Status of recently released WRF 4D-Var

Xin Zhang, Xiang-Yu Huang and Yong-Run Guo
MMM Division, NCAR, Boulder, CO USA
Ling-Feng Hsiao
Taiwan Central Weather Bureau, Taipei

Content of talk

- Why is 4D-Var performing better than 3D-Var?
- Overview of the WRF 4D-Var
- Observations used by the data assimilation system
- Weak constraint with digital filter
- Multi-incremental 4D-Var
- Recent performance of WRF 4D-Var System
- Computational issues
- Further developments

4D-Var versus 3D-Var

(Adopted from ECMWF training Course 2008)

- 4D-Var is comparing observations with background model fields at the correct time
- 4D-Var can use observations from frequently reporting stations
- The dynamics and physics of the forecast model in an integral part of 4D-Var, so observations are used in a meteorologically more consistent way
- 4D-Var combines observations at different times during the 4D-Var window in a way that reduces analysis error
- 4D-Var propagates information horizontally and vertically in a meteorologically more consistent way

Four-dimensional Variational Approach

The general cost function of the variational formulation:

$$J(\mathbf{x}) = \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}^b)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}^b) + J_x$$
$$+ \frac{1}{2}\sum_{k=0}^{K}[\mathbf{h}(\mathbf{x}_k) - \mathbf{y}_k]^T \mathbf{R}_k^{-1}[\mathbf{h}(\mathbf{x}_k) - \mathbf{y}_k]$$

where

- $\mathbf{x} \equiv [\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_K]^T$ **is a 4-dimensional state vector;**
- h **is the nonlinear observation operator;**
- B **and** R_k **are the background, and observation error covariances, respectively;**
- J_x **represents extra constraint (e.g., balance).**

Strong Constraint Incremental 4DVAR

(Courtier, Thépaut and Hollingsworth (1994))

For simplicity consider now the strong constraint case. In incremental 4DVAR the cost function at the j-th iteration is

$$
\begin{aligned}
J_j(\delta \mathbf{x}_j) &= \frac{1}{2}(\delta \mathbf{x}_j - \delta \mathbf{x}_j^b)^T \mathbf{B}^{-1}(\delta \mathbf{x}_j - \delta \mathbf{x}_j^b) \\
&+ \frac{1}{2}\sum_{k=0}^{K}(\mathbf{H}_{j,k}\mathbf{M}_{j,k}\delta \mathbf{x}_j - \mathbf{d}_{j,k})^T \mathbf{R}^{-1}(\mathbf{H}_{j,k}\mathbf{M}_{j,k}\delta \mathbf{x}_j - \mathbf{d}_{j,k})
\end{aligned}
$$

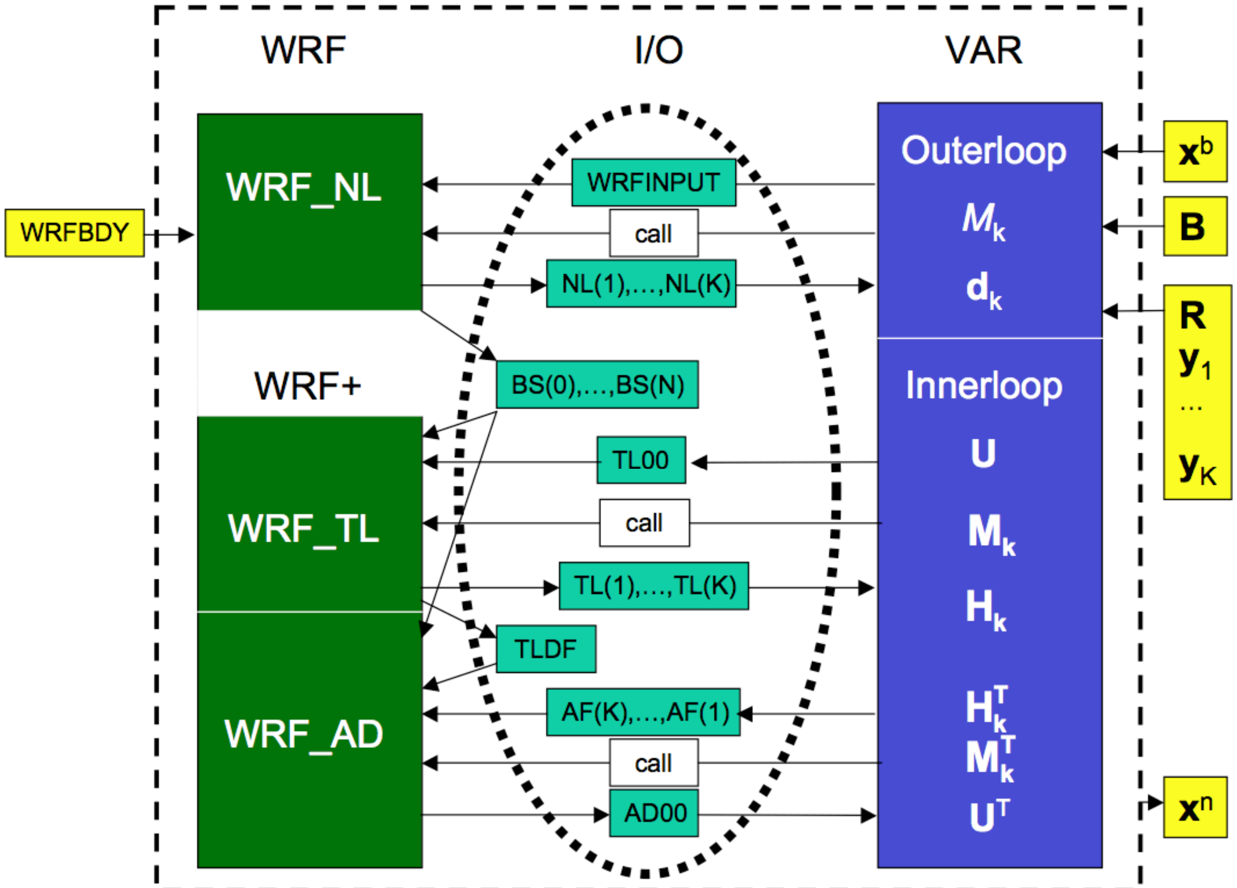
WRF NL

WRF TL (WRF AD)

where $\mathbf{d}_{j,k} \equiv \mathbf{y}_k - \mathbf{h}_k(\mathbf{m}_k(\mathbf{x}^b))$, $\delta \mathbf{x}_j^b \equiv \mathbf{x}^b - \mathbf{x}_{j-1}$ **and**

