# The Advanced Research WRF (ARW) Dynamics Solver

1. Terrain, vertical coordinate
2. Equations and variables
3. Time integration scheme
4. Grid staggering
5. Advection (transport) and conservation
6. Time step parameters
7. Filters
8. Map projections and global configuration


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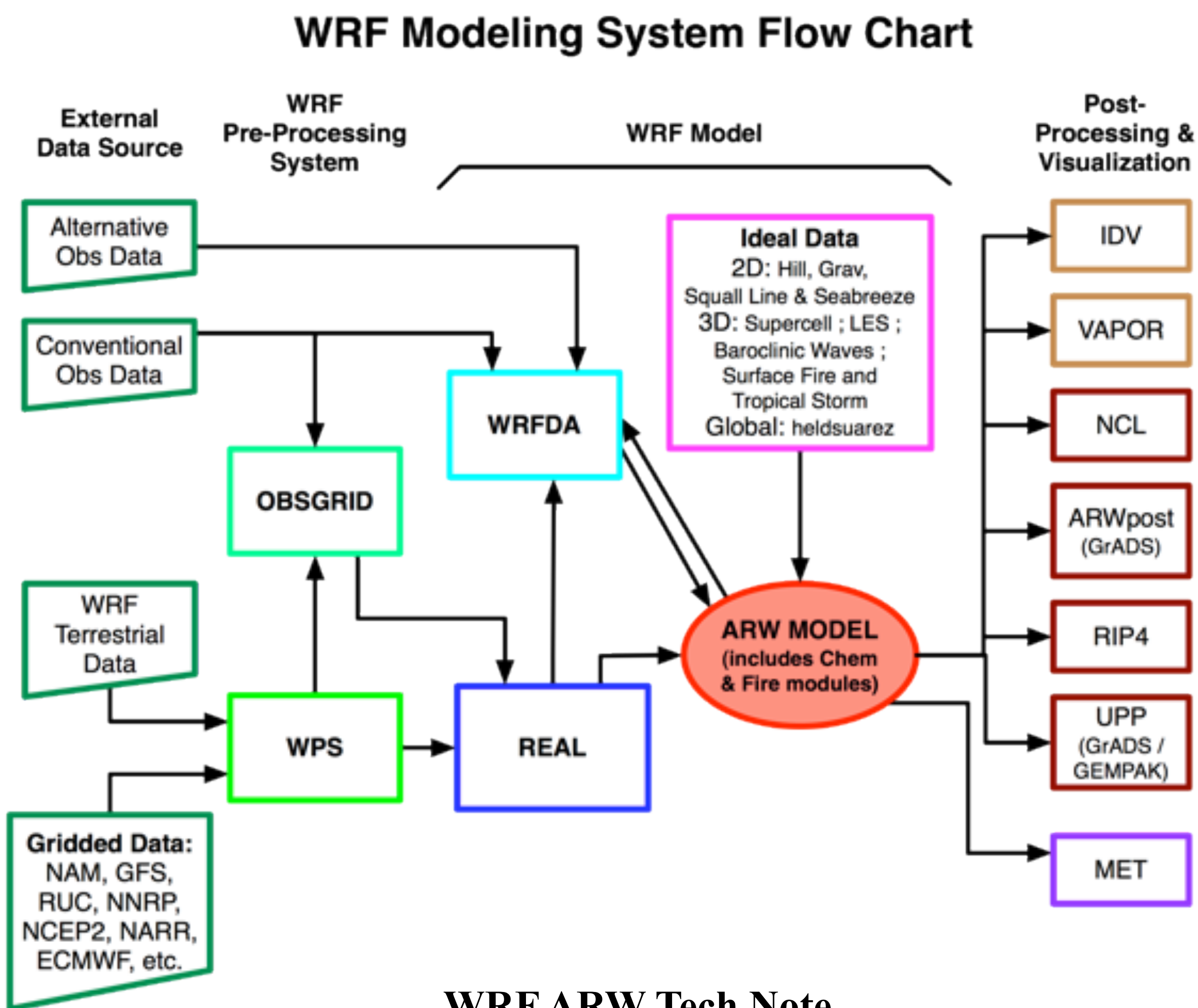
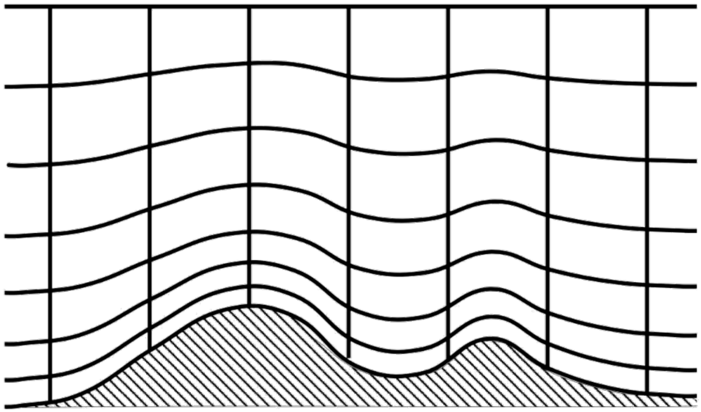



**WRF ARW Tech Note**

A Description of the Advanced Research WRF Version 3

http://www.mmm.ucar.edu/wrf/users/pub-doc.html

# Dynamics: 1. Terrain, vertical coordinate



Hydrostatic pressure $\pi$

Column mass (per unit area) $\mu = \pi_s - \pi_t$

Vertical coordinate $\eta = \frac{(\pi - \pi_t)}{\mu}$

Layer mass (per unit area) $\mu \Delta\eta = \Delta\pi = g\rho\Delta z$

Conserved state (prognostic) variables:

$$\mu, \quad U = \mu u, \quad V = \mu v, \quad W = \mu w, \quad \Theta = \mu\theta$$

Non-conserved state variable: $\phi = gz$

# Dynamics: 2. Equations and variables – moist equations

Moist Equations:

$$\frac{\partial U}{\partial t} + \alpha\mu_d \frac{\partial p}{\partial x} + \frac{\alpha}{\alpha_d}\frac{\partial p}{\partial \eta}\frac{\partial \phi}{\partial x} = -\frac{\partial Uu}{\partial x} - \frac{\partial \Omega u}{\partial \eta}$$

$$\frac{\partial W}{\partial t} + g\left(\mu_d - \frac{\alpha}{\alpha_d}\frac{\partial p}{\partial \eta}\right) = -\frac{\partial Uw}{\partial x} - \frac{\partial \Omega w}{\partial \eta}$$

$$\frac{\partial \mu_d}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial \Omega}{\partial \eta} = 0$$

$$\frac{\partial \Theta}{\partial t} + \frac{\partial U\theta}{\partial x} + \frac{\partial \Omega\theta}{\partial \eta} = \mu Q$$

$$\frac{d\phi}{dt} = gw$$

$$\frac{\partial(\mu_d q_{v,l})}{\partial t} + \frac{\partial(U q_{v,l})}{\partial x} + \frac{\partial(\Omega q_{v,l})}{\partial \eta} = \mu Q_{v,l}$$

Diagnostic relations: $\frac{\partial \phi}{\partial \eta} = -\alpha_d \mu_d, \quad p = \left(\frac{R_d \Theta_m}{p_o \mu_d \alpha_d}\right)^{\gamma}, \quad \Theta_m = \Theta\left(1 + \frac{R_v}{R_d} q_v\right)$

# Dynamics: 2. Equations and variables – velocities

Vertical momentum eqn. $$\frac{\partial W}{\partial t} + g\left(\mu_d - \frac{\alpha}{\alpha_d}\frac{\partial p}{\partial \eta}\right) = -\frac{\partial U w}{\partial x} - \frac{\partial \Omega w}{\partial \eta}$$

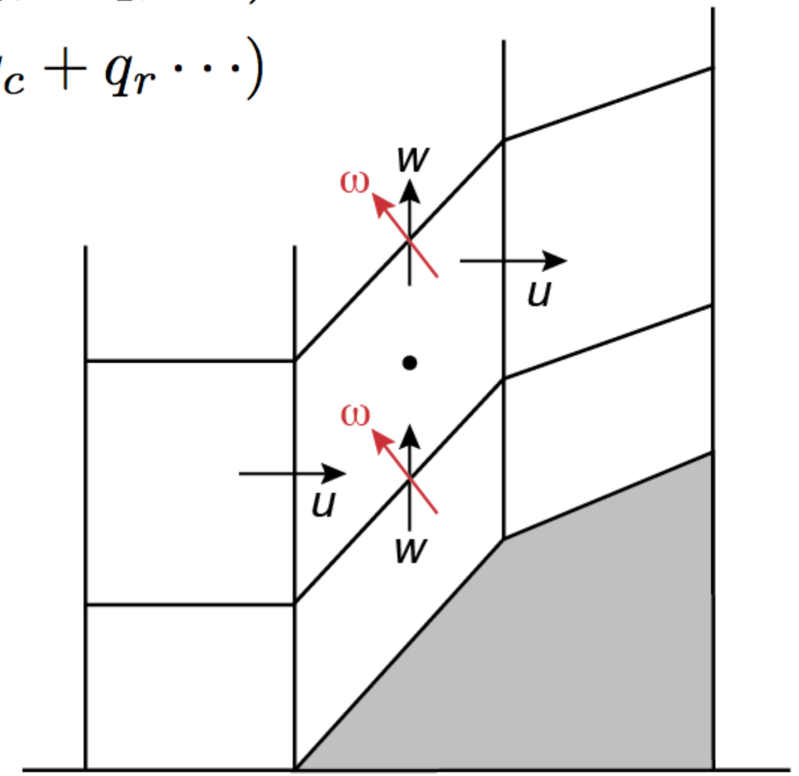
Subscript *d* denotes *dry*, and

$$\alpha_d = \frac{1}{\rho_d} \qquad \begin{aligned} \alpha &= \alpha_d\,(1 + q_v + q_c + q_r \cdots)^{-1} \\ \rho &= \rho_d\,(1 + q_v + q_c + q_r \cdots) \end{aligned}$$

covariant ($u$, $\omega$) and
contravariant $w$ velocities

$$u = \frac{dx}{dt}, \quad w = \frac{dz}{dt}, \quad \omega = \frac{d\eta}{dt}$$

$$U = \mu u, \quad W \mu w, \quad \Omega = \mu\omega$$



## 3rd Order Runge-Kutta time integration

advance $\phi^{t} \rightarrow \phi^{t+\Delta t}$

$$\phi^{*} = \phi^{t} + \frac{\Delta t}{3} R\left(\phi^{t}\right)$$

$$\phi^{**} = \phi^{t} + \frac{\Delta t}{2} R\left(\phi^{*}\right)$$

$$\phi^{t+\Delta t} = \phi^{t} + \Delta t\, R\left(\phi^{**}\right)$$

Amplification factor $\phi_t = i k \phi; \quad \phi^{n+1} = A \phi^{n}; \quad |A| = 1 - \frac{(k\Delta t)^4}{24}$



