Ensemble and Hybrid

WRF-Var Tutorial Presentation

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Why Ensemble and Hybrid?

- Ensemble DA can be directly utilized for ensemble probabilistic forecast

- Background errors are flow-dependent
  - 3D-Var uses static BE
  - 4D-Var implicitly uses flow-dependent information, but still starts from static BE
  - Ensemble DA computes flow-dependent covariances
  - Hybrid: using flow-dependent information from ensemble in WRF-Var

Flow-Dependent Ensemble BE Covariances

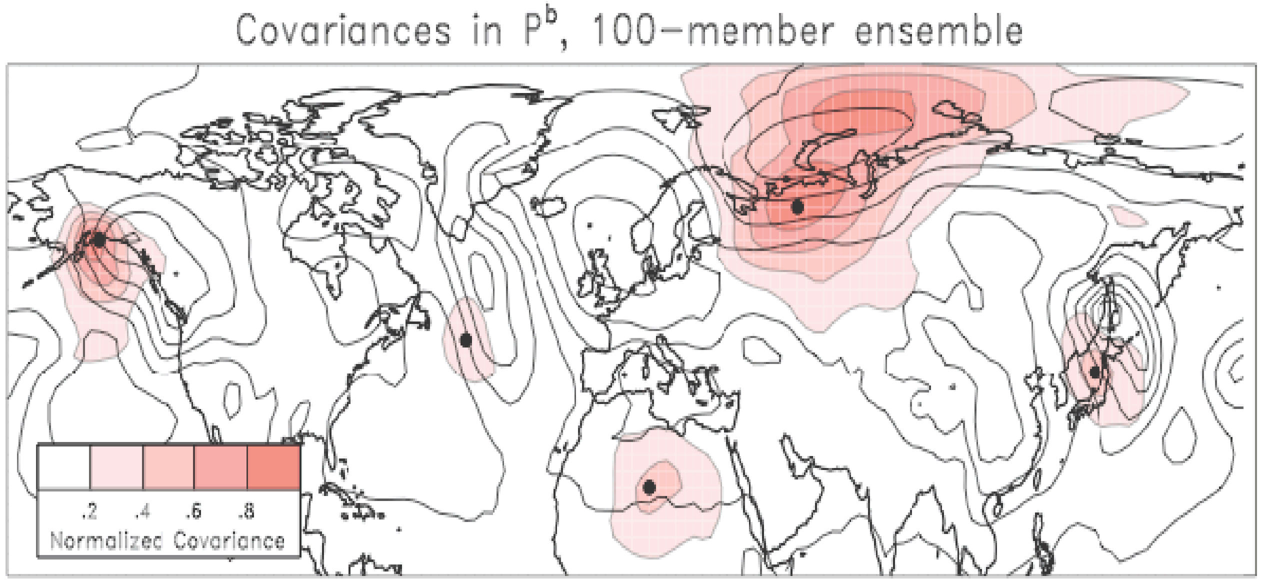
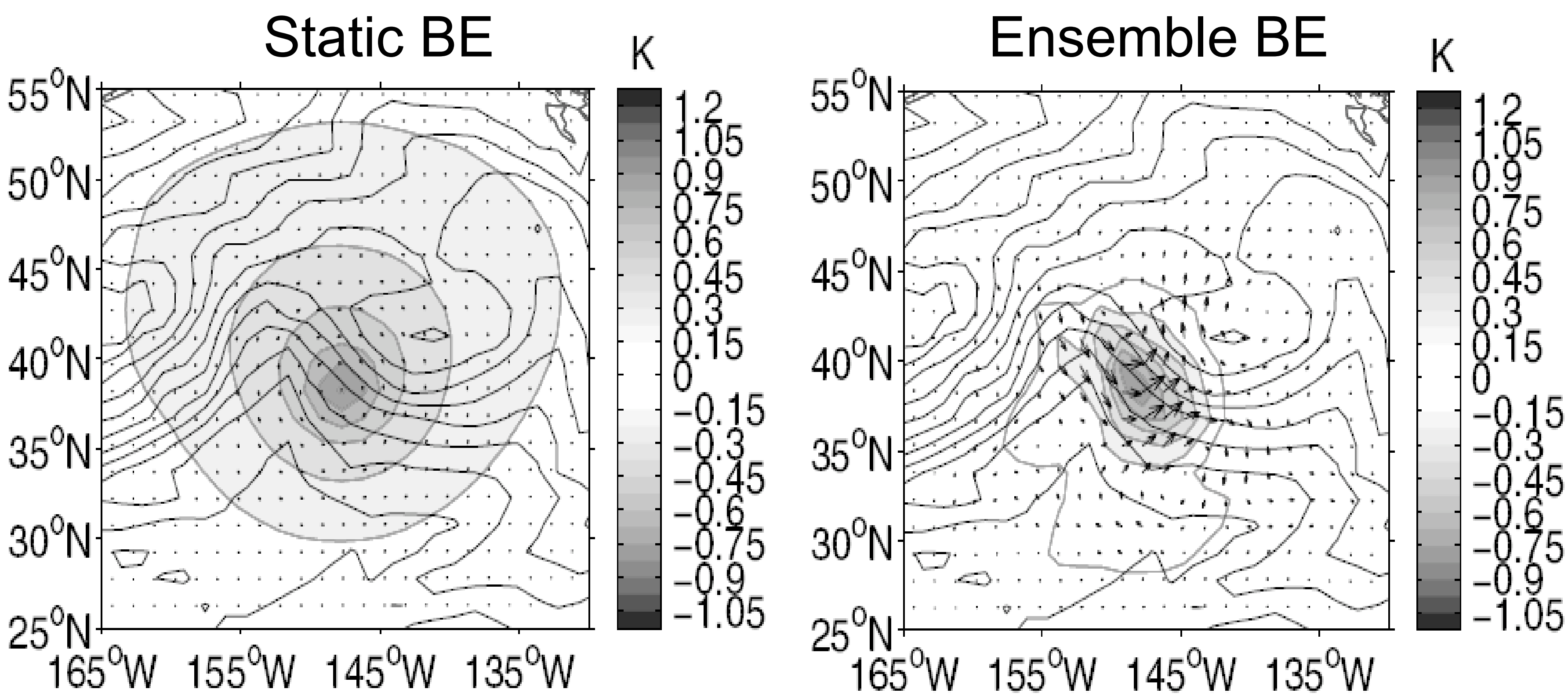


