National Weather Service
National Centers for Environmental Prediction

The WRF NMM Core

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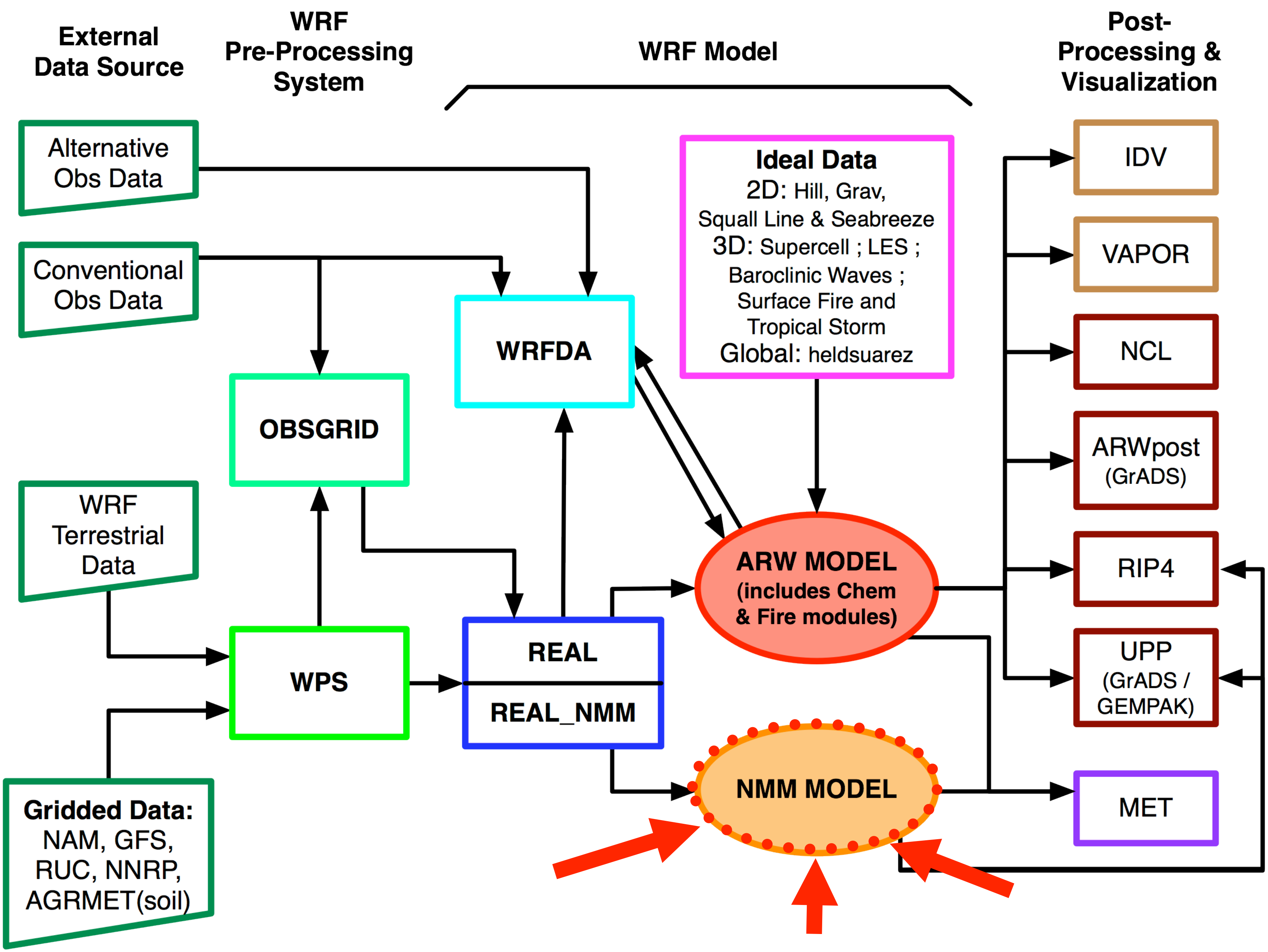
WRF Modeling System Flow Chart



NMM Dynamic Solver

- Basic Principles
- Equations / Variables
- Model Integration
- Horizontal Grid
- Spatial Discretization
- Vertical Grid
- Boundary Conditions
- Dissipative Processes
- Namelist switches
- Summary

Basic Principles

- Use full compressible equations split into hydrostatic and nonhydrostatic contributions
    - Easy comparison of hydro and nonhydro solutions
    - Reduced computational effort at lower resolutions
- Apply modeling principles proven in previous NWP and regional climate applications
- Use methods that minimize the generation of small-scale noise
- Robust, computationally efficient

Mass Based Vertical Coordinate

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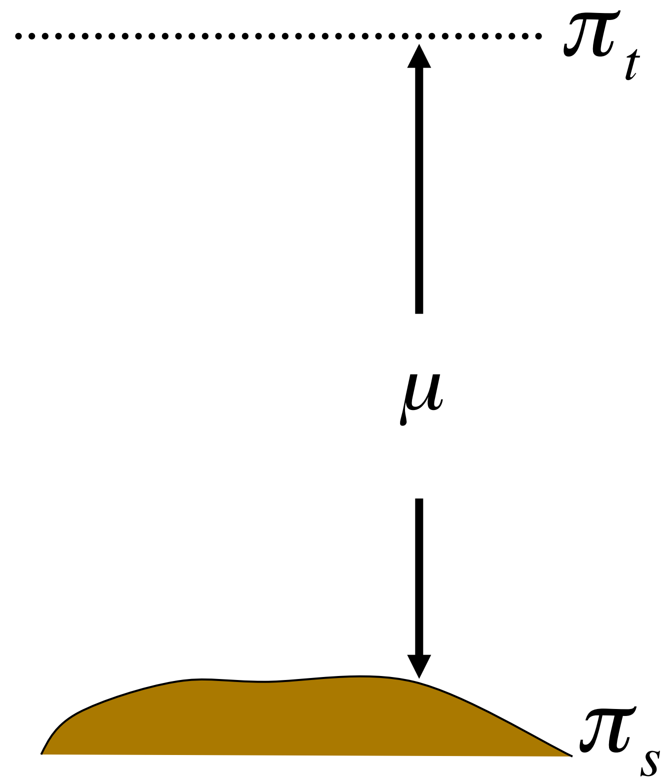
To simplify discussion of the model equations, consider a sigma coordinate to represent a vertical coordinate based on hydrostatic pressure π :

$$\mu = \pi_s - \pi_t$$

$$\sigma = \frac{\pi - \pi_t}{\mu}$$



WRF-NMM dynamical equations

inviscid, adiabatic, sigma form

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Analogous to a hydrostatic system, except for p and ε , where p is the total (nonhydrostatic) pressure and ε is defined below.

Momentum eqn.

$$\frac{\partial \mathbf{v}}{\partial t} = -\mathbf{v} \cdot \nabla_\sigma \mathbf{v} - \dot{\sigma} \frac{\partial \mathbf{v}}{\partial \sigma} - (1+\varepsilon)\nabla_\sigma \Phi - \alpha \nabla_\sigma p + f\mathbf{k} \times \mathbf{v}$$

Thermodynamic eqn.

$$\frac{\partial T}{\partial t} = -\mathbf{v} \cdot \nabla_\sigma T - \dot{\sigma} \frac{\partial T}{\partial \sigma} + \frac{\alpha}{c_p}\left[\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla_\sigma p + \dot{\sigma} \frac{\partial p}{\partial \sigma}\right]$$

Hydrostatic Continuity eqn.

$$\frac{\partial \mu}{\partial t} + \nabla_\sigma \cdot (\mu \mathbf{v}) + \frac{\partial (\mu \dot{\sigma})}{\partial \sigma} = 0$$

$$\varepsilon \equiv \frac{1}{g}\frac{dw}{dt}$$

$$\alpha = RT/p$$

Janjic et al. 2001, *MWR*

*

Hypsometric eqn.

$$\frac{\partial \Phi}{\partial \sigma} = -\mu \frac{RT}{p}$$

Nonhydro var. definition (restated)

$$\varepsilon \equiv \frac{1}{g}\frac{dw}{dt}$$

3rd eqn of motion

$$\frac{\partial p}{\partial \pi} = 1 + \varepsilon$$

ε generally is small. Even a large vertical acceleration of 20 m/s in 1000 s produces ε of only ~0.002, and nonhydrostatic pressure deviations of ~200 Pa.

Nonhydrostatic continuity eqn.

$$w = \frac{1}{g}\frac{d\Phi}{dt} = \frac{1}{g}\left(\frac{\partial \Phi}{\partial t} + \mathbf{v} \cdot \nabla_\sigma \Phi + \dot{\sigma}\frac{\partial \Phi}{\partial \sigma}\right)$$

*

Properties of system

- ϕ , w, and ε are not independent → no independent prognostic equation for w!

- ε << 1 in meso- and large-scale atmospheric flows.

- Generically, the impact of nonhydrostatic dynamics becomes detectable at resolutions < 10 km, and important at ~1 km.

Vertical boundary conditions for model equations*

Top: $\dot{\sigma} = 0$, $p - \pi = 0$

Surface: $\dot{\sigma} = 0$, $\dfrac{\partial(p - \pi)}{\partial \sigma} = 0$

WRF-NMM predictive variables

- Mass variables:
  - **PD** – hydrostatic pressure depth (time/space varying component) (Pa)
  - **PINT** – nonhydrostatic pressure (Pa)
  - **T** – sensible temperature (K)
  - **Q** – specific humidity (kg/kg)
  - **CWM** – total cloud water condensate (kg/kg)
  - **Q2** – 2 * turbulent kinetic energy (m^2/s^2)
- Wind variables:
  - **U,V** – wind components (m/s)

Model Integration

- Explicit time differencing preferred where possible, as allows for better phase speeds and more transparent coding:
  - horizontal advection of u, v, T
  - advection of q, cloud water, TKE (“passive substances”)
- Implicit time differencing for very fast processes that would require a restrictively short time step for numerical stability:
  - vertical advection of u, v, T and vertically propagating sound waves

Model Integration

*

Horizontal advection of u, v, T

2nd order Adams-Bashforth:

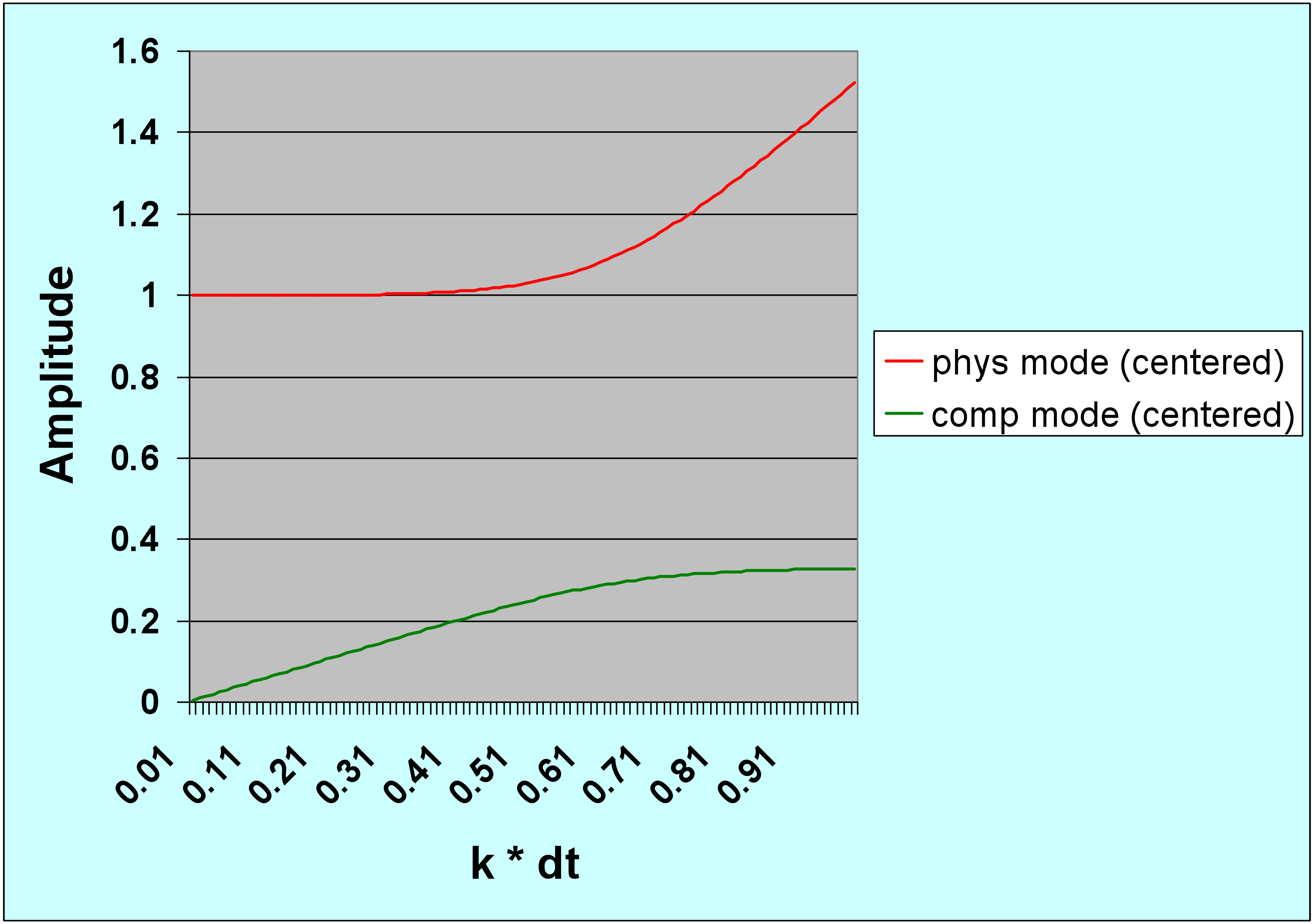
$$\frac{y^{\tau+1} - y^{\tau}}{\Delta t} = \frac{3}{2} f(y^{\tau}) - \frac{1}{2} f(y^{\tau-1})$$