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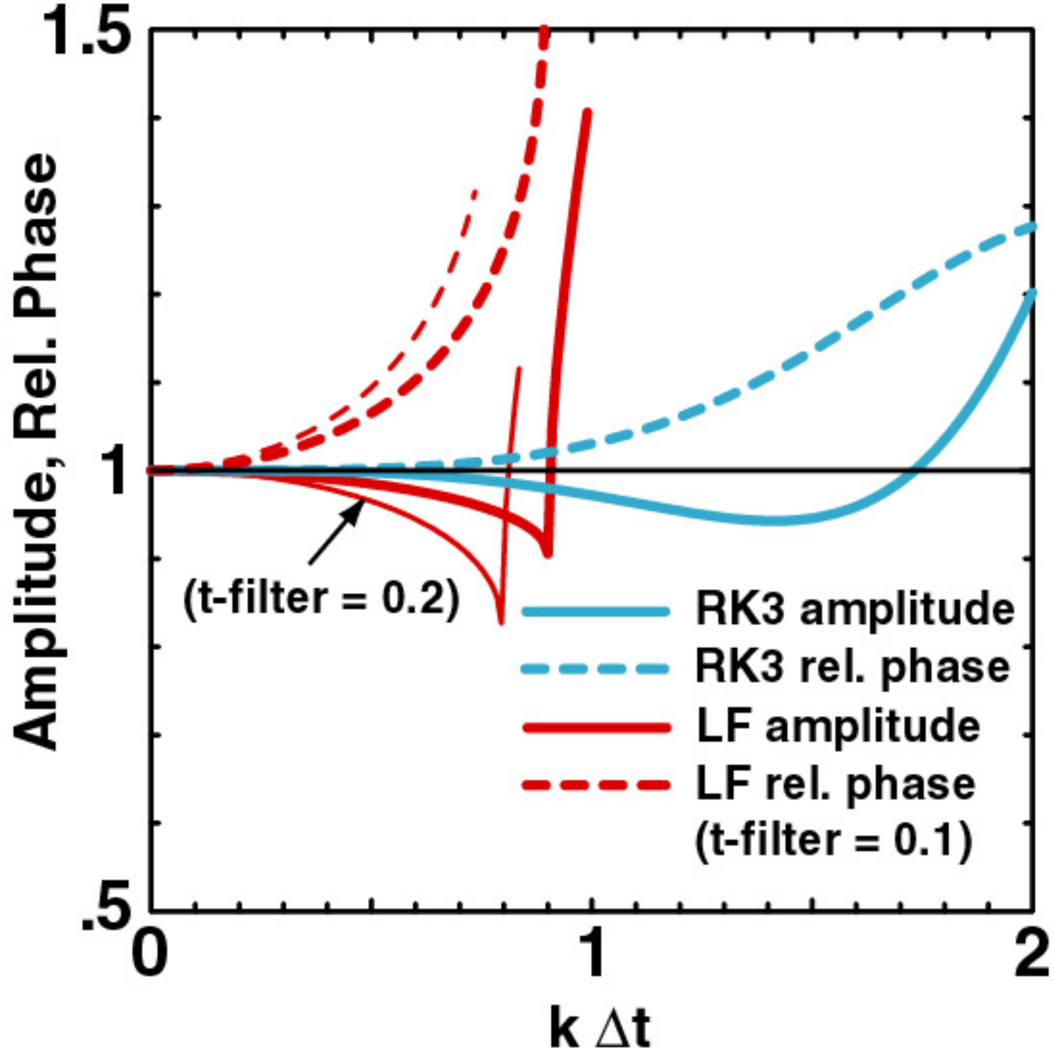
# Phase and amplitude errors for LF, RK3

Oscillation equation analysis

$$\phi_t = ik\phi$$





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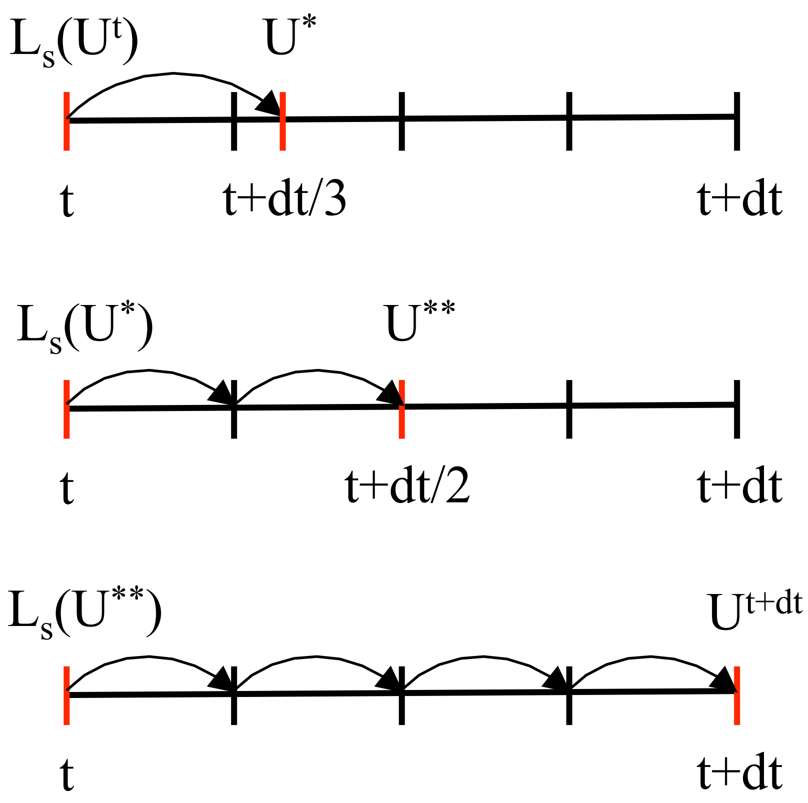
# Dynamics: 3. Time integration scheme – time splitting

$$U_t = L_{fast}(U) + L_{slow}(U)$$

3rd order Runge-Kutta, 3 steps




- RK3 is 3rd order accurate for linear eqns, 2nd order accurate for nonlinear eqns.
- Stable for centered and upwind advection schemes.
- Stable for Courant number $Udt/dx < 1.73$
- Three $L_{slow}(U)$ evaluations per timestep.

# Dynamics: 3. Time integration scheme - implementation

Begin time step

Runge-Kutta loop (steps 1, 2, and 3)

(i) advection, p-grad, buoyancy using ($\phi^t$, $\phi^*$, $\phi^{**}$)

(ii) physics if step 1, save for steps 2 and 3

(iii) mixing, other non-RK dynamics, save…

(iv) assemble dynamics tendencies

Acoustic step loop

(i) advance U,V, then μ, Θ, then w, $\phi$

(ii) time-average U,V, Ω

End acoustic loop

Advance scalars using time-averaged U,V, Ω

End Runge-Kutta loop

Adjustment physics (currently microphysics)

End time step

# Dynamics: 3. Time integration scheme – perturbation variables

Introduce the perturbation variables:

$$\phi = \overline{\phi}(\overline{z}) + \phi', \mu = \overline{\mu}(\overline{z}) + \mu';$$
$$p = \overline{p}(\overline{z}) + p', \alpha = \overline{\alpha}(\overline{z}) + \alpha'$$

Note – $\phi = \overline{\phi}(\overline{z}) = \overline{\phi}(x, y, \eta),$

likewise $\overline{p}(x, y, \eta), \overline{\alpha}(x, y, \eta)$

Reduces horizontal pressure-gradient errors.

For small time steps, recast variables as perturbations from time t

$$U' = U'^{t} + U'', \; V' = V'^{t} + V'', \; W' = W'^{t} + W'',$$
$$\Theta' = \Theta'^{t} + \Theta'', \; \mu' = \mu'^{t} + \mu'', \; \phi' = \phi'^{t} + \phi'';$$
$$p' = p'^{t} + p'', \; \alpha' = \alpha'^{t} + \alpha''$$

Allows vertical pressure gradient to be expressed in terms of $\phi$”.

# Dynamics: 3. Time integration scheme – acoustic step

(Without expanding variables into perturbation form)

$U^{\tau+\Delta\tau}$
$$\frac{\partial U}{\partial t} + \left(\mu_d \alpha \frac{\partial p}{\partial x} + \frac{\alpha}{\alpha_d}\frac{\partial p}{\partial \eta}\frac{\partial \phi}{\partial x}\right)^{\tau} = R_U^t$$

$\mu_d^{\tau+\Delta\tau}$ $\quad \Omega^{\tau+\Delta\tau}$
$$\frac{\partial \mu_d}{\partial t} + \frac{\partial U}{\partial x}^{\tau+\Delta\tau} + \frac{\partial \Omega}{\partial \eta}^{\tau+\Delta\tau} = 0$$

$\Theta^{\tau+\Delta\tau}$
$$\frac{\partial \Theta}{\partial t} + \left(\frac{\partial U \theta^t}{\partial x} + \frac{\partial \Omega \theta^t}{\partial \eta}\right)^{\tau+\Delta\tau} = R_\Theta^t$$

$W^{\tau+\Delta\tau}$, $\phi^{\tau+\Delta\tau}$
$$\begin{cases} \dfrac{\partial W}{\partial t} + g\overline{\left(\mu_d - \dfrac{\alpha}{\alpha_d}\dfrac{\partial p}{\partial \eta}\right)}^{\tau} = R_W^t \\ \mu_d^t \dfrac{\partial \phi}{\partial t} + U^{\tau+\Delta\tau}\dfrac{\partial \phi}{\partial x}^t + \Omega^{\tau+\Delta\tau}\dfrac{\partial \phi}{\partial \eta}^t - g\overline{W}^{\tau} = R_\phi^t \end{cases}$$

- Forward-backward differencing on $U$, $\Theta$, and $\mu$ equations
- Vertically implicit differencing on $W$ and $\phi$ equations

Instead of solving vertically implicit equations for $W$ and $\phi$

Integrate the hydrostatic equation to obtain $p$ ($\pi$):