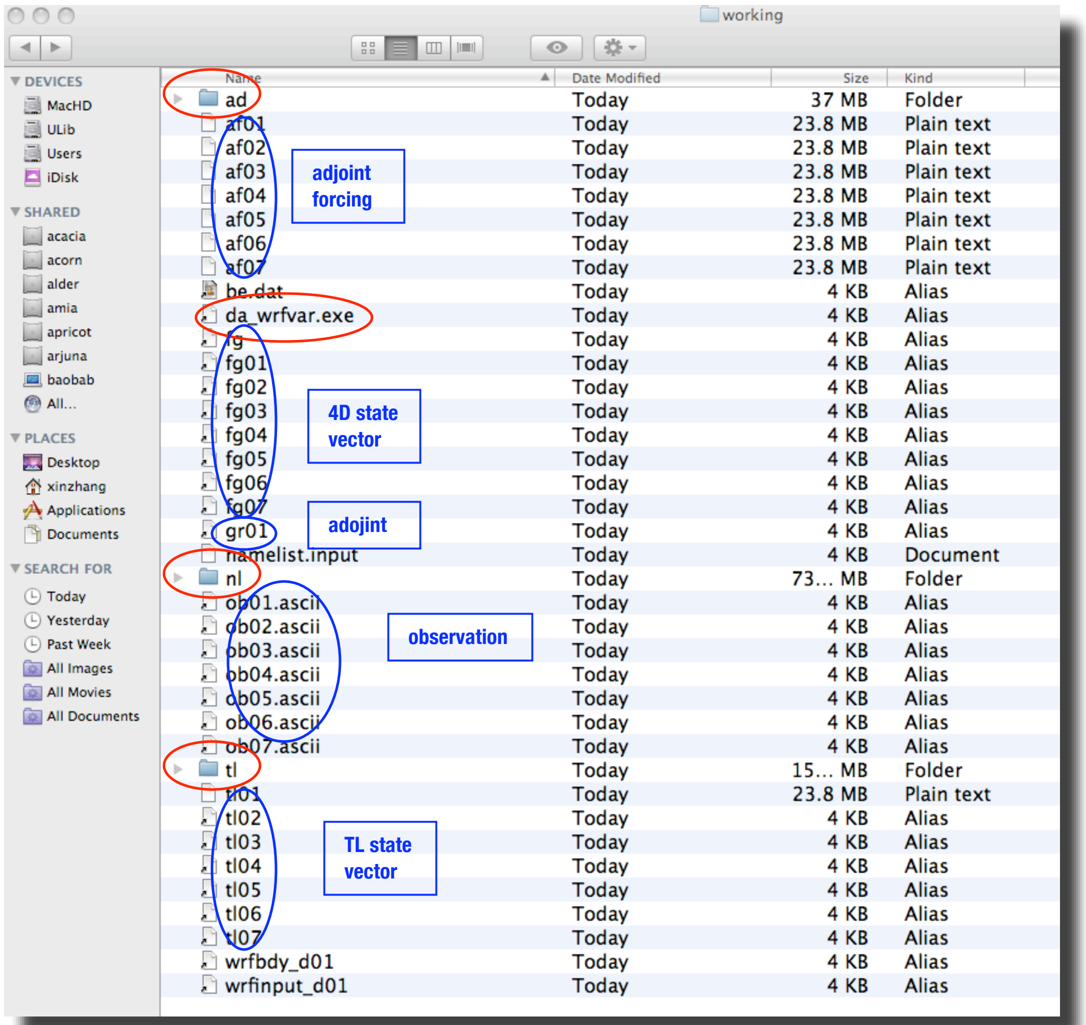
- $\delta \mathbf{x}_j \equiv \mathbf{x}_j - \mathbf{x}_{j-1}$ **is the control variable;**
- **The inner loop minimization of** J_j **can be solved by Conjugate gradient or Lanczos**

