# Fundamentals in Atmospheric Modeling

Song-You Hong

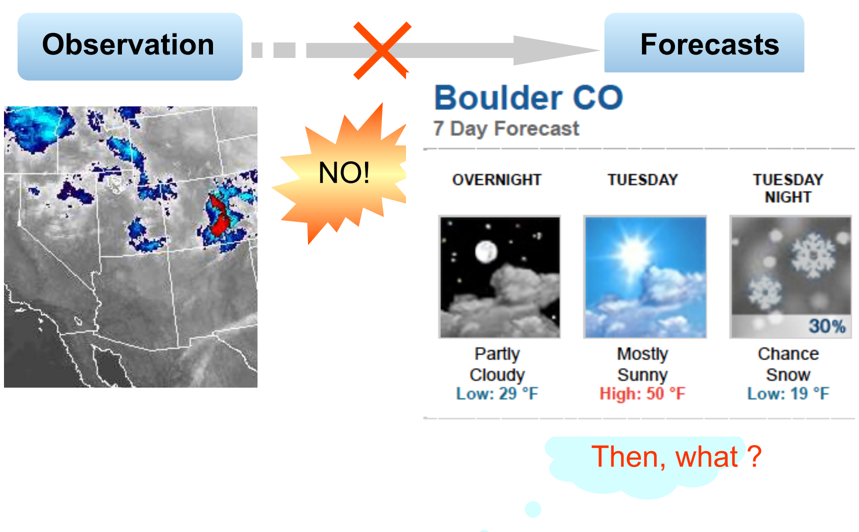
(KIAPS: Korea Institute of Atmospheric Prediction Systems)

(Also NCAR affiliate scientist)