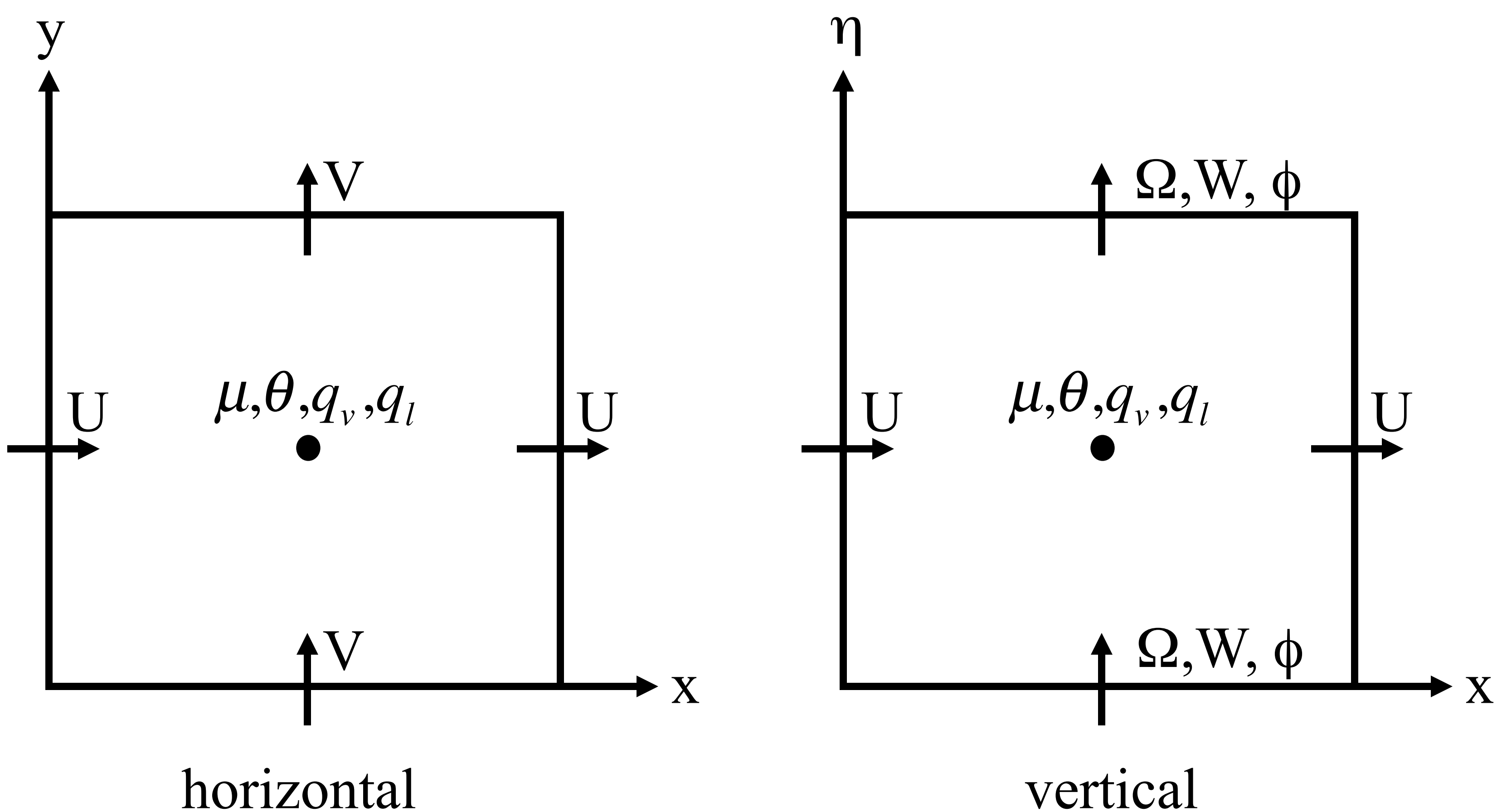
$$\frac{\partial p}{\partial \eta} = \left(\frac{\alpha_d}{\alpha}\right)^t \mu_d$$

Recover $\alpha$ and $\phi$ from: $p = \left(\frac{R_d \Theta_m}{p_o \mu_d \alpha_d}\right)^\gamma$, $\Theta_m = \Theta\left(1 + \frac{R_v}{R_d} q_v\right)$,

and $\frac{\partial \phi}{\partial \eta} = -\mu_d \alpha_d$

*W* is no longer required during the integration.

C-grid staggering



2nd, 3rd, 4th, 5th and 6th order centered and upwind-biased schemes are available in the ARW model.

Example: 5th order scheme

$$\frac{\partial(U\psi)}{\partial x} = \frac{1}{\Delta x}\left( F_{i+\frac{1}{2}}(U\psi) - F_{i-\frac{1}{2}}(U\psi) \right)$$

where

$$F_{i-\frac{1}{2}}(U\psi) = U_{i-\frac{1}{2}}\left\{ \frac{37}{60}(\psi_i + \psi_{i-1}) - \frac{2}{15}(\psi_{i+1} + \psi_{i-2}) + \frac{1}{60}(\psi_{i+2} + \psi_{i-3}) \right\}$$

$$- sign(1,U)\,\frac{1}{60}\left\{ (\psi_{i+2} - \psi_{i-3}) - 5(\psi_{i+1} - \psi_{i-2}) + 10(\psi_i - \psi_{i-1}) \right\}$$

For constant U, the 5th order flux divergence tendency becomes

$$\Delta t \left.\frac{\delta(U\psi)}{\Delta x}\right|_{5th} = \Delta t \left.\frac{\delta(U\psi)}{\Delta x}\right|_{6th} - \underbrace{\left|\frac{U\Delta t}{\Delta x}\right| \frac{1}{60}\left(-\psi_{i-3} + 6\psi_{i-2} - 15\psi_{i-1} + 20\psi_{i} - 15\psi_{i+1} + 6\psi_{i+2} - \psi_{i+3}\right)}_{\frac{Cr}{60}\frac{\partial^6\psi}{\partial x^6} + H.O.T}$$

The odd-ordered flux divergence schemes are equivalent to the next higher ordered (even) flux-divergence scheme plus a dissipation term of the higher even order with a coefficient proportional to the Courant number.

# Maximum Courant Number for Advection

$$C_a = U \Delta t / \Delta x$$

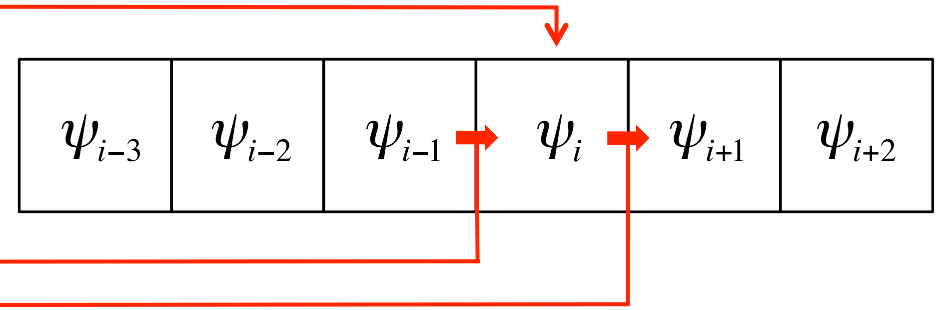
| *Time Integration Scheme* | *Advection Scheme* | | | | |
|---|---|---|---|---|---|
| | *2nd* | *3rd* | *4th* | *5th* | *6th* |
| Leapfrog (γ=0.1) | 0.91 | U | 0.66 | U | 0.57 |
| RK2 | U | 0.90 | U | 0.39 | U |
| RK3 | 1.73 | 1.63 | 1.26 | 1.43 | 1.09 |

U = unstable

(Wicker & Skamarock, 2002)

# *Weighted Essentially Non-Oscillatory scheme*

$$\psi_i^{t+\Delta t} - \psi_i^t = -\frac{\Delta t}{\Delta x}\left( flux_{i+1/2} - flux_{i-1/2} \right)$$




$for \;\; U_{i-1/2} \geq 0$

$$flux_{i-1/2} = U_{i-1/2}\left( \frac{w_0}{\overline{w}} f_0 + \frac{w_1}{\overline{w}} f_1 + \frac{w_2}{\overline{w}} f_2 \right)$$

$$where \;\; \overline{w} = w_0 + w_1 + w_2$$

$$f_0 = \frac{1}{3}\psi_{i-3} - \frac{7}{6}\psi_{i-2} + \frac{11}{6}\psi_{i-1} \qquad w_0 = \frac{0.1}{(\varepsilon + \beta_0)^2}$$

$$f_1 = -\frac{1}{6}\psi_{i-2} + \frac{5}{6}\psi_{i-1} + \frac{1}{3}\psi_i \qquad w_1 = \frac{0.6}{(\varepsilon + \beta_1)^2}$$

$$f_2 = \frac{1}{3}\psi_{i-1} + \frac{5}{6}\psi_i - \frac{1}{6}\psi_{i+1} \qquad w_2 = \frac{0.3}{(\varepsilon + \beta_2)^2}$$

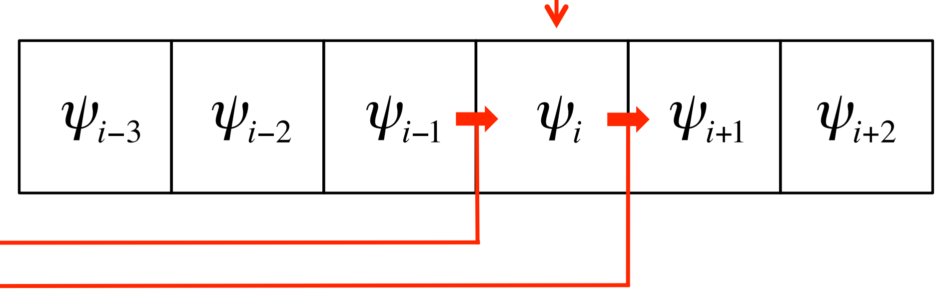
$$\beta_0 = \frac{13}{12}\left(\psi_{i-3} - 2\psi_{i-2} + \psi_{i-1}\right)^2 + \frac{1}{4}\left(\psi_{i-3} - 4\psi_{i-2} + 3\psi_{i-1}\right)^2$$

$$\beta_1 = \frac{13}{12}\left(\psi_{i-2} - 2\psi_{i-1} + \psi_i\right)^2 + \frac{1}{4}\left(\psi_{i-2} - 4\psi_{i-1} + 3\psi_i\right)^2$$

$$\beta_2 = \frac{13}{12}\left(\psi_{i-1} - 2\psi_i + \psi_{i+1}\right)^2 + \frac{1}{4}\left(\psi_{i-1} - 4\psi_i + 3\psi_{i+1}\right)^2$$

*Weighted Essentially Non-Oscillatory scheme*

$$\psi_i^{t+\Delta t} - \psi_i^t = -\frac{\Delta t}{\Delta x}\left(flux_{i+1/2} - flux_{i-1/2}\right)$$



