# Fundamentals in Atmospheric Modeling

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# List of presentations

- Concept of modeling
- Structure of models
- Predictability

# How the today's forecasts were made?



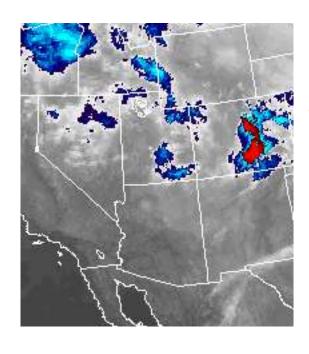




### Forecasts.

### **Boulder CO**

7 Day Forecast





OVERNIGHT



Partly Cloudy Low: 29 °F

TUESDAY



Mostly Sunny High: 50 °F

TUESDAY NIGHT

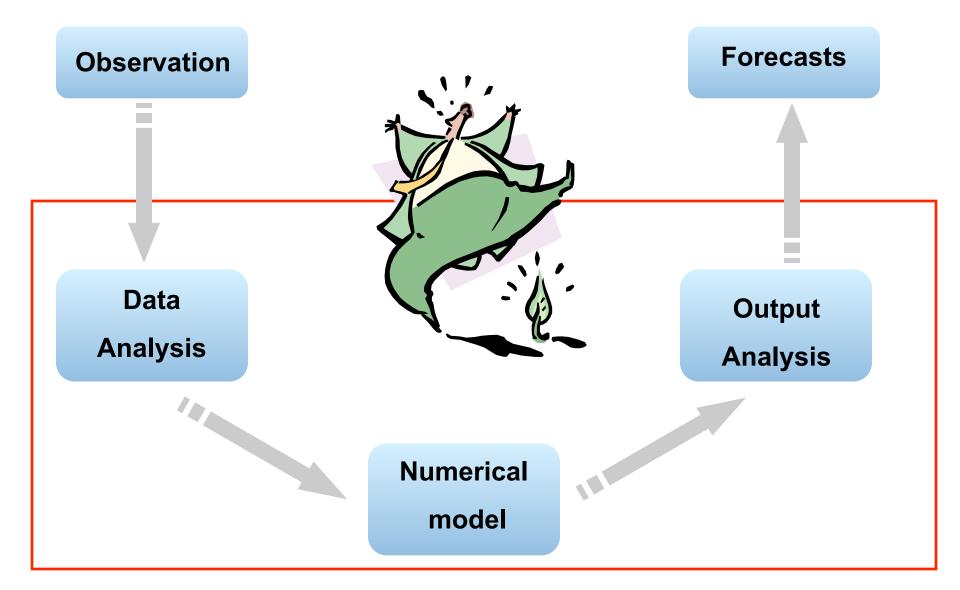


Chance Snow Low: 19 °F

Then, what?

# Weather forecasts ....









Step1: Observation



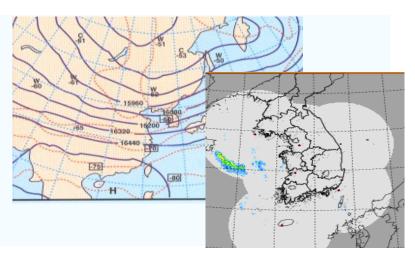








Step2: Data analysis



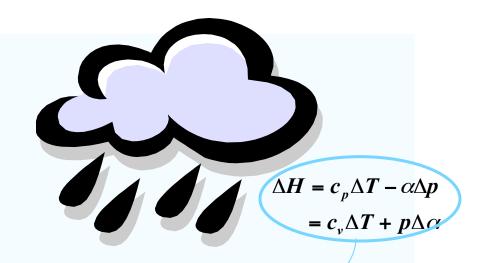


### Theory?



### Thermodynamics

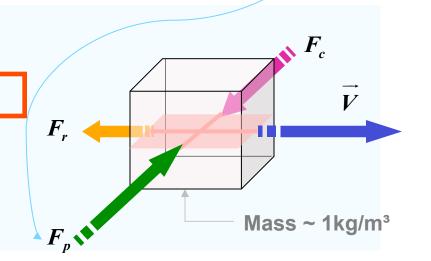
Heat = Energy + Work

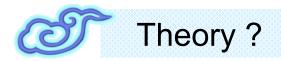


### **Dynamics**

Force =  $Mass \times Acceleration$ 

- Mass ≒ 1 kg/m³
- Force: **PGF**, **CO**, **Friction**...