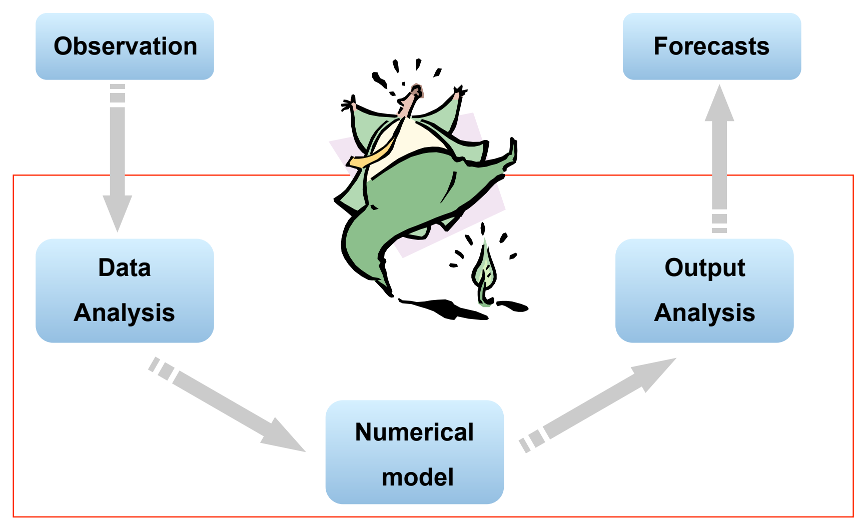
# List of presentations

- Concept of modeling
- Structure of models
- Predictability

## How were the today's forecasts made ?



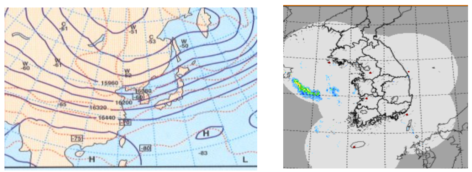

## Numerical model is a crucial component



## Then, how ?

Step1: Observation

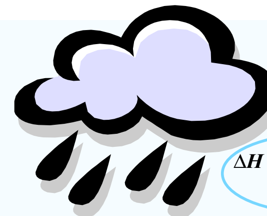
Step2: Data analysis



## Theory of NWP

**Thermodynamics**

Heat = Energy + Work

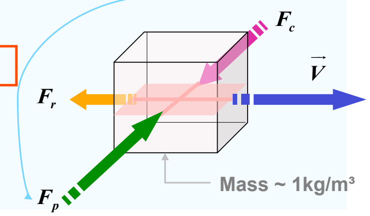


$$\Delta H = c_p \Delta T - \alpha \Delta p = c_v \Delta T + p \Delta \alpha$$

**Dynamics**

Force = Mass × Acceleration

- Mass ≒ 1 kg/m³
- Force: **PGF, CO, Friction**…




## Theory of NWP : Atmosphere is conserved

- **Momentum** $F = ma$
- **Mass** $\frac{1}{M}\frac{dM}{dt} = 0$
- **Moisture** $\frac{dq}{dt} = E - C$
- **Ideal gas** $p\alpha = RT$
- **Energy** $Q = C_v \frac{dT}{dt} + p\frac{d\alpha}{dt}$

CONSERVATION

## The governing equations

*V. Bjerknes (1904) pointed out for the first time that there is a complete set of 7 equations with 7 unknowns that governs the evolution of the atmosphere:*

$$\frac{d\mathbf{v}}{dt} = -\alpha\nabla p - \nabla\phi + \mathbf{F} - 2\Omega\times\mathbf{v} \quad (1\text{-}3)$$

$$\frac{\partial\rho}{\partial t} = -\nabla.(\rho\mathbf{v}) \quad (4)$$

$$p = \rho RT \quad (5)$$

$$\frac{ds}{dt} = C_p \frac{1}{\theta}\frac{d\theta}{dt} = \frac{Q}{T} \quad (6)$$

$$\frac{dq}{dt} = E - C \quad (7)$$

7 equations, 7 unknown (u,v,w,T, p, den and q)

solvable

## History of numerical weather forecasts

**1904** : Norwegian **V.** Bjerknes (1862-1951) :
Setup the governing equations

**1922** : British **L. F.** Richardson (1881-1953) :
Integrate model → failed

**1939** : Swedish **C.-G. Rossby** :

**1948, 1949, J. G. Charney** (1917-1981)

1950 : **Princeton Group**
(Charney, Fjortoft, von Newman)
ENIAC
(Electrical Numerical Integrator and Computer)
→ **first success**

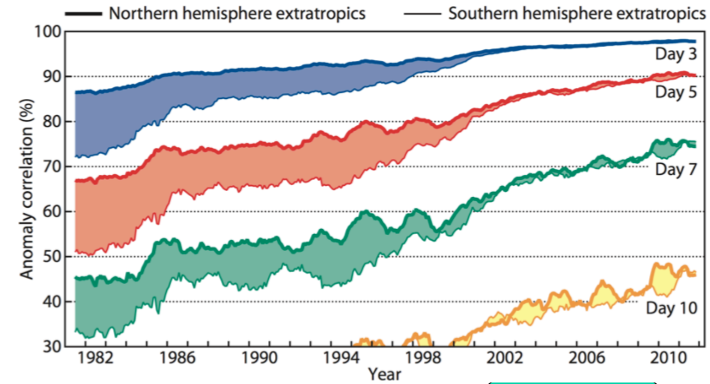
**Computer Age** (1946~)

- von Neumann and Charney
  - Applied ENIAC to weather prediction

- Carl-Gustaf Rossby
  - The Swedish Institute of Meteorology
  - First routine real-time numerical weather forecasting. (1954)
  - ( US in 1958, Japan in 1959 )

