WRFDA Overview

Hans Huang

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NCAR/RAL/JNT/DATC

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Outline of Talk

- Introduction to data assimilation.
- Basics of modern data assimilation.
- WRFDA overview.

Why data assimilation?

- Initial conditions
- Calibration and validation
- Observing system design, monitoring and assessment
- Reanalysis
- Better understanding:
  - Data assimilation methods
  - Model errors
  - Data errors
  - Physical process interactions
  - …

Modern weather forecast (Bjerknes,1904)

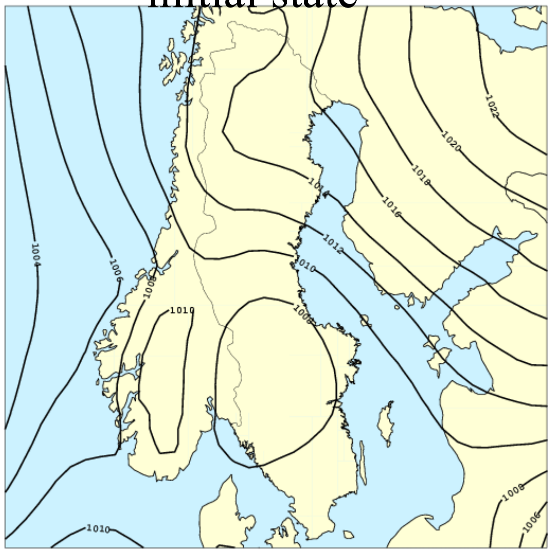
- A sufficiently accurate knowledge of the state of the atmosphere at the initial time
- A sufficiently accurate knowledge of the laws according to which one state of the atmosphere develops from another.

Vilhelm Bjerknes (1862–1951)

- Analysis: using observations and other information, we can specify the atmospheric state at a given initial time: “Today’s Weather”
- Forecast: using the equations, we can calculate how this state will change over time: “Tomorrow’s Weather”

(Peter Lynch)

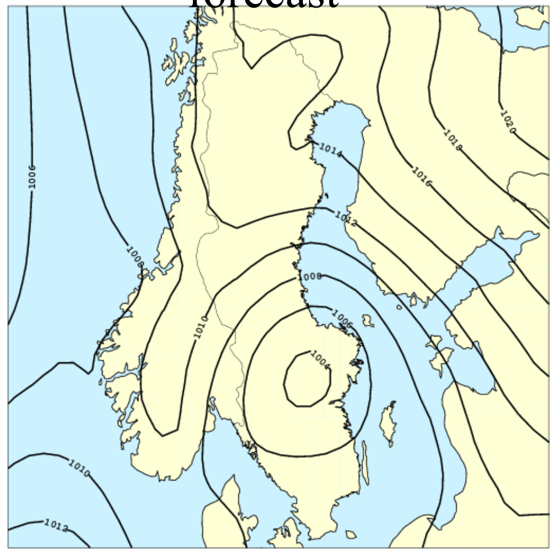
initial state



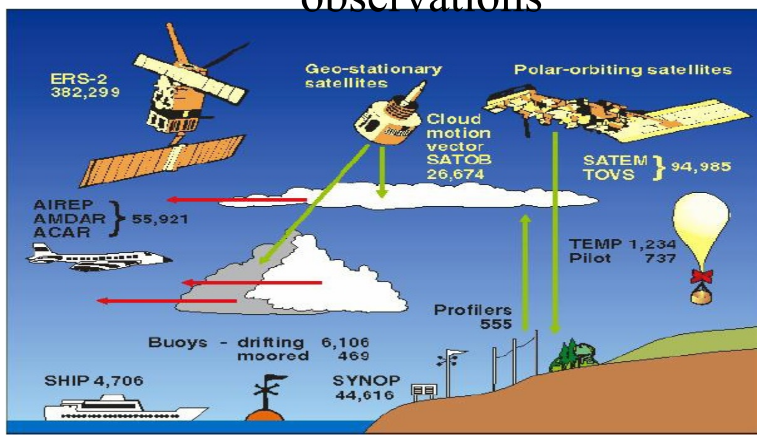
Model

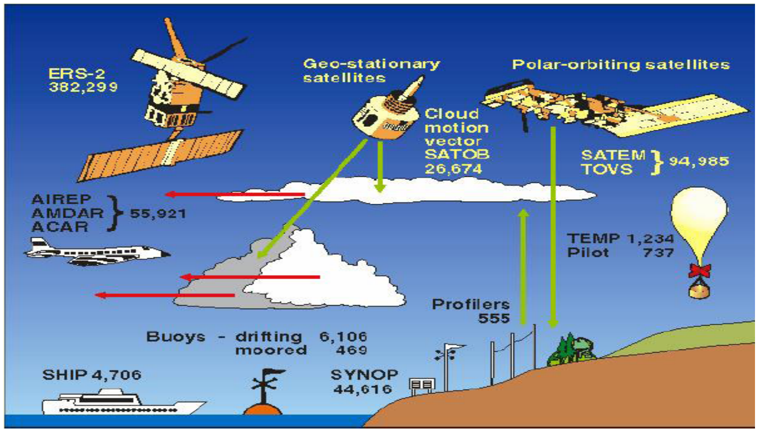
Analysis

forecast



observations



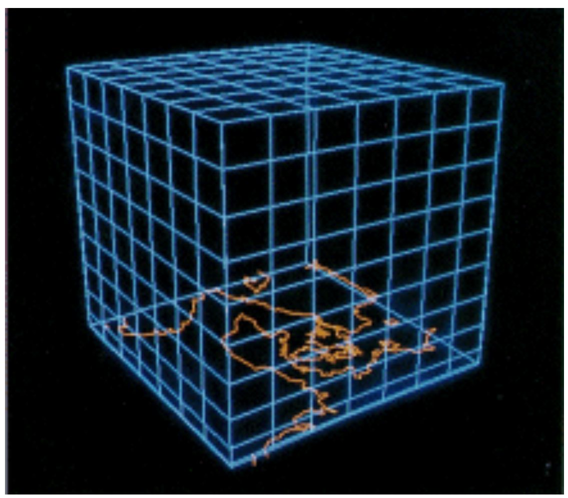


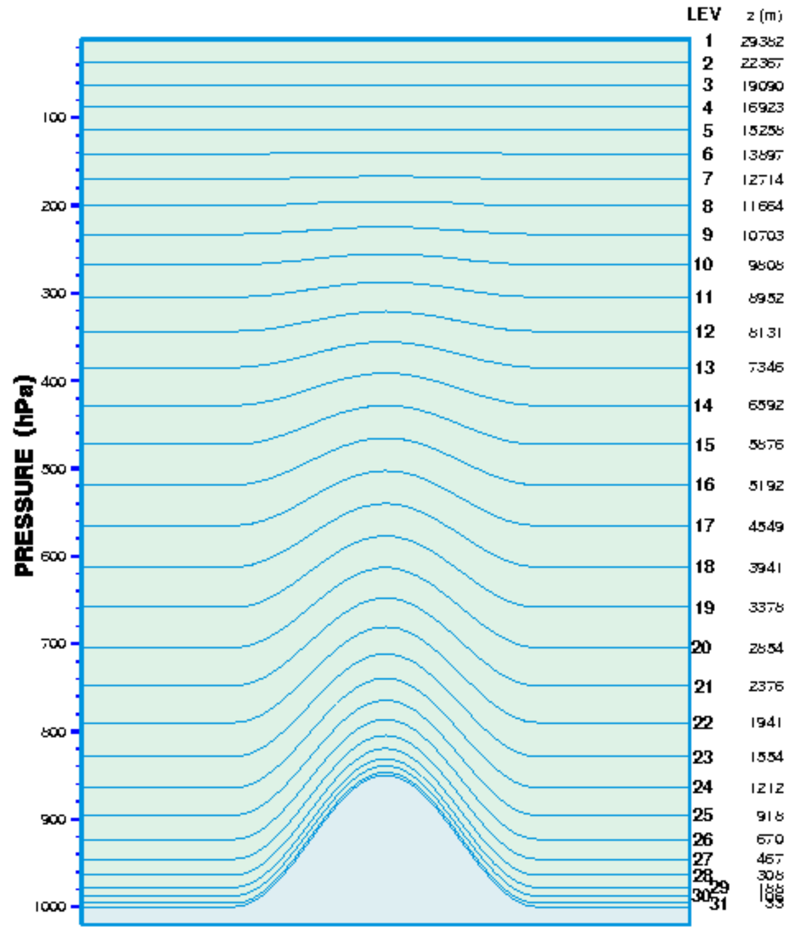
Observations

y^o, $\sim 10^5$-10^6

Model state

x, $\sim 10^7$





Vertical resolution of the DMI-HIRLAM system

Assimilation methods

- Empirical methods
  - Successive Correction Method (SCM)
  - Nudging
  - Physical Initialisation (PI), Latent Heat Nudging (LHN)
- Statistical methods
  - Optimal Interpolation (OI)
  - 3-Dimensional VARiational data assimilation (3DVAR)
  - 4-Dimensional VARiational data assimilation (4DVAR)
- Advanced methods
  - Extended Kalman Filter (EKF)
  - Ensemble Kalman Filter (EnFK)