Momentum

$$F = ma$$

Mass

$$\frac{1}{M}\frac{dM}{dt} = 0$$

# CONSERVATION

Moisture

$$\frac{dq}{dt} = E - C$$

Ideal gas

$$p\alpha = RT$$

Energy

$$Q = C_{v} \frac{dT}{dt} + p \frac{d\alpha}{dt}$$

### The governing equations

V. Bjerknes (1904) pointed out for the first time that there is a complete set of7 equations with 7 unknowns that governs the evolution of the atmosphere:

$$\frac{d\mathbf{v}}{dt} = -\alpha \nabla p - \nabla \phi + \mathbf{F} - 2\Omega \times \mathbf{v} \quad (1-3)$$

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{v}) \quad (4)$$

$$p = \rho RT \quad (5)$$

$$\frac{ds}{dt} = C_p \frac{1}{\theta} \frac{d\theta}{dt} = \frac{Q}{T} \quad (6)$$

$$\frac{dq}{dt} = E - C \quad (7)$$

7 equations, 7 unknown (u,v,w,T, p, den and q)

solvable

### History of numerical weather forecasts

```
1904: Norwegian V. <u>Bierknes</u> (1862-1951):
    Setup the governing equations
1922: British L. F. Richardson (1881-1953):
    Integrate model → failed
1939: Swedish C.-G. Rossby:
1948, 1949, J. G. Charney (1917-1981)
```

### 1950: Princeton Group (Charney, Fjortoft, von Newman) FNIAC (Electrical Numerical Integrator and Computer) → first success

### Computer Age (1946~)

- von Neumann and Charney
  - Applied ENIAC to weather prediction



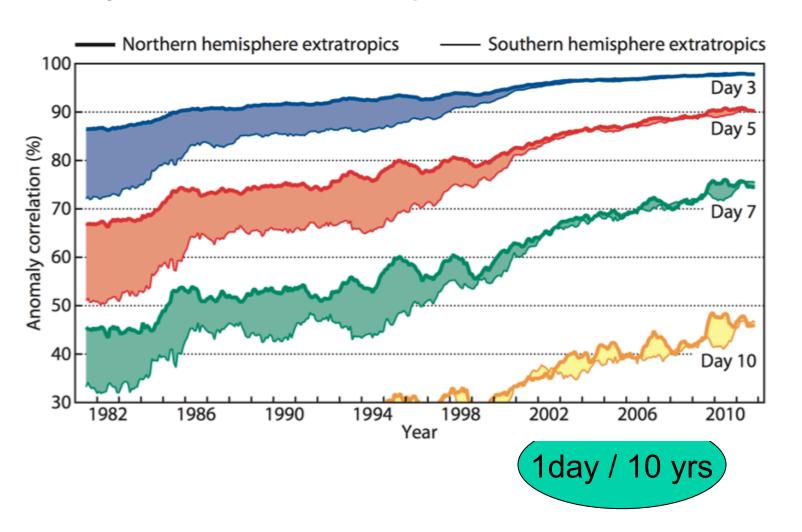


- Carl-Gustaf Rossby
  - The Swedish Institute of Meteorology
  - First routine real-time numerical weather forecasting. (1954) (US in 1958, Japan in 1959)



# **ECMWF MR-Forecast**

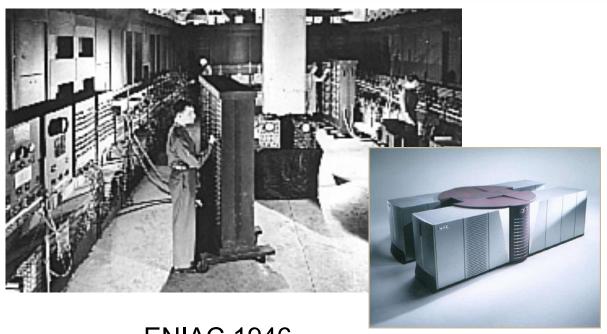
### Anomaly correlation of 500 hPa Geopotential