## History of NWP skill : ECMWF

Anomaly correlation of 500 hPa Geopotential



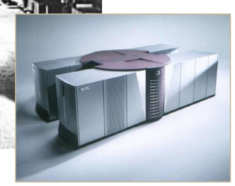

1day / 10 yrs

## Factors for the improvement (Kalnay 2002)

- Supercomputers
- Physical processes
- Initial conditions

## Super-computer for weather models

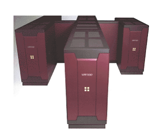
ENIAC,1946



**NEC SX-5**

Cray T3E

Cray SV1

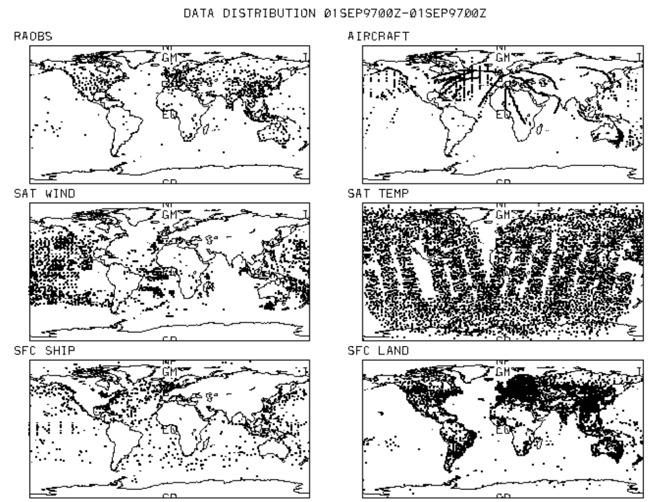


Fujitsu VPP700E

Cray T90

# Initial condition (data assimilation)

## Various observations




heterogeneous in space and time….

## Data Assimilation

- Model 1°X 1° resolution, 20 levels

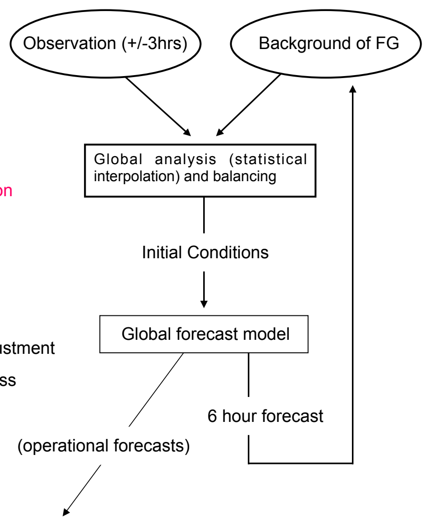
  u, v, T, q, Ps, Tg

  $360 \times 180 \times 20 = 1.3 \times 10^6 \times 4 \ variabl = 5 \times 10^6$

- observation : $10^4 \sim 10^5$ non-uniform distribution

  $\pm 3$ *hour window*

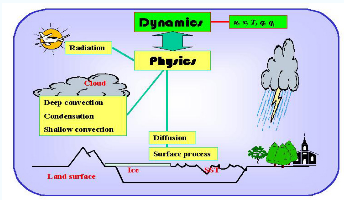
- Data assimilation cycle
  1) data checking
  2) objective analysis
  3) Initialization: dynamical adjustment
  4) short-range fcst for first guess

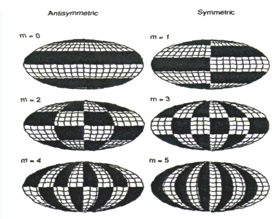


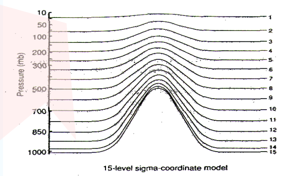

# Model

- Dynamics : Identity (Speed)
- Physics : Components (Predictability)

## Step3: Integration




$$\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \omega\frac{\partial u}{\partial p} - \frac{\partial \Phi}{\partial x} + fv + F_x$$

$$\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \omega\frac{\partial v}{\partial p} - \frac{\partial \Phi}{\partial y} - fu + F_y$$

$$\frac{\partial \Phi}{\partial t} = -\frac{RT}{p}$$

$$\frac{\partial T}{\partial t} = -u\frac{\partial T}{\partial x} - v\frac{\partial T}{\partial y} + \omega\left(\frac{\kappa T}{p} - \frac{\partial T}{\partial p}\right) + \frac{\dot{H}}{c_p}$$

$$\frac{\partial \omega}{\partial p} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$$

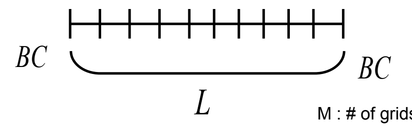
## Dynamics : Numerical method (spatial)

Finite difference method (FDM) :
Spectral method (SPM) :
Finite element method (FEM) :

Ex) $\frac{\partial \phi}{\partial t} = -c\frac{\partial \phi}{\partial x}$ ; advection eq.