Figure 2. Background-error covariances (colors) of sea-level pressure in the vicinity of five selected observation locations, denoted by dots. Covariance magnitudes are normalized by the largest covariance magnitude on the plot. Solid lines denote ensemble mean background sea-level pressure contoured every 8 hPa.

(Hamill 2006)

Using Flow-Dependent BE in Var

T and wind increments from a single T observation



(Buehner 2005)

Ensemble DA in WRF-Var (WRFDA)

1. Ensemble Transform Kalman Filter (ETKF)

2. Hybrid ETKF/Var

3. Ensemble Kalman Filter (EnKF)

ETKF

- The ETKF (Bishop et al. 2001) finds the transformation matrix $\mathbf{T}$ so that

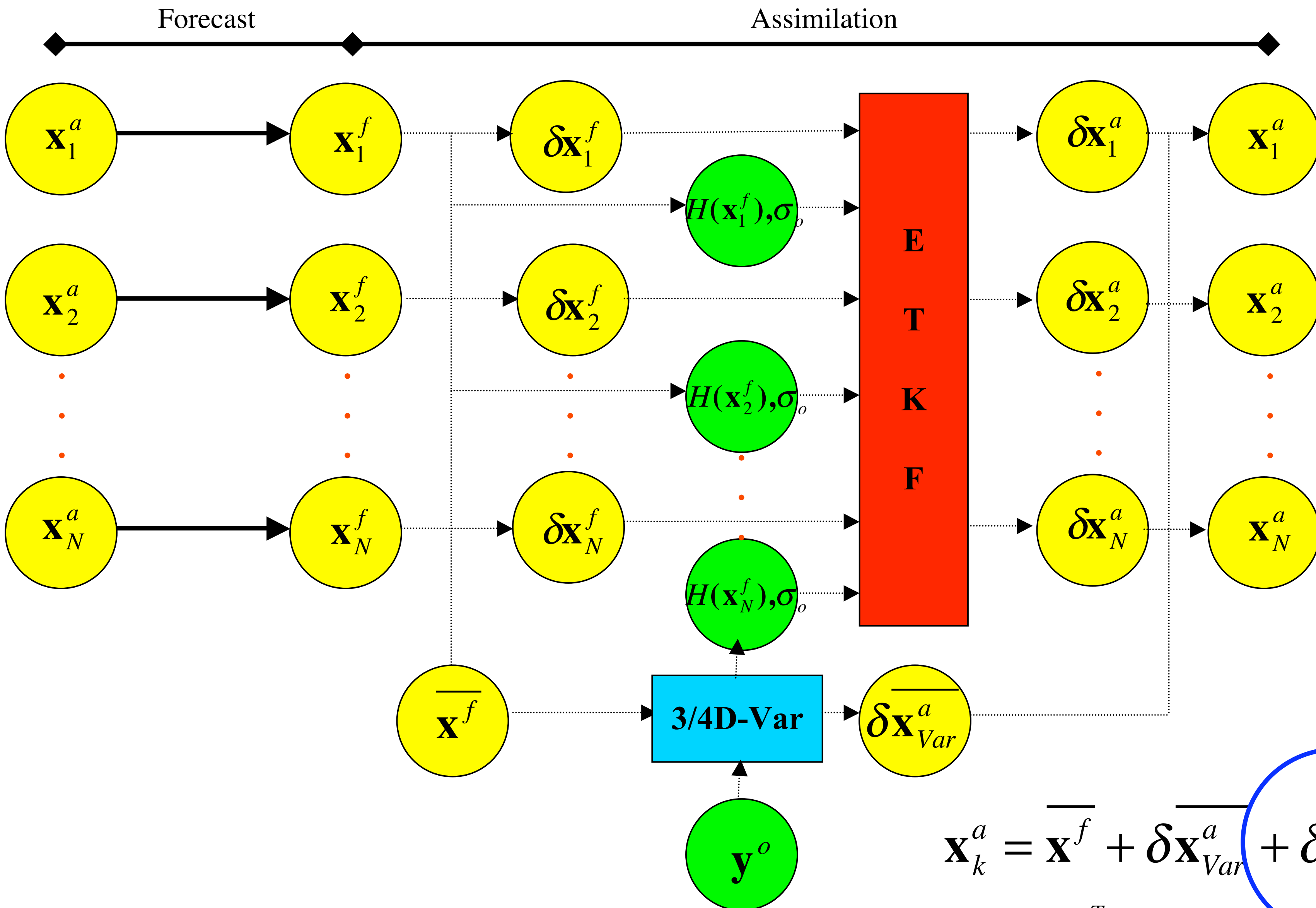
$$\mathbf{P}^a_{ens} = \frac{1}{N-1}(\delta\mathbf{X}^f\mathbf{T})(\delta\mathbf{X}^f\mathbf{T})^{\mathrm{T}}$$