# Factors for the improvement (Kalnay 2002)

- Supercomputers
- Physical processes
- Initial conditions

### **Super-computer for weather models**





Cray T3E

**ENIAC,1946** 

**NEC SX-5** 



Cray SV1



Fujitsu VPP700E

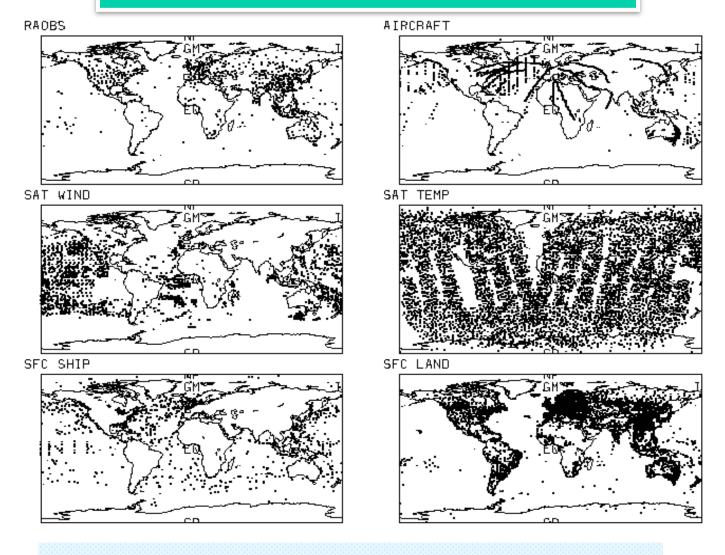


Cray T90



# Initial condition (data assimilation)

# Various observations



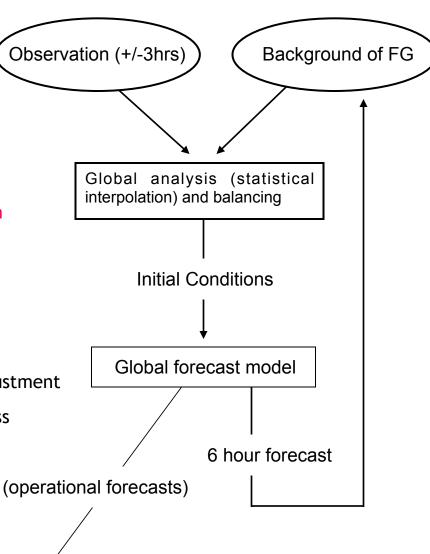
heterogeneous in space and time....

### **Data Assimilation**

• Model 1°X 1° resolution, 20 levels

$$360 \times 180 \times 20 = 1.3 \times 10^6 \times 4 \text{ var } iabl = 5 \times 10^6$$

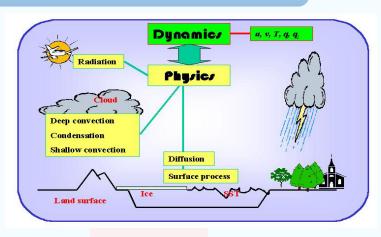
- observation :  $10^4 \sim 10^5$  non-uniform distribution  $\pm 3 \ hour \ window$
- Data assimilation cycle 1) data checking
  - 2) objective analysis
  - 3) Initialization: dynamical adjustment
  - 4) short-range fcst for first guess



## Model

- Dynamics: Identity (Speed)
- Physics : Components (Predictability)

### Step3: Integration



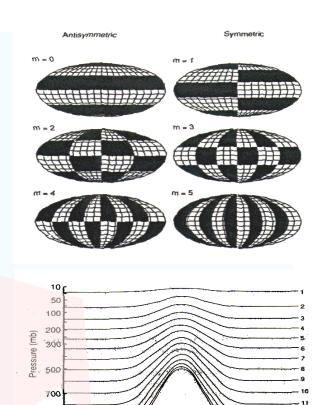
$$\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - \omega \frac{\partial u}{\partial p} - \frac{\partial \Phi}{\partial x} + fv + F_{x}$$

$$\frac{\partial v}{\partial t} = -u \frac{\partial v}{\partial x} - v \frac{\partial v}{\partial y} - \omega \frac{\partial v}{\partial p} - \frac{\partial \Phi}{\partial y} - fu + F_{y}$$

$$\frac{\partial \Phi}{\partial t} = -\frac{RT}{p}$$

$$\frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} + \omega \left(\frac{\kappa T}{p} - \frac{\partial T}{\partial p}\right) + \frac{\dot{H}}{c_{p}}$$

$$\frac{\partial \omega}{\partial p} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$



15-level sigma-coordinate model

850 1000

### Dynamics: model frame

Finite difference method (FDM):

Spectral method (SPM):

Finite element method (FEM):

Ex) 
$$\frac{\partial \phi}{\partial t} = -c \frac{\partial \phi}{\partial x}$$
; advection eq.

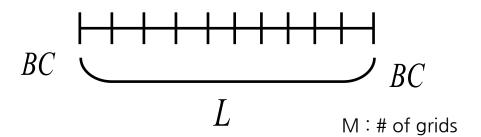
1) FDM (Finite difference)

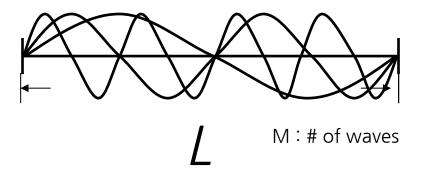
$$\frac{\Delta \phi}{\Delta t} = \frac{\phi_2 - \phi_1}{t_2 - t_1}$$