1) FDM (Finite difference)

$$\frac{\Delta\phi}{\Delta t} = \frac{\phi_2 - \phi_1}{t_2 - t_1}$$



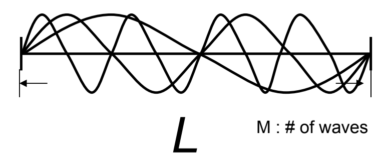

M : # of grids

2) Spectral Method

- Determine basis function to get $H(\phi(x))$
- Expand $\phi$ in terms of a time series

$e_m(x)$ (basis funct ), $m = m_1 L\ m_n$ → infinite

$$\Rightarrow \phi(x,t) = \sum_{m=m_1}^{m_2} \phi_m(t) e_m(x)$$




M : # of waves

* Resolution Increases $\begin{cases} \Delta x \rightarrow decreases \\ m \rightarrow increases \end{cases}$



## Dynamics : Numerical method (temporal)

a) $\frac{u^{n+1} - u^{n-1}}{2\Delta t} = F(u^n)$ : leap-frog **good for hyperbolic**
**unstable for parabolic**

b) $\frac{u^{n+1} - u^n}{\Delta t} = F(u^n)$ : Euler-forward **good for diffusion**
**unstable for hyperbolic**

c) $\frac{u^{n+1} - u^n}{\Delta t} = F\left(\frac{u^n + u^{n+1}}{2}\right)$ : **Crank-Nicholson**

d) $\frac{u^{n+1} - u^n}{\Delta t} = F(u^{n+1})$ : **Fully implicit, backward**

e) $\frac{u^* - u^n}{\Delta t} = F(u^n)$ : $\frac{u^{n+1} - u^n}{\Delta t} = F(u^*)$ : **Euler-backward (Matzuno)**

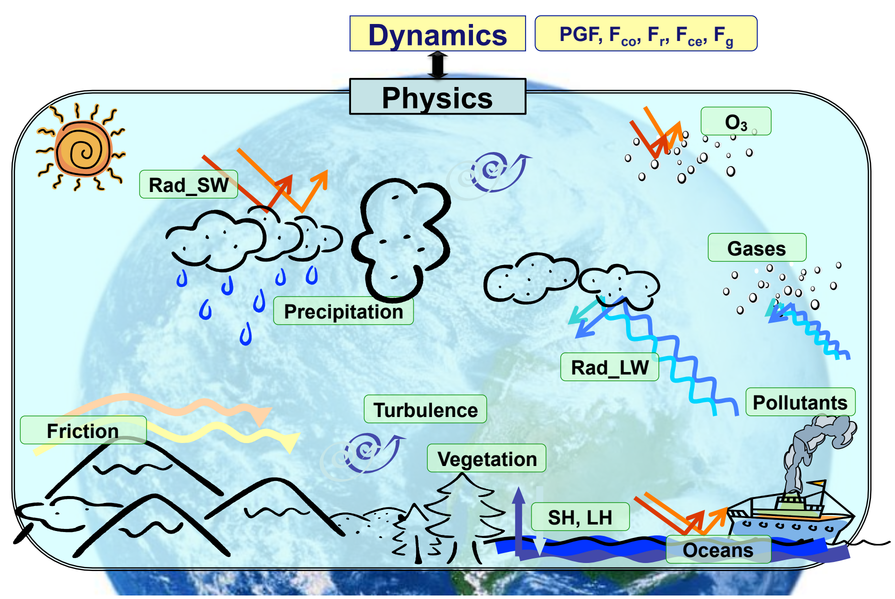
f) $\frac{u^{n+\frac{1}{2}*} - u^n}{\Delta t/2} = F(u^n)$ : $\frac{u^{n+\frac{1}{2}**} - u^n}{\Delta t/2} = F\left(u^{n+\frac{1}{2}*}\right)$

$\frac{u^{n+1} - u^n}{\Delta t} = \frac{1}{6}\left[F(u^n) + 2F\left(u^{n+\frac{1}{2}*}\right) + 2F\left(u^{n+\frac{1}{2}**}\right) + F\left(u^{n+1*}\right)\right]$ : **RK(Runge-Kuta)-4th order**

g) $\frac{u^{n+1} - u^{n-1}}{2\Delta t} = F_1(u^n) + F_2\left(\frac{u^{n+1} - u^{n-1}}{2}\right)$ : **Semi-Implicit**