Structure of WRF 4D-Var



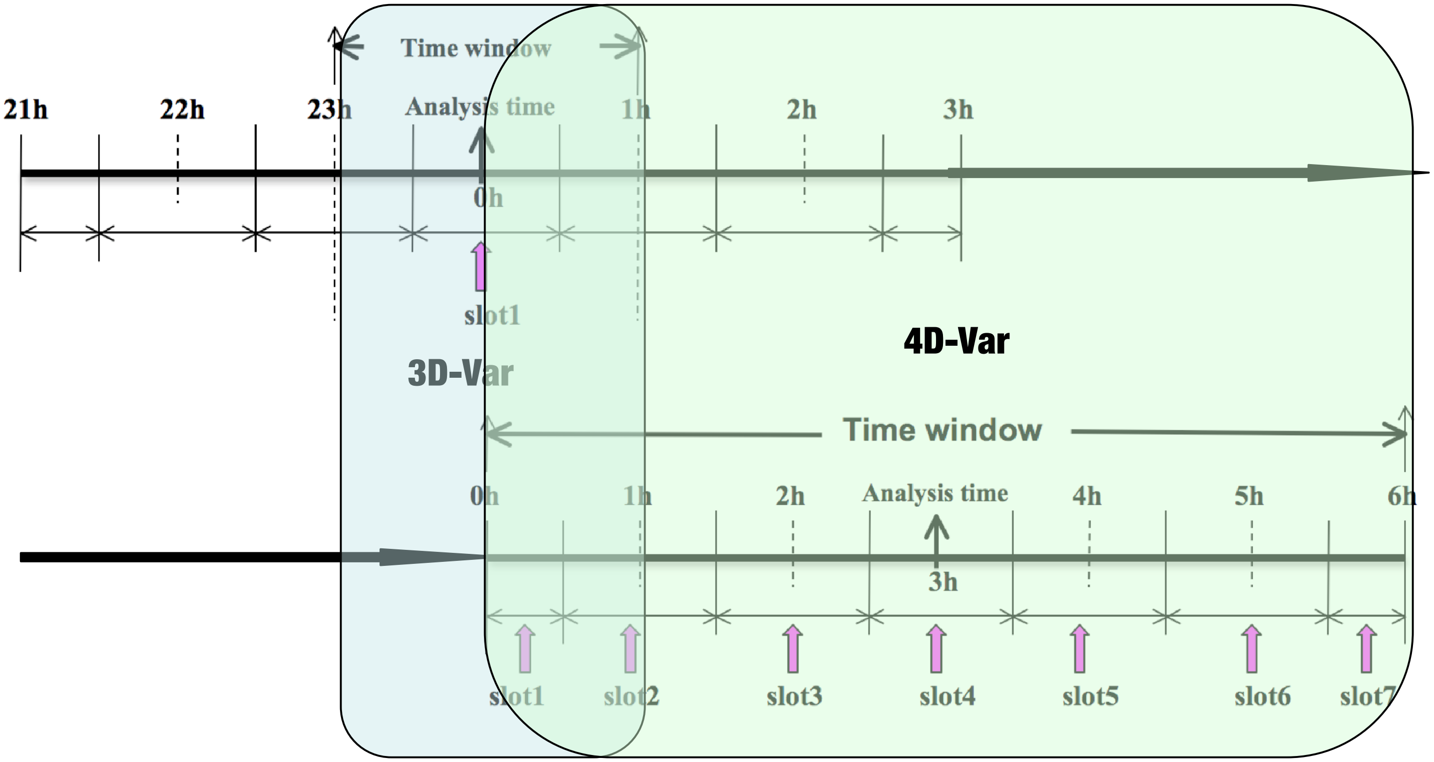
Hans Huang: WRF 4D-Var MMM Seminar, 13 December 2007

Structure of WRF 4D-Var (Cont'd)



Observations used by the 4D-Var

- Conventional observation data
- Radar radial velocity
- Radiance satellite data (under testing)



Weak constraint with digital filter

$$J = J_b + J_o + J_c$$

$$J_b(\mathbf{x}_0) = \frac{1}{2}\left[(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1} (\mathbf{x}_0 - \mathbf{x}_b)\right]$$

$$J_o(\mathbf{x}_0) = \frac{1}{2}\sum_{k=1}^{K}\left[(\mathbf{H}_k\mathbf{x}_k - \mathbf{y}_k)^T \mathbf{R}^{-1} (\mathbf{H}_k\mathbf{x}_k - \mathbf{y}_k)\right]$$

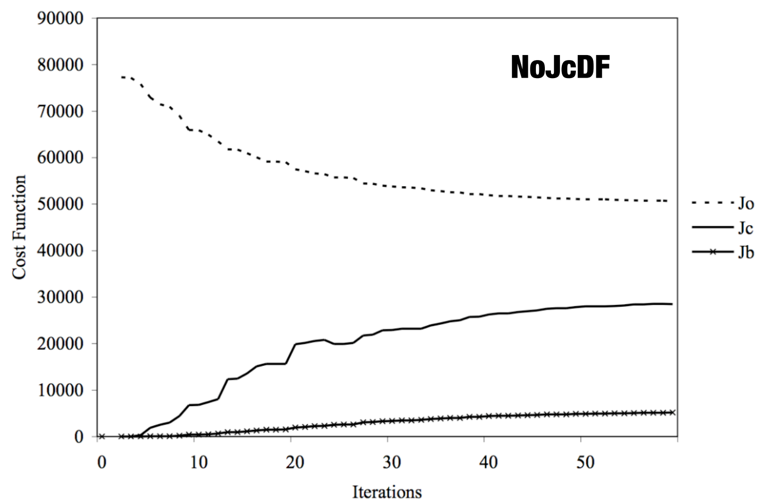
$$J_c(\mathbf{x}_0) = \frac{\gamma_{df}}{2}\left[(\delta\mathbf{x}_{N/2} - \delta\mathbf{x}_{N/2}^{df})^T \mathbf{C}^{-1} (\delta\mathbf{x}_{N/2} - \delta\mathbf{x}_{N/2}^{df})\right]$$

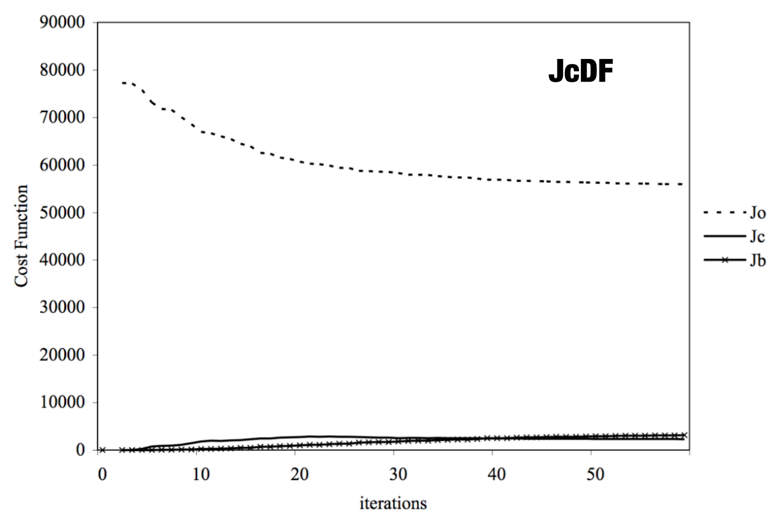
$$= \frac{\gamma_{df}}{2}\left[\left(\delta\mathbf{x}_{N/2} - \sum_{i=0}^{N} f_i \delta\mathbf{x}_i\right)^T \mathbf{C}^{-1} \left(\delta\mathbf{x}_{N/2} - \sum_{i=0}^{N} f_i \delta\mathbf{x}_i\right)\right]$$

$$= \frac{\gamma_{df}}{2}\left[\left(\sum_{i=0}^{N} h_i \delta\mathbf{x}_i\right)^T \mathbf{C}^{-1} \left(\sum_{i=0}^{N} h_i \delta\mathbf{x}_i\right)\right]$$

where:

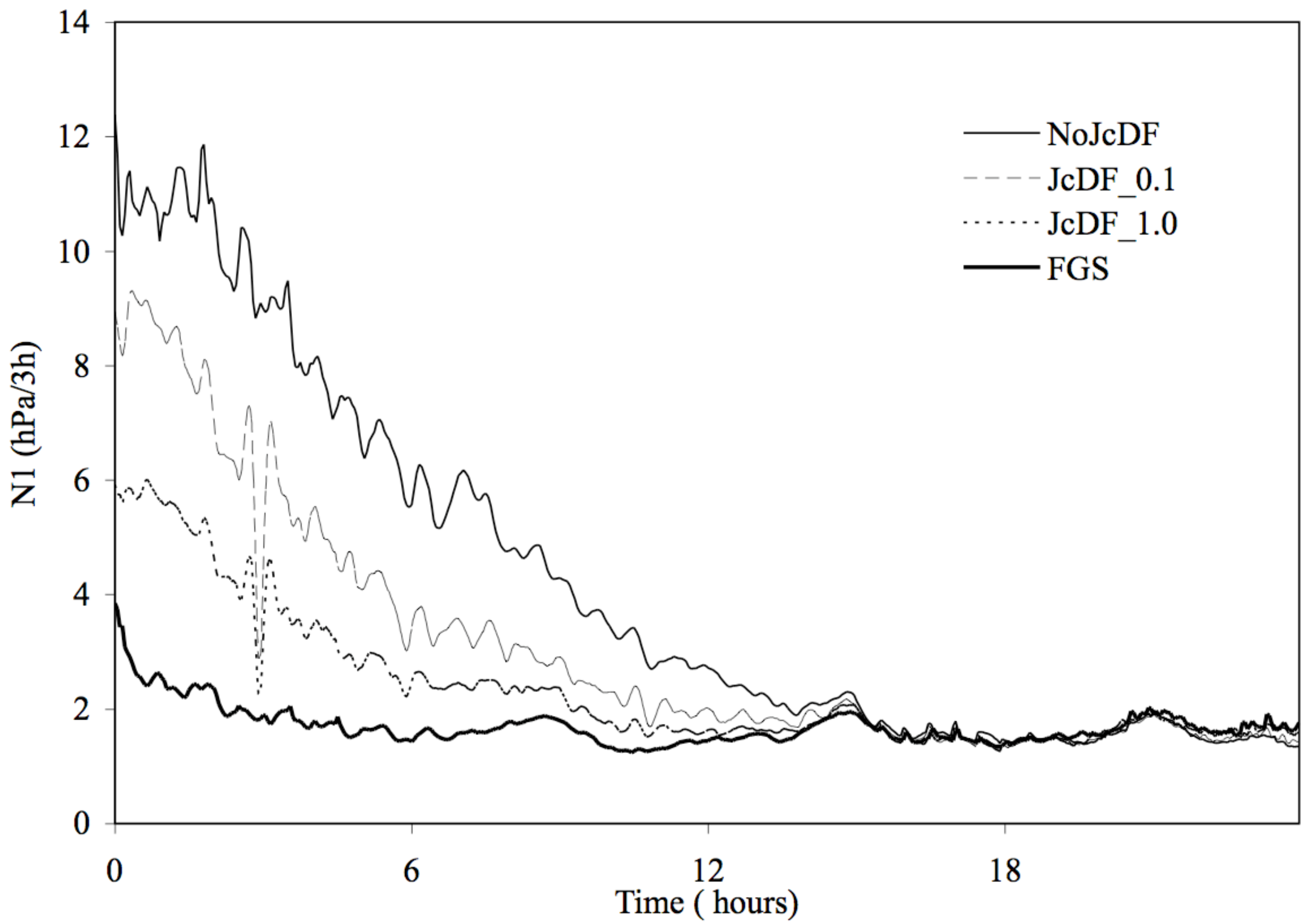
$$h_i = \begin{cases} -f_i, & \text{if } i \neq N/2 \\ 1 - f_i, & \text{if } i = N/2 \end{cases}$$