- 2) Spectral Method
- Determine basis function to get  $H(\phi(x))$
- Expand  $\phi$  in terms of a time series

$$e_m(x)$$
 (basis funct),  $m = m_1 L m_n \rightarrow \text{infinite}$ 

$$\Rightarrow \phi(x,t) = \sum_{m=m_1}^{m_2} \phi_m(t) e_m(x)$$



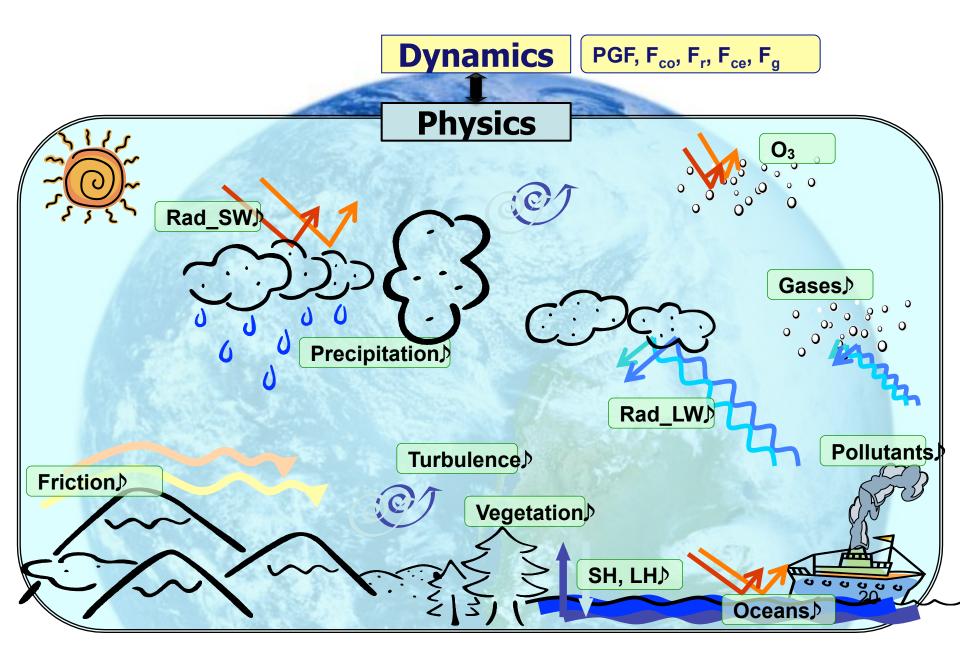


\* Resolution Increases 
$$\begin{pmatrix} \Delta x \to decreases \\ m \to increases \end{pmatrix}$$
 18

- Integration scheme ...
  a)  $\frac{u^{n+1}-u^{n-1}}{2\Lambda t} = F(u^n)$  : leap-frog good for hyperbolic
  - b)  $\frac{u^{n+1} u^n}{\Lambda_{+}} = F(u^n)$ : Euler-forward good for diffusion unstable for hyperbolic
  - c)  $\frac{u^{n+1}-u^n}{\Delta t} = F\left(\frac{u^n+u^{n+1}}{2}\right)$  : Crank-Nicholson
  - d)  $\frac{u^{n+1}-u^n}{1-u} = F(u^{n+1})$  : Fully implicit, backward
  - e)  $\frac{u^* u^n}{\Delta t} = F(u^n)$  :  $\frac{u^{n+1} u^n}{\Delta t} = F(u^*)$  : Euler-backward (Matzuno)
  - f)  $\frac{u^{n+\frac{1}{2}*}-u^n}{\Delta t/2} = F(u^n) : \frac{u^{n+\frac{1}{2}**}-u^n}{\Delta t/2} = F(u^{n+\frac{1}{2}*})$

$$\frac{u^{n+1} - u^n}{\Delta t} = \frac{1}{6} \left[ F(u^n) + 2F(u^{n+\frac{1}{2}*}) + 2F(u^{n+\frac{1}{2}**}) + F(u^{n+1}*) \right] : \text{RK(Runge-Kuta)-4th order}$$

- g)  $\frac{u^{n+1} u^{n-1}}{2\Delta t} = F_1(u^n) + F_2(\frac{u^{n+1} u^{n-1}}{2})$  : Semi-Implicit
- h)  $\frac{u^* u^n}{\Delta_4} = F_1(u^n);$   $\frac{u^{n+1} u^*}{\Delta_4} = F_2(u^*)$ : Fractional steps



# Example : Cloud and precipitation



Real atmosphere

Theory

Model (computer program)

### Formulation

```
💢 login1
  ODULE module_mp_wsm3
       💢 login1
           -----
            compute internal functions
              cpmcal(x) = cpd*(1.-max(x,qmin))+max(x,qmin)*cpv
    REREERE REERE
                      compute the minor time steps.
                      loops = max(nint(delt/dtcldcr),1)
                      dtcld = delt/loops
                      if(delt,le,dtcldcr) dtcld = delt
                      do loop = 1,loops
                      initialize the large scale variables
                      do i = its, ite
                        mstep(i) = 1
                        flgcld(i) = .true.
                      do k = kts. kte
                        CALL vsrec(tvec1(its),den(its,k),ite -its+1)
                        do i = its, ite
                          tvec1(i) = tvec1(i)*den0
                        CALL vssqrt(denfac(its,k),tvec1(its),ite -its+1)
                      cvap = cpv
                      hvap=x1v0
                      hsub=xls
                      dldt=cvap-cliq
                      xa=-dldt/rv
                      xb=xa+hvap/(rv*ttp)
                      dldti=cvap-cice
                      xai=-dldti/rv
                      xbi=xai+hsub/(rv*ttp)
                       do k = kts, kte
                          o i = its, ite
                          tr=ttp/t(i.k)
                          if(t(i,k).lt.ttp) then
                            qs(i,k) = psat*(exp(log(tr)*(xai)))*exp(xbi*(1,-tr))
                            qs(i,k) = psat*(exp(log(tr)*(xa)))*exp(xb*(1.-tr))
                          qsO(i,k) = psat*(exp(log(tr)*(xa)))*exp(xb*(1.-tr))
                          qs0(i,k) = (qs0(i,k)-qs(i,k))/qs(i,k)
                          qs(i,k) = ep2 * qs(i,k) / (p(i,k) - qs(i,k))
                          qs(i,k) = max(qs(i,k),qmin)
rh(i,k) = max(q(i,k) / qs(i,k),qmin)
```