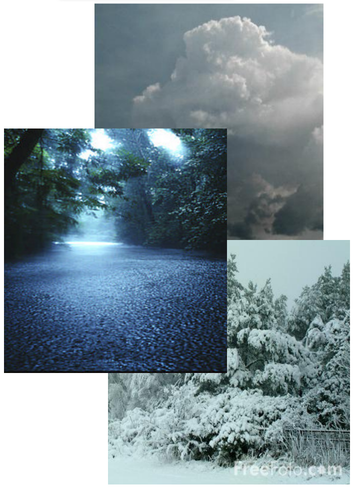
h) $\frac{u^* - u^n}{\Delta t} = F_1(u^n)$; $\frac{u^{n+1} - u^*}{\Delta t} = F_2(u^*)$: **Fractional steps**

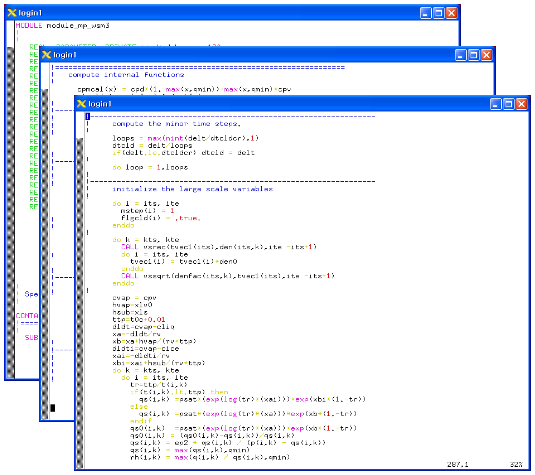
## Physics modules : Branches of atmospheric sciences



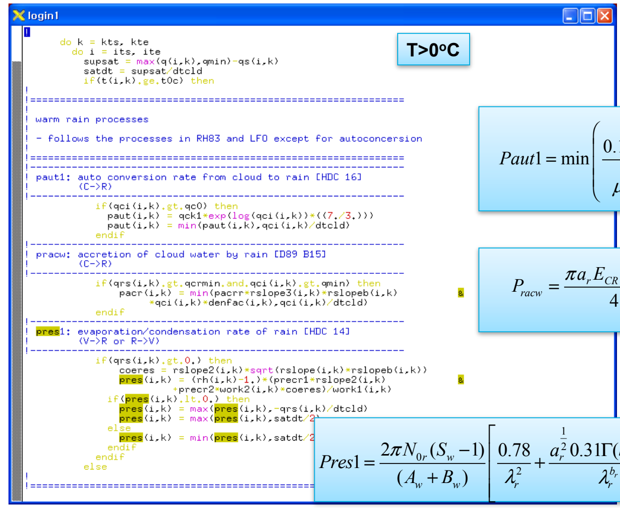
## Physics module (example): Cloud and precipitation

Real atmosphere → Theory / Formulation → Model (computer program)





## Physics module (example): Cloud and precipitation




T>0°C

$$Paut1 = \min\left( \frac{0.104 g E_C \rho^{\frac{4}{3}}}{\mu (N_c \rho_w)^{\frac{1}{3}}} q_c^{\frac{7}{3}},\ \frac{q_c}{dt} \right)$$

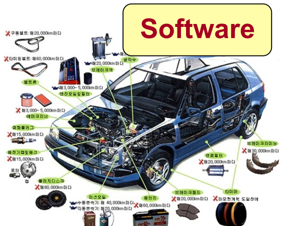
$$P_{racw} = \frac{\pi a_r E_{CR} N_{0r} q_c}{4} \left( \frac{\rho_0}{\rho} \right)^{\frac{1}{2}} \frac{\Gamma(3+b_r)}{\lambda_r^{3+b_r}}$$

$$Pres1 = \frac{2\pi N_{0r}(S_w - 1)}{(A_w + B_w)} \left[ \frac{0.78}{\lambda_r^2} + \frac{a_r^{\frac{1}{2}} 0.31 \Gamma(b_r/2 + 5/2)}{\lambda_r^{b_r/2+5/2}} \left(\frac{\mu}{D}\right)^{\frac{1}{3}} \left(\frac{1}{\mu}\right)^{\frac{1}{2}} \left(\frac{\rho_0}{\rho}\right)^{\frac{1}{4}} \right]$$