- The perturbations are updated, but not ensemble mean

$$\delta\mathbf{X}^a = \delta\mathbf{X}^f\mathbf{T}$$

- ETKF is fast, but also prone to sampling error if lack of localization
- Use ETKF to generate ensemble forecast IC perturbations, and flow-dependent errors in hybrid DA

1. Cycling WRF/WRF-Var/ETKF System



$$\mathbf{x}_k^a = \overline{\mathbf{x}^f} + \overline{\delta\mathbf{x}_{Var}^a} + \delta\mathbf{x}_k^a$$

$$J = \frac{1}{2}\delta\mathbf{x}_0{}^T\mathbf{B}_o^{-1}\delta\mathbf{x}_0 + \frac{1}{2}\sum_i\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]^T\mathbf{R}_i^{-1}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]$$

ETKF Ensemble Perturbations

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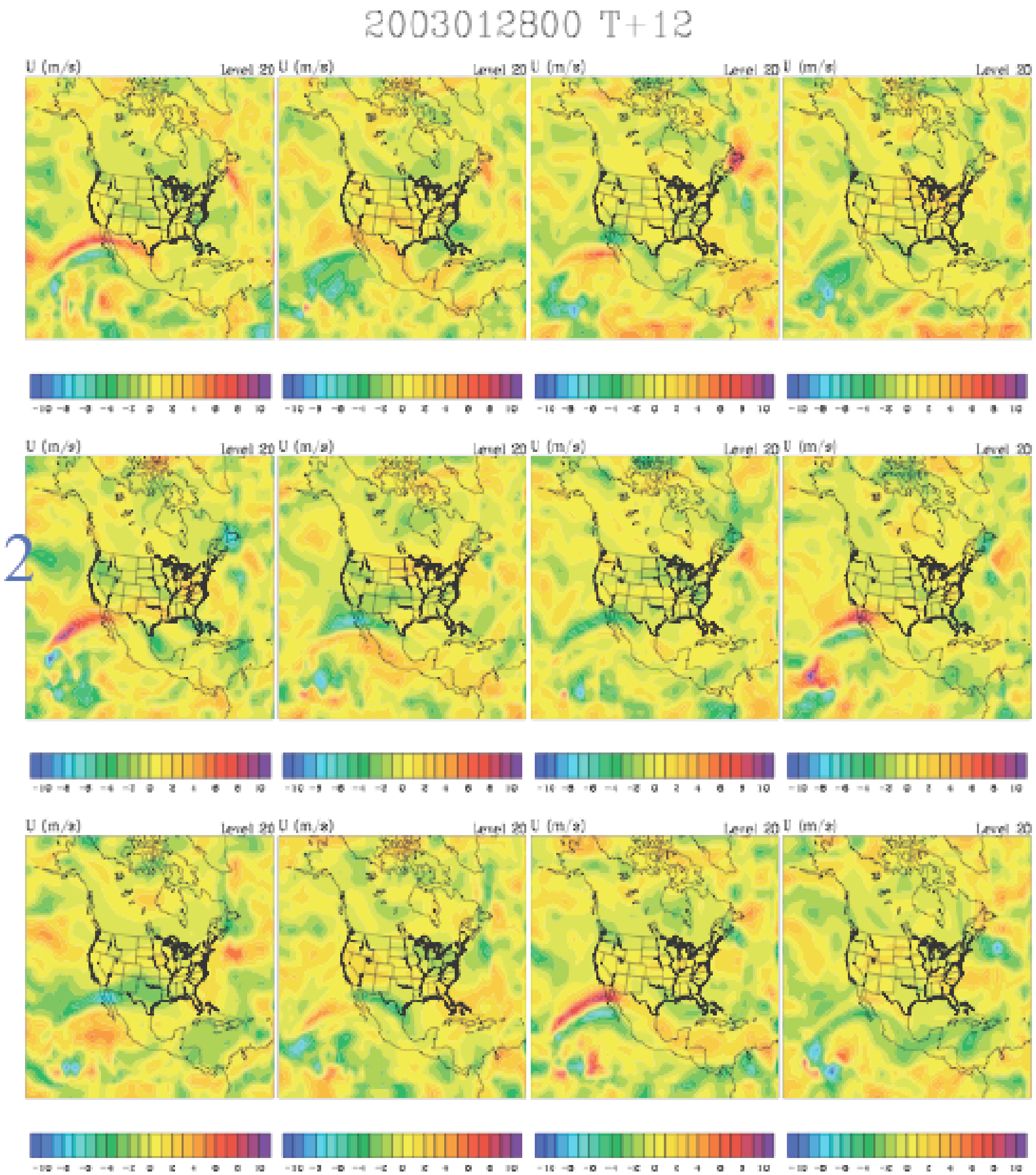