Stability/Amplification:

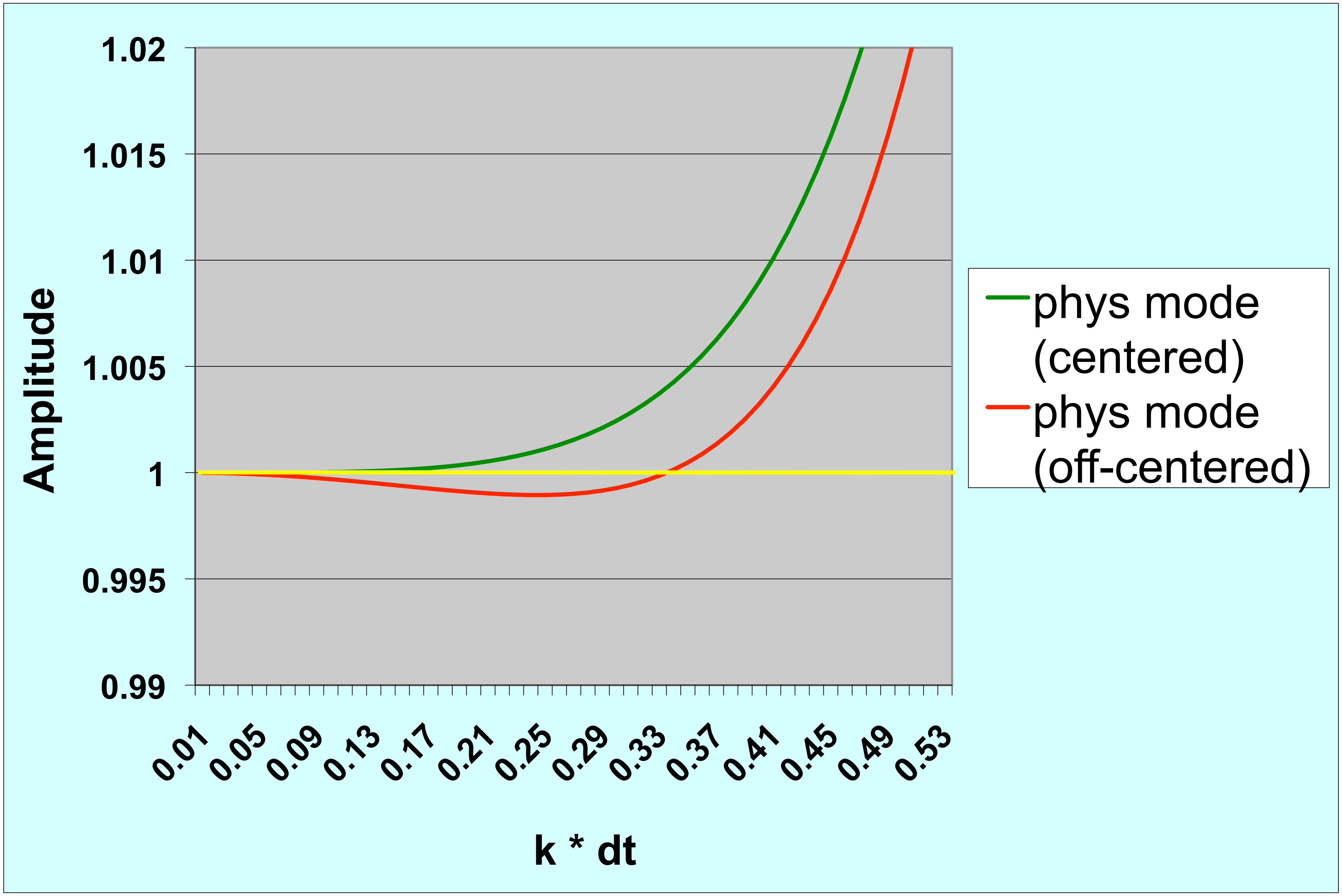
A-B has a weak linear instability (amplification) which either can be tolerated or can be stabilized by a slight off-centering as is done in the WRF-NMM.

$$\frac{y^{\tau+1} - y^{\tau}}{\Delta t} = 1.533 f(y^{\tau}) - 0.533 f(y^{\tau-1})$$

Adams-Bashforth amplification factor derived from oscillation equation ($d\psi/dt=ik\psi$)



Adams-Bashforth amplification factor, off-centering impact



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Model Integration

Vertical advection of u, v, & T

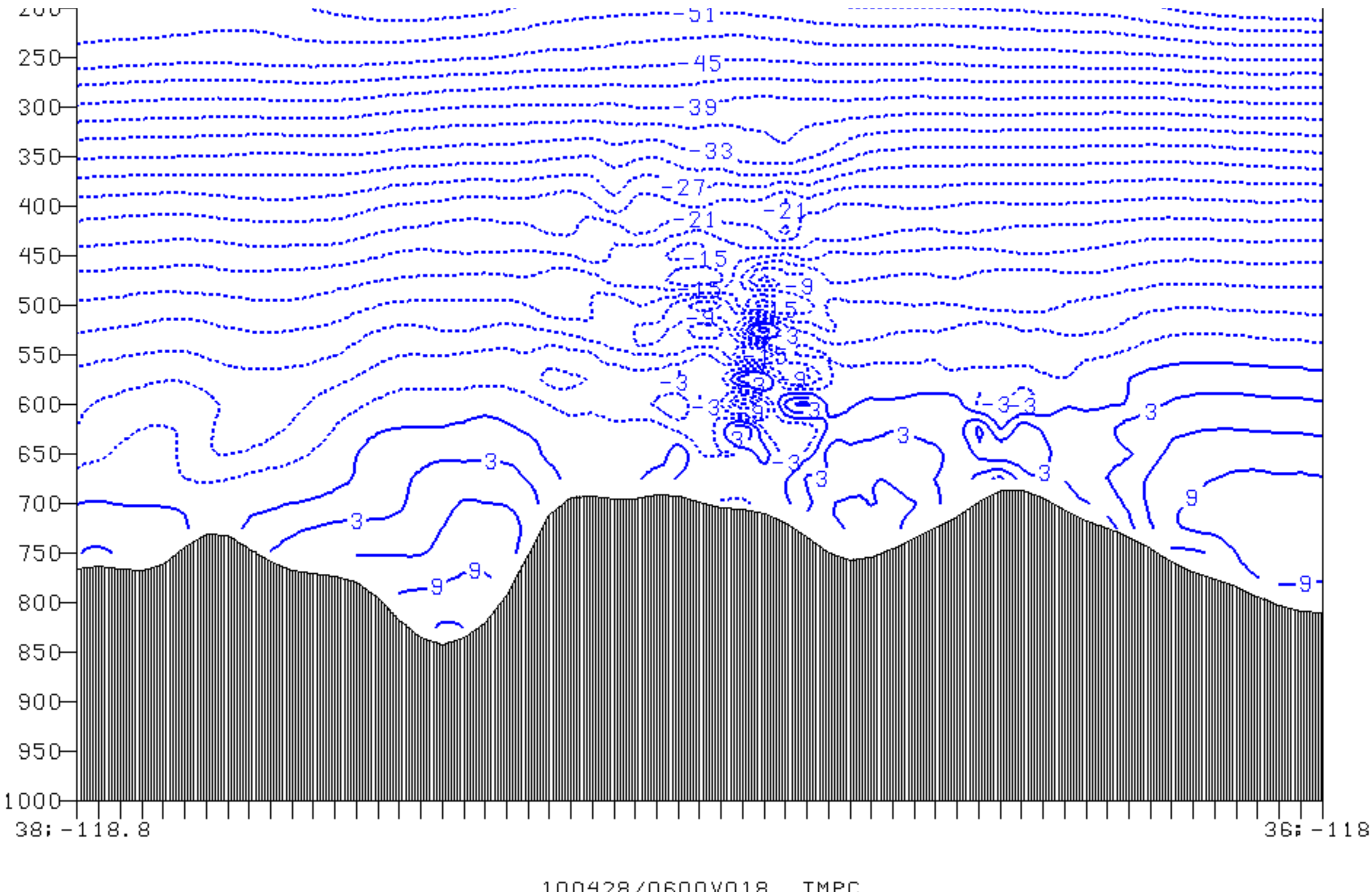
Crank-Nicolson (w/ off centering in time):

$$\frac{y^{\tau+1} - y^{\tau}}{\Delta t} = \frac{1}{2}[1.1 f(y^{\tau+1}) + 0.9 f(y^{\tau})]$$

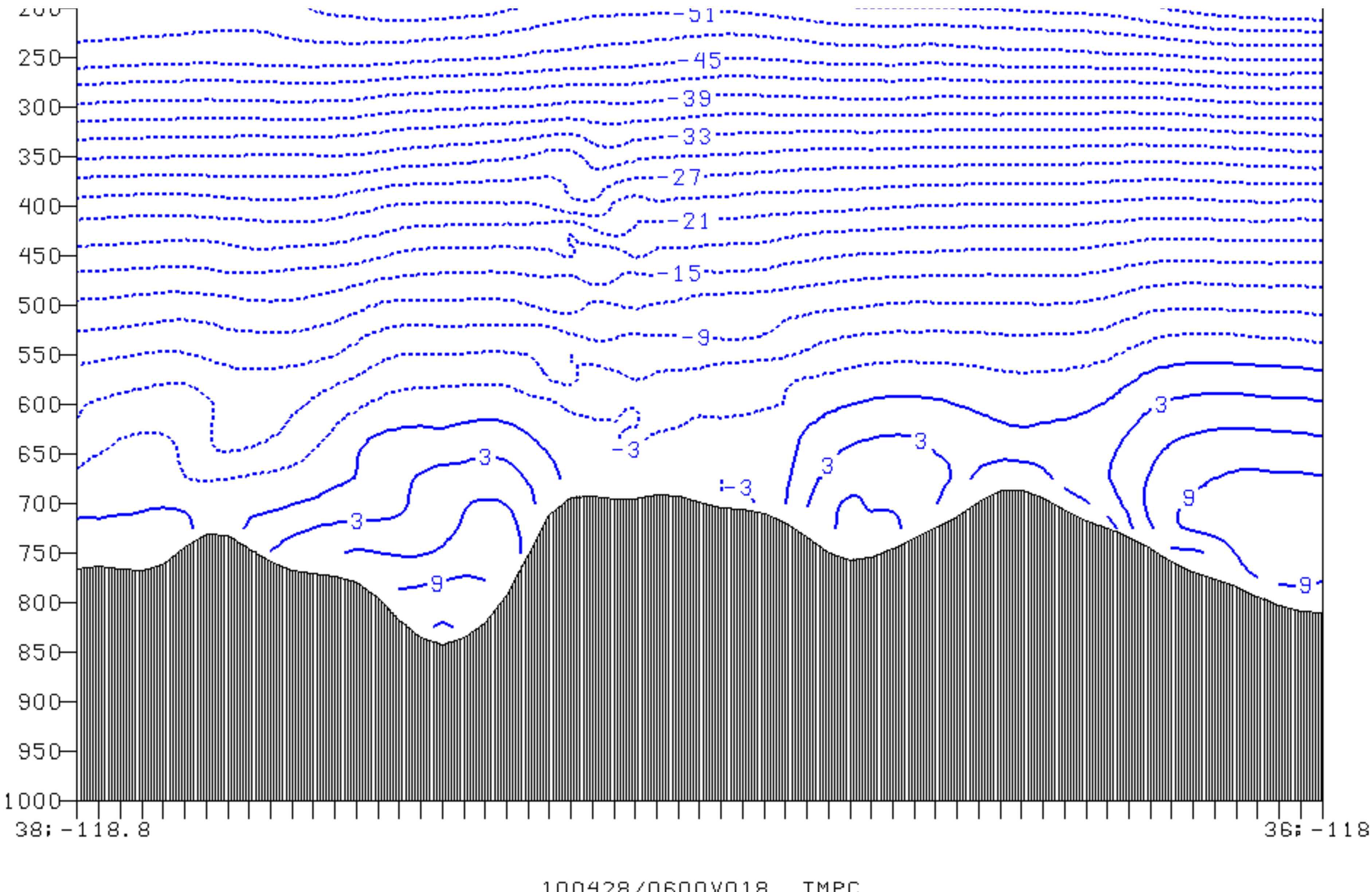
Stability:

An implicit method, it is absolutely stable numerically. Short time steps still needed for *accuracy.*

Cross-section of temperatures 18 h into an integration experiencing strong orographically-forced vertical motion and using **centered in time** C-N vertical advection



Cross-section of temperatures 18 h into an integration experiencing strong orographically-forced vertical motion and using **off-centered in time** C-N vertical advection.



