: 1-D advection equation

Try: 2-D shallow water equation

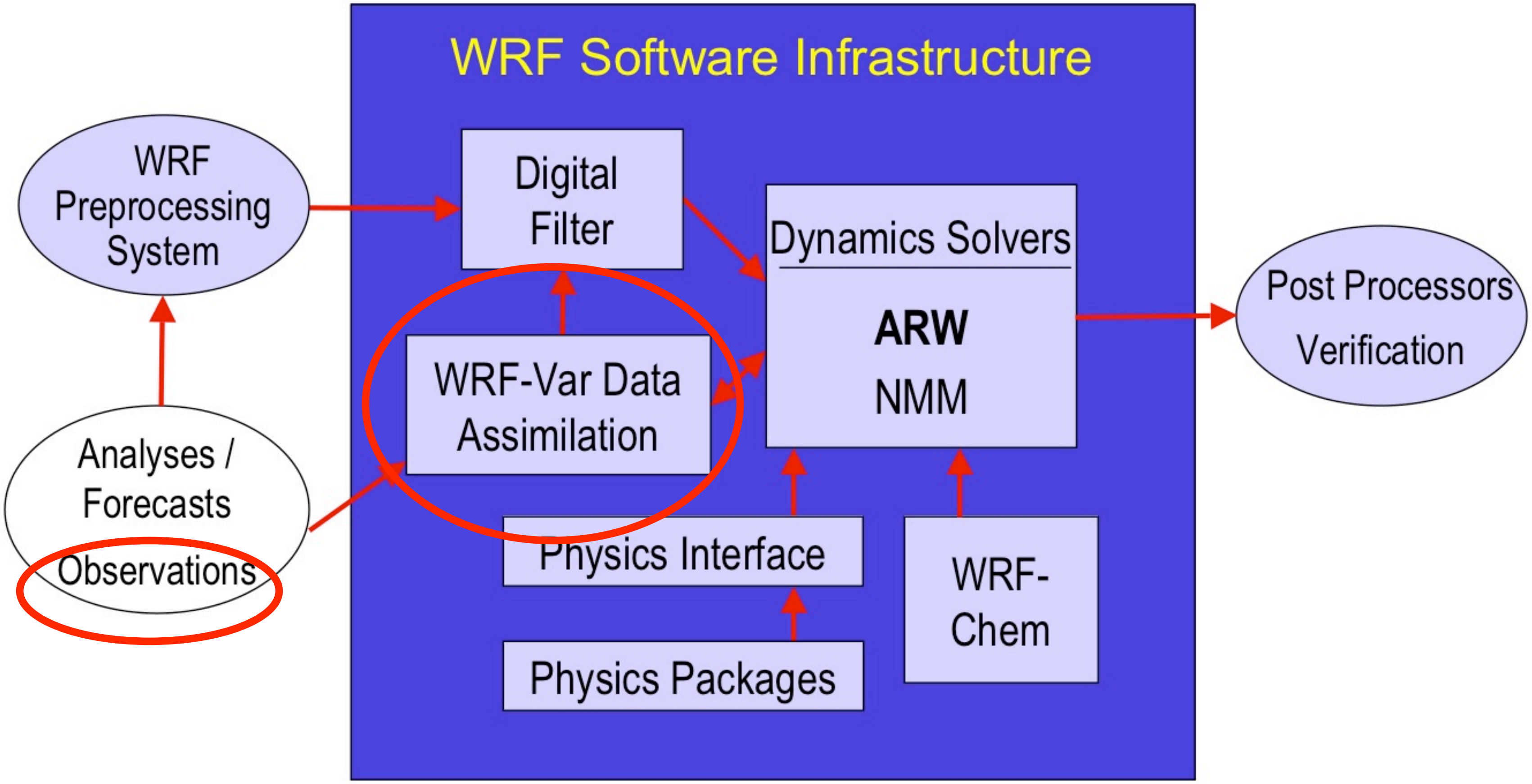
…

Try: ARW!!!

4. WRF-Var

…WRF-Var is a **Var**iational data assimilation system built within the software framework of **WRF**, used for application in both research and operational environments….