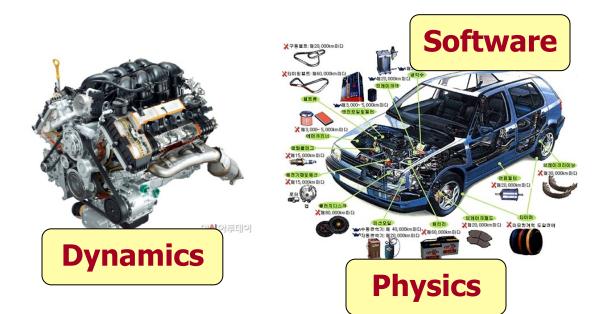


### Cloud and precipitation



```
💢 login 1
         do k = kts, kte
                                                                                        T>0°C
            do i = its, ite
               supsat = max(q(i,k),qmin)-qs(i,k)
               satdt = supsat/dtcld
               if(t(i,k).ge.t0c) then
   warm rain processes
                                                                                                           Paut1 = min \left| \frac{0.104gE_C \rho^{\frac{1}{3}}}{\mu(N_c \rho_w)^{\frac{1}{3}}} q_c^{\frac{7}{3}}, \frac{q_c}{dt} \right|
   - follows the processes in RH83 and LFO except for autoconcersion
    paut1: auto conversion rate from cloud to rain [HDC 16]
                 if(qci(i,k).gt.qc0) then
                    paut(i,k) = qck1*exp(log(qci(i,k))*((7./3.)))
                    paut(i,k) = min(paut(i,k),qci(i,k)/dtcld)
    pracw: accretion of cloud water by rain [D89 B15]
                                                                                                              P_{racw} = \frac{\pi a_r E_{CR} N_{0r} q_c}{4} \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{2}} \frac{\Gamma(3+b_r)}{\lambda^{3+b_r}}
                 if(qrs(i,k).gt.qcrmin.and.qci(i,k).gt.qmin) then
                       pacr(i,k) = min(pacrr*rslope3(i,k)*rslopeb(i,k)
                              *qci(i,k)*denfac(i,k),qci(i,k)/dtcld)
    pres1: evaporation/condensation rate of rain [HDC 14]
             (V-\dot{>}R \text{ or } R-\dot{>}V)
                 if(qrs(i,k).gt.O.) then
                       coeres = rslope2(i.k)*sqrt(rslope(i.k)*rslopeb(i.k))
                       pres(i,k) = (rh(i,k)-1,)*(precr1*rslope2(i,k)
                                   +precr2*work2(i,k)*coeres)/work1(i,k)
                    if(pres(i,k).lt.0.) then
                       pres(i,k) = max(pres(i,k),-qrs(i,k)/dtcld)
                       pres(i,k) = max(pres(i,k),satdt/2
                       pres(i,k) = min(pres(i,k),satdt/2
                                                                   Pres1 = \frac{2\pi N_{0r}(S_w - 1)}{(A_w + B_w)} \left| \frac{0.78}{\lambda_r^2} + \frac{a_r^{\frac{1}{2}} 0.31 \Gamma(b_r / 2 + 5 / 2)}{\lambda_r^{b_r / 2 + 5 / 2}} \left(\frac{\mu}{D}\right)^{\frac{1}{3}} \left(\frac{1}{\mu}\right)^{\frac{1}{2}} \right|
                 endif
               else
```

# Car and model

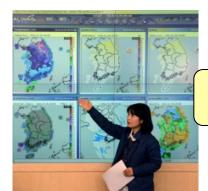




**Data assimilation** 

**Driver** 





**Forecaster** 

# Classification of models

### • Dynamic frame

Hydrostatic	Non-hydrostatic
Large-scale	Small-scale (heavy rainfall, complex mountain)

### Scale

Global	Regional
10 km - 100 km	1 km-10 km

### Purpose

Initial data-> FORECAST	Forcing → RESPONSE
NWP : upto 2 weeks	GCM (General circulation model)