Model Integration

Advection of TKE (Q2) and moisture (Q, CWM)

- Traditionally has taken an approach similar to the Janjic (1997) scheme used in Eta model:
  - Starts with an initial upstream advection step
  - Anti-diffusion/anti-filtering step applied to reduce dispersiveness
  - Conservation enforced after each anti-filtering step
    - maintain global sum of advected quantity
    - prevent generation of new extrema
- Proved inadequate for atmospheric chemistry applications, which inspired….

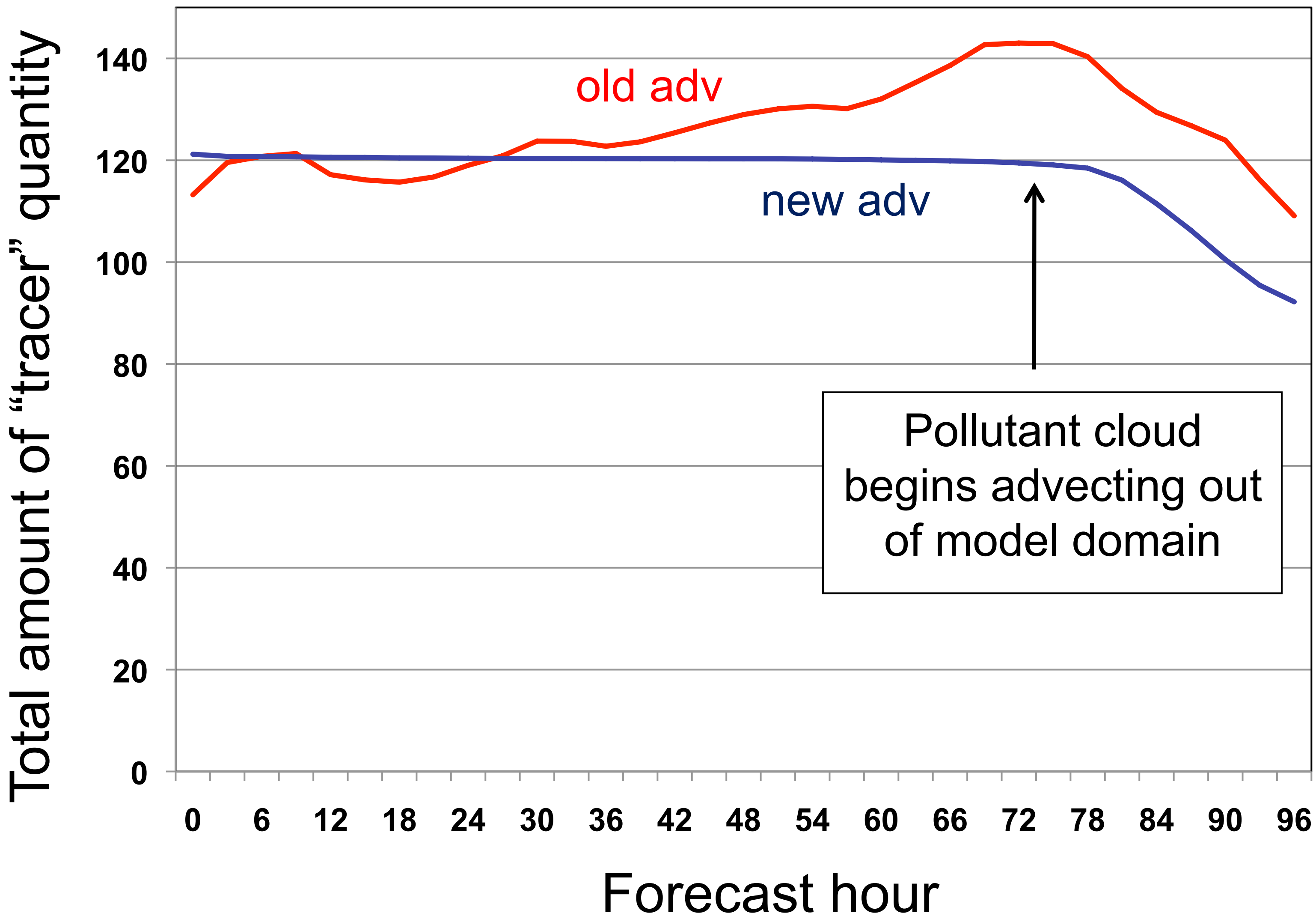
Model Integration

Advection of TKE (Q2) and moisture (Q, CWM)

- …a new "Eulerian" advection option for the NMM:
  - Improved conservation of advected species, and more consistent with remainder of the NMM dynamics.
  - Advects sqrt(quantity) to ensure positive-definiteness.
  - Reduces precipitation bias in warm season.
  - Is the default as of WRFV3.3, but the old option can be invoked in the model namelist by adding:

```
&dynamics
 euler_adv                        = .false.,
 idtadt                           = 2,
```

Advection only experiments of a prescribed pollutant tracer in a real atmospheric flow



Courtesy Youhua Tang

Model Integration *

Fast adjustment processes - gravity wave propagation

Forward-Backward: Mass field computed from a forward time difference, while the velocity field comes from a backward time difference.

In a shallow water equation sense:

$$\frac{\partial u}{\partial t} = -g\frac{\partial h}{\partial x};\frac{\partial h}{\partial t} = -H\frac{\partial u}{\partial x}$$

$$h^{\tau+1} = h^{\tau} - \Delta t\, H\frac{\partial u^{\tau}}{\partial x}$$

$$u^{\tau+1} = u^{\tau} - \Delta t\, g\frac{\partial h^{\tau+1}}{\partial x}$$

Mass field forcing to update wind from $\tau + 1$ time

Ames, 1968; Janjic and Wiin-Nielsen, 1977; Janjic 1979

Model Integration

- Subroutine sequence within solve_nmm (ignoring physics):
  - (3%) PDTE – integrates mass flux divergence, computes vertical velocity and updates hydrostatic pressure.
  - (21%) ADVE – horizontal and vertical advection of T, u, v, Coriolis and curvature terms applied.
  - (32%) ADV2 (typically every other step) – vertical/horizontal advection of q, CWM, TKE
  - (1%) VTOA – updates nonhydrostatic pressure, applies $\omega\alpha$ term to thermodynamic equation
  - (6%) VADZ/HADZ – vertical/horizontal advection of height. w=dz/dt updated.

(*approximate* relative % of dynamics time spent in these subroutines)

Model Integration

- Subroutine sequence within solve_nmm (cont):

(9%) ▪ EPS – vertical and horizontal advection of dz/dt, vertical sound wave treatment.

(11%) ▪ HDIFF – horizontal diffusion

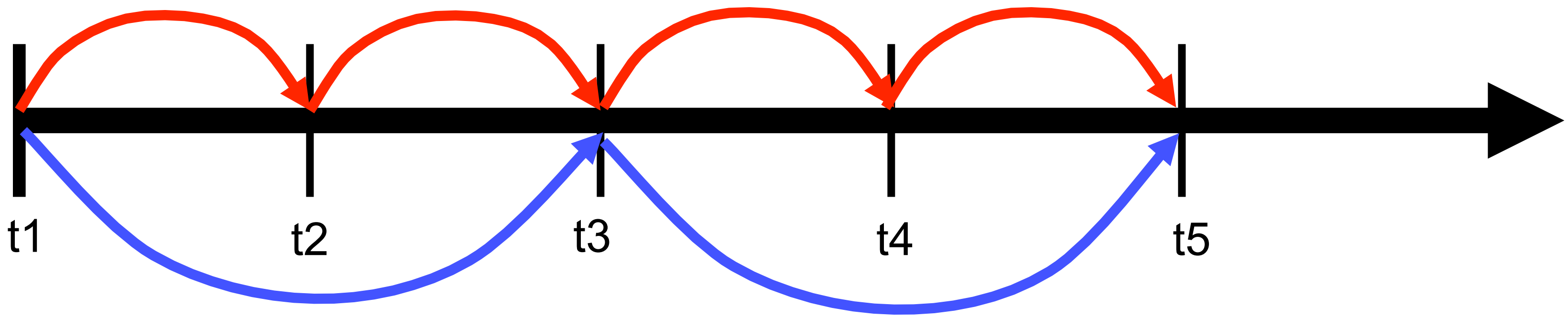
(<1%) ▪ BOCOH – boundary update at mass points

(14%) ▪ PFDHT – calculates PGF, updates winds due to PGF, computes divergence.

(1%) ▪ DDAMP – divergence damping

(<1%) ▪ BOCOV – boundary update at wind points

All dynamical processes every fundamental time step, except….



…passive substance advection, every other time step

Model time step "dt" specified in model namelist.input is for the fundamental time step.

Generally about 2.25x** the horizontal grid spacing (km), or 350x the namelist.input "dy" value (degrees lat).

** runs w/o parameterized convection may benefit from limiting the time step to about 1.9-2.0x the grid spacing.

Now we'll take a look at two items specific to the WRF-NMM horizontal grid:

- Rotated latitude-longitude map projection (only projection used with the WRF-NMM)
- The Arakawa E-grid stagger

Rotated Latitude-Longitude

- Rotates the earth's latitude & longitude such that the intersection of the equator and prime meridian is at the center of the model domain.

- This rotation:
  - minimizes the convergence of meridians.
  - maintains more uniform earth-relative grid spacing than exists for a regular lat-lon grid.