WRF Modeling System



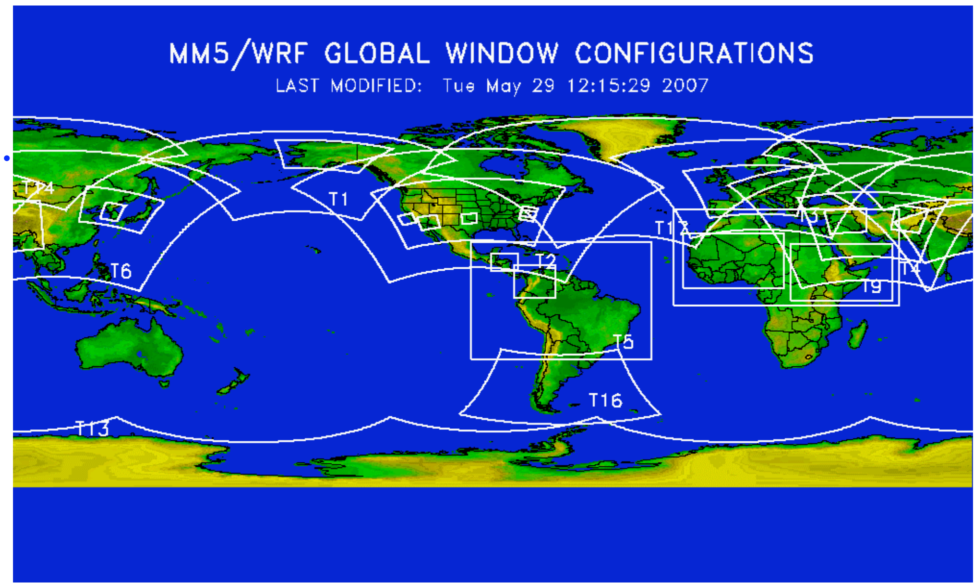
ARW = Advanced Research WRF (NCAR) Core

NMM = Nonhydrostatic Mesoscale Model (NCEP) Core

WRF-Var (WRFDA) Data Assimilation Overview

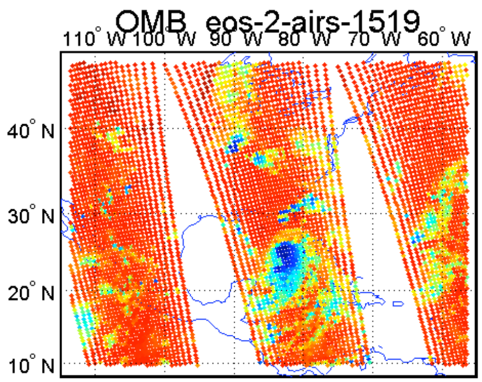
- **Goal:** Community WRF DA system for regional/global, research/operations, and deterministic/probabilistic applications.
- **Techniques:**
  - 3D-Var
  - 4D-Var (regional)
  - Ensemble DA,
  - Hybrid Variational/Ensemble DA.
- **Models:** WRF, MM5, KMA global.
- **Support:**
  - NCAR/ESSL/MMM/DAG
  - NCAR/RAL/JNT/DATC
- **Observations:** Conv.+Sat.+Radar

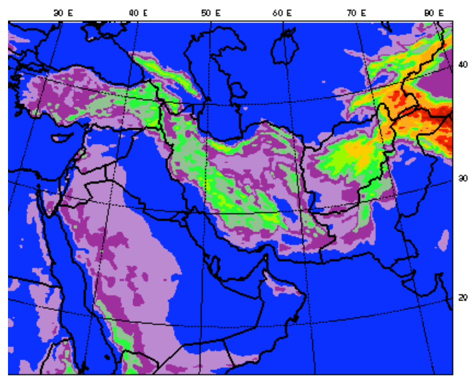
AFWA Theaters:

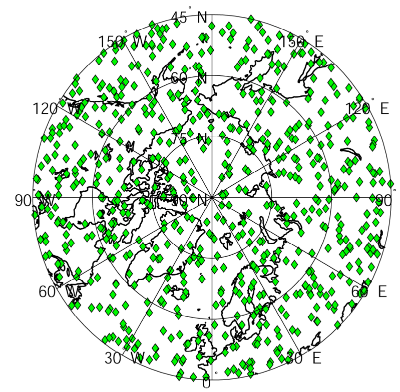


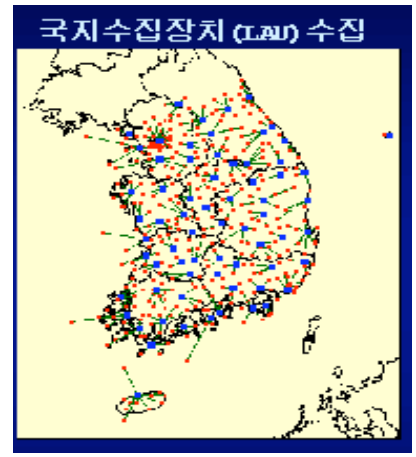
The WRF-Var Program

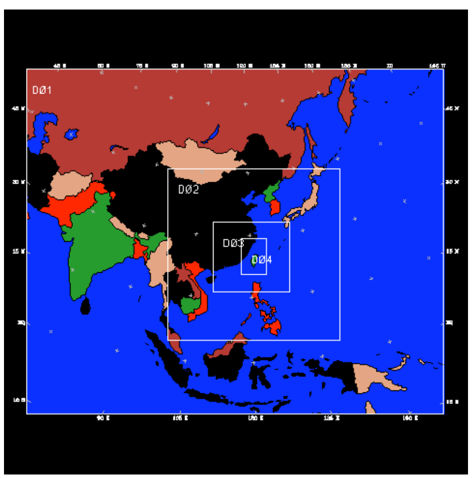
- NCAR staff: 23FTE, ~12 projects.
- Non-NCAR collaborators (AFWA, KMA, CWB, BMB, etc): ~10FTE.
- Community users: ~30 (more in 4000 general WRF downloads?).