Outline of Talk

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The analysis problem for a given time

Consider a scalar x.

The background (normally a short-range forecast):

$$x^b = x^t + b.$$

The observation:

$$x^r = x^t + r.$$

The error statistics are assumed to be known:

$$
\begin{aligned}
< b > &= 0, \text{ mean error (unbiased)},\\
< r > &= 0, \text{ mean error (unbiased)},\\
< b^2 > &= B, \text{ background error variance},\\
< r^2 > &= R, \text{ observation error variance},\\
< br > &= 0, \text{ no correlation between } b \text{ and } r,
\end{aligned}
$$

where $<\cdot>$ is ensemble average.

BLUE: the Best Liner Unbiased Estimate

The analysis: $x^a = x^t + a$.

Search for the best estimate: $x^a = \alpha x^b + \beta x^r$

Substitute the definitions, we have:

$$\alpha + \beta = 1.$$

$$< a > = 0$$

The variance:

$$A = < a^2 > = B - 2\beta B + \beta^2 \left(B + R \right)$$

To determine β: $\dfrac{dA}{d\beta} = -2B + 2\beta \left(B + R \right) = 0$

we have $\beta = \dfrac{B}{B+R}$

The analysis: $x^a = x^b + \dfrac{B}{B+R}\left(x^r - x^b \right)$

The analysis error variance: $A^{-1} = B^{-1} + R^{-1}$

The analysis: $x^a = x^b + \frac{B}{B+R}\left(x^r - x^b\right)$

$0 < \frac{B}{B+R} < 1$	The analysis value should be between background and observation.
$\lim_{B \to 0} x^a = x^b$	If B is too small, observations are less useful.
$\lim_{R \to 0} x^a = x^r$	If R can be tuned, analysis can fit observations as close as one wants!

The analysis error variance: $A^{-1} = B^{-1} + R^{-1}$

$A < B$	Statistically, analyses are better than background.
$A < R$	Statistically, analyses are better than observations!

3D-Var

The analysis is obtained by minimizing the cost function J, defined as :

$$J = \frac{1}{2}\left(x - x^b\right)^T B^{-1}\left(x - x^b\right) + \frac{1}{2}\left(x - x^r\right)^T R^{-1}\left(x - x^r\right)$$

The gradient of J with respect to x:

$$J' = B^{-1}\left(x - x^b\right) + R^{-1}\left(x - x^r\right)$$

At the minimum, $J' = 0$, we have:

$$x^a = x^b + \frac{B}{B+R}\left(x^r - x^b\right)$$

the same as BLUE.

Sequential data assimilation (I)

True states : ..., x_{i-1}^{t}, x_{i}^{t}, x_{i+1}^{t}, ...

Observations : ..., x_{i-1}^{r}, x_{i}^{r}, x_{i+1}^{r}, ...

Forecasts : ..., x_{i-1}^{f}, x_{i}^{f}, x_{i+1}^{f}, ...

Analyses : ..., x_{i-1}^{a}, x_{i}^{a}, x_{i+1}^{a}, ...

Sequential data assimilation (II)

Forecast model:

$$x_{i+1}^t = M\left(x_i^t\right) + q_i$$

Where q_i is the model error.

As q_i is unknown and x_i^a is the best estimate of x_i^t the forecast model usually takes the form:

$$x_{i+1}^f = M\left(x_i^a\right)$$

OI (and 3DVAR):

$$x_i^a = x_i^f + \frac{B}{B+R}\left(x_i^r - x_i^f\right)$$

Sequential data assimilation (III)

4DVAR analysis is obtained by minimizing the cost function J, defined as:

$$J(x_i) = \frac{1}{2}\left(x_i - x_i^f\right)^T B^{-1}\left(x_i - x_i^f\right)$$

$$+\frac{1}{2}\sum_{k=0}^{K}\left[M_{k-1}(x_i) - x_{i+k}^r\right]^T R^{-1}\left[M_{k-1}(x_i) - x_{i+k}^r\right]$$

where, K is the assimilation window and

$$M_{-1}(x_i) = x_i$$

$$M_0(x_i) = M(x_i)$$

$$M_{k-1}(x_i) = \underbrace{M(M(\ldots M}_{k}(x_i)\ldots))$$

Sequential data assimilation (IV)
4DVAR (continue)

The gradient of J with respect to x:

$$J' = B^{-1}\left(x_i - x_i^f\right) + \sum_{k=0}^{K} \prod_{j=0}^{k-1} M_{i+j}^T R^{-1}\left[M_{k-1}\left(x_i\right) - x_{i+k}^r\right]$$

where, M_{i+j}^T is the adjoint model of M at time step $i+j$.

Sequential data assimilation (V)

Extended Kalman Filters:

True states: $x_{i+1}^t = M\left(x_i^t\right) + q_i$

Model states: $x_{i+1}^f = M\left(x_i^a\right)$

Forecast error: $x_{i+1}^f - x_{i+1}^t = M\left(x_i^a\right) - M\left(x_i^t\right) - q_i \approx \mathbf{M}_i\left(x_i^a - x_i^t\right) - q_i$

Forecast error covariance matrix:

$$P_{i+1}^f = \left\langle \left(x_{i+1}^f - x_{i+1}^t\right)\left(x_{i+1}^f - x_{i+1}^t\right)^T \right\rangle$$

$$\approx \mathbf{M}_i \left\langle \left(x_i^a - x_i^t\right)\left(x_i^a - x_i^t\right)^T \right\rangle \mathbf{M}_i^T + \left\langle q_i\, q_i^T \right\rangle$$

$$= \mathbf{M}_i\, P_i^a \mathbf{M}_i^T + Q_i$$

Sequential data assimilation (VI)

Extended Kalman Filters (continue):

For the analysis step:

$$K_i = P_i^f \left(P_i^f + R \right)^{-1}$$

$$x_i^a = x_i^f + K_i \left(x_i^r - x_i^f \right)$$

$$P_i^a = \left(I - K_i \right) P_i^f$$

For the forecast step:

$$x_{i+1}^f = M\left(x_i^a \right)$$

$$P_{i+1}^f = \mathrm{M}_i P_i^a \mathrm{M}_i^T + Q_i$$

(Ensemble KF use ensembles to calculate P^f)

Sequential data assimilation (VII)

From scalar to vector:

Number of grid points $\approx 10^7$:

$$x \rightarrow \mathbf{x}$$

$$x^b \rightarrow \mathbf{x}^{\mathrm{b}}$$

Dimension of **B, P** $\approx 10^7 \mathrm{x}\ 10^7$

Number of observations, 10^6 :

$$x^r \rightarrow \mathbf{y}^o$$

$$x - x^r \rightarrow H(\mathbf{x}) - \mathbf{y}^{\mathrm{b}}$$

Dimension of $\mathbf{R} \approx 10^6 \mathrm{x}\ 10^6$

Sequential data assimilation (VIII)