## Car and model

Dynamics

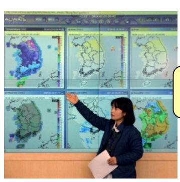
Software



Physics

Data assimilation

Driver



Forecaster

## Classification of models

- **Dynamic frame**

| Hydrostatic | Non-hydrostatic |
|---|---|
| Large-scale | Small-scale<br>(heavy rainfall, complex mountain) |

- **Scale**

| Global | Regional |
|---|---|
| 10 km – 100 km | 1 km-10 km |

- **Purpose**

| Initial data-> FORECAST | Forcing → RESPONSE |
|---|---|
| **NWP** : upto 2 weeks | **GCM** (General circulation model) |

# Predictability

## Chaos theory (Lorenz)

**Charney (1951) : Uncertainties in initial condition and model**

↓

**Lorenz (1962,1963) : Unstable nature of atmosphere**

↓

**Purpose : NWP is better than statistical forecast**
**Tool : 4 K memory computer**
**Model : 12 variables (heating and dissipation forcing)**
**Results : differences -> non-periodicity**

↓

**Initial condition (3 decimal point) : different after 2 month**

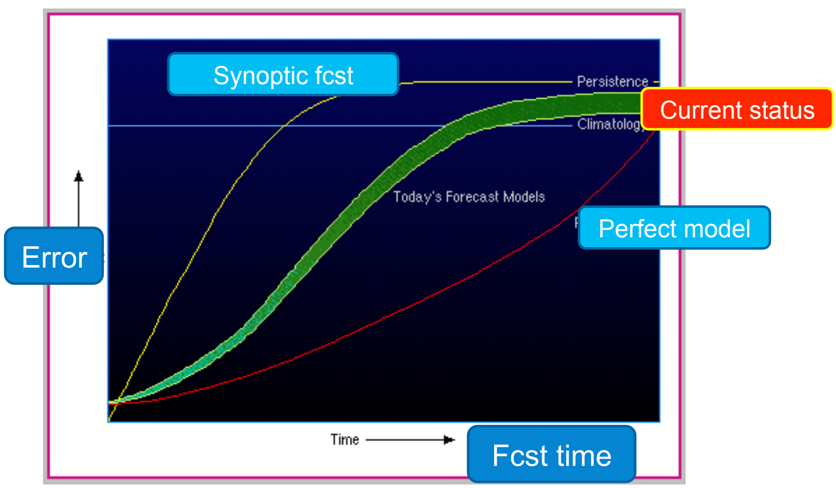