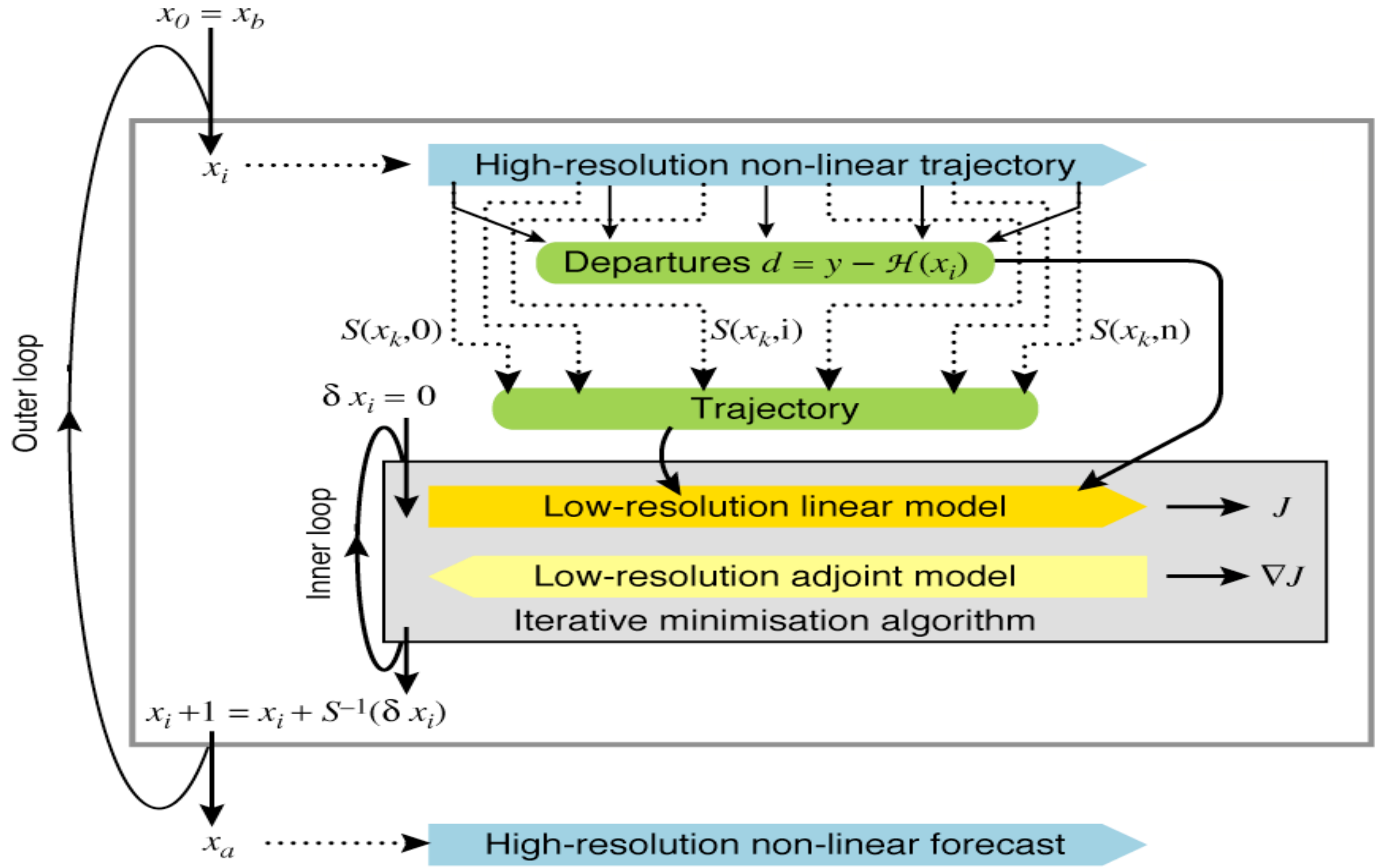
Weak constraint with digital filter

(domain averaged surface pressure variation)



Multi-incremental WRF 4D-Var

(Adopted from ECMWF training course 2008)



Multi-incremental WRF 4D-Var procedure

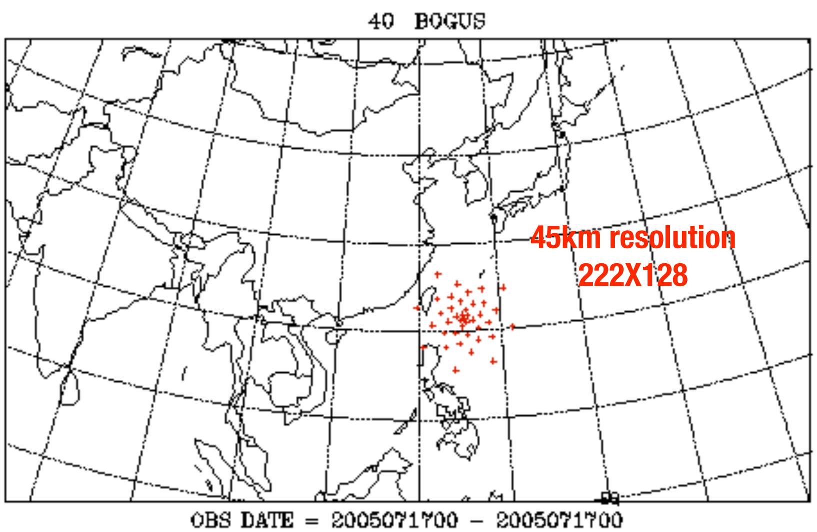
(Under testing)

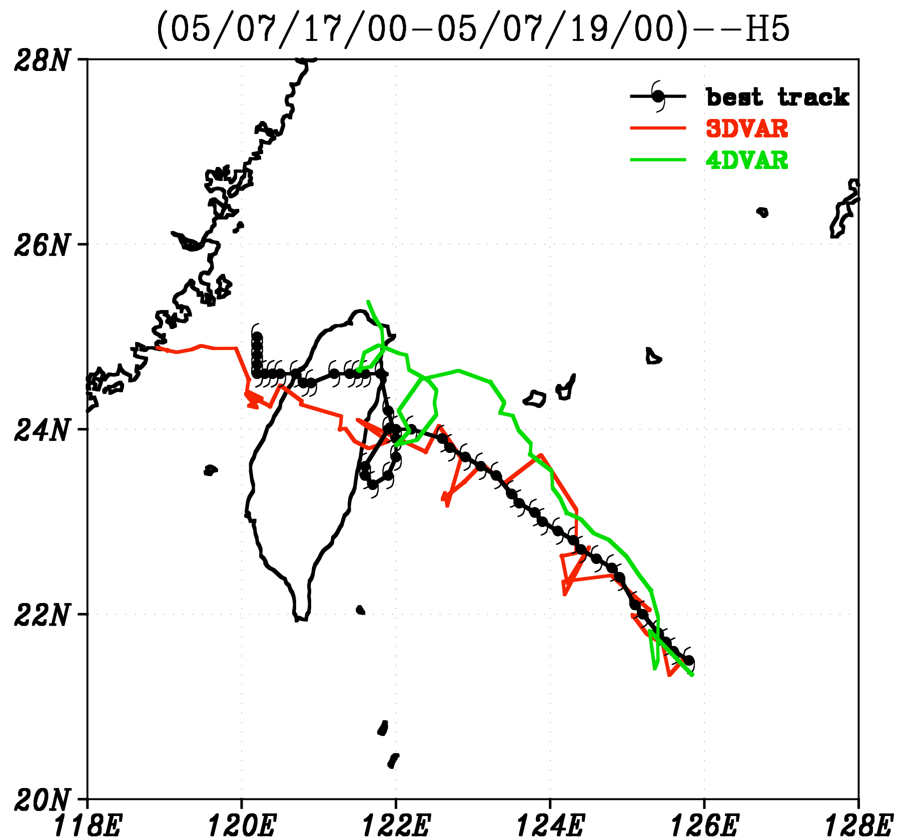
Use all data in a 6-hour window (0900-1500 UTC for 1200 UTC analysis)