OI:

$$\mathbf{x}_i^a = \mathbf{x}_i^f + \mathbf{B}\mathbf{H}^T\left(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R}\right)^{-1}\left[\mathbf{y}^o - H\left(\mathbf{x}_i^f\right)\right]$$

$$\mathbf{x}_{i+1}^f = M\left(\mathbf{x}_i^a\right)$$

4D-Var:

$$J\left(\mathbf{x}_i\right) = \frac{1}{2}\left(\mathbf{x}_i - \mathbf{x}_i^f\right)^T \mathbf{B}^{-1}\left(\mathbf{x}_i - \mathbf{x}_i^f\right)$$

$$+\frac{1}{2}\sum_{k=0}^{k}\left[H\left(M_{k-1}\left(\mathbf{x}_i\right)\right) - \mathbf{y}_{i+k}^o\right]^T \mathbf{R}^{-1}\left[H\left(M_{k-1}\left(\mathbf{x}_i\right)\right) - \mathbf{y}_{i+k}^o\right]$$

$$J' = \mathbf{B}^{-1}\left(\mathbf{x}_i - \mathbf{x}_i^f\right) + \sum_{k=0}^{K}\prod_{j=0}^{k-1}\mathbf{M}_{i+j}^T\mathbf{H}^T\mathbf{R}^{-1}\left[H\left[M_{k-1}\left(\mathbf{x}_i\right)\right] - \mathbf{y}_{i+k}^o\right]$$

Sequential data assimilation (IX)

The Extended Kalman Filter:

For the analysis step i:

$$\mathbf{K}_i = \mathbf{P}_i^f \mathbf{H}_i^T \left(\mathbf{H}_i \mathbf{P}_i^f \mathbf{H}_i^T + \mathbf{R} \right)^{-1}$$

$$\mathbf{x}_i^a = \mathbf{x}_i^f + \mathbf{K}_i \left[\mathbf{y}^o - H\left(\mathbf{x}_i^f \right) \right]$$

$$\mathbf{P}_i^a = \left(\mathbf{I} - \mathbf{K}_i \mathbf{H}_i \right) \mathbf{P}_i^f$$

For the forecast step, from i to $i+1$:

$$\mathbf{x}_{i+1}^f = M\left(\mathbf{x}_i^a \right)$$

$$\mathbf{P}_{i+1}^f = \mathbf{M}_i \mathbf{P}_i^a \mathbf{M}_i^T + \mathbf{Q}_i$$

(Ensemble KF use ensembles to calculate P^a and P^f)

Sequential data assimilation (X)

EKF

$$\mathbf{K}_i = \mathbf{P}_i^f \mathbf{H}_i^T \left(\mathbf{H}_i \mathbf{P}_i^f \mathbf{H}_i^T + \mathbf{R} \right)^{-1}$$

$$\mathbf{x}_i^a = \mathbf{x}_i^f + \mathbf{K}_i \left[\mathbf{y}^o - H\left(\mathbf{x}_i^f \right) \right]$$

$$\mathbf{P}_i^a = \left(\mathbf{I} - \mathbf{K}_i \mathbf{H}_i \right) \mathbf{P}_i^f$$

$$\mathbf{x}_{i+1}^f = M\left(\mathbf{x}_i^a \right)$$

$$\mathbf{P}_{i+1}^f = \mathbf{M}_i \mathbf{P}_i^a \mathbf{M}_i^T + \mathbf{Q}_i$$

(Ensemble KF use ensembles to calculate P^a and P^f)

$\left(\mathbf{B} = \mathbf{P}_{i+1}^f \right)$ EKF → OI or VAR

$$\left(\mathbf{K} = \mathbf{B}\mathbf{H}^T \left(\mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R} \right)^{-1} \right)$$

OI or VAR

$$\mathbf{x}_i^a = \mathbf{x}_i^f + \mathbf{K} \left[\mathbf{y}^o - H\left(\mathbf{x}_i^f \right) \right]$$

$$\mathbf{x}_{i+1}^f = M\left(\mathbf{x}_i^a \right)$$

Issues on data assimilation

- Observations $\mathbf{y}^o$
- Observation operator H
- Observation errors $\mathbf{R}$
- Backgound $\mathbf{x}^b$
- Size of $\mathbf{B}$: statistical model and tuning
- Minimization algorithm $\left(\text{Quasi}-\text{Newton; Conjugate Gradient; }\ldots\right)$
- $\mathbf{M}$ and $\mathbf{M}^T$: development and validity
- Size of $\mathbf{P}^f$ and $\mathbf{P}^a$: simplifications
- Model errors $\mathbf{Q}$

H - Observation operator

H maps variables from “model space” to “observation space”

$$\mathbf{x} \Longrightarrow \mathbf{y}$$

– Interpolations from model grids to observation locations

– Extrapolations using PBL schemes

– Time integration using full NWP models

– Transformations of model variables (u, v, T, q, p_s, etc.) to “indirect” observations (e.g. satellite radiance, radar radial winds, etc.)

- Simple relations like PW, radial wind, refractivity, …
- Radar reflectivity $Z = Z(T, IWC, LWC, RWC, SWC)$
- Radiative transfer models $L(\nu) \approx \int_0^\infty B(\nu, T(z)) \left[\frac{dTR(\nu)}{dz}\right] dz$
- Precipitation using simple or complex models
- …

!!! Need *H*, **H** and $\mathbf{H}^T$, not $\mathbf{H}^{-1}$!!!

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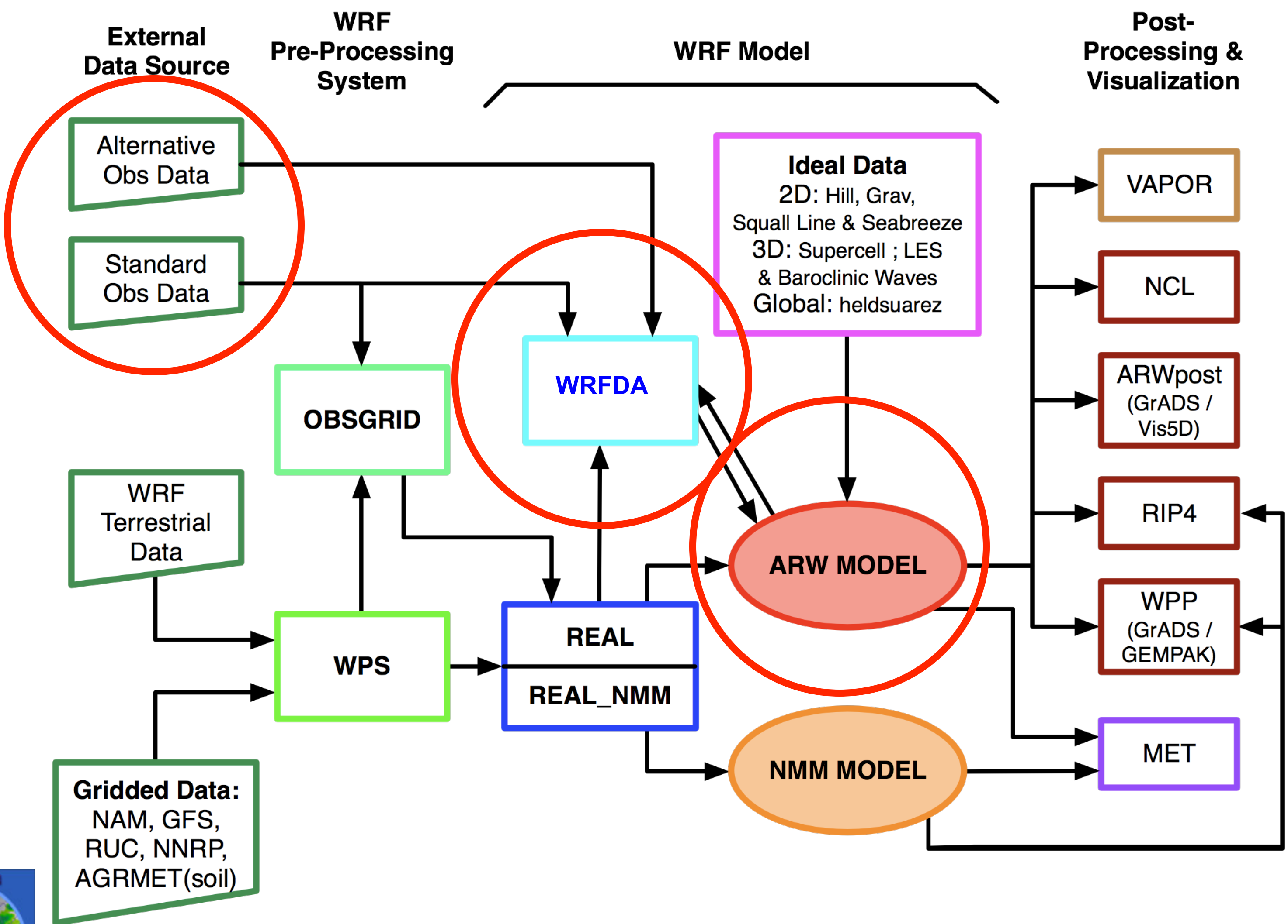
- Introduction to data assimilation.
- Basics of modern data assimilation.
- WRFDA overview…