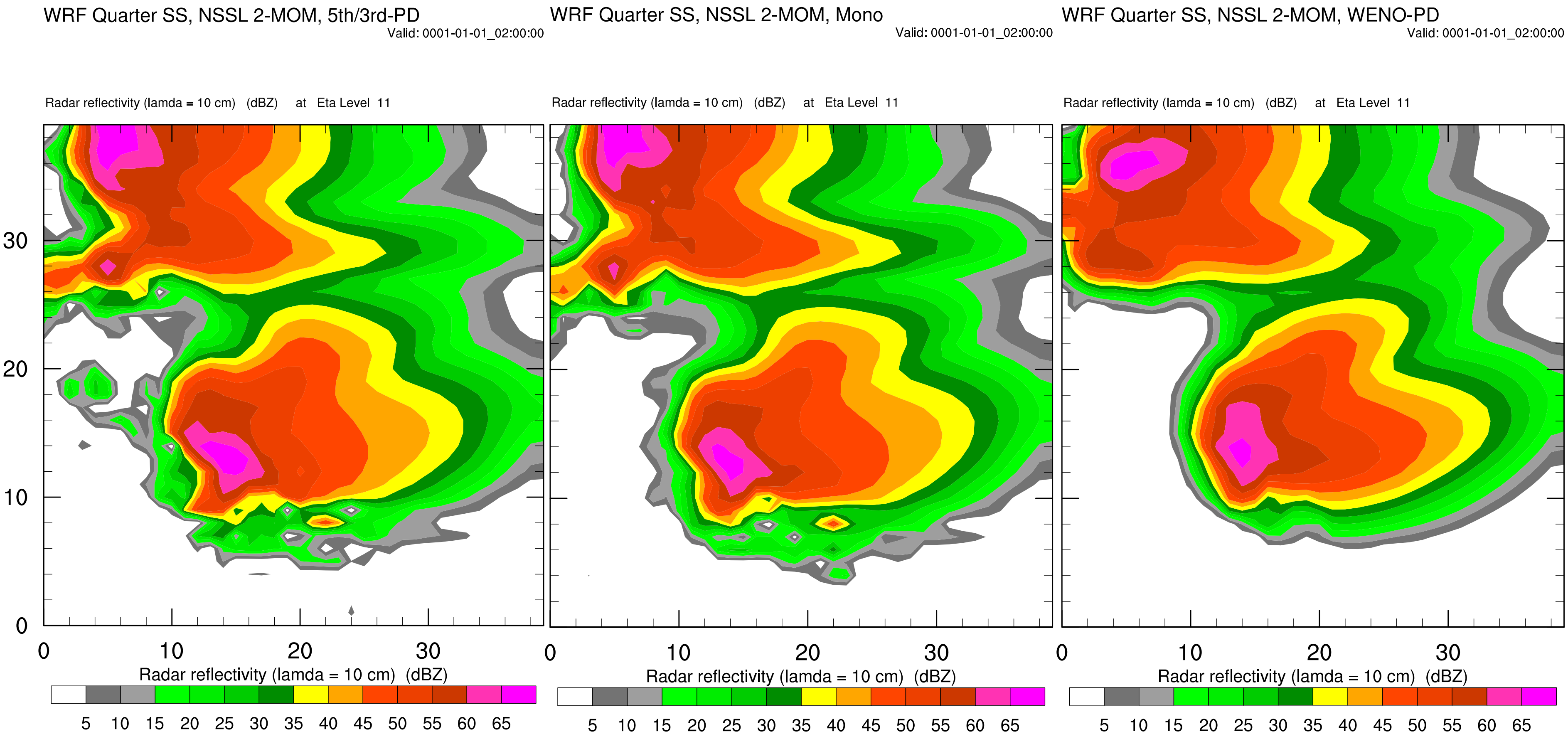

$$for \;\; U_{i-1/2} \geq 0$$

$$flux_{i-1/2} = U_{i-1/2}\left(\frac{w_0}{\overline{w}} f_0 + \frac{w_1}{\overline{w}} f_1 + \frac{w_2}{\overline{w}} f_2\right)$$

Positive-definite option uses same flux-renormalization as the standard ARW transport schemes.

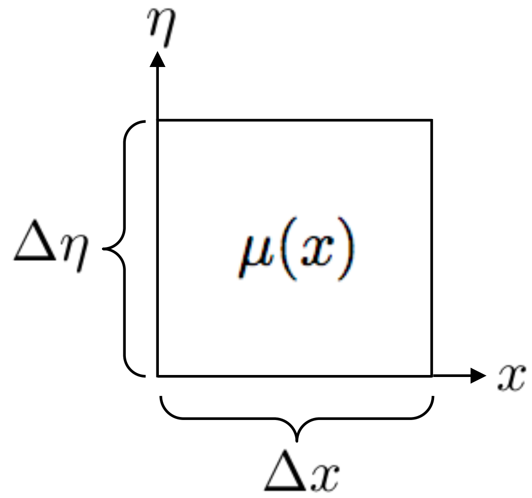
Can be used on momentum in addition to scalars.

# Dynamics: 5. Advection (transport) and conservation – WENO scheme




One should consider WENO for some double-moment microphysics scheme applications where noise is a problem.
Application to momentum may not provide much benefit.

Results courtesy of Ted Mansell (NOAA/NSSL) control volume (2D example)




Mass in a control volume is proportional to

$$(\Delta x \Delta\eta)(\mu)^t$$

since $\mu(x)\Delta\eta = \Delta\pi = -g\rho\Delta z$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# Dynamics: 5. Advection (transport) and conservation – dry-air mass

Mass in a control volume $(\Delta x \Delta \eta)(\mu)^t$

2D example

Mass conservation equation

$$\Delta t^{-1}(\Delta x \Delta \eta) \cdot \left[(\mu)^{t+\Delta t} - (\mu)^t\right] = \left[(\mu u \Delta \eta)_{x-\Delta x/2,\eta} - (\mu u \Delta \eta)_{x+\Delta x/2,\eta}\right] + \left[(\mu \omega \Delta x)_{x,\eta-\Delta\eta/2} - (\mu \omega \Delta x)_{x,\eta+\Delta\eta/2}\right]$$

Change in mass over a time step

mass fluxes through control volume faces



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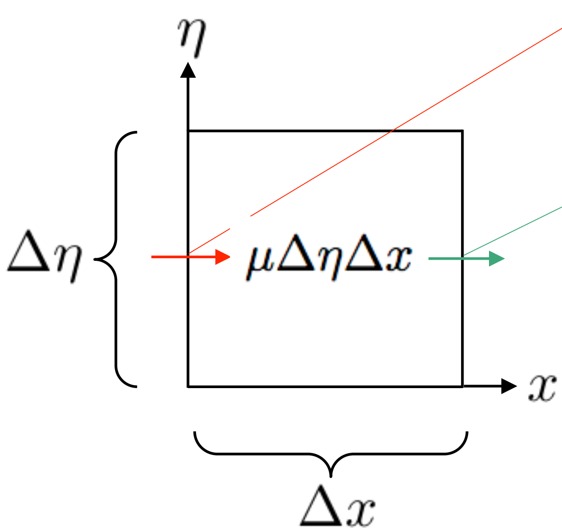
Mass in a control volume $(\Delta x \Delta \eta)(\mu)^t$

Mass conservation equation

$$\Delta t^{-1}(\Delta x \Delta \eta)\cdot[(\mu)^{t+\Delta t} - (\mu)^t)] = [(\mu u \Delta\eta)_{x-\Delta x/2,\eta} - (\mu u \Delta\eta)_{x+\Delta x/2,\eta})] + [(\mu\omega\Delta x)_{x,\eta-\Delta\eta/2} - (\mu\omega\Delta x)_{x,\eta+\Delta\eta/2})]$$



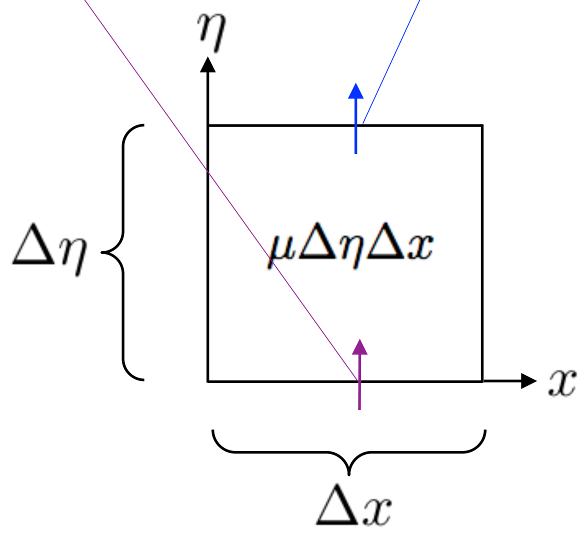

Horizontal fluxes through the vertical control-volume faces

Mass in a control volume $(\Delta x \Delta\eta)(\mu)^t$

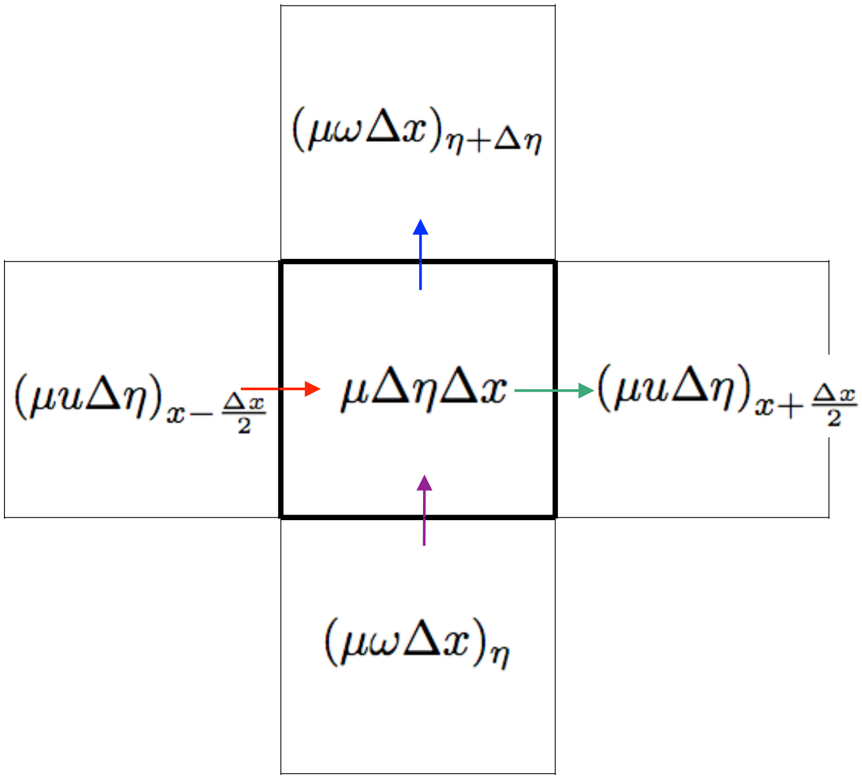
Mass conservation equation

$$\Delta t^{-1}(\Delta x \Delta\eta) \cdot \left[(\mu)^{t+\Delta t} - (\mu)^t)\right] = \left[(\mu u \Delta\eta)_{x-\Delta x/2,\eta} - (\mu u \Delta\eta)_{x+\Delta x/2,\eta})\right] + \left[(\mu\omega\Delta x)_{x,\eta-\Delta\eta/2} - (\mu\omega\Delta x)_{x,\eta+\Delta\eta/2})\right]$$

Vertical fluxes through the horizontal control-volume faces



The same mass fluxes are used for neighboring grid cells - hence mass is conserved locally and globally.