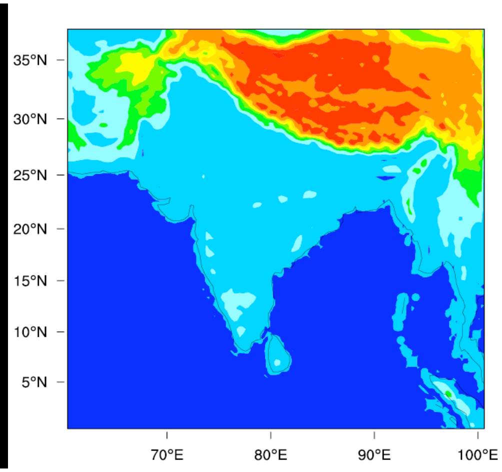


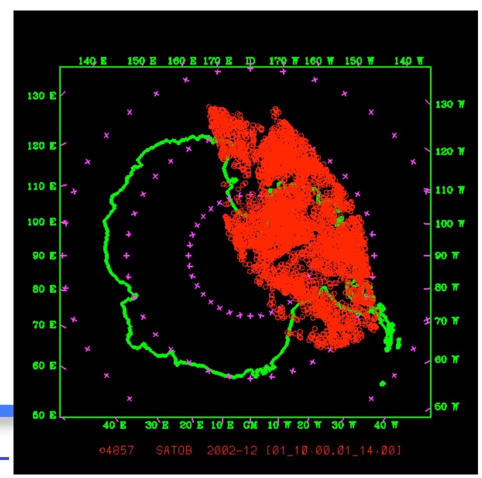


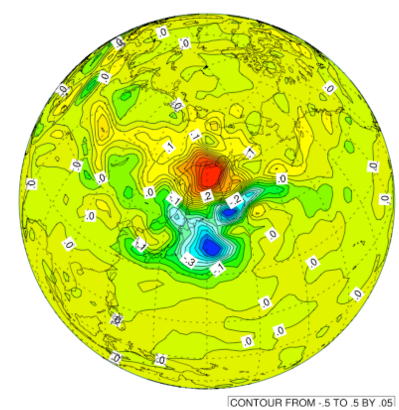












WRF-Var (Latest Code)

- Major new features:
  - Direct assimilation of satellite radiances (AMSU, AIRS, SSMI/S, etc.).
  - Four-Dimensional Variational Data Assimilation (4D-Var).
  - Ensemble Transform Kalman Filter (ETKF).
  - Hybrid variational/ensemble DA.
  - Enhanced forecast error covariances (e.g. ensemble-based).
  - Major software engineering reorganization.
  - Remove obsolete features (e.g. MM5/GFS-based errors).
  - Verification package
  - Utilization of observations packed in NCEP PREPBUFR format
  - Inclusion of various scripts and NCL based graphics
- Unified WRF/WRF-Var code repository.
- Unified WRF/WRF-Var namelist
- Extended wiki-based documentation.

WRF-Var Version 3.0 (Release April 2008)

- Major new features:
  - Direct assimilation of satellite radiances (AMSU, AIRS, SSMI/S, etc.).
  - Four-Dimensional Variational Data Assimilation (4D-Var).
  - Ensemble Transform Kalman Filter (ETKF).
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- Unified WRF/WRF-Var code repository.
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- Extended wiki-based documentation.

Not included in public release due to lack of funding.

Collaborations welcome!

Future Plans

General Goals:

- Unified, multi-technique WRF DA system.
- Retain flexibility for research, multi-applications.
- Leverage international WRF community efforts.