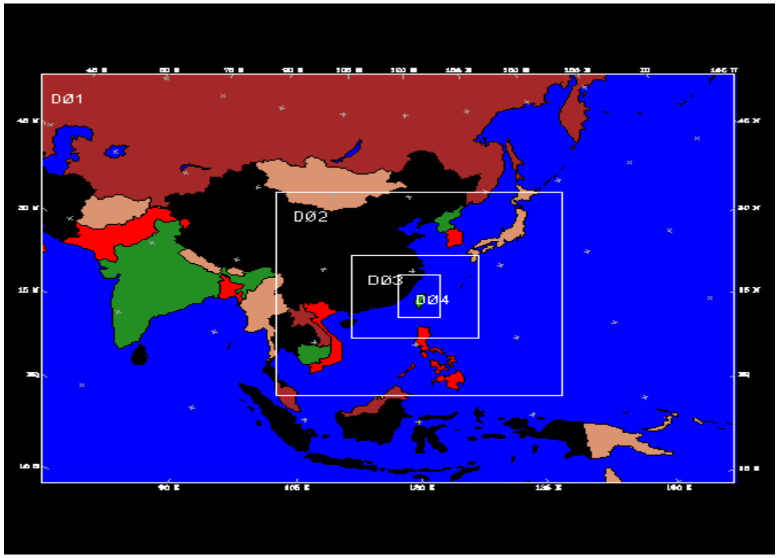
WRFDA is a **D**ata **A**ssimilation system built within the **WRF** software framework, used for application in both research and operational environments….

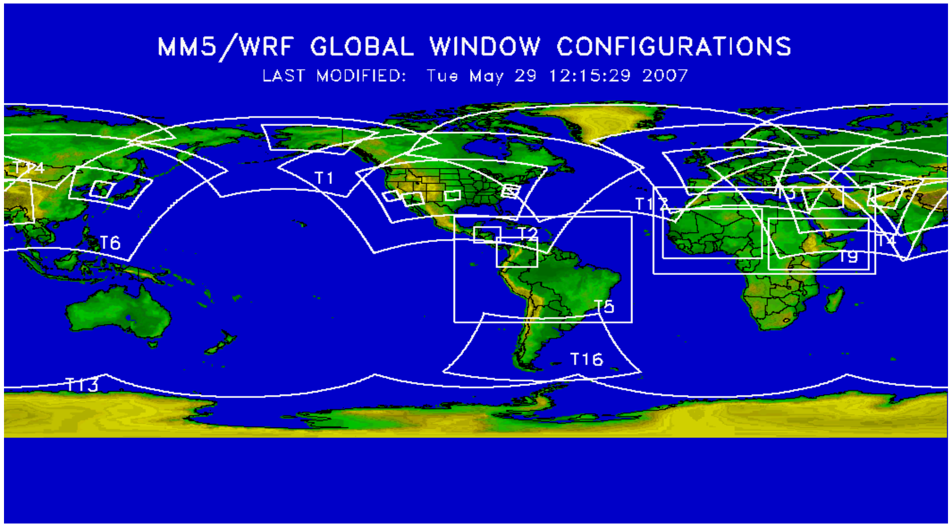
WRFDA in WRF Modeling System



WRFDA

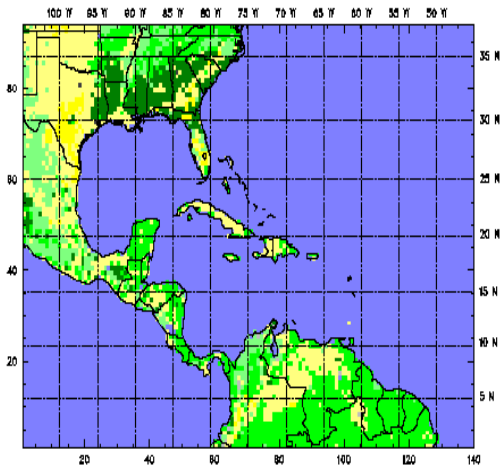
- **Goal:** Community WRF DA system for
  - regional/global,
  - research/operations, and
  - deterministic/probabilistic applications.
- **Techniques:**
  - 3D-Var
  - 4D-Var (regional)
  - Ensemble DA,
  - Hybrid Variational/Ensemble DA.
- **Model:** WRF (ARW, NMM, Global)
- **Support:**
  - NCAR/ESSL/MMM/DAG
  - NCAR/RAL/JNT/DATC
- **Observations:** Conv.+Sat.+Radar (+bogus)

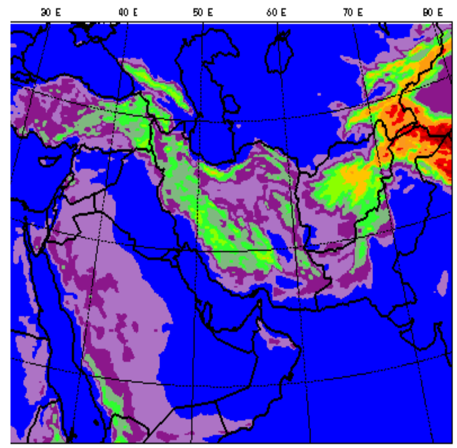


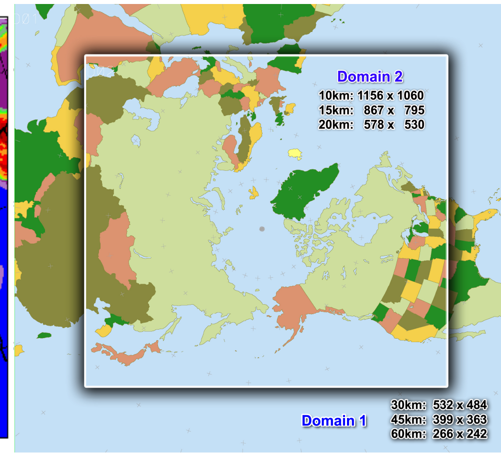


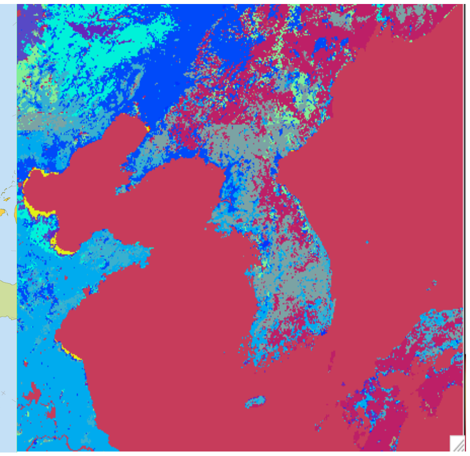
The WRFDA Program

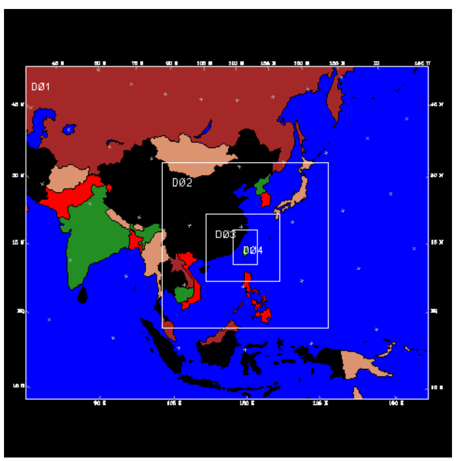
- NCAR staff (DAG,DATC): 20FTE, ~10 projects.
- Non-NCAR collaborators (AFWA, KMA, CWB, BMB, etc): ~10FTE.
- Community users: ~30 (more in 10,000 general WRF downloads?).