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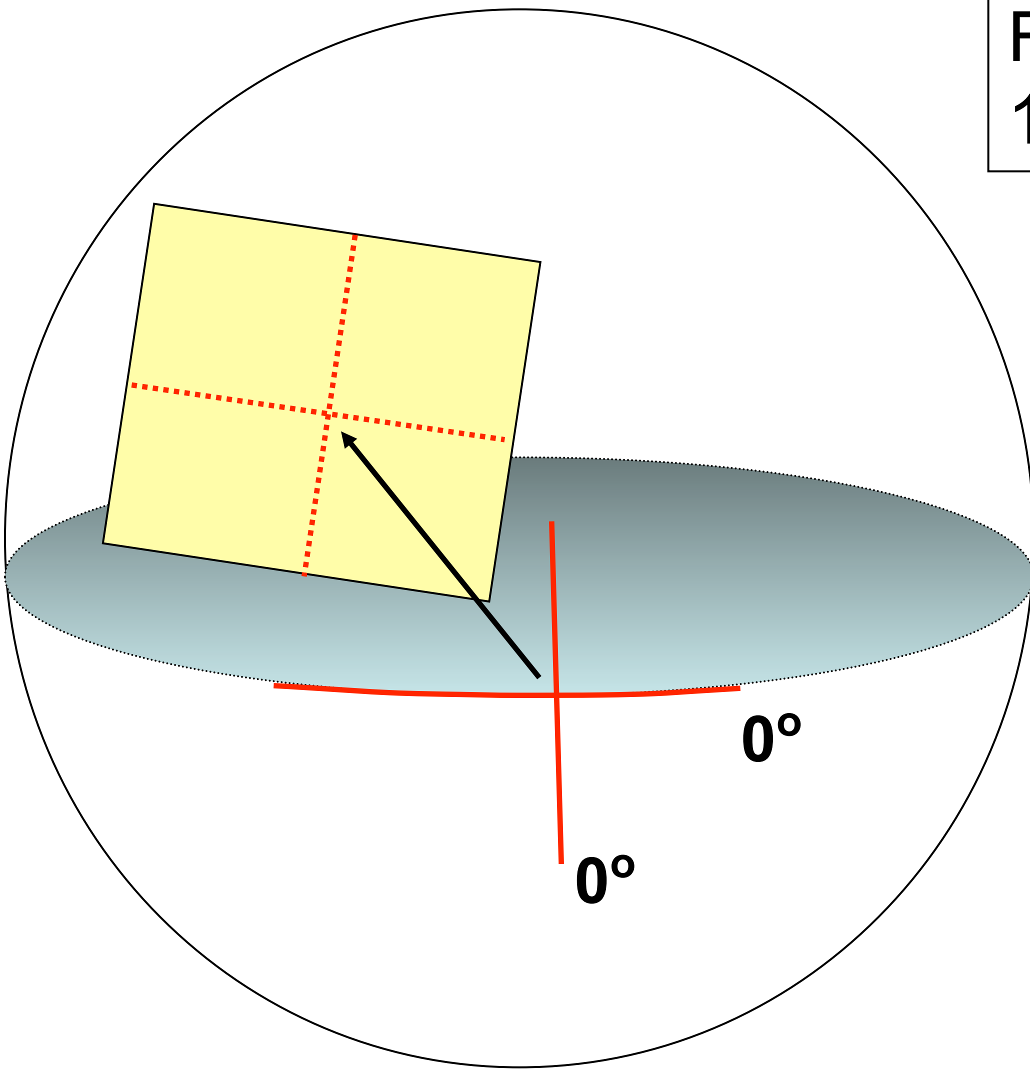
Impact on variation of Δx over domain



For a domain spanning 10N to 70N:

$$\Delta x \propto \cos(lat)$$

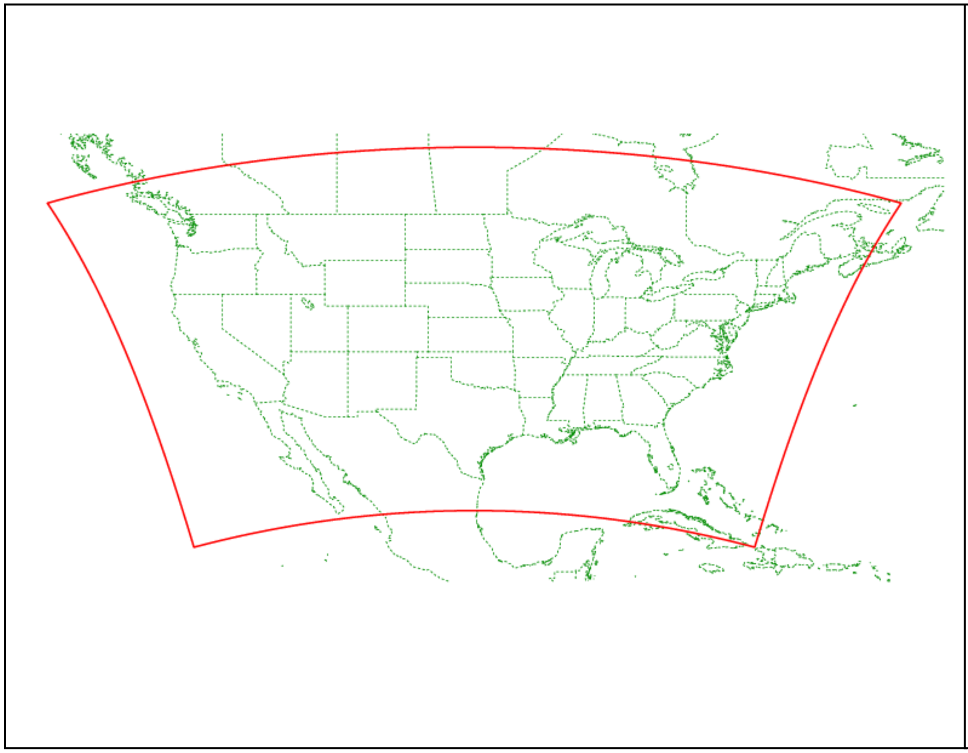
Regular lat-lon grid

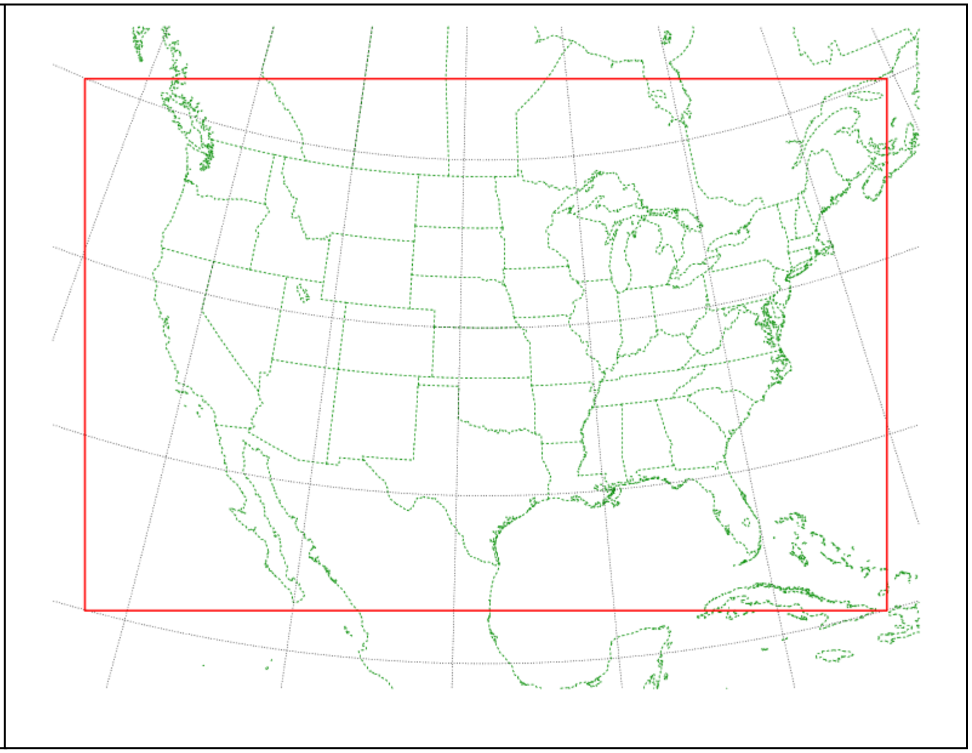
$$\cos(70^0)/\cos(10^0) = 0.347$$

Rotated lat-lon grid

$$\cos(30^0)/\cos(0^0) = 0.866$$

Sample rotated lat-lon domain





On a regular lat-lon map background

On a rotated lat-lon map background (same rotation as model grid).

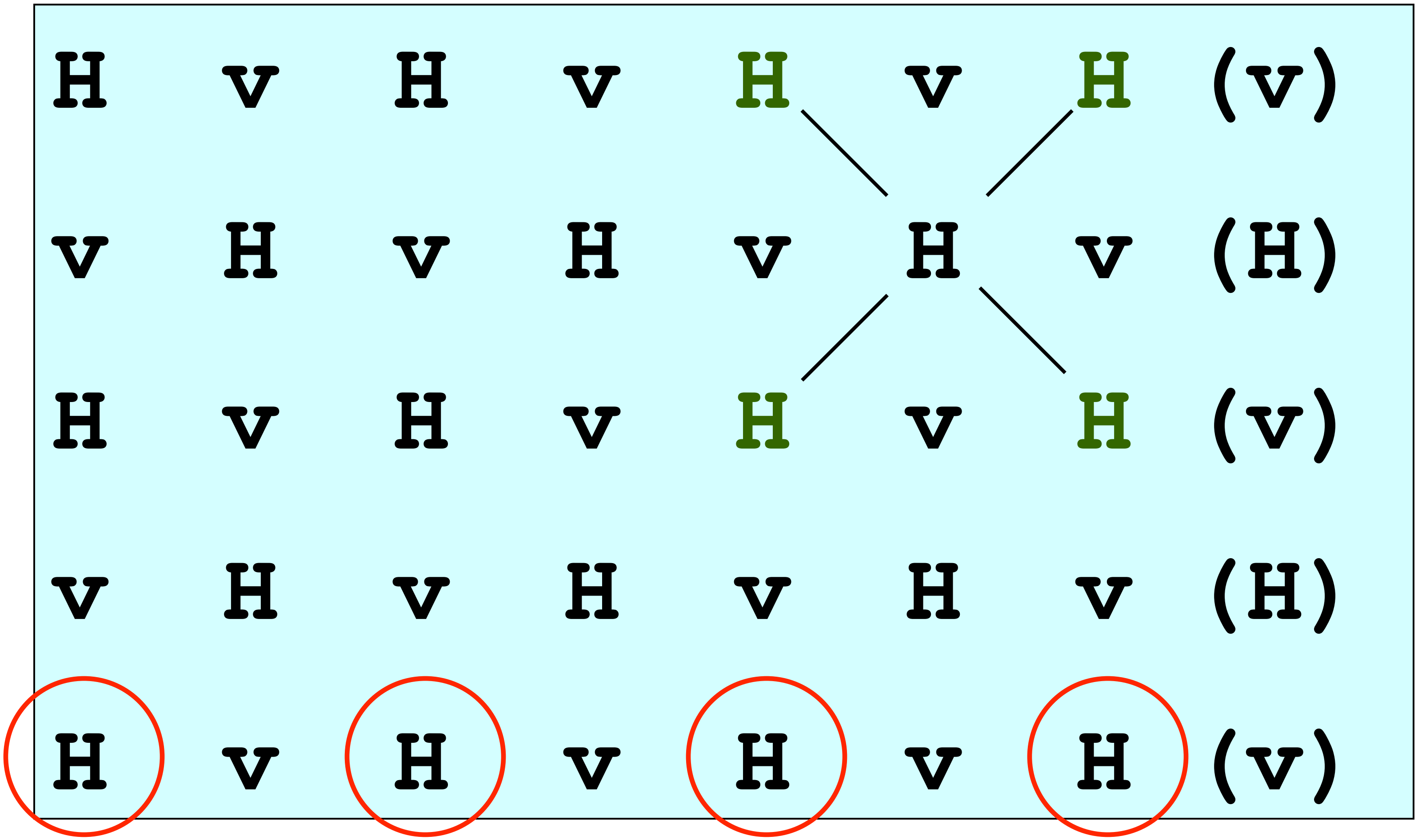
The E-grid Stagger

H	v	H	v	H	v	H	(v)
v	H	v	H	v	H	v	(H)
H	v	H	v	H	v	H	(v)
v	H	v	H	v	H	v	(H)
H	v	H	v	H	v	H	(v)

H=mass point, v=wind point

red=(1,1) ; blue=(1,2)

The E-grid Stagger



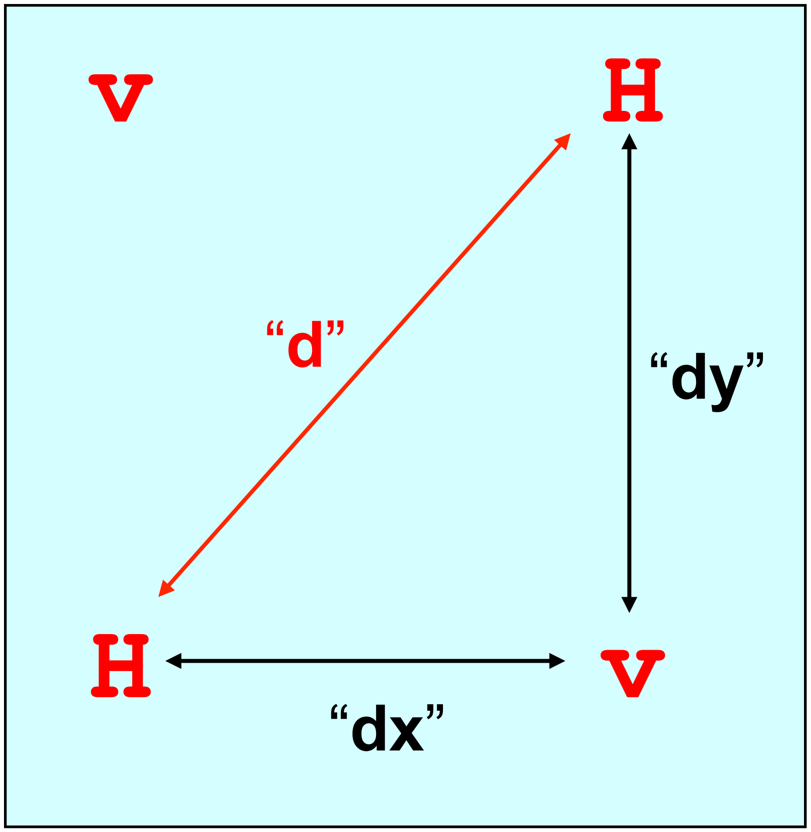
XDIM=4 (# of mass points on odd numbered row)
YDIM=5 (number of rows)

The E-grid Stagger - properties

- Due to the indexing convention, the X-dimension is half as large as would be expected from a C-grid domain (typically XDIM < YDIM for the E-grid).
- “Think diagonally” –the shortest distance between adjacent like points is along the diagonals of the grid.