1. **Group observations into 1 hour time slots**
2. **Run the (15km) high resolution forecast from the previous analysis and compute "observation"- "model" differences**
3. **Adjust the model fields at the start of assimilation window (0900 UTC) so the 6-hour forecast better fits the observations. This is an iterative process using a lower resolution linearized model (45km) and its adjoint model**
4. **Rerun the (15km) high resolution model from the modified (improved) initial state and calculate new observation departures**
5. **The 3-4 loop is repeated two times to produce a good high resolution estimate of the atmospheric state – the result is the WRF 4D-Var analysis**

Typhoon Haitang 2005

Analysis time	2005071700
Data assimilation window	2005071700~ 2005071706
Background	6 hour forecast from 2005071618 NCEP GFS 1x1 analysis
Observations	SOUND, SOUNDEsfc, SYNOP, SIREP, PILOT, METAR, SHIP, SATEM, BOUY, BOGUS (1000, 925, 850, 700, 500, 400hPa)





First radar data assimilation with WRF 4D-Var

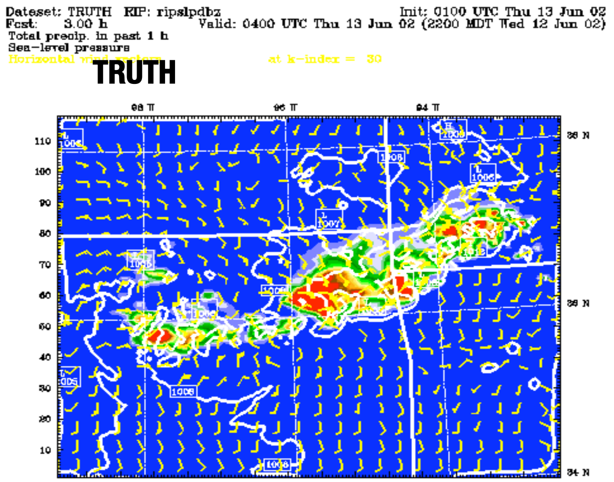
- TRUTH ----- Initial condition from TRUTH (13-h forecast initialized at 2002061212Z from AWIPS 3-h analysis) run cutted by ndown, boundary condition from NCEP GFS data.

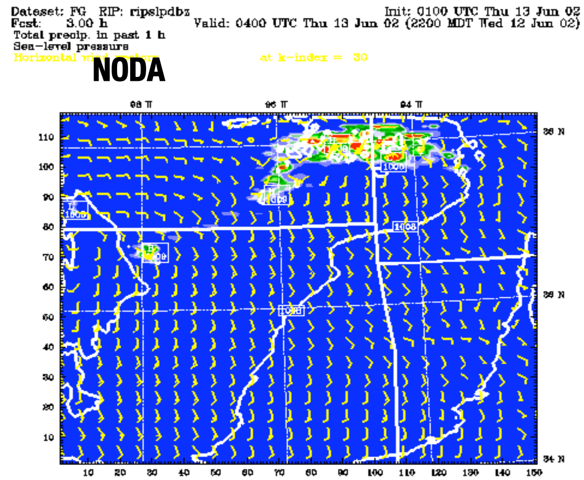
- NODA ----- Both initial condition and boundary condition from NCEP GFS data.

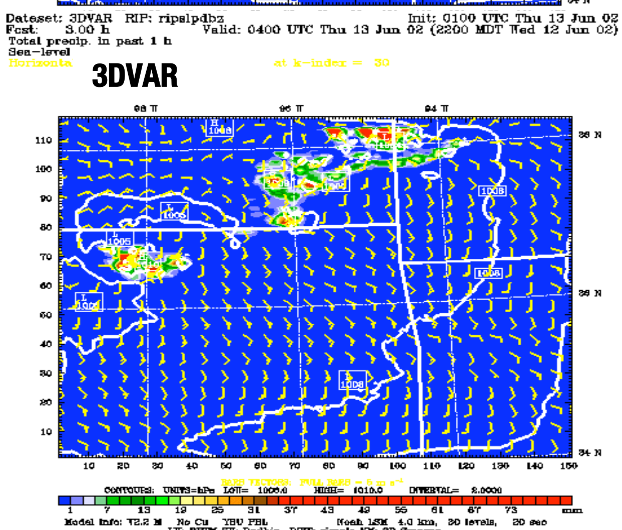
- 3DVAR -----3DVAR analysis at 2002061301Z used as the initial condition, and boundary condition from NCEP GFS. Only Radar radial velocity at 2002061301Z assimilated (total # of data points = 65,195).