Hybrid ETKF/Var

- Hybrid 3/4D-Var analysis increments (ensemble mean)

$$\overline{\delta\mathbf{x}^a_{\mathrm{var}}} = \overline{\delta\mathbf{x}^a_{\mathrm{var\_only}}} + \sum_{k=1}^{N} \mathbf{a}_k \circ \delta\mathbf{x}^f_k$$

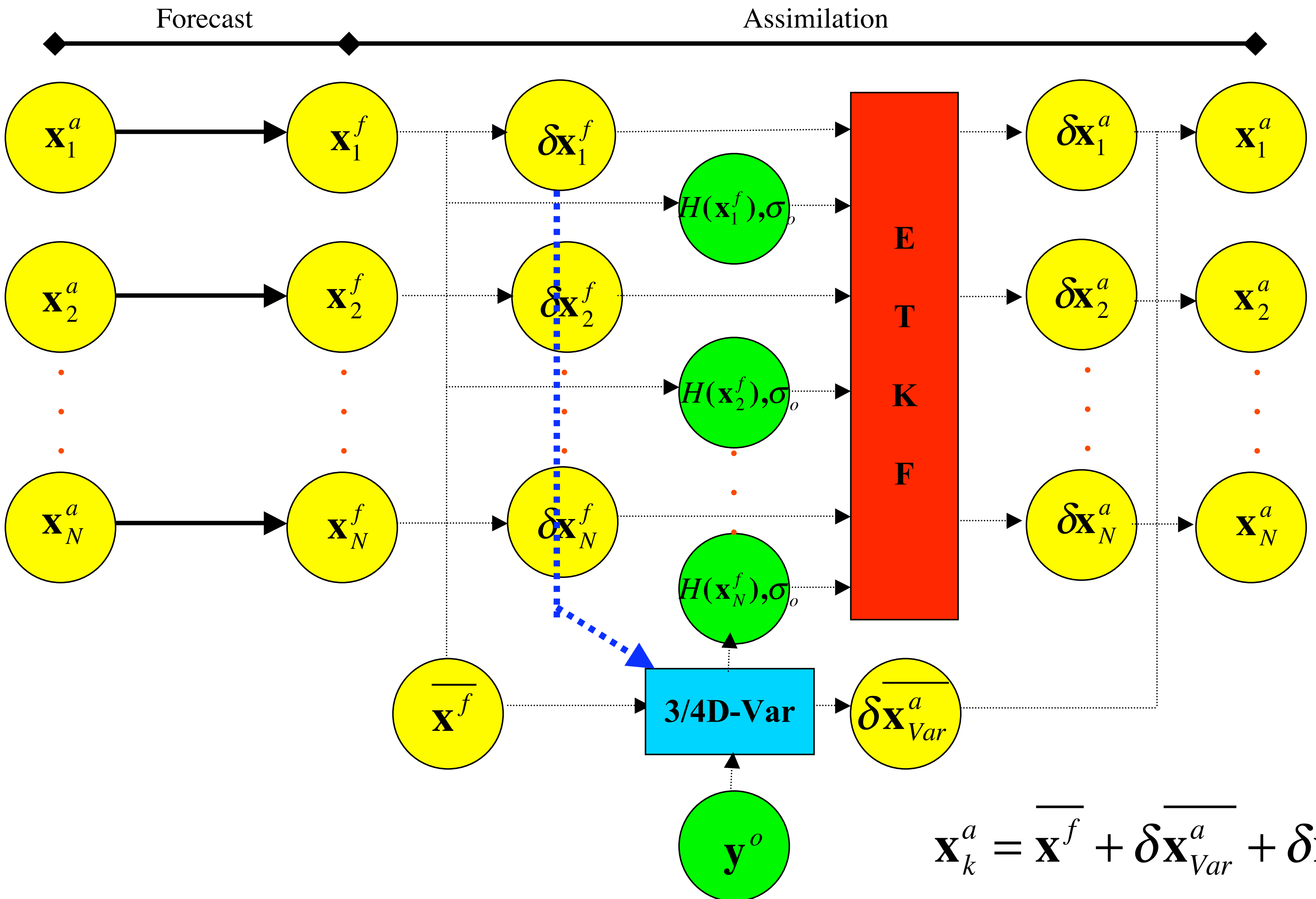
- The perturbations are updated using ETKF as before

- Flow-dependence is constrained by an additional cost-function:

$$J = \frac{W_b}{2}\delta\mathbf{x}_0{}^T\mathbf{B}_o^{-1}\delta\mathbf{x}_0 + \boxed{\frac{W_\alpha}{2}\mathbf{a}^T\mathbf{A}^{-1}\mathbf{a}} + \frac{1}{2}\sum_i\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]^T\mathbf{R}_i^{-1}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]$$

- Hybrid may be more robust for small ensemble size and/or large model errors (Wang et al. 2008a,b)