- E-grid energy and enstrophy conserving momentum advection scheme (Janjic, 1984, MWR) controls the spurious nonlinear energy cascade (accumulation of small scale computational noise due to nonlinearity) more effectively than schemes on the C grid – an argument in favor of the E grid.

The E-grid Stagger

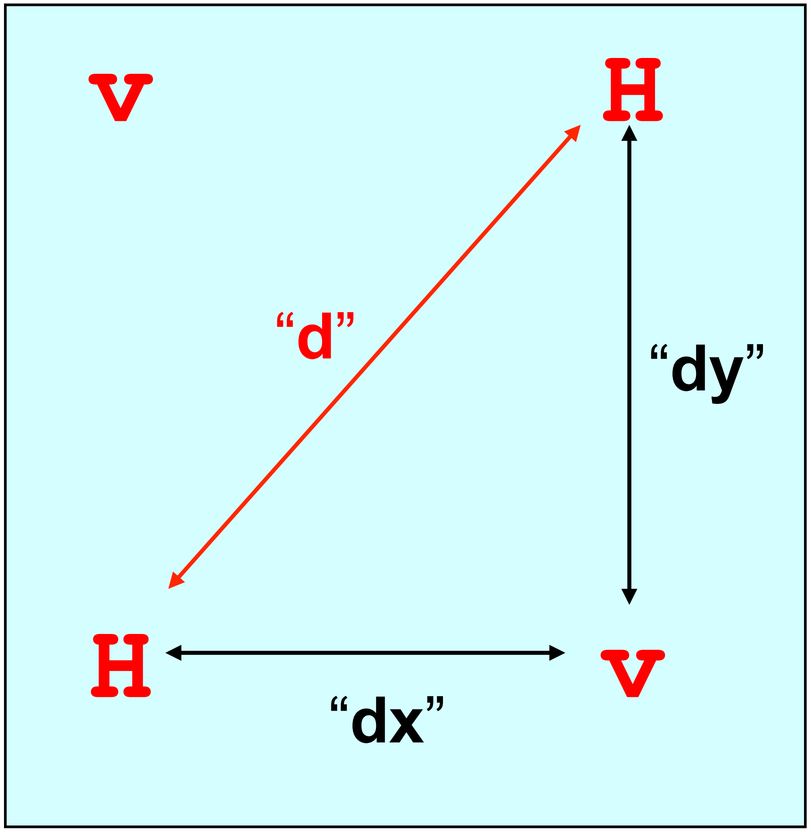


- Conventional grid spacing is the diagonal distance "**d**".

- Grid spacings in the WPS and WRF namelists are the "**dx**" and "**dy**" values, *specified in fractions of a degree for the WRF-NMM*.

- "WRF domain wizard" takes input grid spacing "**d**" in km and computes the angular distances "**dx**" and "**dy**" for the namelist.

The E-grid Stagger - examples



"d" (km)	"dx" (deg)	"dy" (deg)	~dt (s)
4	.026	.0256	7.5
12	.078	.0768	26.66
32	.208	.2048	72

Note that dx > dy traditionally:

- Helps offset the slight convergence in the x-direction
- More important for domains covering a large latitudinal expanse.

Spatial Discretization

- Basic discretization principle is conservation of important properties of the continuous system.
  - “Mimetic” approach

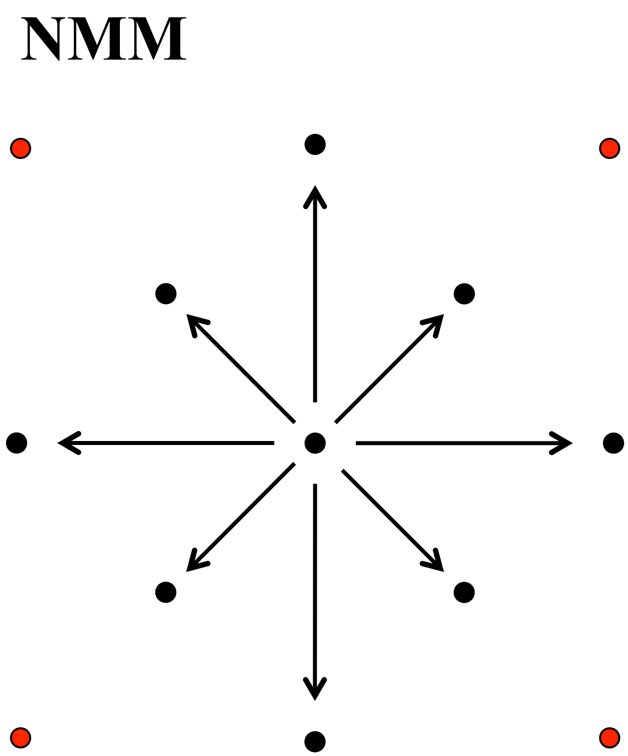
    http://www.math.unm.edu/~stanly/mimetic/mimetic.html

  - Something of a novelty in applied mathematics, …
  - … but well established in atmospheric modeling (Arakawa, 1966, 1972, 1977 …; Sadourny, 1975 …; Janjic, 1977, 1984 …; Tripoli, 1992, …)

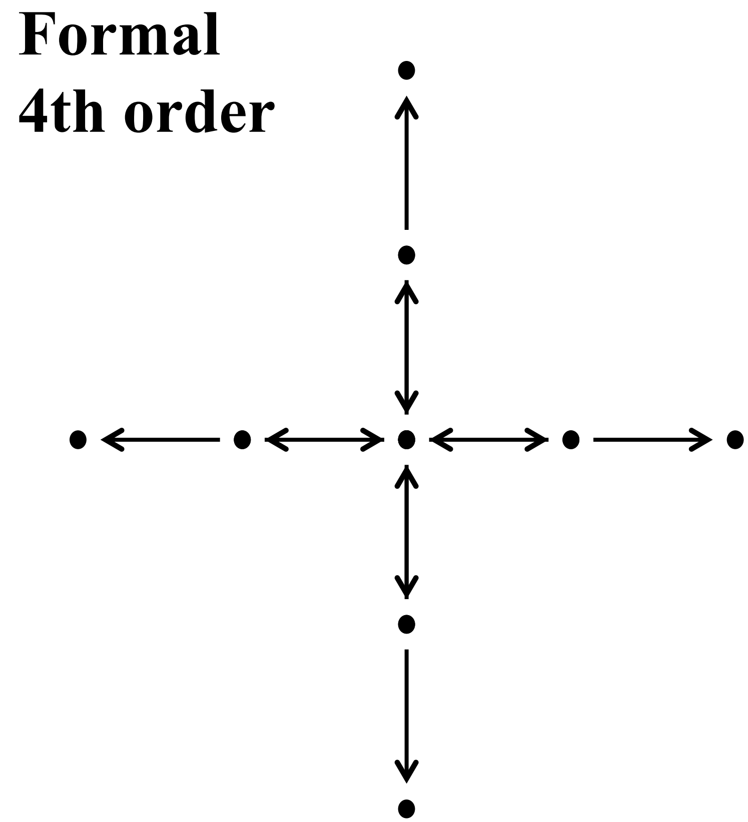
Spatial Discretization

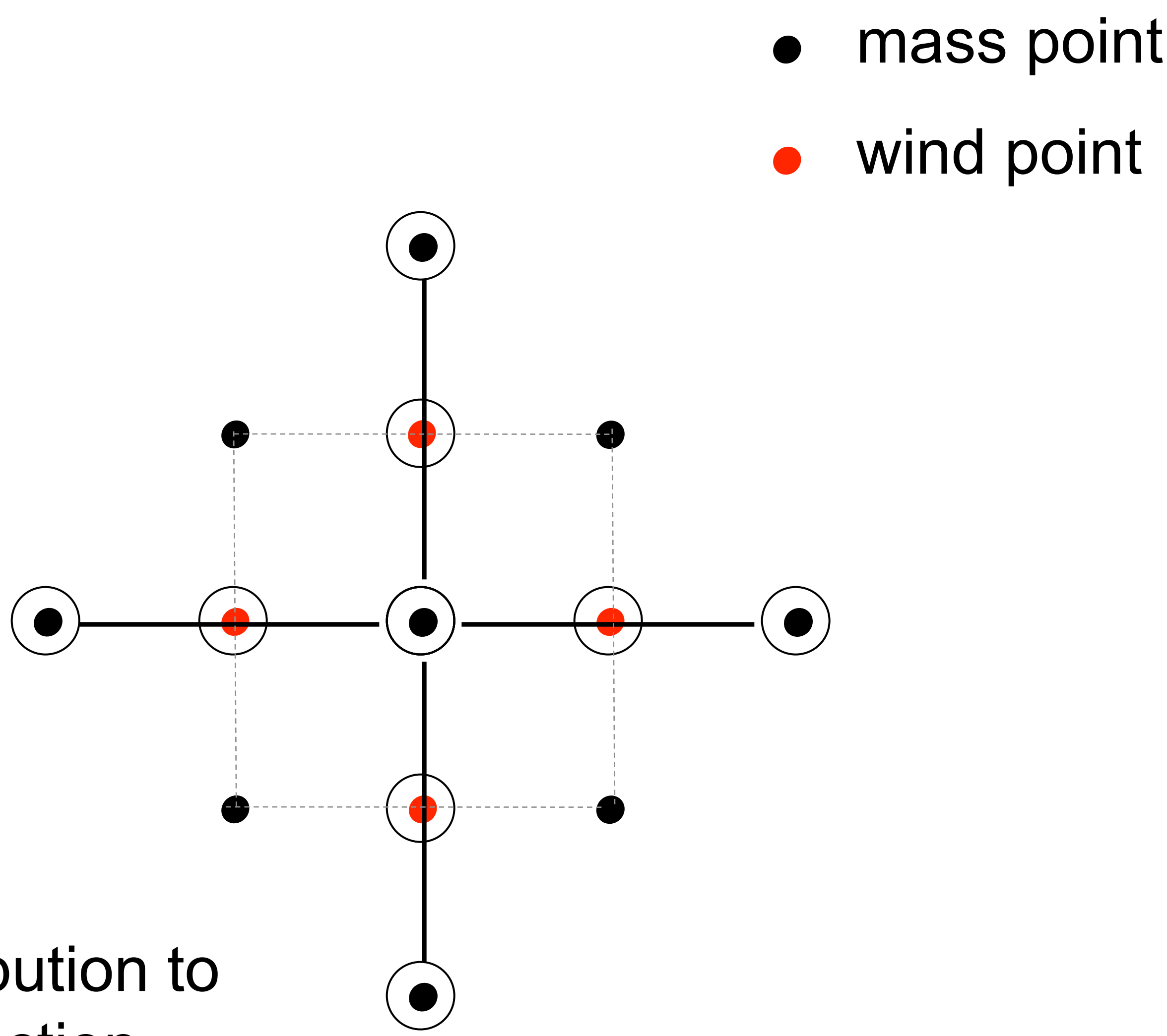
General Philosophy

- Conserve energy and enstrophy in order to control nonlinear energy cascade; eliminate the need for numerical filtering to the extent possible.
- Conserve a number of first order and quadratic quantities (mass, momentum, energy, ...).
- Use consistent order of accuracy for advection and divergence operators and the omega-alpha term; consistent transformations between KE and PE in the hydrostatic limit.
- Preserve properties of differential operators.