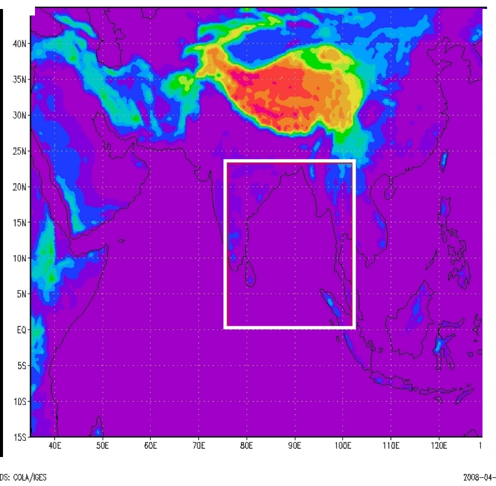


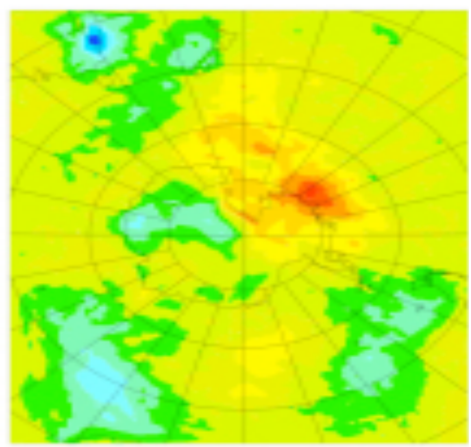


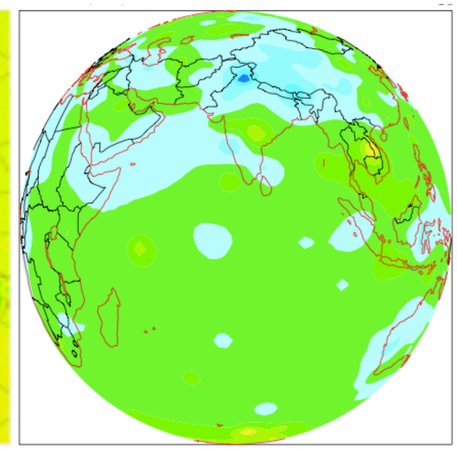












WRFDA Observations

- **In-Situ:**
  - Surface (SYNOP, METAR, SHIP, BUOY).
  - Upper air (TEMP, PIBAL, AIREP, ACARS, TAMDAR).
- **Remotely sensed retrievals:**
  - Atmospheric Motion Vectors (geo/polar).
  - SATEM thickness.
  - Ground-based GPS Total Precipitable Water/Zenith Total Delay.
  - SSM/I oceanic surface wind speed and TPW.
  - Scatterometer oceanic surface winds.
  - Wind Profiler.
  - Radar radial velocities and reflectivities.
  - Satellite temperature/humidity/thickness profiles.
  - GPS refractivity (e.g. COSMIC).
- **Radiative Transfer (RTTOV or CRTM):**
  - HIRS from NOAA-16, NOAA-17, NOAA-18, METOP-2
  - AMSU-A from NOAA-15, NOAA-16, NOAA-18, EOS-Aqua, METOP-2
  - AMSU-B from NOAA-15, NOAA-16, NOAA-17
  - MHS from NOAA-18, METOP-2
  - AIRS from EOS-Aqua
  - SSMIS from DMSP-16

- **Bogus:**
  - TC bogus.
  - Global bogus.

3D-Var

$$J = \frac{1}{2}\left(\mathbf{x}-\mathbf{x}^{b}\right)^{T}\mathbf{B}^{-1}\left(\mathbf{x}-\mathbf{x}^{b}\right)+\frac{1}{2}\left(\mathbf{y}-H(\mathbf{x})\right)^{T}\mathbf{R}^{-1}\left(\mathbf{y}-H(\mathbf{x})\right)$$

The incremental formulation (in the general form, $\mathbf{x}^{g} \neq \mathbf{x}^{b}$!)

$$J = \frac{1}{2}\left(\mathbf{x}-\mathbf{x}^{g}+\mathbf{x}^{g}-\mathbf{x}^{b}\right)^{T}\mathbf{B}^{-1}\left(\mathbf{x}-\mathbf{x}^{g}+\mathbf{x}^{g}-\mathbf{x}^{b}\right)+\frac{1}{2}\left(\mathbf{y}-H(\mathbf{x}^{g})+H(\mathbf{x}^{g})-H(\mathbf{x})\right)^{T}\mathbf{R}^{-1}\left(\mathbf{y}-H(\mathbf{x}^{g})+H(\mathbf{x}^{g})-H(\mathbf{x})\right)$$

$\delta\mathbf{x} = \mathbf{x}-\mathbf{x}^{g}$

$\mathbf{d} = \mathbf{y}-H(\mathbf{x}^{g}) \quad H(\mathbf{x})-H(\mathbf{x}^{g}) \approx \mathbf{H}\delta\mathbf{x}$

$$J = \frac{1}{2}\left(\delta\mathbf{x}+\mathbf{x}^{g}-\mathbf{x}^{b}\right)^{T}\mathbf{B}^{-1}\left(\delta\mathbf{x}+\mathbf{x}^{g}-\mathbf{x}^{b}\right)+\frac{1}{2}\left(\mathbf{d}-\mathbf{H}\delta\mathbf{x}\right)^{T}\mathbf{R}^{-1}\left(\mathbf{d}-\mathbf{H}\delta\mathbf{x}\right)$$

The first outer-loop: $\mathbf{x}^{g} = \mathbf{x}^{b}$

$$J = \frac{1}{2}\delta\mathbf{x}^{T}\mathbf{B}^{-1}\delta\mathbf{x}+\frac{1}{2}\left(\mathbf{d}-\mathbf{H}\delta\mathbf{x}\right)^{T}\mathbf{R}^{-1}\left(\mathbf{d}-\mathbf{H}\delta\mathbf{x}\right)$$

Outer-loop:

- **d** (and QC, etc) … nonlinear!
- Inner-loop: minimization
- update $\mathbf{x}^{g}$

3D-Var

$$J = \frac{1}{2}\delta\mathbf{x}^T\mathbf{B}^{-1}\delta\mathbf{x} + \frac{1}{2}\left(\mathbf{d} - \mathbf{H}\delta\mathbf{x}\right)^T\mathbf{R}^{-1}\left(\mathbf{d} - \mathbf{H}\delta\mathbf{x}\right)$$

$$\nabla J = \mathbf{B}^{-1}\delta\mathbf{x} - \mathbf{H}^T\mathbf{R}^{-1}\left(\mathbf{d} - \mathbf{H}\delta\mathbf{x}\right)$$

$$\mathbf{d} = \mathbf{y} - H\left(\mathbf{x}^g\right)$$

4D-Var

replace *H* by *HM*

replace **H** by **HM**

replace $\mathbf{H}^T$ by $\mathbf{M}^T\mathbf{H}^T$

$$J = \frac{1}{2}\delta\mathbf{x}^T\mathbf{B}^{-1}\delta\mathbf{x} + \frac{1}{2}\left(\mathbf{d} - \mathbf{HM}\delta\mathbf{x}\right)^T\mathbf{R}^{-1}\left(\mathbf{d} - \mathbf{HM}\delta\mathbf{x}\right)$$

$$\nabla J = \mathbf{B}^{-1}\delta\mathbf{x} - \mathbf{M}^T\mathbf{H}^T\mathbf{R}^{-1}\left(\mathbf{d} - \mathbf{HM}\delta\mathbf{x}\right)$$

$$\mathbf{d} = \mathbf{y} - H\left(M(\mathbf{x}^g)\right)$$

FGAT:

From 3D-Var,

only replace *H* by *HM*

From 4D-Var,

set $\mathbf{M}=\mathbf{M}^T=\mathbf{I}$, but keep *M*

$$J = \frac{1}{2}\delta\mathbf{x}^T\mathbf{B}^{-1}\delta\mathbf{x} + \frac{1}{2}\left(\mathbf{d} - \mathbf{H}\delta\mathbf{x}\right)^T\mathbf{R}^{-1}\left(\mathbf{d} - \mathbf{H}\delta\mathbf{x}\right)$$

$$\nabla J = \mathbf{B}^{-1}\delta\mathbf{x} - \mathbf{H}^T\mathbf{R}^{-1}\left(\mathbf{d} - \mathbf{H}\delta\mathbf{x}\right)$$

$$\mathbf{d} = \mathbf{y} - H\left(M(\mathbf{x}^g)\right)$$

4D-Var

$$J = \frac{1}{2}\delta\mathbf{x}^T\mathbf{B}^{-1}\delta\mathbf{x} + \frac{1}{2}(\mathbf{d} - \mathbf{HM}\delta\mathbf{x})^T\mathbf{R}^{-1}(\mathbf{d} - \mathbf{HM}\delta\mathbf{x})$$

$$\nabla J = \mathbf{B}^{-1}\delta\mathbf{x} - \mathbf{M}^T\mathbf{H}^T\mathbf{R}^{-1}(\mathbf{d} - \mathbf{HM}\delta\mathbf{x})$$

$$\mathbf{d} = \mathbf{y} - H\left(M(\mathbf{x}^{\mathbf{g}})\right)$$

The U transform:

$$\delta\mathbf{x} = \mathbf{Uv}$$

$$\mathbf{B} = \mathbf{UU}^{\mathbf{T}}$$

4D-Var in control variable space:

$$J = \frac{1}{2}\mathbf{v}^T\mathbf{v} + \frac{1}{2}(\mathbf{d} - \mathbf{HMUv})^T\mathbf{R}^{-1}(\mathbf{d} - \mathbf{HMUv})$$

$$\nabla J = \mathbf{v} - \mathbf{U}^T\mathbf{M}^T\mathbf{H}^T\mathbf{R}^{-1}(\mathbf{d} - \mathbf{HMUv})$$

$$\mathbf{d} = \mathbf{y} - H\left(M(\mathbf{x}^{\mathbf{g}})\right)$$

B in WRF-Var (and most Var systems)

- Partly for preconditioning reasons:

  $\mathbf{B} = \mathbf{U}\,\mathbf{U}^T$ with $\mathbf{U} = \mathbf{U_p}\mathbf{U_v}\mathbf{U_h}$

- Horizontal transformation ($\mathbf{U_h}$) is via

  Regional - recursive filters

  Global - power spectrum

- Vertical transformation ($\mathbf{U_v}$) is

  cv3 - recursive filters

  cv5, cv6 - via EOF

- Physical transformation ($\mathbf{U_p}$) depends upon the choice of the analysis control variable

The U_p transform

- WRF-Var control variables

  Stream function (ψ)

  Unbalanced velocity potential ($\chi_u = \chi - \chi_b$)

  Unbalanced temperature ($T_u = T - T_b$)

  Relative Humidity (q)

  Unbalanced surface pressure ($p_{s\_u} = p_s - p_{s\_b}$)
- With this choice of analysis control variables off-diagonal elements of B are ASSUMED to be negligible.

The WRFDA "balance" relations

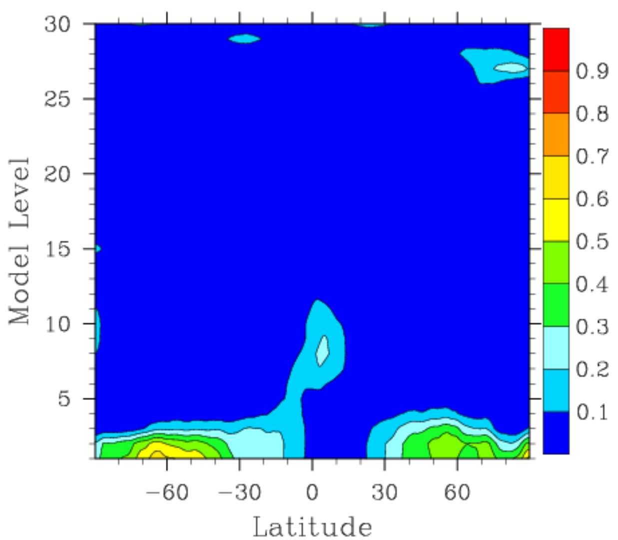
(confusing relations!)