2. Cycling WRF/WRF-Var/ETKF System (Hybrid DA)



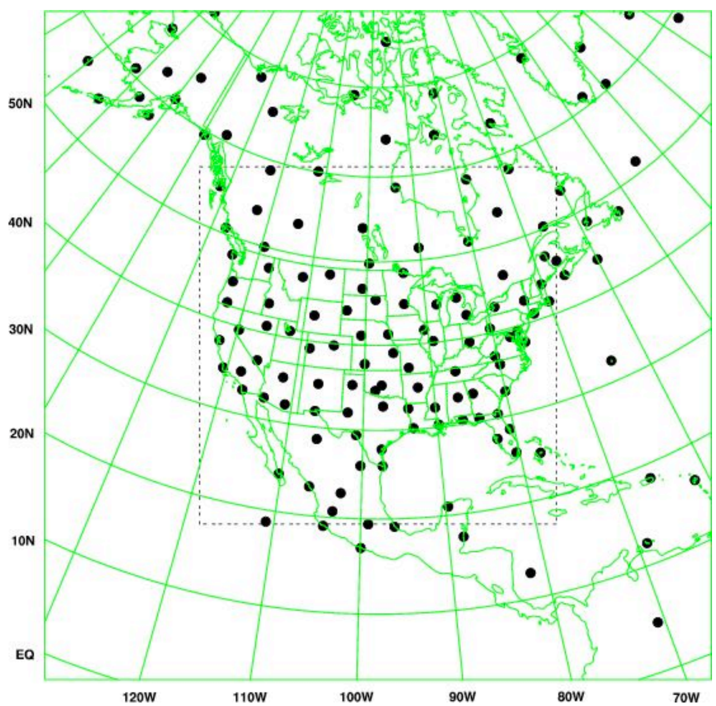
$$\mathbf{x}_k^a = \overline{\mathbf{x}^f} + \delta\overline{\mathbf{x}_{Var}^a} + \delta\mathbf{x}_k^a$$

$$J = \frac{W_b}{2}\delta\mathbf{x}_0{}^T\mathbf{B}_o^{-1}\delta\mathbf{x}_0 + \frac{W_\alpha}{2}\mathbf{a}^T\mathbf{A}^{-1}\mathbf{a} + \frac{1}{2}\sum_i\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]^T\mathbf{R}_i^{-1}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]$$

Hybrid Example

OSSE and real obs. experiment with WRF

WRF domain, observation locations and verification region



Wang et al. 2008ab

WRF domain: North America; 200km resolution; 45x45x28 grid

Observation: simulated (OSSE) and real (real obs. Exp) U,V,T radiosondes

Cycle: every 12h during Jan 2003.

ETKF ensemble: 50-member

LBC ensemble: AVN analysis + 50 perturbations drawn from default 3DVAR static covariance.

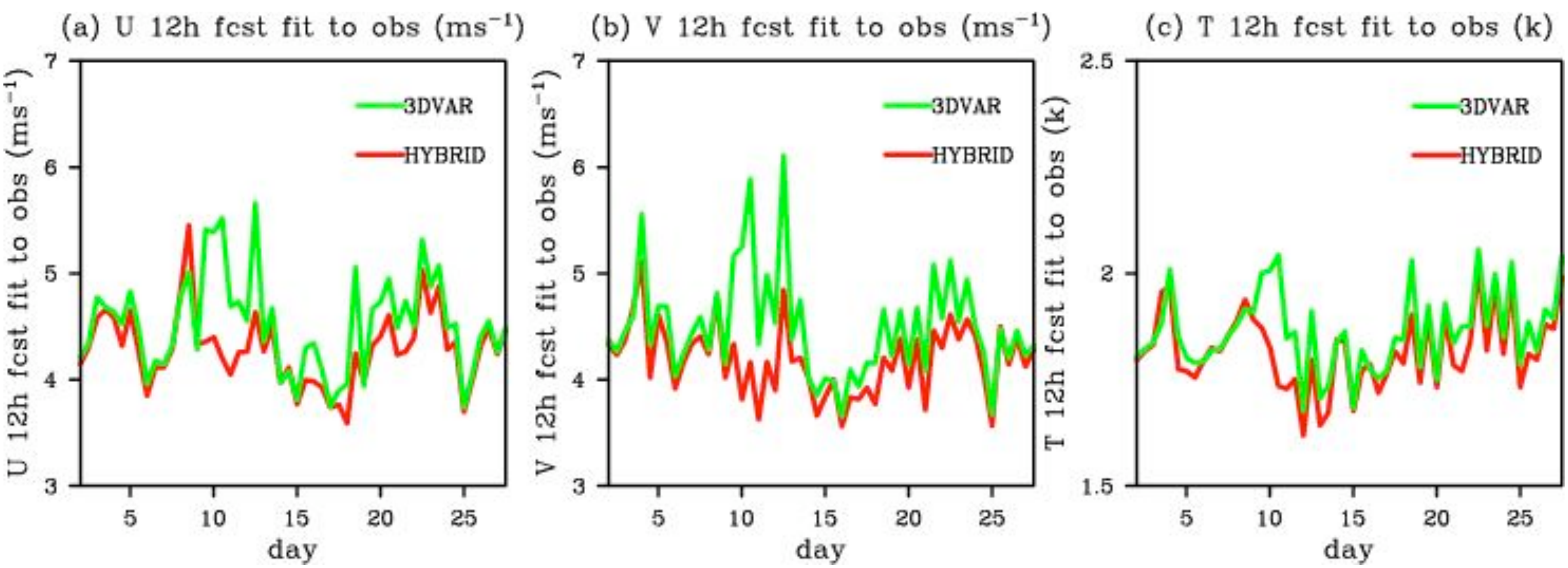
Verifications:

OSSE: against "truth" at all grids and variables within the verification domain

REAL: fitting 12h forecasts to obs. within the verification domain.