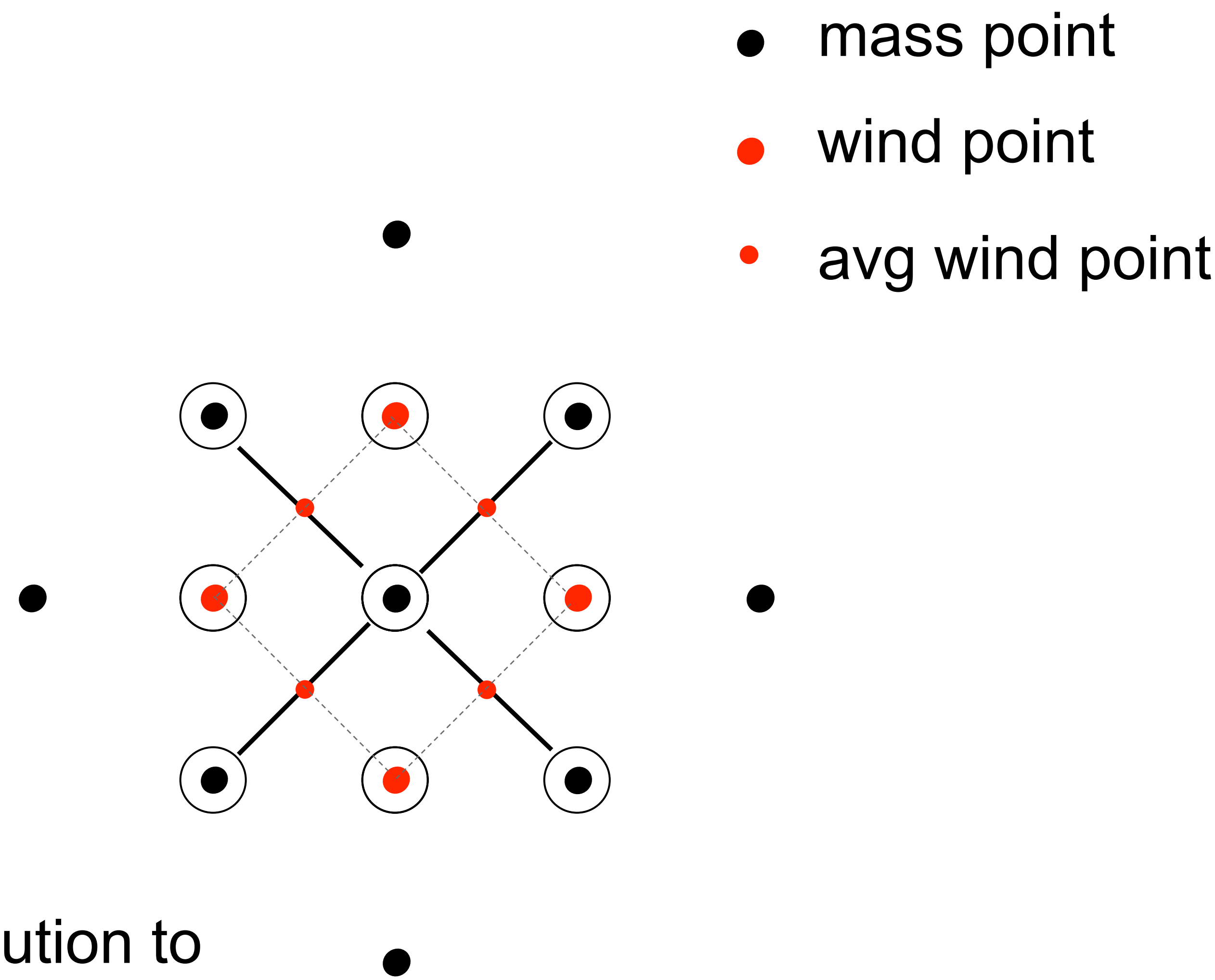


Advection and divergence operators – each point talks to all eight neighboring points (isotropic)



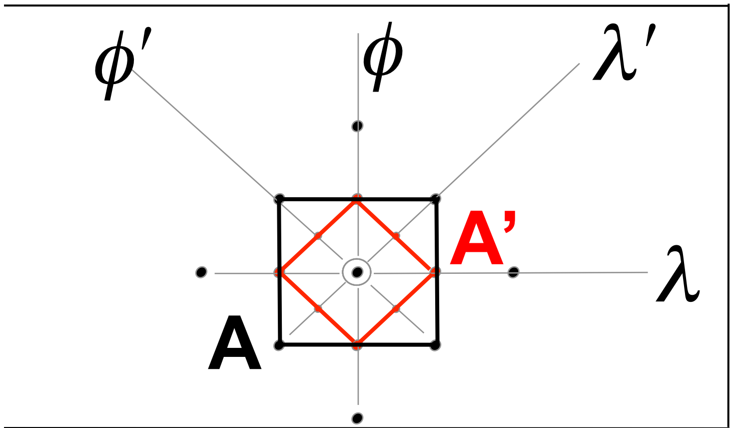


1/3 of the contribution to divergence/advection comes from these N/S and E/W fluxes.



2/3 of the contribution to divergence/advection comes from these diagonal fluxes.

Horizontal temperature advection detail (mathematical)



$\Delta\pi$ is hydrostatic layer thickness

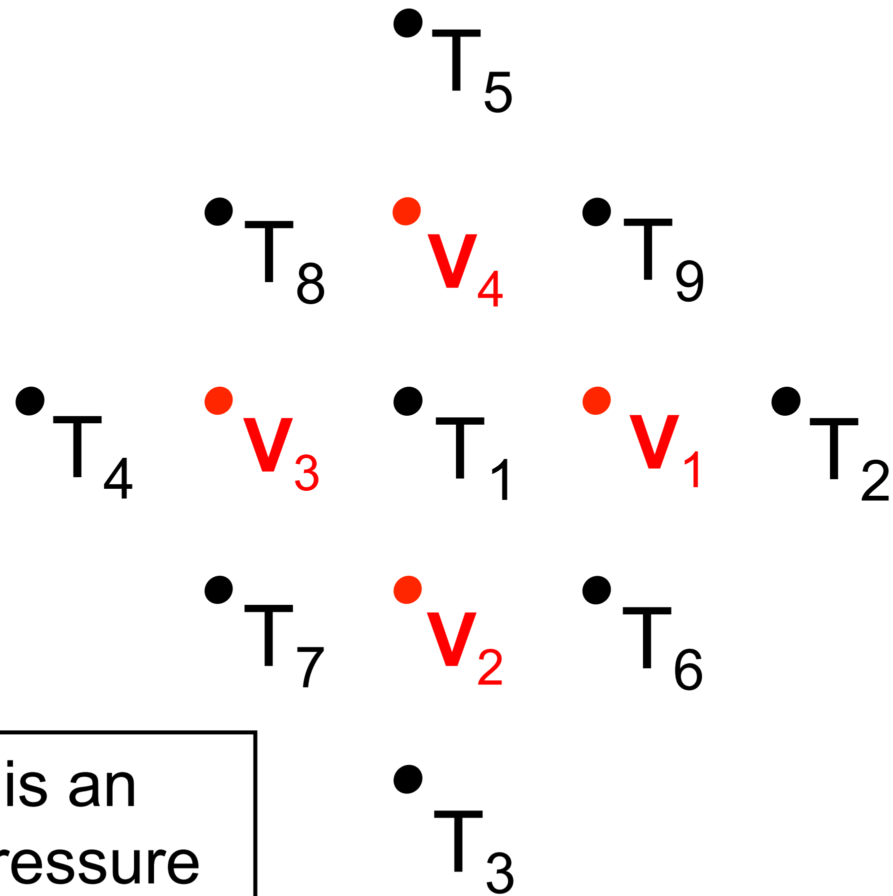
$$U = \overline{\Delta\pi}^{\lambda} u 2\Delta y \qquad U' = \overline{\Delta\pi}^{\lambda'} \left(\overline{u\Delta y + v\Delta x}^{\phi'} \right)$$

$$V = \overline{\Delta\pi}^{\phi} v 2\Delta x \qquad V' = \overline{\Delta\pi}^{\phi'} \left(\overline{-u\Delta y + v\Delta x}^{\lambda'} \right)$$

$$A = 4\Delta x \Delta y \qquad A' = 2\Delta x \Delta y$$

$$-v \cdot \nabla T =$$
$$-\frac{1}{\Delta\pi}\left[\frac{1}{3}\frac{1}{A}\left(\overline{U\Delta_{\lambda}T}^{\lambda} + \overline{V\Delta_{\phi}T}^{\phi}\right) + \frac{2}{3}\frac{1}{A'}\left(\overline{U'\Delta_{\lambda'}T}^{\lambda'} + \overline{V'\Delta_{\phi'}T}^{\phi'}\right)\right]$$

Horizontal temperature advection detail (computerese)



For each T_n, there is an associated layer pressure depth (here denoted dp_n). There also is a dx_n specific to each point

Horizontal temperature advection detail (computerese)

Temperature fluxes in E/W, N/S, and diagonal directions:

TEW $=u_3dy\ (dp_1+dp_4)(T_1-T_4) + u_1dy\ (dp_1+dp_2)(T_2-T_1)$

TNS $=v_2dx_2\ (dp_1+dp_3)(T_1-T_3) + v_4dx_4\ (dp_1+dp_5)(T_5-T_1)$

TNE $= [(u_1dy + v_1dx_1 + u_4dy + v_4dx_4)\ (dp_1+dp_9)\ (T_9-T_1)$
$+ \ (u_3dy + v_3dx_3 + u_2dy + v_2dx_2)\ (dp_1+dp_7)\ (T_1-T_7)]$

TSE $= [(u_1dy - v_1dx_1 + u_2dy - v_2dx_2)\ (dp_1+dp_6)\ (T_6-T_1)$
$+ \ (u_3dy - v_3dx_3 + u_4dy - v_4dx_4)\ (dp_1+dp_8)\ (T_1-T_8)]$

Advective tendency, ADT, combines the fluxes:

ADT = (**TEW** + **TNS** + **TNE** + **TSE**) $* (-dt/24) * (1/dx_1*dy*dp_1)$

NMM Vertical Coordinate

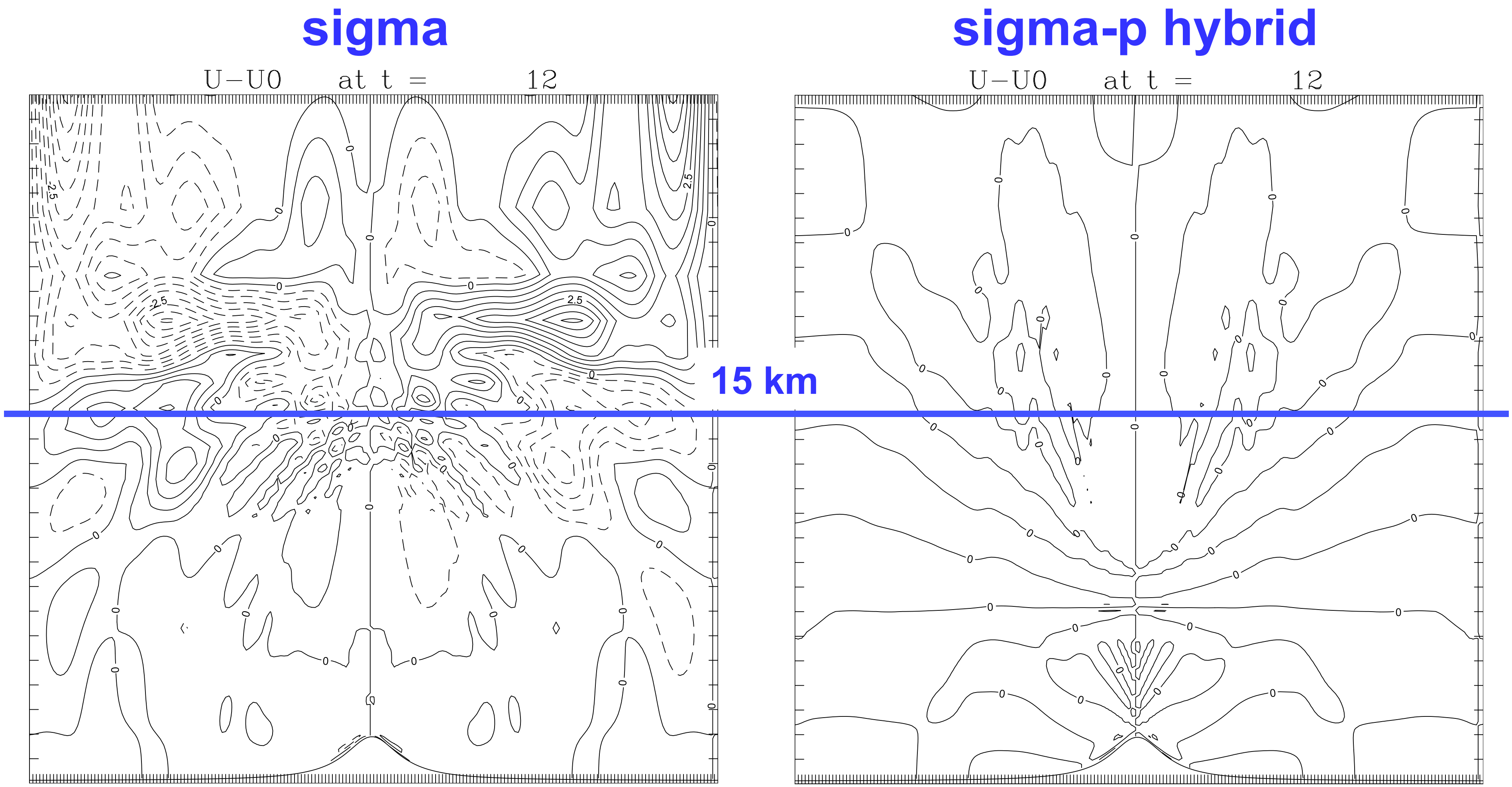
Pressure-sigma hybrid (Arakawa and Lamb, 1977)