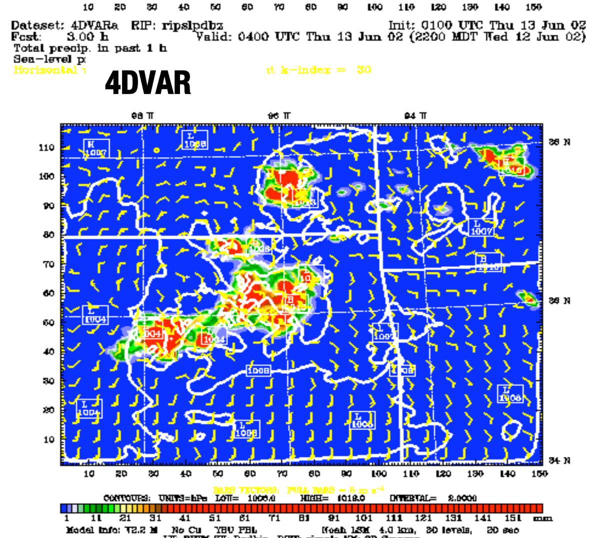
- 4DVAR ----- 4DVAR analysis at 2002061301Z used as initial condition, and boundary condition from NCEP GFS. The radar radial velocity at 4 times: 200206130100, 05, 10, and 15, are assimilated (total # of data points = 262,445).