Hybrid Example (cont.)

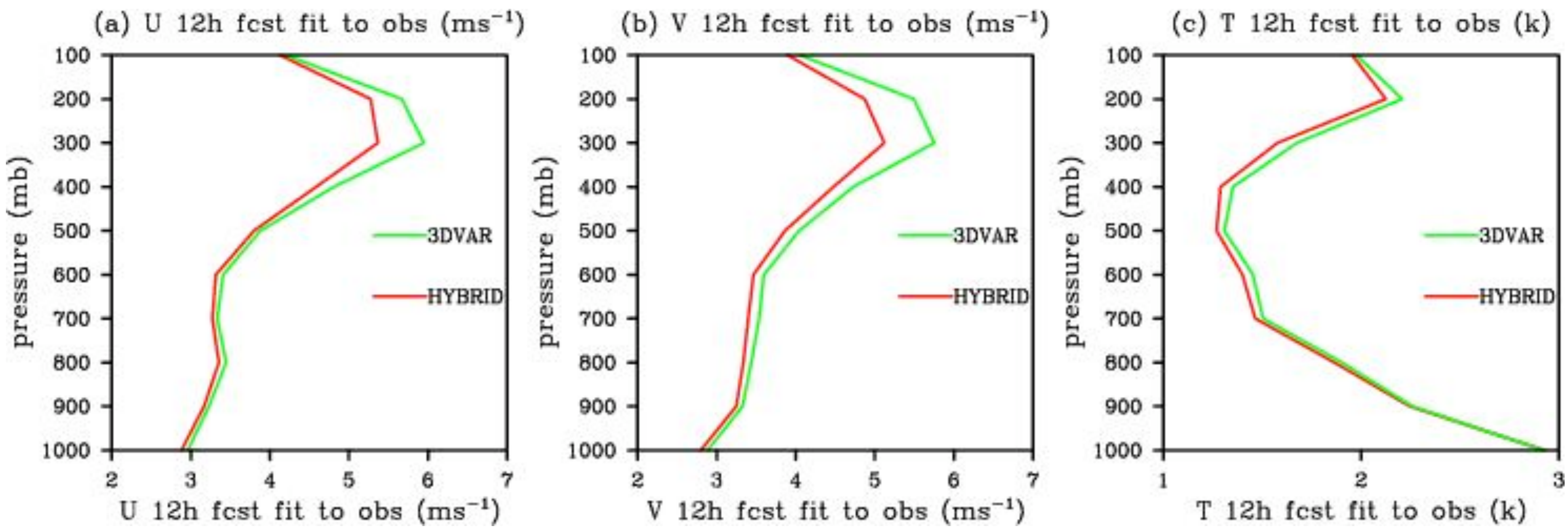
Real obs. experiment: 12h forecast fit to obs.



- Hybrid 12h forecast is more accurate than the 3DVAR for most time.

Hybrid Example (cont.)

Real obs. experiment: 12h forecast fit to obs.



- **Wind:** Hybrid has the largest improvement at 200mb-300mb;
- **Temperature:** Improvement smaller than wind. No improvement at lower troposphere (significant bias).

EnKF

- Kalman filter equations

$$\overline{\mathbf{x}^a} = \overline{\mathbf{x}^f} + \mathbf{K}[y^o - \overline{H(\mathbf{x}^f)}]$$

$$\mathbf{K} = \mathbf{P}^f\mathbf{H}^T(\mathbf{H}\mathbf{P}^f\mathbf{H}^T + \mathbf{R})^{-1}$$