# Predictability

# Chaos theory (Lorenz)



### Charney (1951): Uncertainties in initial condition and model

Lorenz (1962,1963): Unstable nature of atmosphere

**Purpose: NWP is better than statistical forecast** 

Tool: 4 K memory computer

Model: 12 variables (heating and dissipation forcing)

Results: differences -> non-periodicity

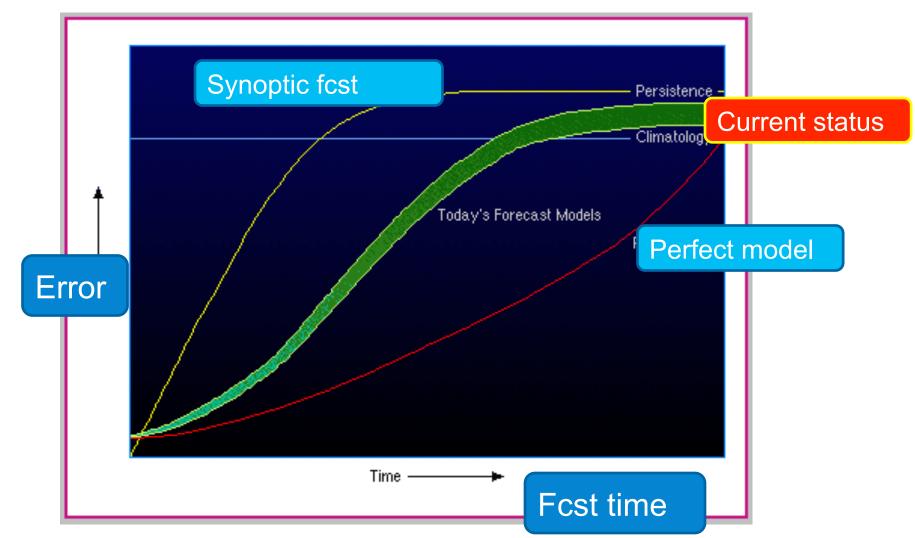
Initial condition (3 decimal point): different after 2 month