Has the desirable properties of a terrain-following pressure coordinate:

- Exact mass (etc.) conservation
- Nondivergent flow remains on pressure surfaces
- No problems with weak static stability
- No discontinuities or internal boundary conditions

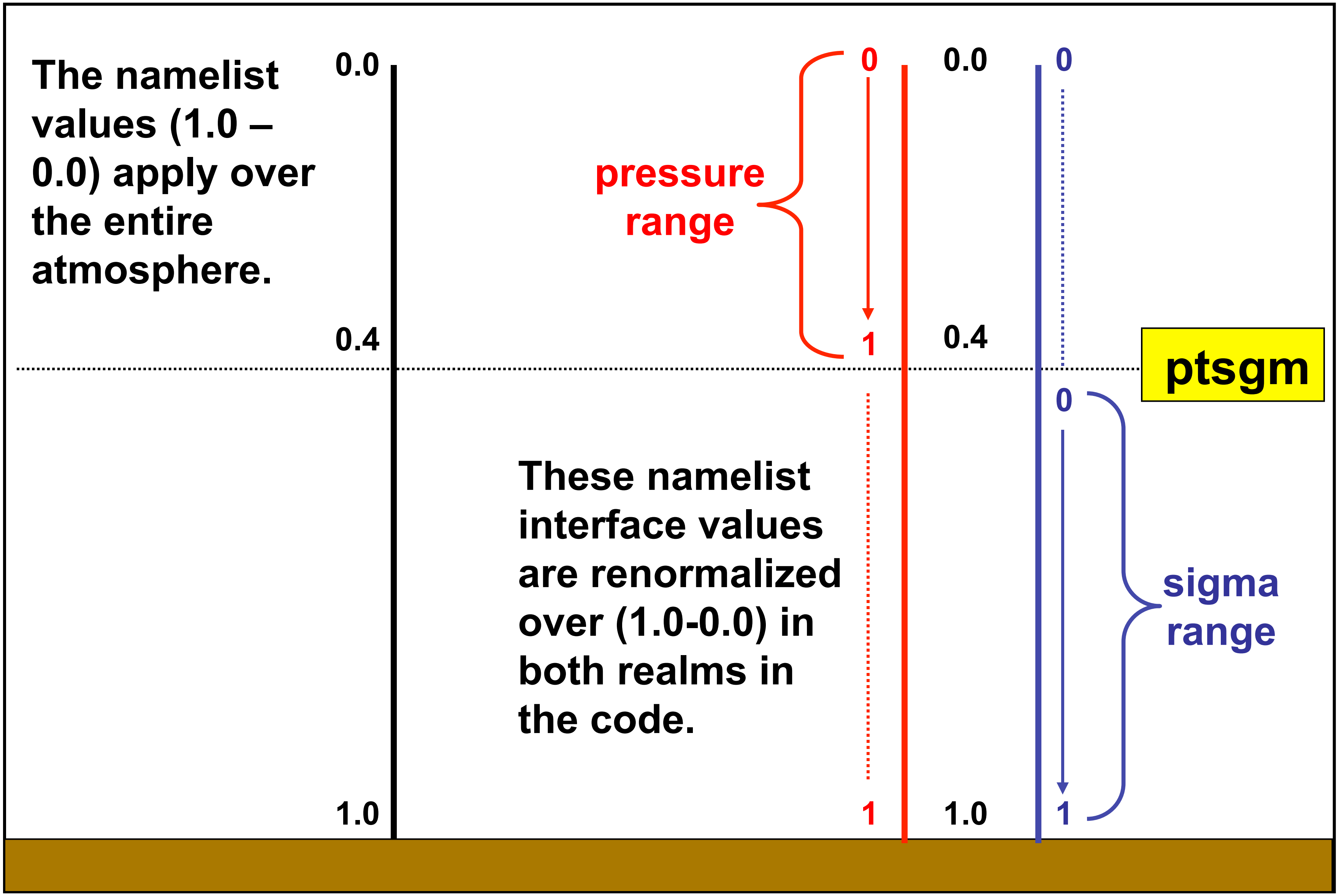
And an additional benefit from the hybrid:

- Flat coordinate surfaces at high altitudes where sigma problems worst (e.g., Simmons and Burridge, 1981)

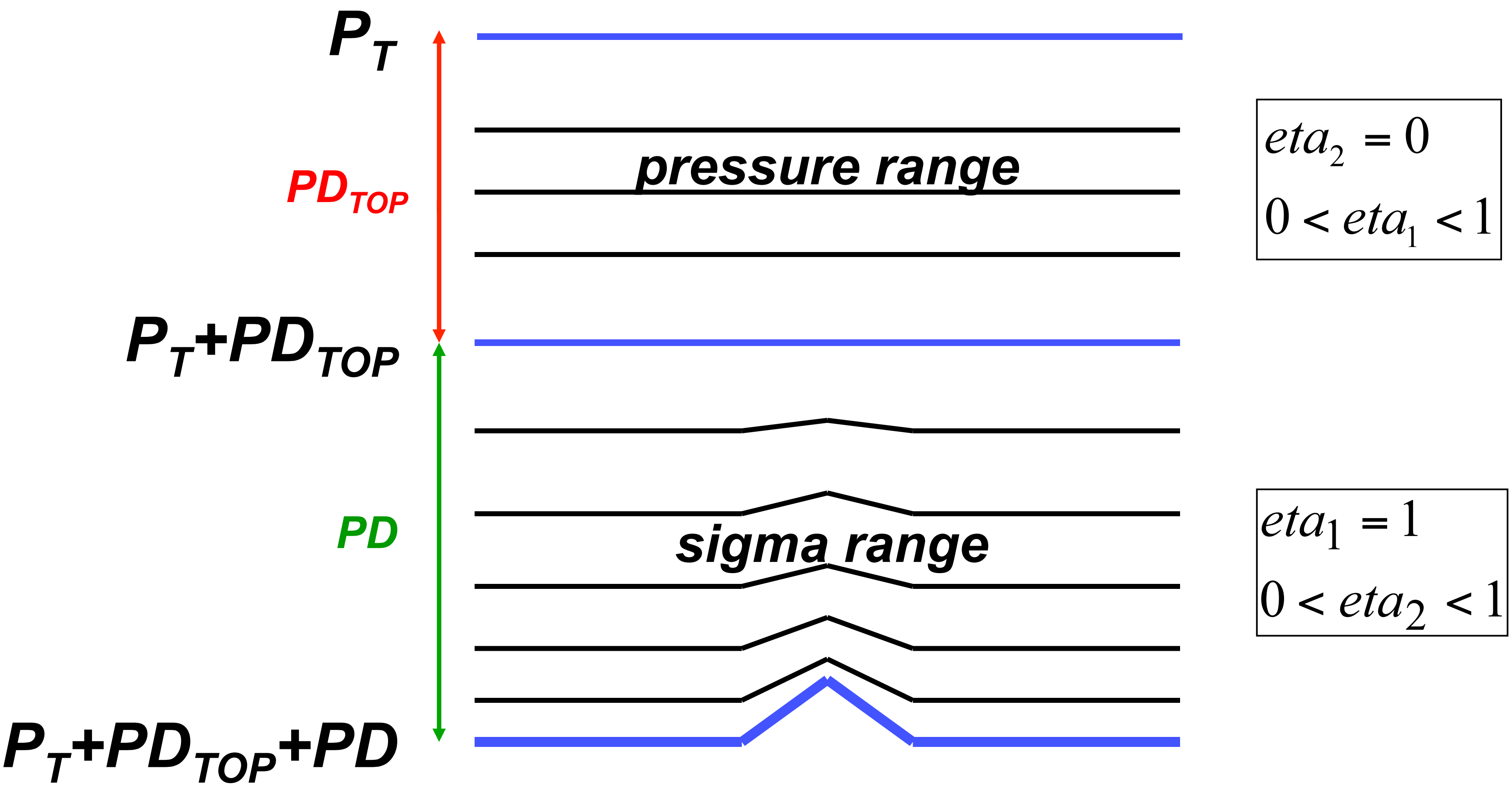


Wind developing due to the spurious pressure gradient force in an idealized integration. The hybrid coordinate boundary between the pressure and sigma domains is at ~400 hPa.

Pressure-Sigma Hybrid Vertical Coordinate



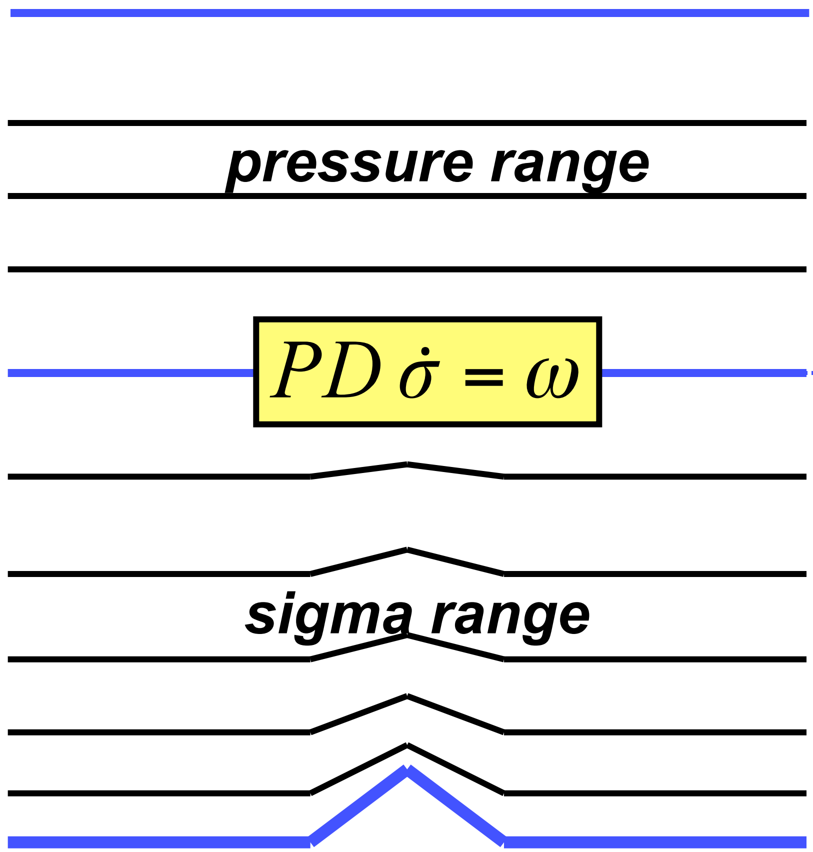
Pressure-Sigma Hybrid Vertical Coordinate



$$\pi = eta_1 * PD_{TOP} + eta_2 * PD + P_T$$

*

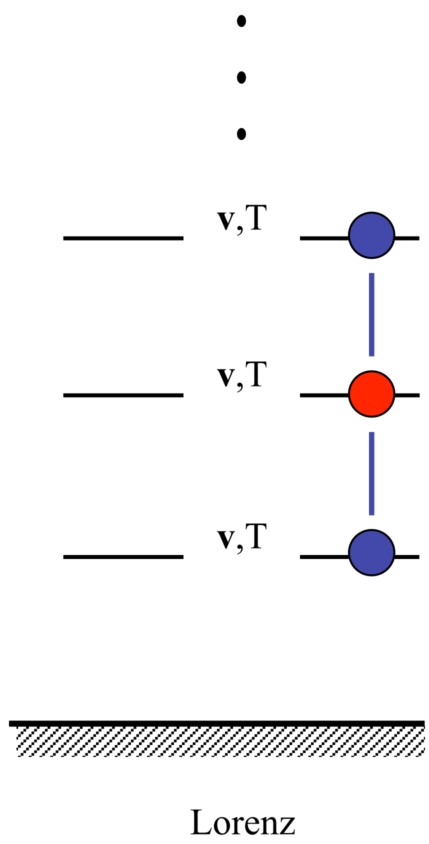
Equations in Hybrid Coordinate



$$\nabla_p \cdot (\mathbf{v}) + \frac{\partial \omega}{\partial p} = 0$$

$$\frac{\partial PD}{\partial t} + \nabla_\sigma \cdot (PD\ \mathbf{v}) + \frac{\partial (PD\,\dot{\sigma})}{\partial \sigma} = 0$$