# Dynamics: 5. Advection (transport) and conservation – scalars

| | |
|---|---|
| Mass in a control volume | $(\Delta x \Delta \eta)(\mu)^t$ |
| Scalar mass | $(\Delta x \Delta \eta)(\mu \phi)^t$ |

Mass conservation equation:

$$\Delta t^{-1}(\Delta x \Delta \eta) \cdot \left[(\mu)^{t+\Delta t} - (\mu)^t)\right] = \left[(\mu u \Delta \eta)_{x-\Delta x/2,\eta} - (\mu u \Delta \eta)_{x+\Delta x/2,\eta})\right] + \left[(\mu \omega \Delta x)_{x,\eta-\Delta \eta/2} - (\mu \omega \Delta x)_{x,\eta+\Delta \eta/2})\right]$$

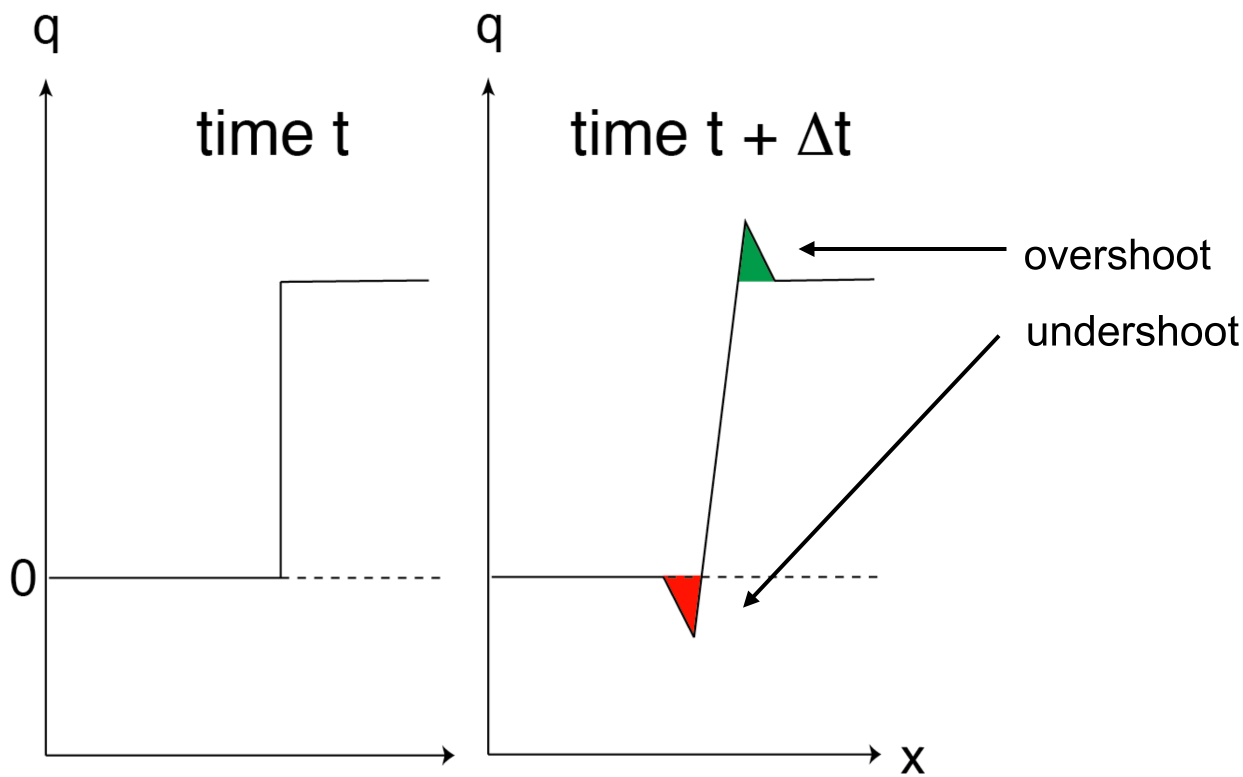
change in mass over a time step — mass fluxes through control volume faces

Scalar mass conservation equation:

$$\Delta t^{-1}(\Delta x \Delta \eta) \cdot \left[(\mu \phi)^{t+\Delta t} - (\mu \phi)^t)\right] = \left[(\mu u \phi \Delta \eta)_{x-\Delta x/2,\eta} - (\mu u \phi \Delta \eta)_{x+\Delta x/2,\eta})\right] + \left[(\mu \omega \phi \Delta x)_{x,\eta-\Delta \eta/2} - (\mu \omega \phi \Delta x)_{x,\eta+\Delta \eta/2})\right]$$

change in tracer mass over a time step — tracer mass fluxes through control volume faces

## 1D advection




ARW transport is conservative,
but not positive definite nor monotonic.
Removal of negative q ■
results in spurious source of q ■ .

# Dynamics: 5. Advection (transport) and conservation – shape preserving

Scalar update, last RK3 step

$$(\mu\phi)^{t+\Delta t} = (\mu\phi)^t - \Delta t \sum_{i=1}^{n} \delta_{x_i}[f_i] \qquad (1)$$

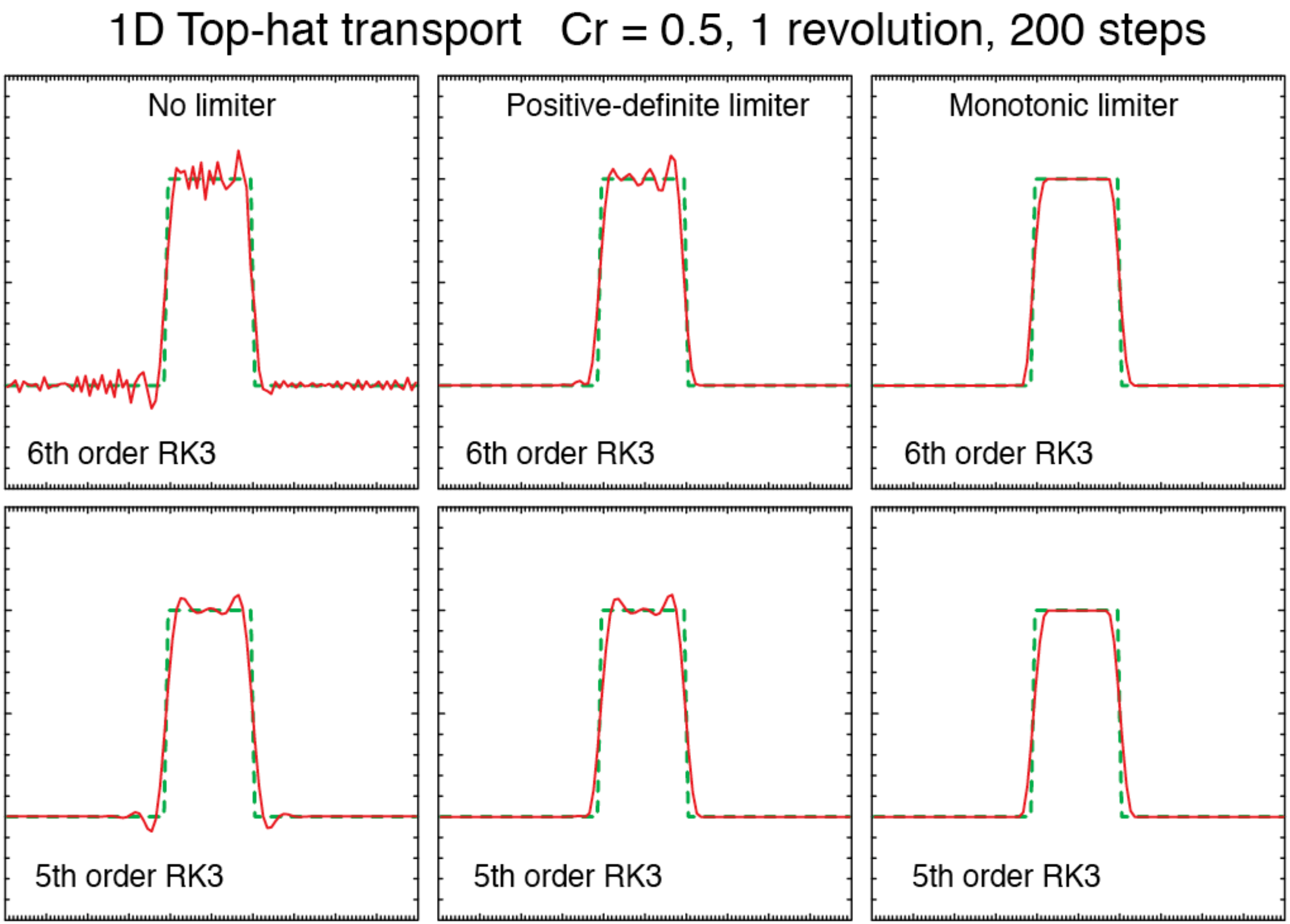
(1) Decompose flux: $f_i = f_i^{upwind} + f_i^c$

(2) Renormalize high-order correction fluxes $f_i^c$ such that solution is positive definite or monotonic: $f_i^c = R(f_i^c)$