WRF-Var Development (MMM Division):

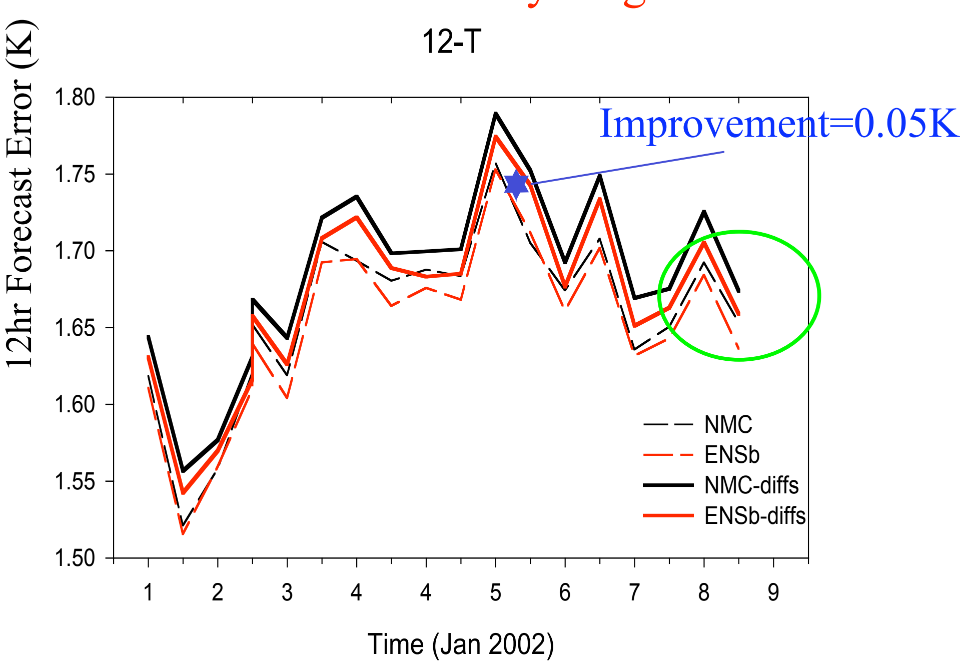
- 4D-Var (additional physics, optimization).
- Sensitivities tools (adjoint, ensemble, etc.).
- EnKF within WRF-Var -> WRFDA.
- Instrument-specific radiance QC, bias correction, etc.

Data Assimilation Testbed Center (DATC):

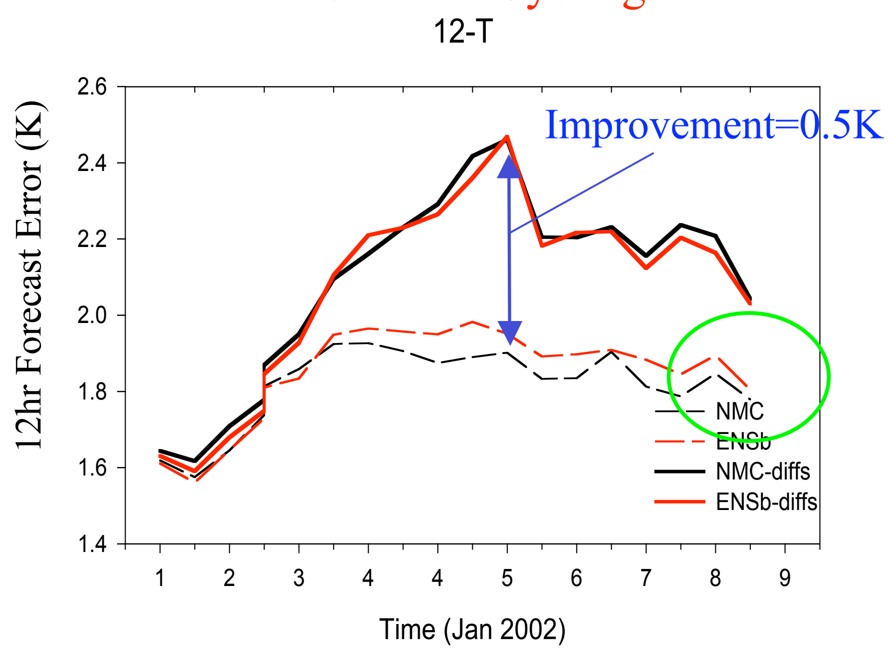
- Technique intercomparison: 3/4D-Var, EnKF, Hybrid
- Obs. impact: AIRS, TMI, SSMI/S, METOP.
- New Regional testbeds: US, India, Arctic, Tropics.

Importance of Data Assimilation For General WRF Development/Testing

Without DA Cycling



With DA Cycling



Warning: Cycling with insufficient observations leads to degradation (1.8K vs. 1.7K)

Experiment (Mi-Seon Lee, KMA)

5. Important issues covered by this tutorial

- y^o observations - collection, quality control, bias correction, thinning, …
- H observation operator, including the tangent linear operator **H** and the adjoint operator $\mathbf{H}^T$.
- M forecast model, including the tangent linear model **M** and adjoint model $\mathbf{M}^T$.
- **B** background error covariance (~ 10^7x10^7).
- **R** observation error covariance which includes the representative error (10^6x10^6).
- Minimization algorithm
- $\mathbf{P}^a$ and $\mathbf{P}^f$ analysis and forecast error covariances