Hourly precipitation at 03h forecast

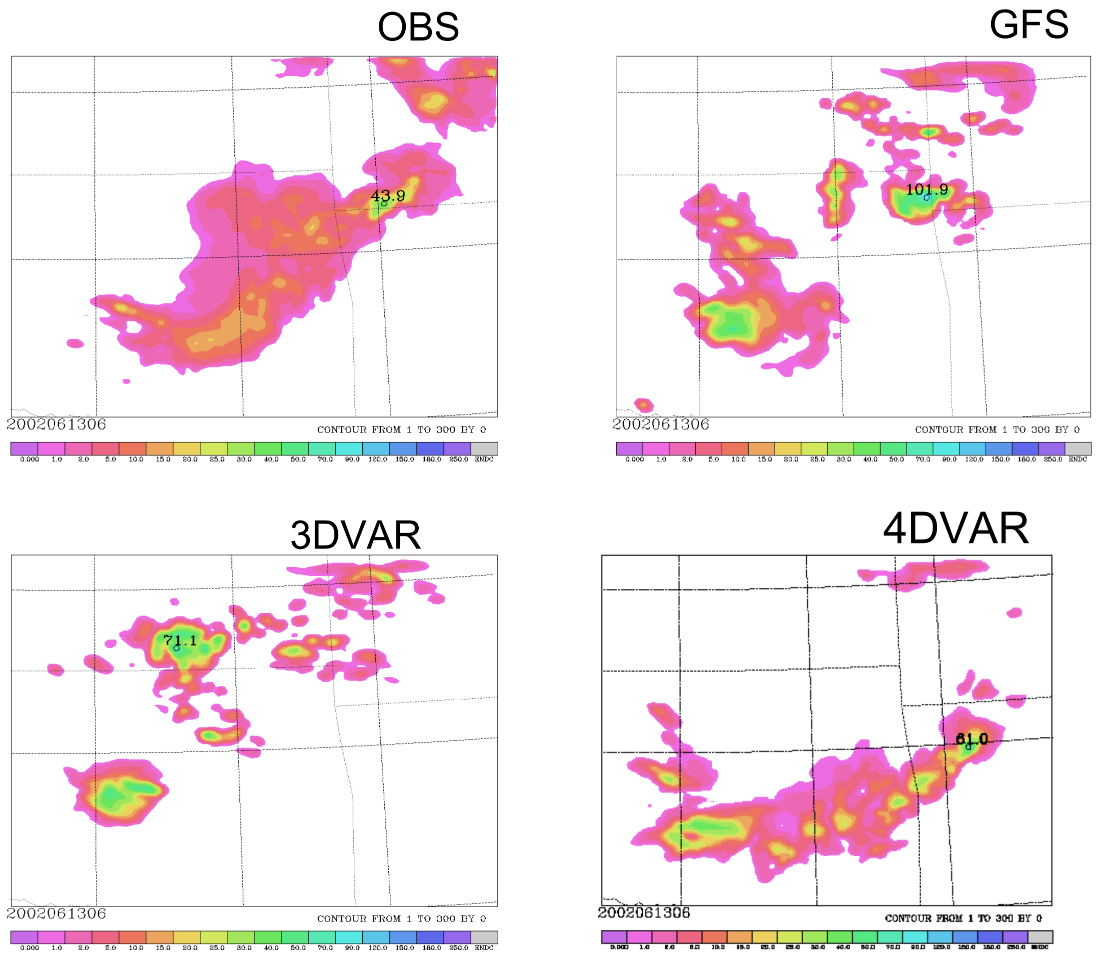




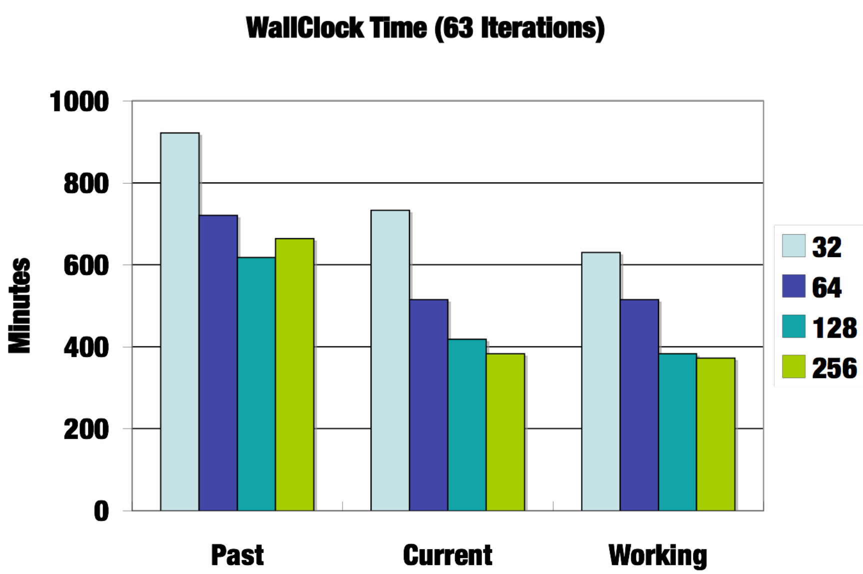


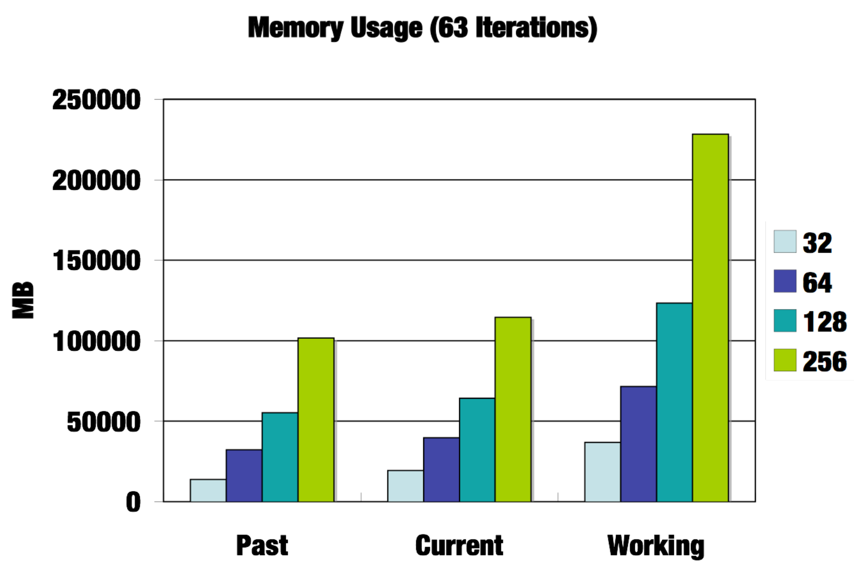


Radar data assimilation (cont'd)
Real data experiments



Computational Efficiency of IKE hurricane case on NCAR Bluefire





Past: Before optimization

Current: Eliminate the disk IO for basic states

Working: Reorganizing adjoint codes,reduce re-computation.

Radar Assimilation Case on IBM bluefire

Domain size:151x118x31

Resolution:4km

Time-step: 20s

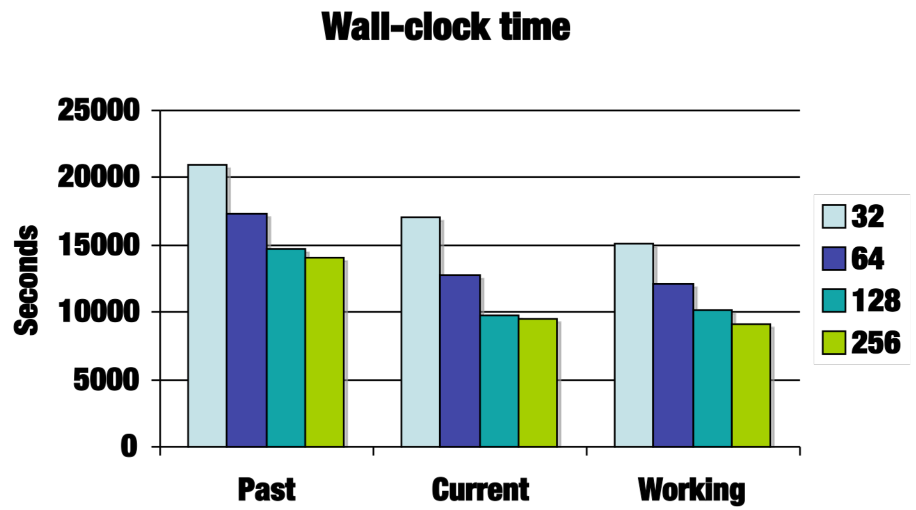
Time window:30m

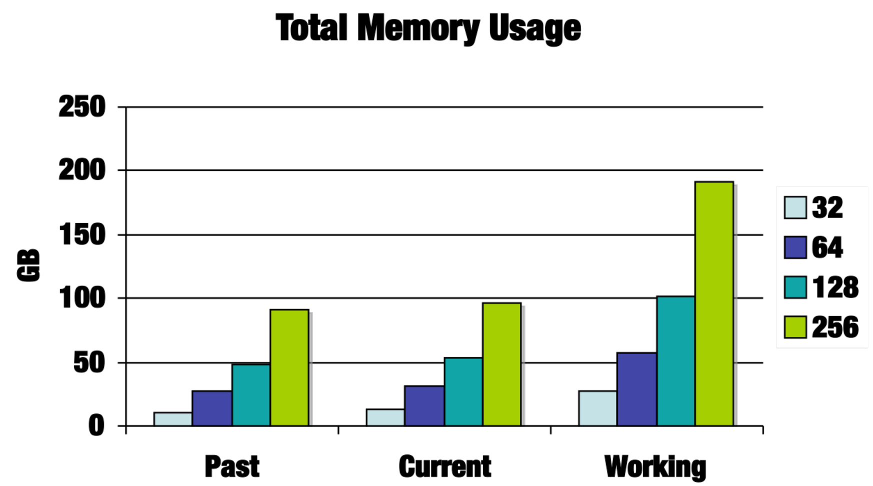
# of iterations: 60

Obs.: OSSE radar wind

# of obs.: 262,517

Obs Freq: 5m





Further Developments

- ESMF coupling: ESMF will be used to couple WRFNL, WRFPLUS and WRFDA together.
- Add boundary conditions as a weak constraint in cost function.
- Improve the current WRF adjoint and tangent Linear codes.

Thank You !