(3) Update scalar eqn. (1) using $f_i = f_i^{upwind} + R(f_i^c)$

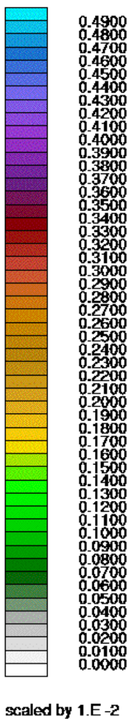
Skamarock, MWR 2006, 2241-2250

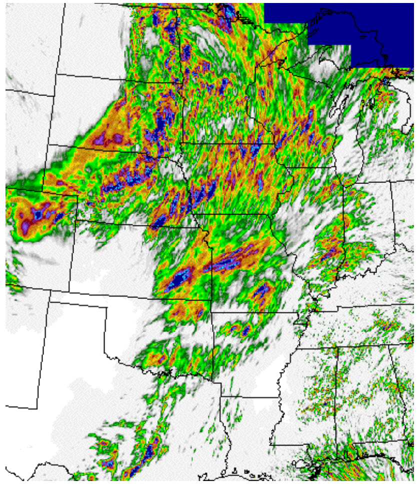
# 1D Example: Top-Hat Advection



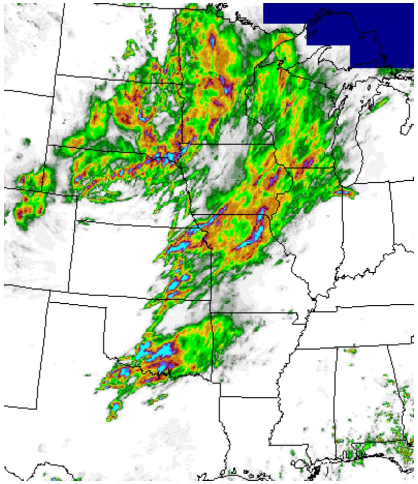
# Dynamics: 5. Advection (transport) and conservation – examples

Initialized 00 UTC 04 June 2005



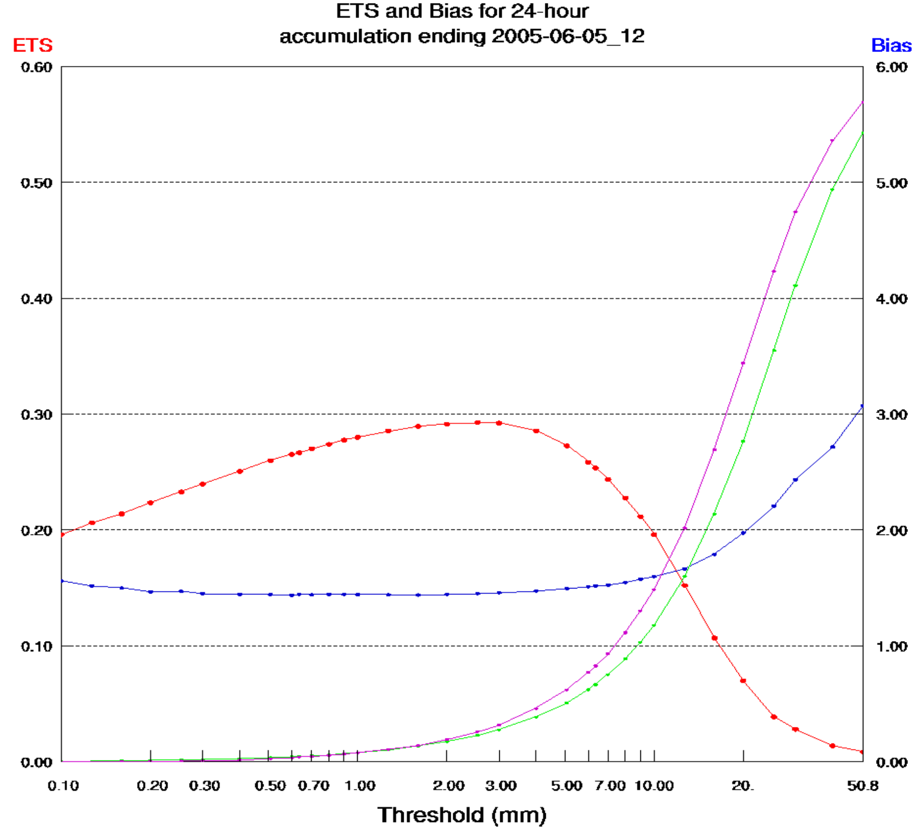



ARW 12-36 h Forecast Precip



24 h Observed (ST4) Precip

## 2005 ARW 4 km Forecasts (standard advection)





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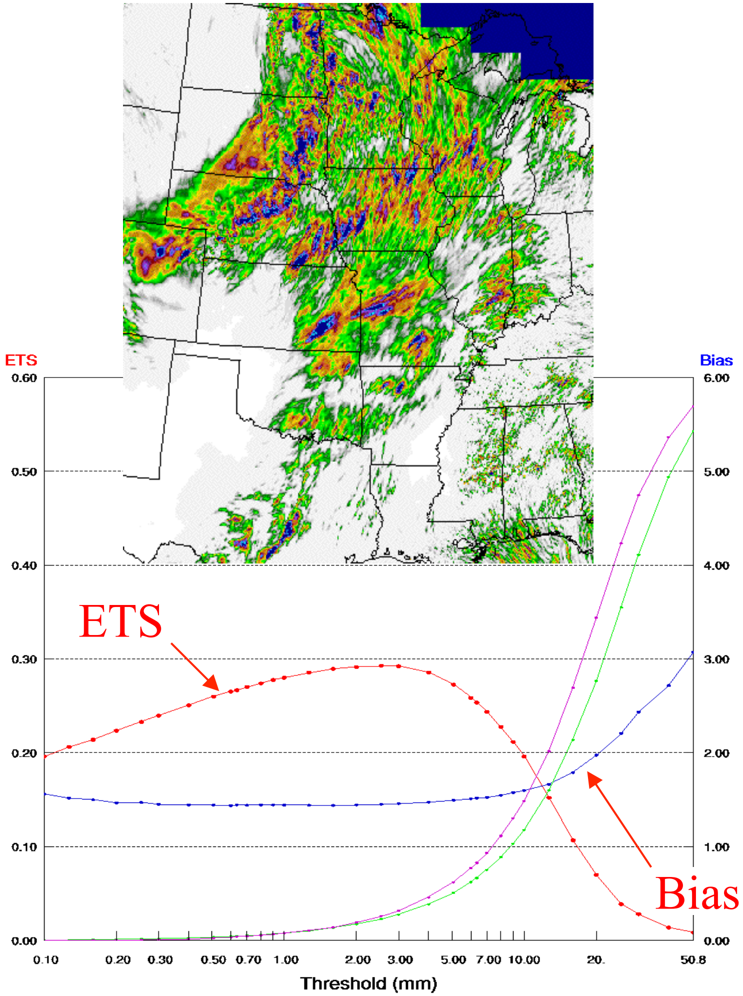
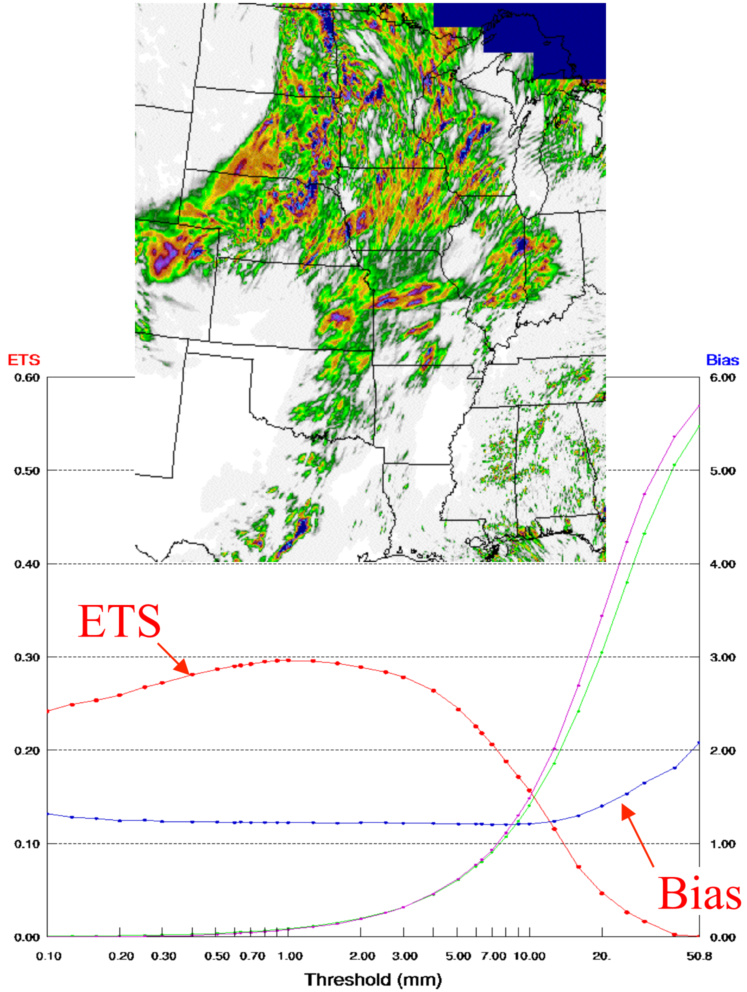
# Dynamics: 5. Advection (transport) and conservation – examples

Initialized 00 UTC 04 June 2005

Standard advection




Positive-definite advection



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# Dynamics: 5. Advection (transport) and conservation

*Where are the transport-scheme parameters?*

The namelist.input file:

&dynamics

*h_mom_adv_order*
*v_mom_adv_order*
*h_sca_adv_order*
*v_sca_adv_order*

scheme order (2, 3, 4, or 5)
defaults:
horizontal (h_*) = 5
vertical (v_*) = 3

*momentum_adv_opt* →

= 1 standard scheme
= 3 5$^{th}$ order WENO
default: 1

*moist_adv_opt*
*scalar_adv_opt*
*chem_adv_opt*
*tracer_adv_opt*
*tke_adv_opt*

options:
= 1, 2, 3 : no limiter,
positive definite (PD),
montonic
= 4 : 5$^{th}$ order WENO
= 5 : 5$^{th}$ order PD WENO

3rd order Runge-Kutta time step $\Delta t_{RK}$

Courant number limited, 1D: $C_r = \frac{U\Delta t}{\Delta x} < 1.43$

Generally stable using a timestep approximately twice as large as used in a leapfrog model.

*Where?*

The namelist.input file:

&domains

*time_step (integer seconds)*

*time_step_fract_num*

*time_step_fract_den*

3rd order Runge-Kutta time step $\Delta t_{RK}$ (&domains *time_step*)

Acoustic time step

2D horizontal Courant number limited: $C_r = \frac{C_s \Delta\tau}{\Delta h} < \frac{1}{\sqrt{2}}$

$$\Delta\tau_{sound} = \Delta t_{RK} / (\text{number of acoustic steps})$$

*Where?*

The namelist.input file:

&dynamics

*time_step_sound* (integer)

3rd order Runge-Kutta time step $\Delta t_{RK}$ (&domains *time_step*)

Acoustic time step [&dynamics *time_step_sound* (integer)]

Guidelines for time step

$\Delta t_{RK}$ in seconds should be about $6*\Delta x$ (grid size in kilometers). Larger $\Delta t$ can be used in smaller-scale dry situations, but *time_step_sound* (default = 4) should increase proportionately if larger $\Delta t$ is used.