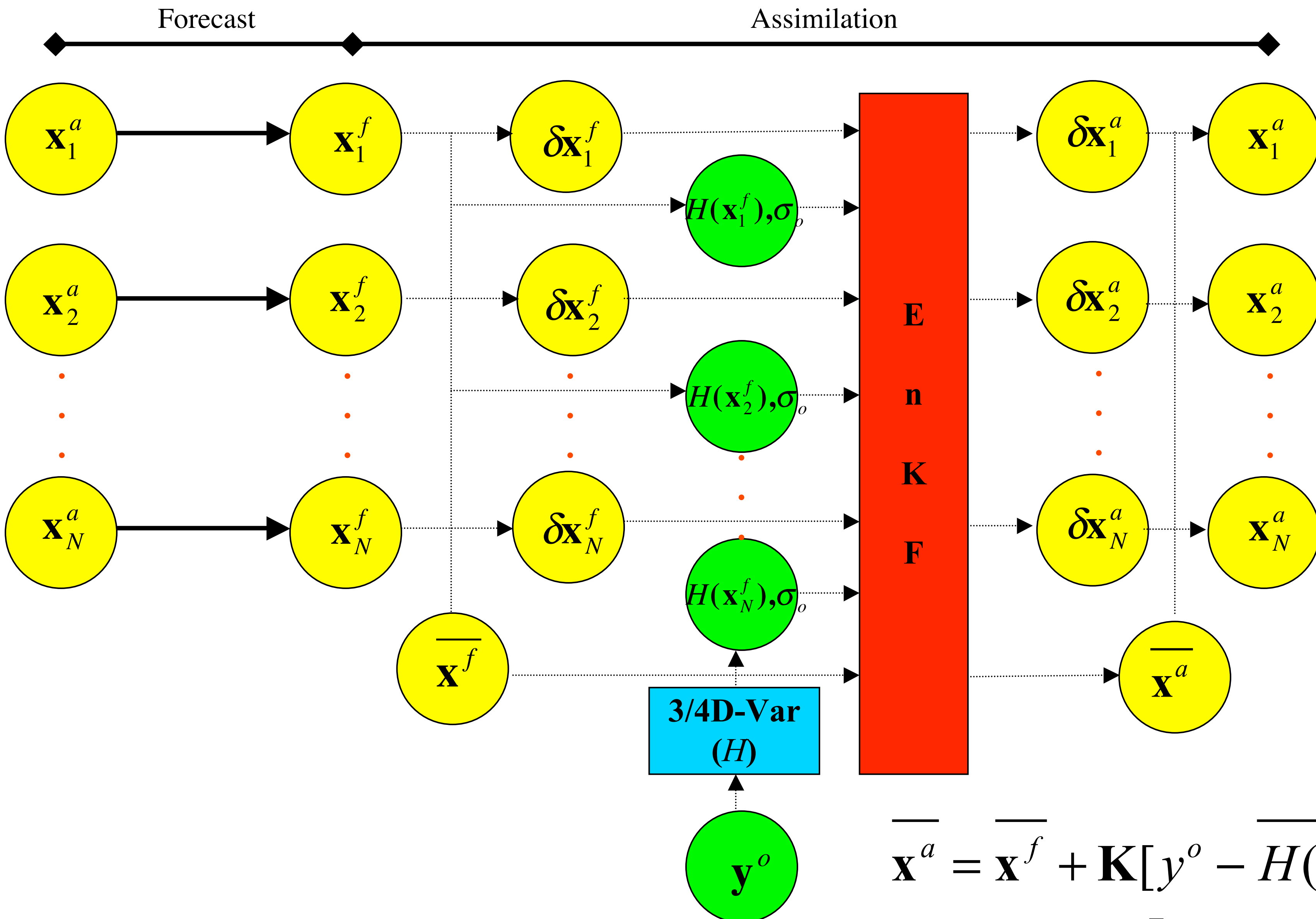
$$\mathbf{P}^a = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}^f$$

- Use ensemble of model forecasts to compute sample covariances

$$\mathbf{P}^f\mathbf{H}^T = \mathrm{cov}(\mathbf{x}^f, \mathbf{H}\mathbf{x}^f) = \frac{1}{N-1}\sum_{k=1}^{N}(\mathbf{x}_k^f - \overline{\mathbf{x}^f})[H(\mathbf{x}_k^f) - \overline{H(\mathbf{x}_k^f)}]$$

$$\mathbf{H}\mathbf{P}^f\mathbf{H}^T = \mathrm{cov}(\mathbf{H}\mathbf{x}^f, \mathbf{H}\mathbf{x}^f) = \frac{1}{N-1}\sum_{k=1}^{N}[H(\mathbf{x}_k^f) - \overline{H(\mathbf{x}_k^f)}][H(\mathbf{x}_k^f) - \overline{H(\mathbf{x}_k^f)}]$$