Round-off error -> cause of non-periodcity

Chaos theory- two weeks

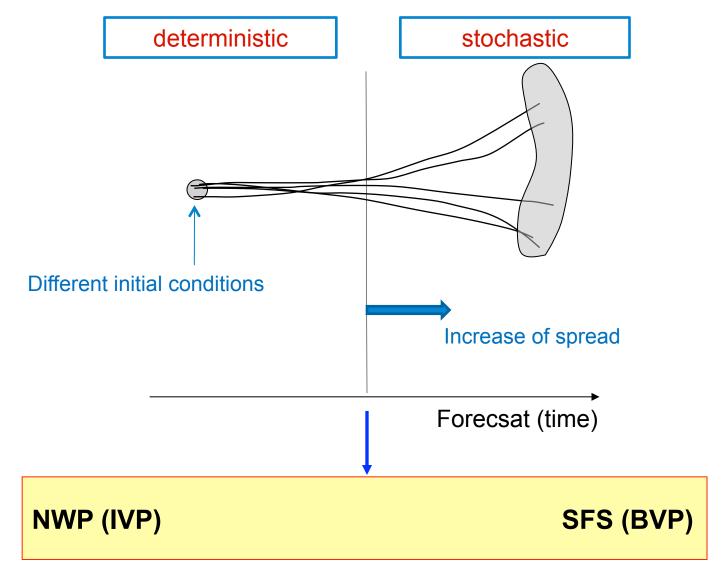
# Predictability



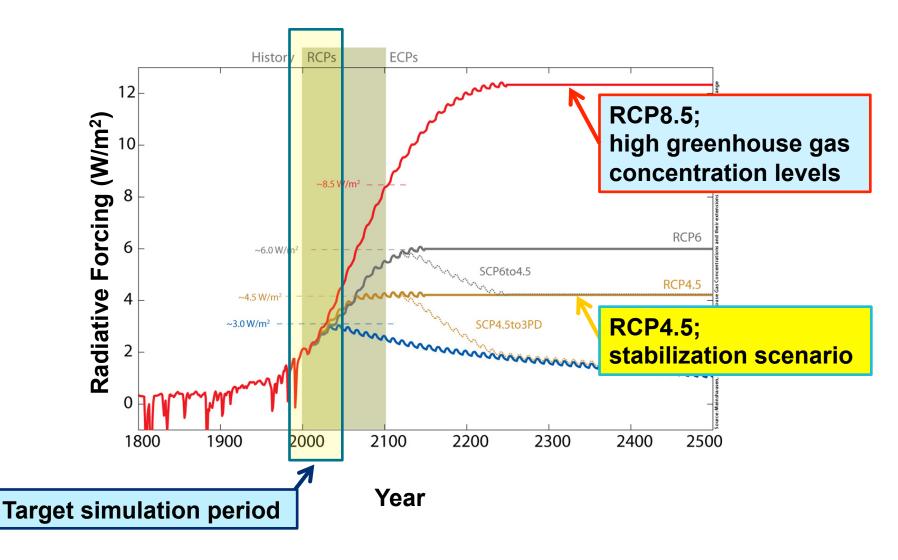


# Ensemble forecasts

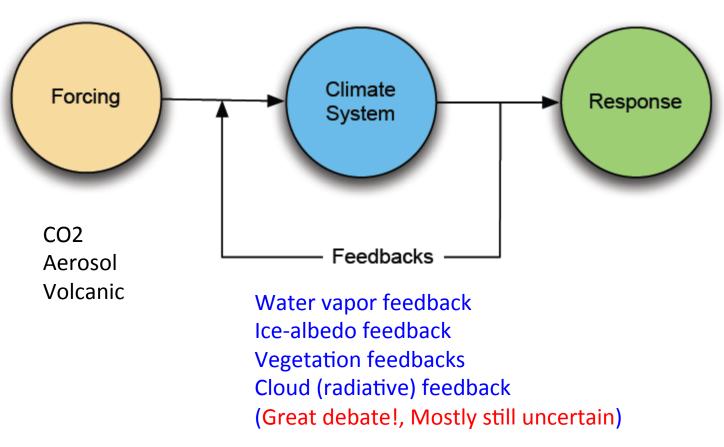




### **RCP** scenarios

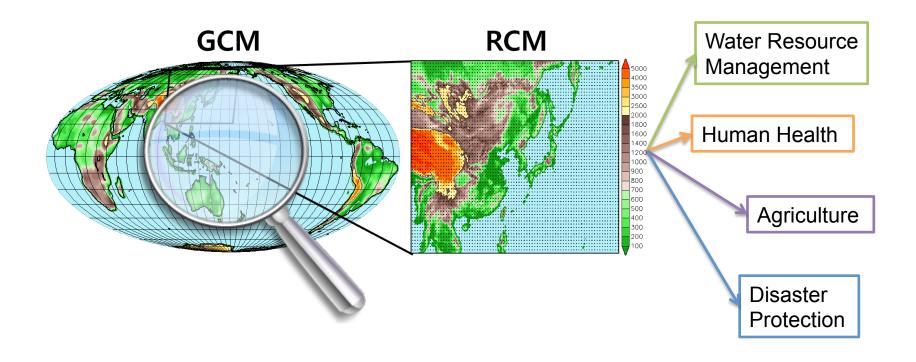


# Climate system sensitivity



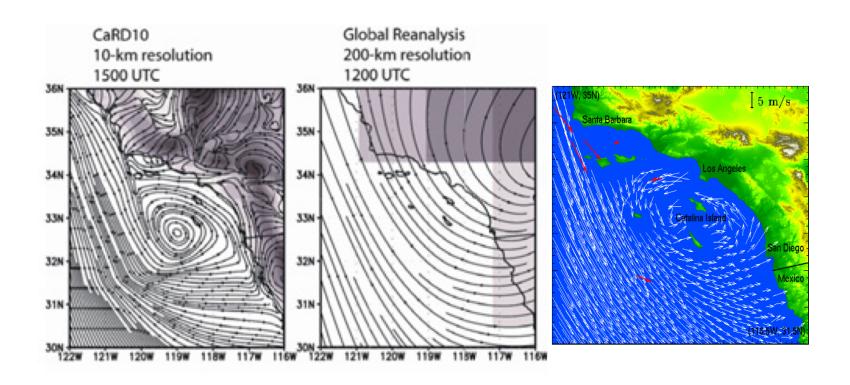
•••

## Global versus Rregional



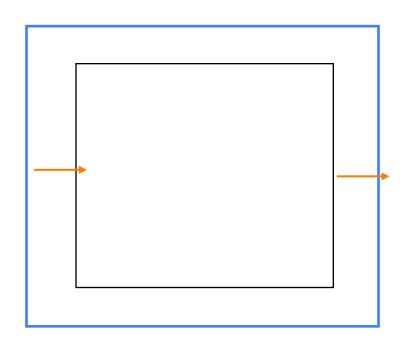
Regional model is a magnifying glass

# Benefit? ---- Very clear!

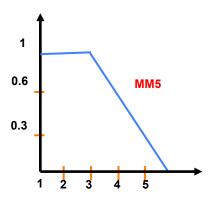


### But, there is another issue on lateral boundary treatment

### Buffer zone



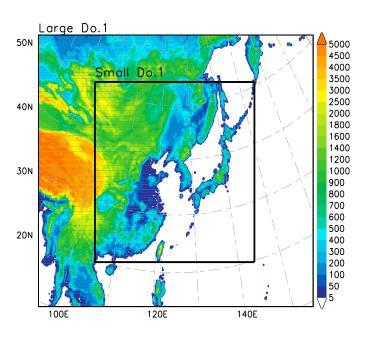
F(n): weighting of global



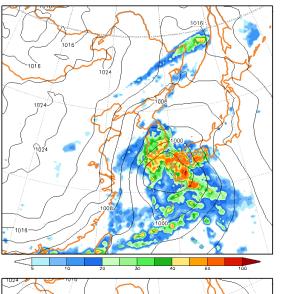
$$\frac{\partial A}{\partial t}\Big|_{n} = F(n)F_{1}(A_{CM} - A_{FM}) - F(n)F_{2}\nabla^{2}(A_{CM} - A_{FM})$$