$$\chi_b = C\,\psi$$

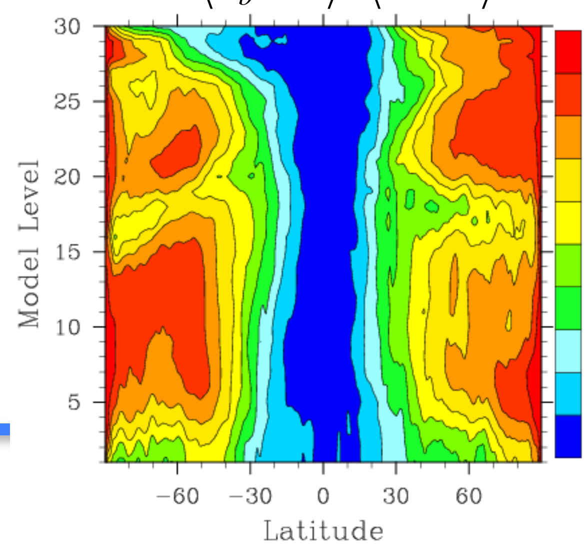
$$T_b(k) = \Sigma_l\, G(k,l)\ \psi(l)$$

$$p_{s\_b} = \Sigma_l W(k)\ \psi(k)$$

$$\langle \chi_b \bullet \chi \rangle / \langle \chi \bullet \chi \rangle$$

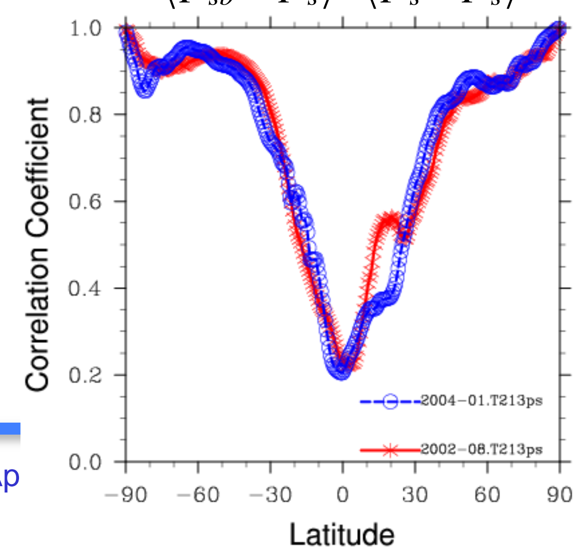


$$\langle T_b \bullet T \rangle / \langle T \bullet T \rangle$$

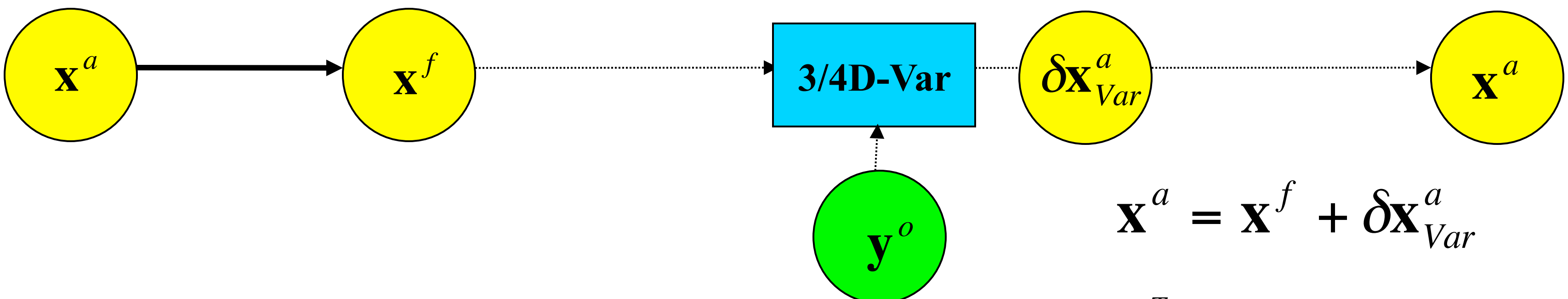


Ap

$$\langle p_{sb} \bullet p_s \rangle / \langle p_s \bullet p_s \rangle$$

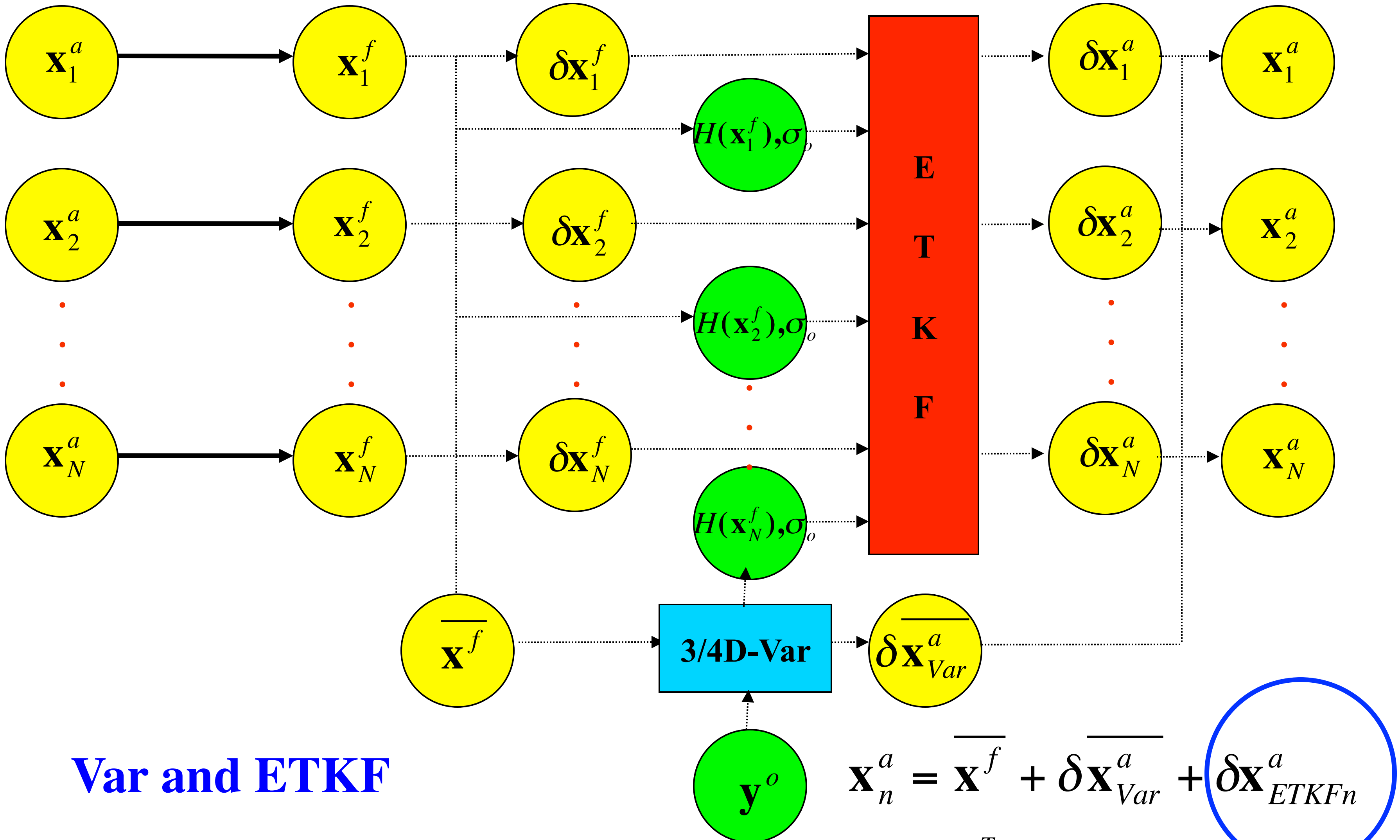


WRFDA
3/4D-Var



$$\mathbf{x}^a = \mathbf{x}^f + \delta\mathbf{x}^a_{Var}$$

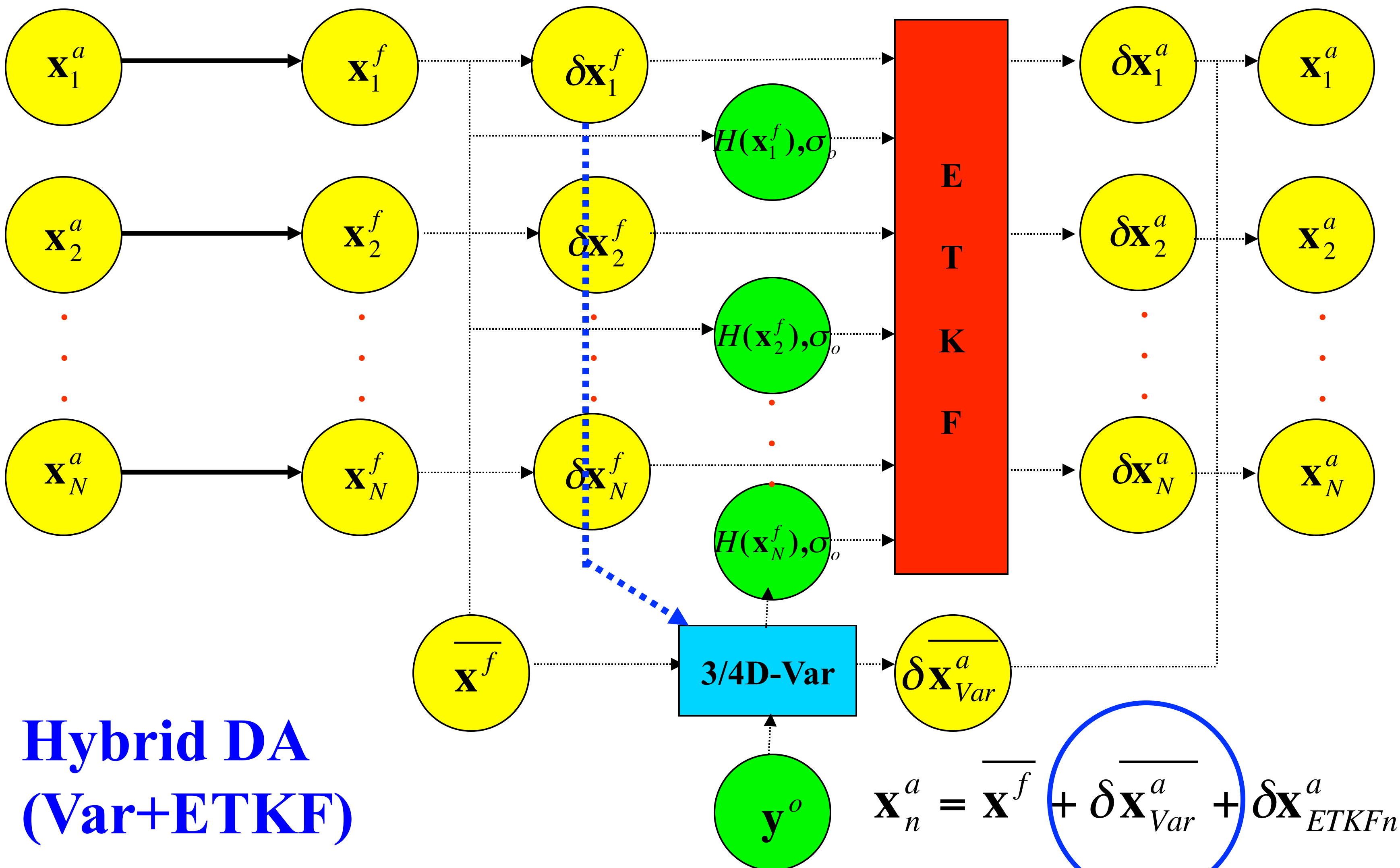
$$J = \frac{1}{2}\delta\mathbf{x}_0{}^T\mathbf{B}_o^{-1}\delta\mathbf{x}_0 + \frac{1}{2}\sum_{i=0}^{n}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]^T\mathbf{R}_i^{-1}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]$$