↓

**Round-off error -> cause of non-periodcity**

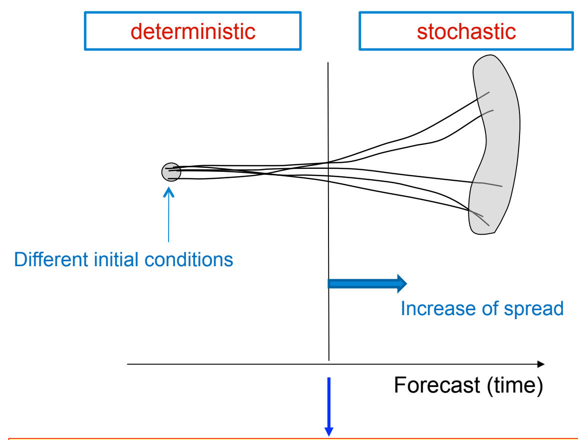
↓

**Chaos theory– two weeks**

## **Predictability :** Atmosphere is unstable

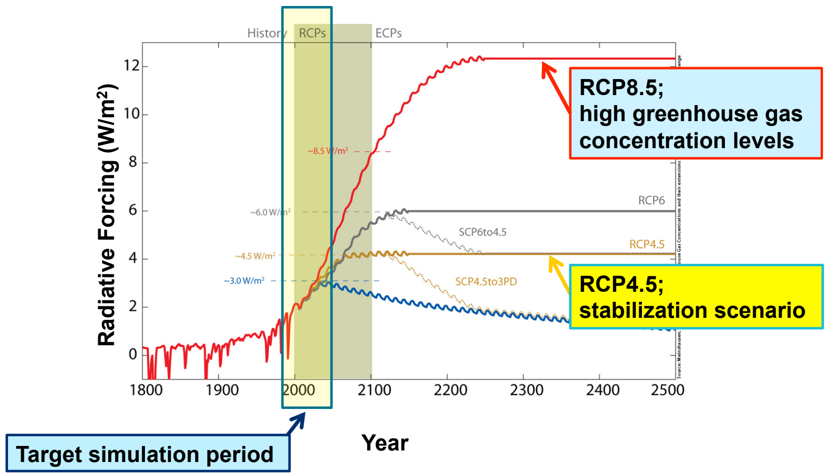



## **Ensemble forecasts :** Seasonal and beyond



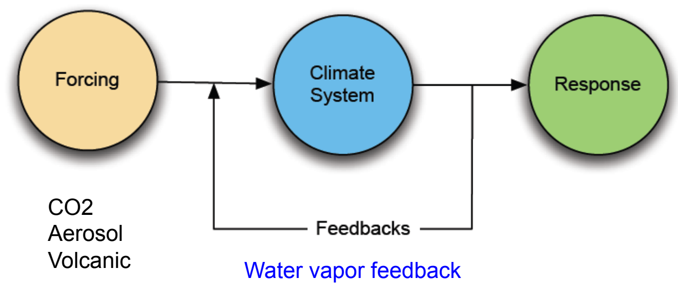

| | |
|---|---|
| **NWP**<br>**(Initial value)** | **Seasonal forecasts**<br>**(External forcing such as SST)** |

## Climate prediction : RCP scenarios



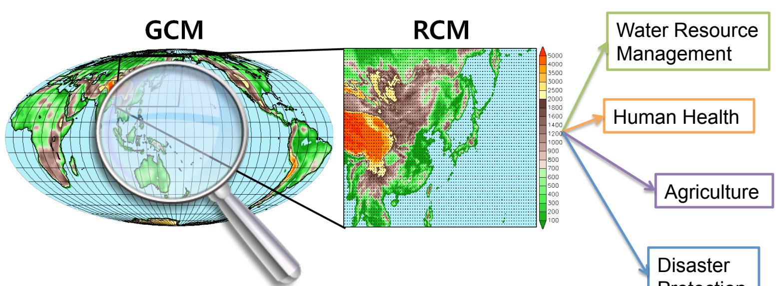

(Meinshausen et al. 2011)

## Climate prediction : Climate system sensitivity



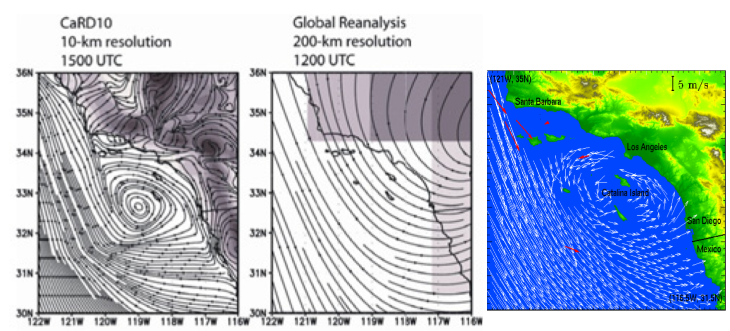

Water vapor feedback
Ice-albedo feedback
Vegetation feedbacks
Cloud (radiative) feedback
(Great debate!, Mostly still uncertain)

## Global versus Regional




Regional model is a magnifying glass

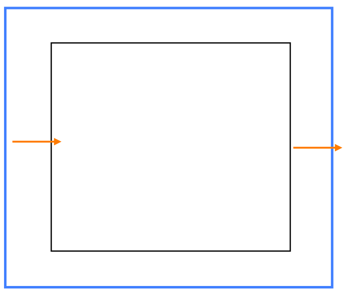
## Benefit ? ---- Very clear !