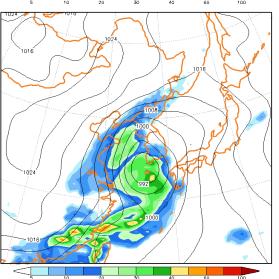
So, empirical

# Domain size sensitivity



Mid-latitude cyclon e on April 6<sup>th</sup>, 2013

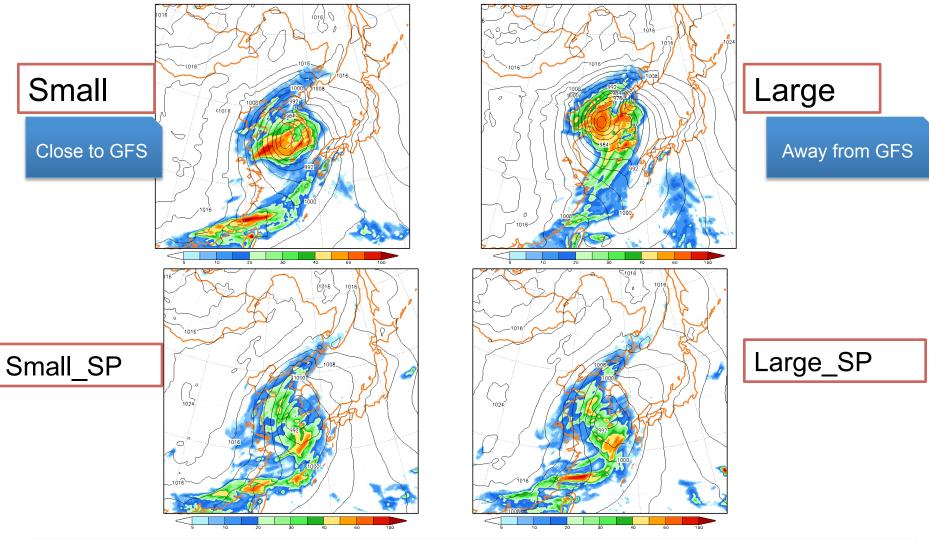




TMPA and FNL

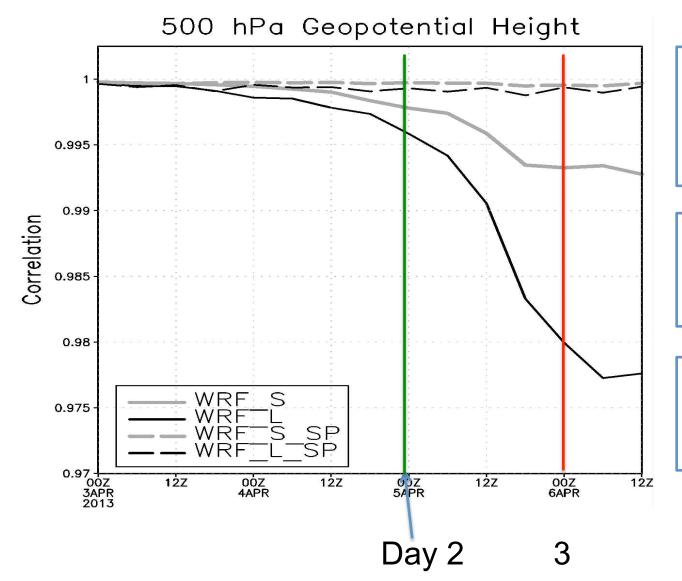
GFS 72 hr fcst

# WRF fcst driven by GFS fcst



Spectral nudging (SP): WRF approaches to GFS forecasts ...but loses regional details

# Domain-averaged PC



Fundamental limit of the regional model: low resolution global and mathematically ill-posed setup

Small domain keeps the large-scale from the global but loses its freedom

Spectral nudging keeps the large-scale, but may lose the regional details

# Thanks for your attention! songyouhong@gmail.com

Hong, S.-Y., and M. Kanamitsu, 2014: Dynamical downscaling: Fundamental issues from an NWP point of view and recommendations. *Asia-Pac. J. Atmos. Sci.*, **50**, 83-104, doi: 10.1007/s13143-014-0029-2.

Dudhia, J., 2014: A history of mesoscale model Development. *Asia-Pac. J. Atmos. Sci.*, **50**, 121-131.