If ARW blows up (aborts) quickly, try:

Decreasing $\Delta t_{RK}$ (that also decreases $\Delta t_{sound}$),

Or increasing time_step_sound (that decreases $\Delta t_{sound}$ but does not change $\Delta t_{RK}$ )

# Dynamics: 7. Filters – divergence damping

*Purpose: filter acoustic modes (3-D divergence,* $D = \nabla \cdot \rho \mathbf{V}$ *)*

$$\left\{ \frac{\partial \rho \mathbf{V}}{\partial t} + \nabla p + \ldots = \gamma_d' \nabla D \right\}$$

$$\nabla \cdot \left\{ \quad \right\} \quad \rightarrow \quad \frac{\partial D}{\partial t} + \nabla^2 p + \ldots = \gamma_d' \nabla^2 D$$

From the pressure equation: $p_t \simeq c^2 D$

$$\frac{\partial \rho \mathbf{V}}{\partial t} + \nabla [p_\tau + \gamma_d (p^\tau - p^{\tau - \Delta\tau})] + \ldots = 0$$

$\gamma_d = 0.1$ recommended (default) (&dynamics *smdiv*)

(Illustrated in height coordinates for simplicity)

# Dynamics: 7. Filters – time off-centering the vertical acoustic modes

*Purpose: damp vertically-propagating acoustic modes*

$$\frac{\partial W}{\partial t} + g\overline{\left(\mu_d - \frac{\alpha}{\alpha_d}\frac{\partial p}{\partial \eta}\right)}^{\tau} = \ldots$$

$$\frac{\partial \phi}{\partial t} - \frac{g}{\mu_d^t}\overline{W}^{\tau} = \ldots$$

$$\overline{(\quad)}^{\tau} = \frac{1+\beta}{2}\overline{(\quad)}^{\tau+\Delta\tau} + \frac{1-\beta}{2}\overline{(\quad)}^{\tau}$$

Slightly forward centering the vertical pressure gradient damps 3-D divergence as demonstrated for the divergence damper

$\beta = 0.1$ recommended (default) [&dynamics *epssm*]

*Purpose: filter the external mode*

Vertically integrated horizontal divergence, $D_h = \int_1^0 (\nabla_\eta \cdot \mu \mathbf{V}_h) d\eta$

$$\left\{ \frac{\partial \mu \mathbf{V}_h}{\partial t} + \ldots = -\gamma_e \nabla_\eta D_h \right\}$$

$$\int_1^0 \nabla_\eta \cdot \left\{ \quad \right\} d\eta \quad \rightarrow \quad \frac{\partial D_h}{\partial t} + \ldots = \gamma_e \nabla^2 D_h$$

Continuity equation: $\frac{\partial \mu}{\partial t} = -\nabla_\eta \cdot \mu \mathbf{V}_h - \frac{\partial \mu \dot{\eta}}{\partial \eta} = D_h$

$$\frac{\partial \mu \mathbf{V}_h}{\partial \tau} + \ldots = -\gamma_e \frac{\Delta x^2}{\Delta \tau^2} \nabla_\eta (\mu^\tau - \mu^{\tau - \Delta \tau})$$

$\gamma_e = 0.01$ recommended (default) [&dynamics *emdiv*]

(Primarily for real-data applications)

Purpose: damp anomalously-large vertical velocities

(usually associated with anomalous physics tendencies)

Additional term:

$$\partial_t W = \ldots - \mu_d \,\mathrm{sign}(W)\gamma_w (Cr - Cr_\beta)$$

$$Cr = \left| \frac{\Omega dt}{\mu d\eta} \right|$$

$Cr_\beta$ = 1.0 typical value (default)

[share/module_model_constants.F *w_beta*]

$\gamma_w$ = 0.3 m/s$^2$ recommended (default)

[share/module_model_constants.F *w_alpha*]

[&dynamics *w_damping* 0 (off; default) 1 (on)]

# 2nd-Order Horizontal Mixing, Horizontal-Deformation-Based $K_h$

Purpose: mixing on horizontal coordinate surfaces (real-data applications) [&dynamics *diff_opt=1, km_opt=4*]

$$K_h = C_s^2\, l^2 \left[ 0.25(D_{11} - D_{22})^2 + \overline{D_{12}^2}^{xy} \right]^{\frac{1}{2}}$$

where $l = (\Delta x \Delta y)^{1/2}$

$$D_{11} = 2\, m^2 \left[ \partial_x (m^{-1} u) - z_x \partial_z (m^{-1} u) \right]$$
$$D_{22} = 2\, m^2 \left[ \partial_y (m^{-1} v) - z_y \partial_z (m^{-1} v) \right]$$
$$D_{12} = m^2 \left[ \partial_y (m^{-1} u) - z_y \partial_z (m^{-1} u) + \partial_x (m^{-1} v) - z_x \partial_z (m^{-1} v) \right]$$

$C_s$ = 0.25 (Smagorinsky coefficient, default value) [&dynamics *c_s*]

## Implicit Rayleigh *w* Damping Layer for Split-Explicit Nonhydrostatic NWP Models (gravity-wave absorbing layer)

*Modification to small time step:*

- Step horizontal momentum, continuity, and potential temperature equations to new time level: $U^{\tau+\Delta\tau}$ $\mu^{\tau+\Delta\tau}$ $\Omega^{\tau+\Delta\tau}$ $\Theta^{\tau+\Delta\tau}$
- Step vertical momentum and geopotential equations (implicit in the vertical): $W^{*\tau+\Delta\tau}$ $\phi^{*\tau+\Delta\tau}$
- Apply implicit Rayleigh damping on *W* as an adjustment step: $$W^{\tau+\Delta\tau} = W^{*\tau+\Delta\tau} - \Delta\tau R_w(\eta) W^{\tau+\Delta\tau}$$
- Update final value of geopotential at new time level: $\phi^{\tau+\Delta\tau}$

## Implicit Rayleigh w Damping Layer for Split-Explicit Nonhydrostatic NWP Models (gravity-wave absorbing layer)

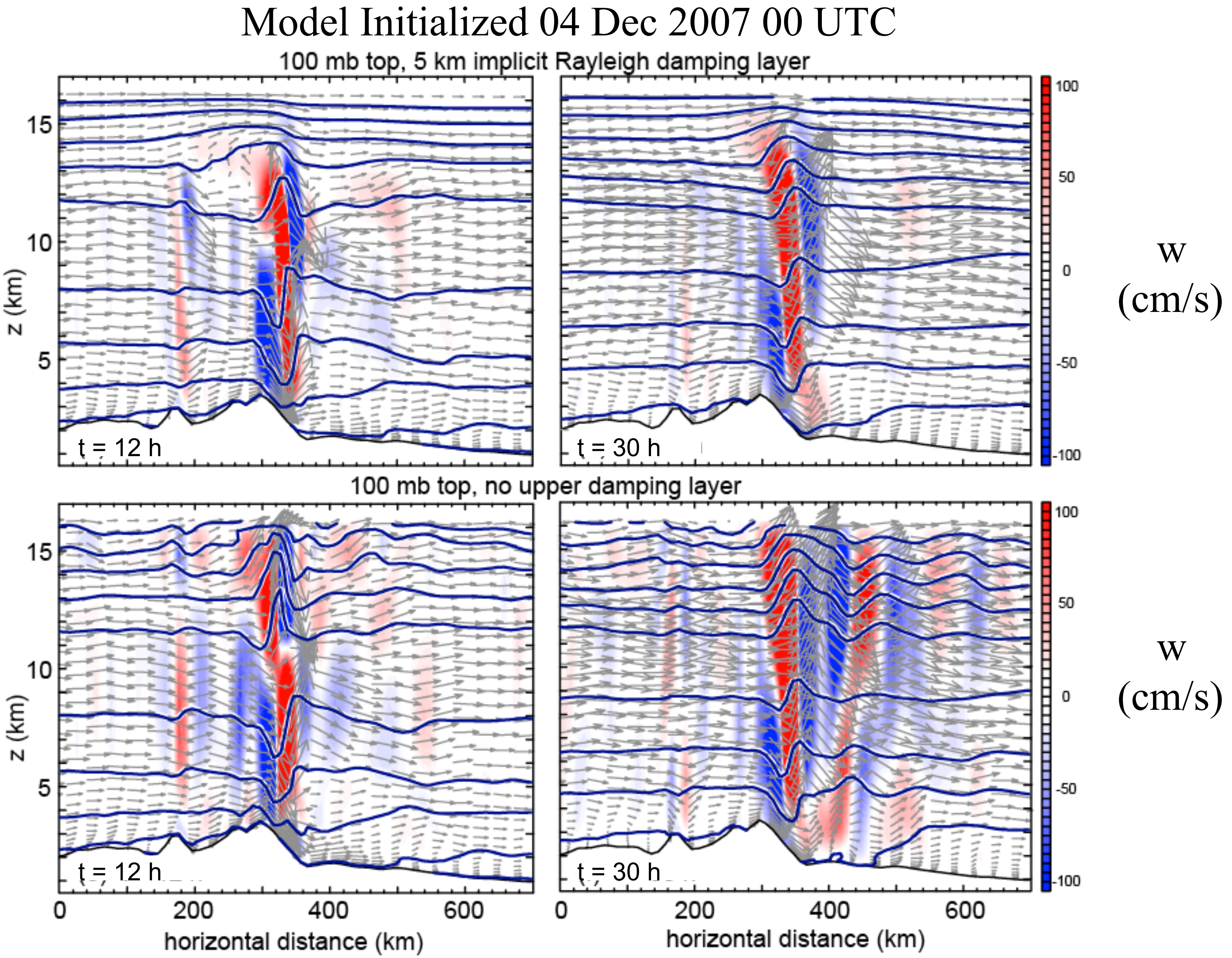
$$W^{\tau+\Delta\tau} = W^{*\tau+\Delta\tau} - \Delta\tau R_w(\eta) W^{\tau+\Delta\tau}$$

$$R_w(\eta) = \begin{cases} \gamma_r \sin^2\left[\frac{\pi}{2}\left(1 - \frac{z_{top}-z}{z_d}\right)\right] & \text{for } z \geq (z_{top} - z_d); \\ 0 & \text{otherwise,} \end{cases}$$

$R_w(\eta)$ - damping rate ($t^{-1}$)
$z_d$ - depth of the damping layer
$\gamma_r$ - damping coefficient

[&dynamics *damp_opt = 3* (default = 0)]
[&dynamics *damp_coef = 0.2* (recommended, = 0. default)]
[&dynamics *zdamp = 5000.* ($z_d$ *(meters)*; default); height below model top where damping begins]