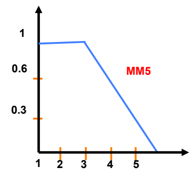
## Another inherent issue in regional modeling

: lateral boundary treatment is empirical

Buffer zone



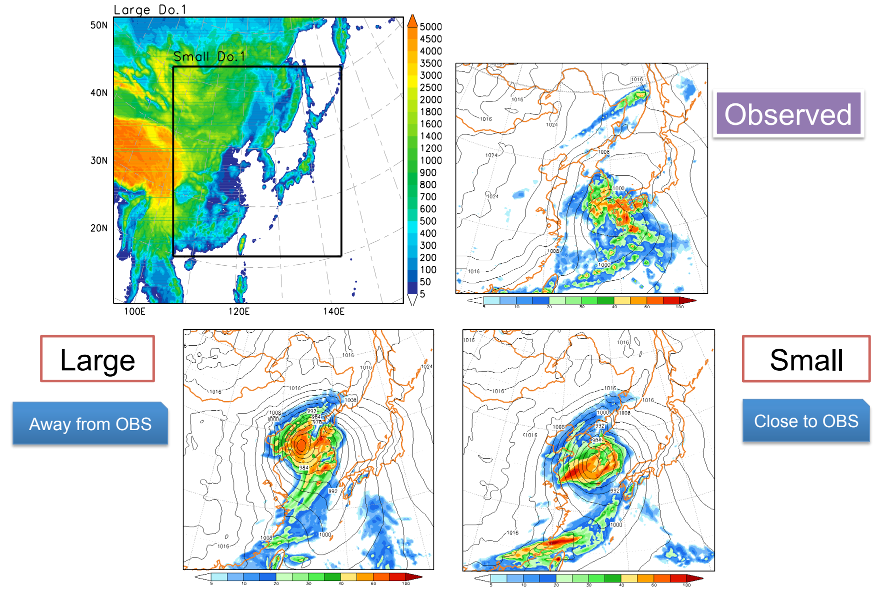
F(n) : weighting of global



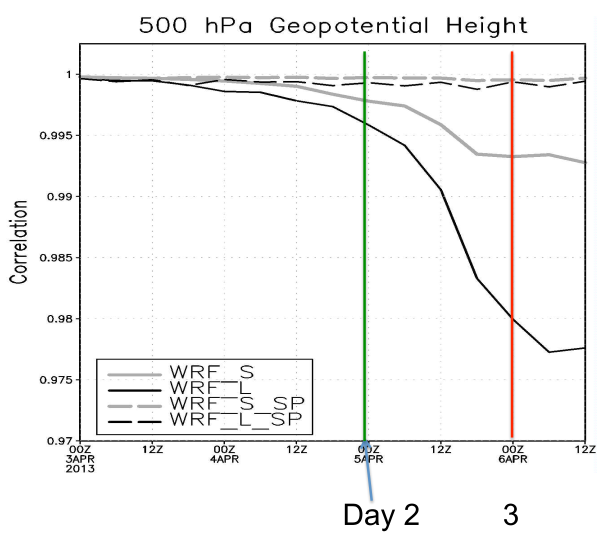

$$\left.\frac{\partial A}{\partial t}\right|_n = F(n)F_1(A_{CM} - A_{FM}) - F(n)F_2\nabla^2(A_{CM} - A_{FM})$$

So, empirical

## Domain size sensitivity : A mid-latitude cyclone




## Domain size sensitivity : Pattern correlation with global




Fundamental limit of the regional model : low resolution global and mathematically ill-posed setup

Small domain keeps the large-scale from the global but loses its freedom

Spectral nudging keeps the large-scale, but may lose the regional details

Thanks for your attention !

songyouhong@gmail.com

Hong, S.-Y., and M. Kanamitsu, 2014: Dynamical downscaling: Fundamental issues from an NWP point of view and recommendations. *Asia-Pac. J. Atmos. Sci.*, **50,** 83-104, doi: 10.1007/s13143-014-0029-2.

Dudhia, J., 2014: A history of mesoscale model Development. *Asia-Pac. J. Atmos. Sci.*, **50,** 121-131.