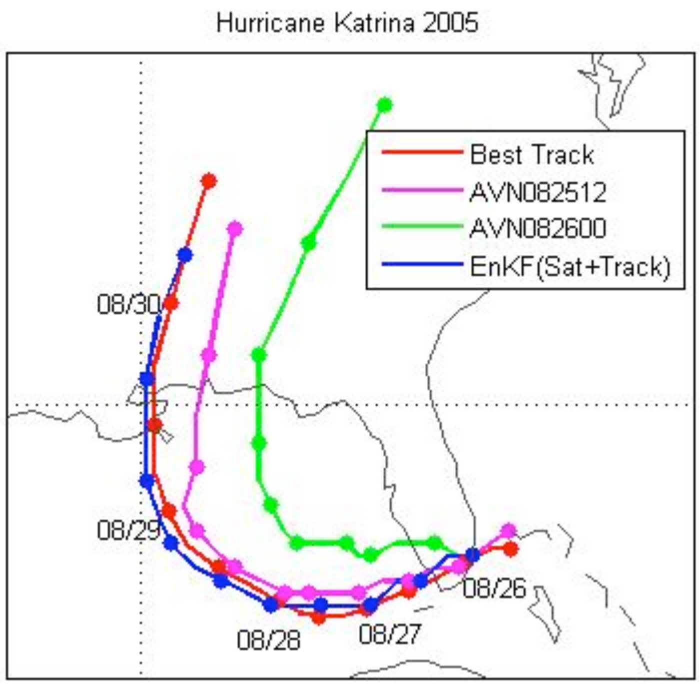
3. Cycling WRF/EnKF System

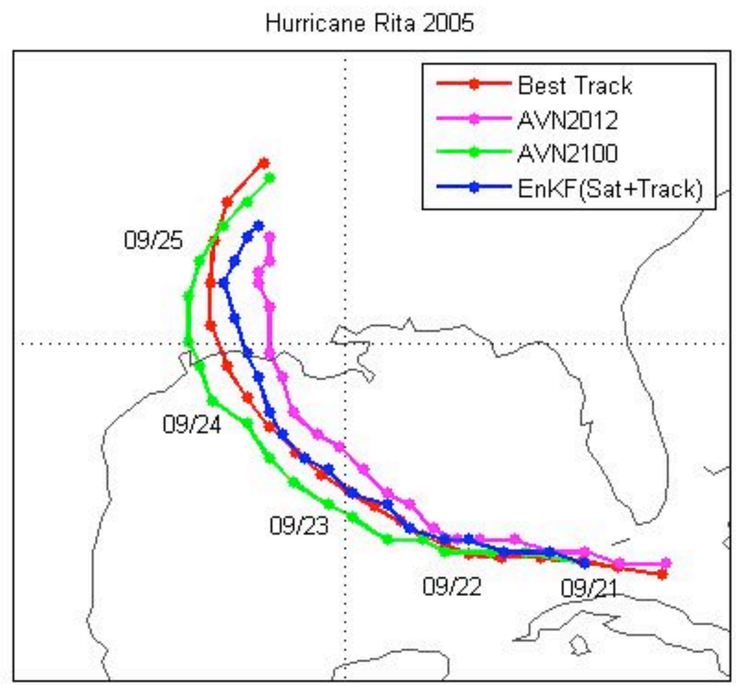


$$\overline{\mathbf{x}^a} = \overline{\mathbf{x}^f} + \mathbf{K}[y^o - \overline{H(\mathbf{x}^f)}]$$

$$\delta\mathbf{x}_i^a = \delta\mathbf{x}_i^f - \alpha\mathbf{K}\left[H(\mathbf{x}_i^f) - \overline{H(\mathbf{x}_i^f)}\right]$$

EnKF Example

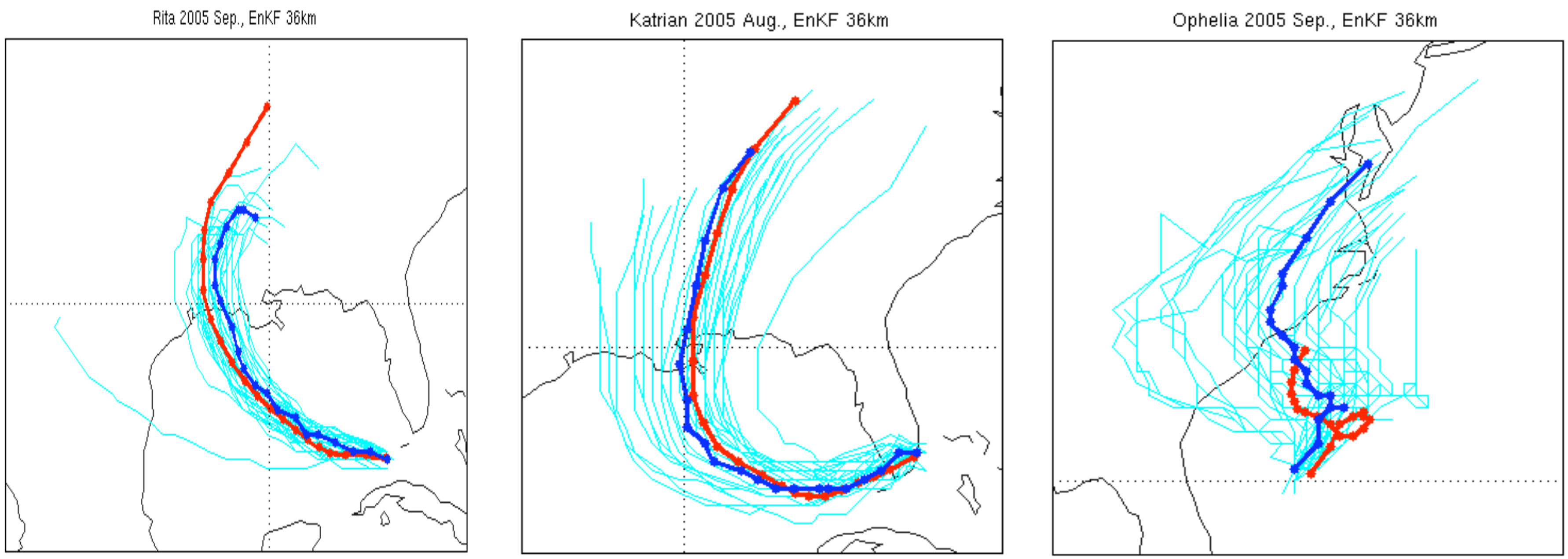




- 36-km horizontal resolution, 26 ensemble members
- Assimilate position, intensity, and satellite winds every hour for a total of 12 hours
- Compare forecasts initialized from the EnKF analysis and from the AVN forecasts.

Ensemble Forecast Example

5-day ensemble track forecast



Challenges in Ensemble DA

- Computational cost
- Sampling error - localization
- Covariance deficiency - inflation