Var and ETKF

$$\mathbf{x}_n^a = \overline{\mathbf{x}^f} + \delta\overline{\mathbf{x}_{Var}^a} + \delta\mathbf{x}_{ETKFn}^a$$

$$J = \frac{1}{2}\delta\mathbf{x}_0{}^T\mathbf{B}_o^{-1}\delta\mathbf{x}_0 + \frac{1}{2}\sum_{i=0}^{n}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]^T\mathbf{R}_i^{-1}\left[\mathbf{H}_i\delta\mathbf{x}(t_i) - \mathbf{d}_i\right]$$

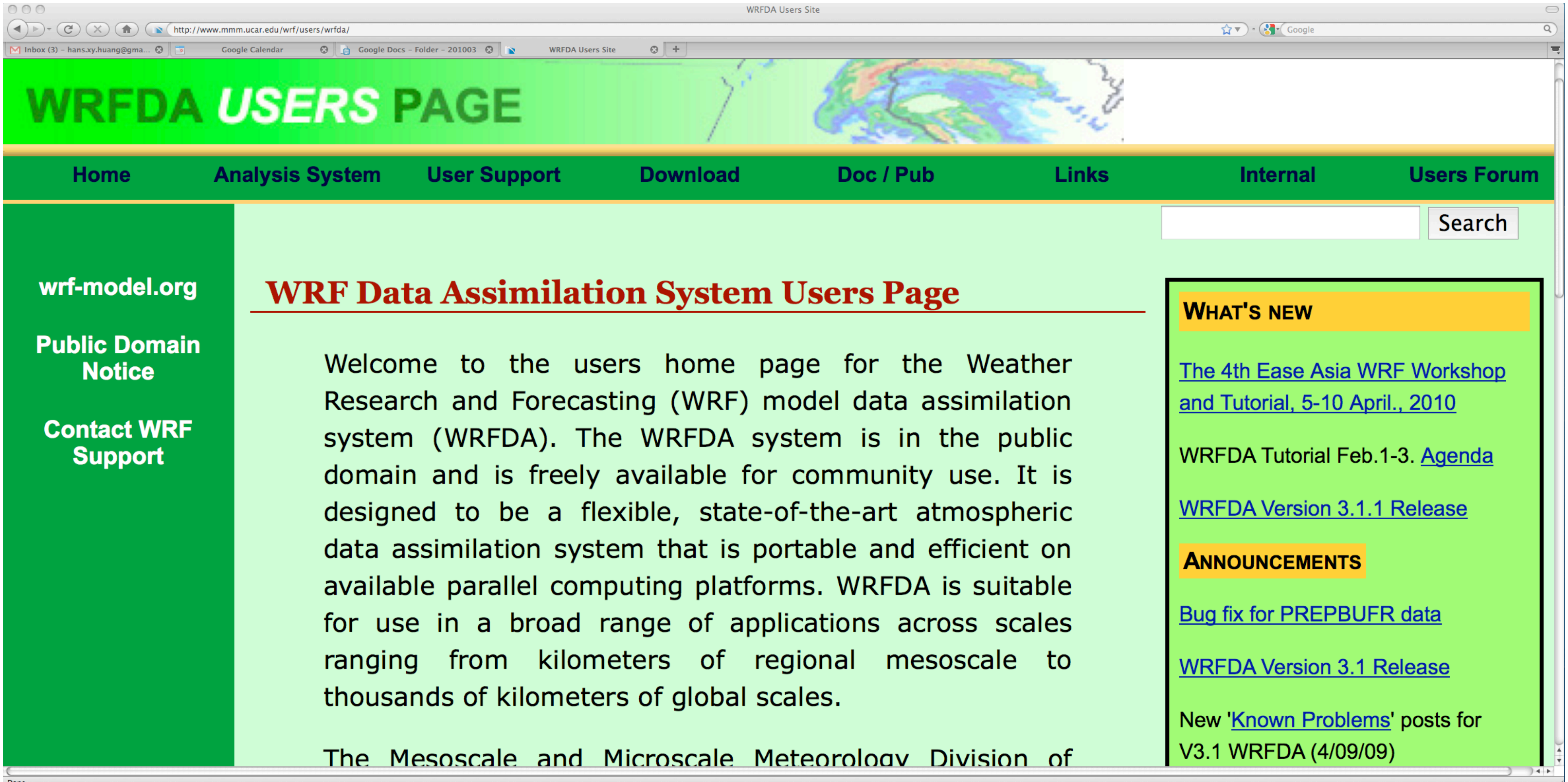


Hybrid DA (Var+ETKF)

$$\mathbf{x}_n^a = \overline{\mathbf{x}^f} + \overline{\delta\mathbf{x}_{Var}^a} + \delta\mathbf{x}_{ETKFn}^a$$

$$J = \frac{W_b}{2}\delta\mathbf{x}_0^T\mathbf{B}_o^{-1}\delta\mathbf{x}_0 + \frac{W_\alpha}{2}\mathbf{a}^T\mathbf{A}^{-1}\mathbf{a} + \frac{1}{2}\sum_{i=0}^{n}\left[\mathbf{H}_i\delta\mathbf{x}(t_i)-\mathbf{d}_i\right]^T\mathbf{R}_i^{-1}\left[\mathbf{H}_i\delta\mathbf{x}(t_i)-\mathbf{d}_i\right]$$

www.mmm.ucar.edu/wrf/users/wrfda



WRFDA tutorials

21-22 July, 2008. NCAR.

2-4 Feb, 2009. NCAR.

17-24 Feb, 2009. Kunming, Yunnan, China.

18 April, 2009. South Korea.

20-22 July, 2009. NCAR.

15-31 Oct, 2009. Nanjing, China.

1-3 Feb, 2010. NCAR.

10 April, 2010. Seoul, South Korea.

3-5 August 2010. NCAR.

WRFDA online tutorial and user guide

http://www.mmm.ucar.edu/wrf/users/wrfda

Issues on data assimilation

- Observations $\mathbf{y}^o$
- Observation operator H
- Observation errors $\mathbf{R}$
- Backgound $\mathbf{x}^b$
- Size of $\mathbf{B}$: statistical model and tuning
- Minimization algorithm $\left(\text{Quasi}-\text{Newton; Conjugate Gradient; …}\right)$
- $\mathbf{M}$ and $\mathbf{M}^T$: development and validity
- Size of $\mathbf{P}^f$ and $\mathbf{P}^a$: simplifications
- Model errors $\mathbf{Q}$

This tutorial

- Introduction and overview
- WRFDA system ($\mathbf{y_o}$, H, $\mathbf{R}$, $\mathbf{B}$, CG,…)
- WRFDA setup, run

Tutorials at NCAR

1. WRFDA Overview
2. Observation Pre-processing
3. WRFDA System
4. WRFDA Set-up, Run
5. WRFDA Background Error Estimations
6. Radar Data
7. Satellite Data
8. WRF 4D-Var
9. WRF Hybrid Data Assimilation System
10. WRFDA Tools and Verification
11. Observation Sensitivity

Practice

1. obsproc
2. wrfda (3D-Var)
3. Single-ob tests
4. Gen_be
5. Radar
6. Radiance
7. 4D-Var
8. Hybrid
9. Advanced (optional)