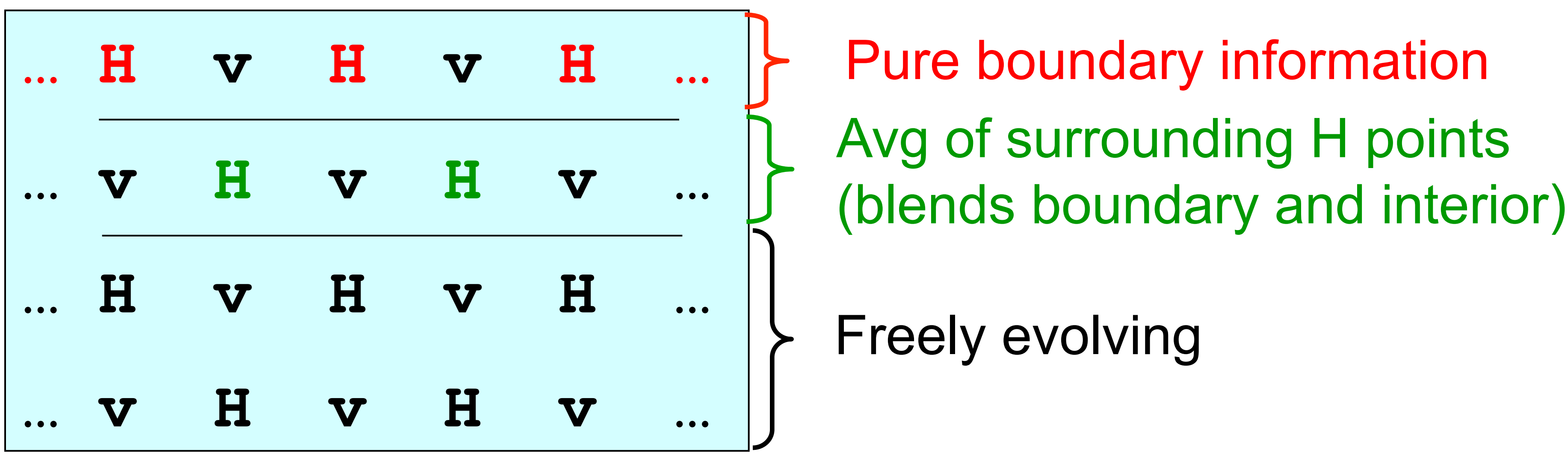
Vertical discretization



Vertical advection combines the advective fluxes computed above and below the layer of interest.

Lateral Boundary Conditions

- Lateral boundary information prescribed only on outermost row:



- Upstream advection in three rows next to the boundary
  - No computational outflow boundary condition for advection
- Enhanced divergence damping close to the boundaries.

Dissipative Processes – lateral diffusion

A 2nd order, nonlinear Smagorinsky-type horizontal diffusion is utilized:

- Diffusion strength a function of the local TKE, deformation of the 3D flow, and a namelist-specified diffusion strength variable (*coac*).
- Lateral diffusion is zeroed for model surfaces sloping more than 4.5 m per km (0.0045) by default.
- This slope limit can be adjusted with the namelist variable *slophc*. *slophc* is expressed as sqrt(2) times the true slope (making the 0.0045 default ~0.00636)

Dissipative Processes - divergence damping *

- Internal mode damping (on each vertical layer)

$$\mathrm{v}_j = \mathrm{v}_j + \frac{(\nabla \cdot dp_{j+1}\vec{\mathbf{v}}_{j+1} - \nabla \cdot dp_{j-1}\vec{\mathbf{v}}_{j-1})}{(dp_{j+1} + dp_{j-1})} \cdot DDMPV$$

- External mode damping (vertically integrated)

$$\mathrm{v}_j = \mathrm{v}_j + \frac{(\int \nabla \cdot dp_{j+1}\vec{\mathbf{v}}_{j+1} - \int \nabla \cdot dp_{j-1}\vec{\mathbf{v}}_{j-1})}{(\int dp_{j+1} + \int dp_{j-1})} \cdot DDMPV$$

$$DDMPV \approx \sqrt{2} \cdot dt \cdot CODAMP$$

CODAMP is a namelist controlled variable = 6.4 by default.

New namelist switches in WRFV3.3

```
&dynamics
wp          = 0.00
coac        = 0.75,
codamp      = 6.4,
slophc      = 0.006364
```

Defaults

- **WP** - off-centering weight in nonhydrostatic computation (value of ~0.1 improves stability of some sub-1.5 km grid forecasts).
- **COAC** - diffusion strength (larger → more diffusive smoothing)
- **CODAMP** - divergence damping strength (larger → more damping, fewer small-scale regions of divergence).
- **SLOPHC** - max surface slope for diffusion (larger value applies lateral diffusion over more mountainous terrain).

A corrected namelist switch for WRFV3.4

```
&dynamics
non_hydrostatic                    = .true.
```

Default

- If .false., will run the model as a hydrostatic system (may make sense for > ~20 km grid spacings where nonhydrostatic effects are minimal).

  Can be specified in current release code namelist, but value of switch has no impact.

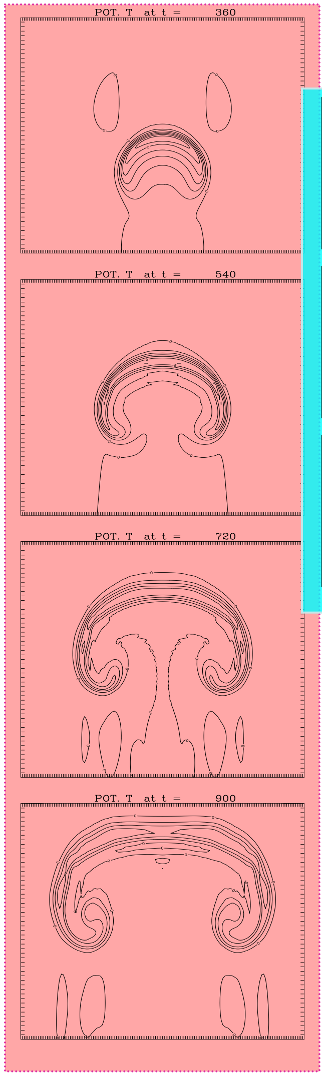
Gravity Wave Drag & Mountain Blocking

- Accounts for sub-grid scale mountain effects: mountain waves (GWD) and stability-dependent blocking of low-level flow around topography (MB).
- More important for coarser grid spacing (> ~10 km) and longer (multi-day) integrations.
- gwd_opt=2 in physics namelist to invoke for the WRF-NMM.
- Benefits overall synoptic patterns and near-surface wind and temperature forecasts.
- Based on the GFS model package for GWD (Alpert et al., 1988, 1996; Kim & Arakawa,1995) and MB (Lott & Miller, 1997).

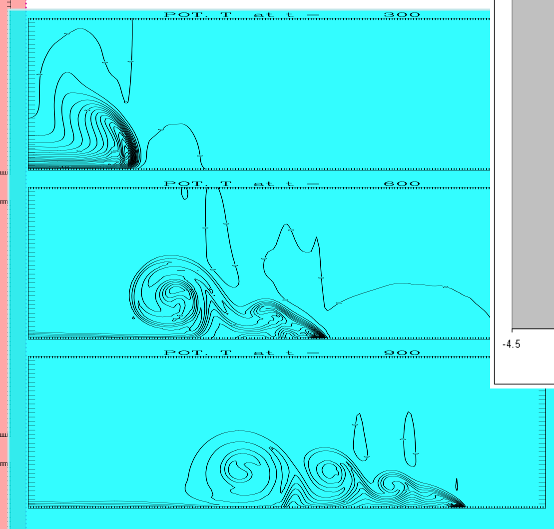
Courtesy of Brad Ferrier

Dynamics formulation tested on various scales

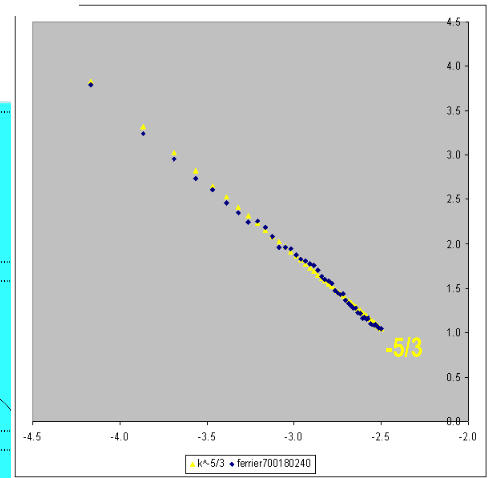
Warm bubble



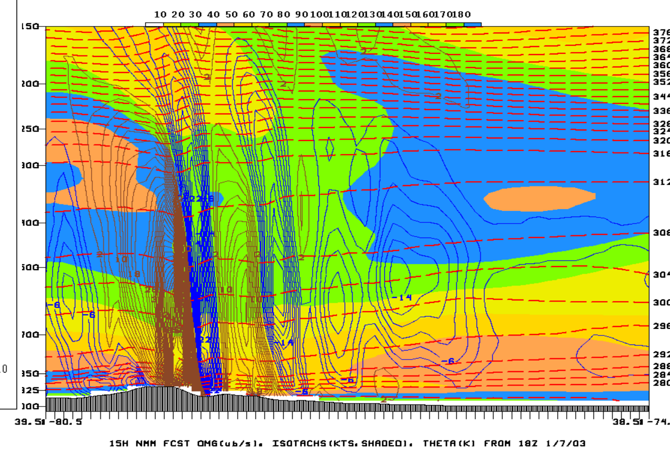
Cold bubble



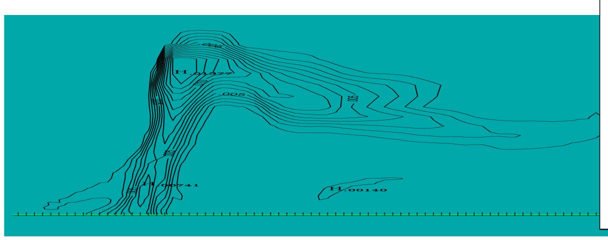
Decaying 3D turbulence



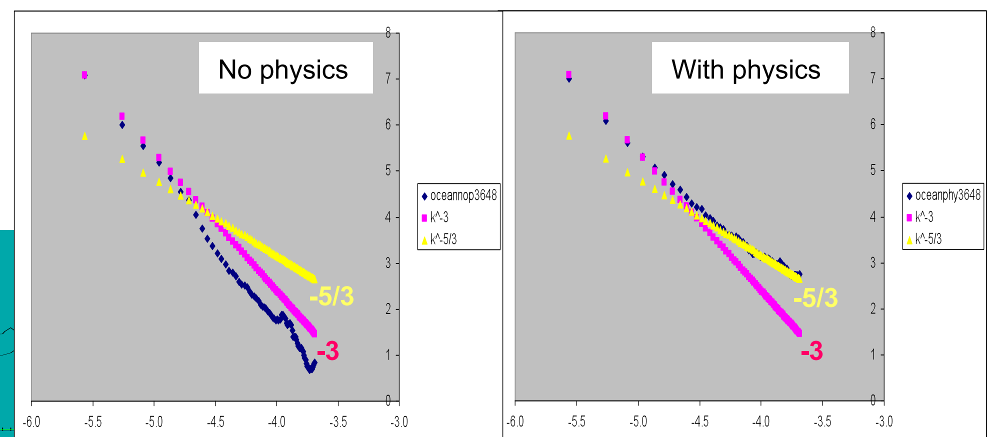
Mountain waves



Convection



Atmospheric spectra



Summary

- Robust, reliable, fast
- Represents an extension of NWP methods developed and refined over a decades-long period into the nonhydrostatic realm.
- Utilized at NCEP in the HWRF, Hires Window* and Short Range Ensemble Forecast (SREF*) operational systems.

* = WRF-ARW used in these systems as well