# Dynamics: 7. Filters – gravity-wave absorbing layer example



# ARW Model: projection options

1. Cartesian geometry:
   idealized cases
2. Lambert Conformal:
   mid-latitude applications
3. Polar Stereographic:
   high-latitude applications
4. Mercator:
   low-latitude applications
5. Latitude-Longitude global, regional

Projections 1-4 are isotropic ($m_x = m_y$)
Latitude-longitude projection is anistropic ($m_x \neq m_y$)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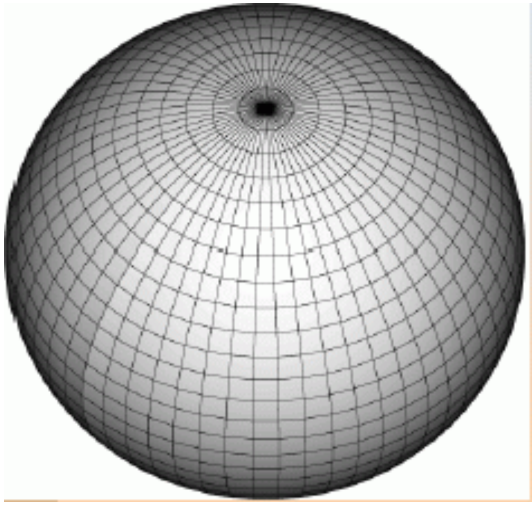
# Global ARW – Polar filters

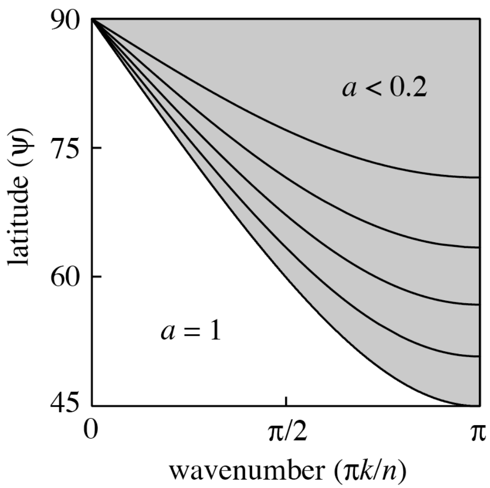


Converging gridlines severely limit timestep.
The polar filter removes this limitation.

Filter procedure - Along a grid latitude circle:

1. Fourier transform variable.
2. Filter Fourier coefficients.
3. Transform back to physical space.

Filter Coefficient $a(k)$, $\psi_o = 45^o$




$$\hat{\phi}(k)_{filtered} = a(k)\,\hat{\phi}(k), \quad \text{for all } k$$

$$a(k) = \min\left[1., \max\left(0., \left(\frac{\cos\psi}{\cos\psi_o}\right)^2 \frac{1}{\sin^2(\pi k/n)}\right)\right]$$

$k =$ dimensionless wavenumber
$\hat{\phi}(k) =$ Fourier coefficients from forward transform
$a(k) =$ filter coefficients
$\psi =$ latitude $\psi_o =$ polar filter latitude, filter when $|\psi| > \psi_o$

# ARW integration with polar filtering

Begin time step

- Runge-Kutta loop (steps 1, 2, and 3)
  - (i) advection, p-grad, buoyancy, physics
  - Acoustic step loop
    - (i) advance U,V (Fourier Filter U,V)
    - (ii) advance μ, Θ (Fourier Filter μ, Θ)
    - (iii) advance w, ϕ (Fourier Filter w, ϕ)
  - End acoustic loop
  - Advance scalars (Fourier Filter Sc)
- End Runge-Kutta loop
- Adjustment physics (currently microphysics) (Fourier Filter Sc)

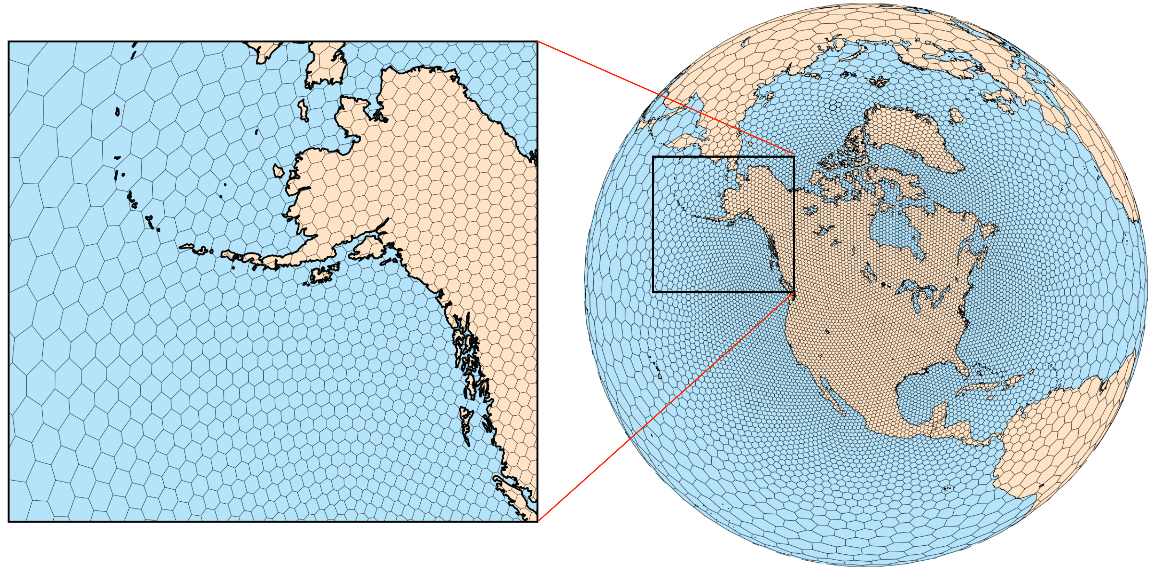
End time step

Timestep limited by minimum Δx outside of polar-filter region.
Monotonic and PD transport is not available for global model.

# Dynamics: 8. Map projections and global configuration

## An alternative to global ARW…

MPAS
Model for Prediction Across Scales



- Global, nonhydrostatic, C-grid Voronoi mesh
- Numerics similar to WRF; WRF-NRCM physics
- No pole problems
- Variable-resolution mesh – no nested BC problems

*Available at: http://mpas-dev.github.io/*

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# ARW Model: Boundary Condition Options

## Lateral boundary conditions

1. Specified (Coarse grid, real-data applications).
2. Open lateral boundaries (gravity-wave radiative).
3. Symmetric lateral boundary condition (free-slip wall).
4. Periodic lateral boundary conditions.
5. Nested boundary conditions (specified).

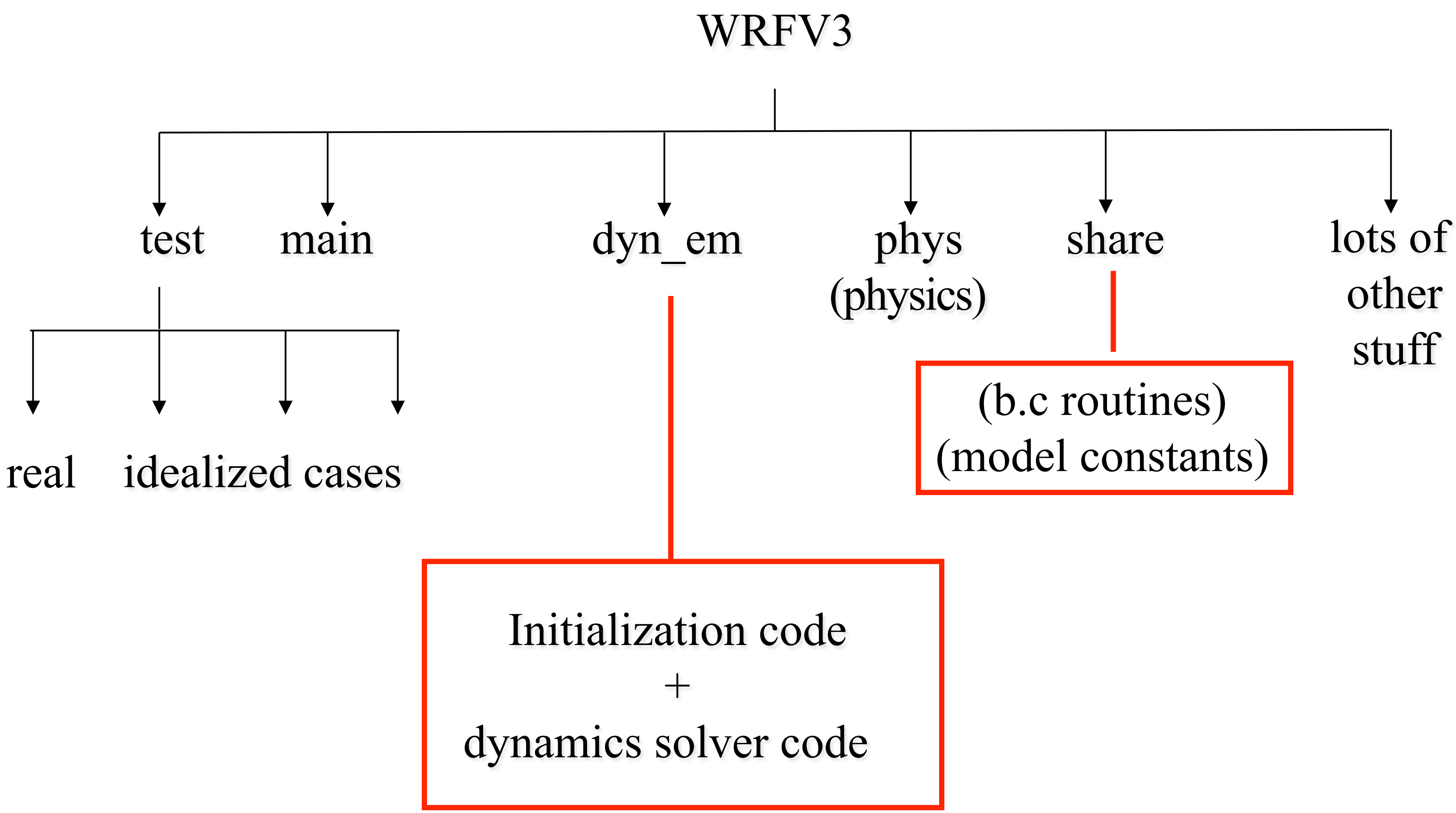
## Top boundary conditions

1. Constant pressure.

## Bottom boundary conditions

1. Free slip.
2. Various B.L. implementations of surface drag, fluxes.

# Dynamics: Where are things?




**WRF ARW Tech Note**

A Description of the Advanced Research WRF Version 3 (June 2008, 2012 update)

http://www.mmm.ucar.edu/wrf/users/pub-doc.html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