Overview of Physical Parameterizations

Song-You Hong
(KIAPS: Korea Institute of Atmospheric Prediction Systems)
(Also, an NCAR affiliate scientist)

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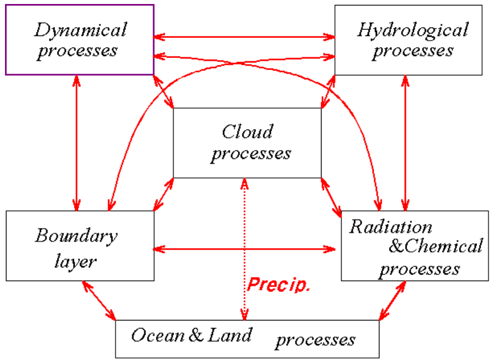
1) Concept



*** Physical processes in the atmosphere**
: Specification of heating, moistening and frictional terms in terms of dependent variables of prediction model
→ Each process is a specialized branch of atmospheric sciences.

*** Parameterization**

The formulation of physical process in terms of the model variables as parameters, i.e., constants or functional relations.

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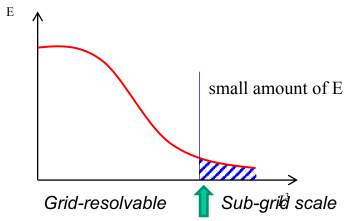
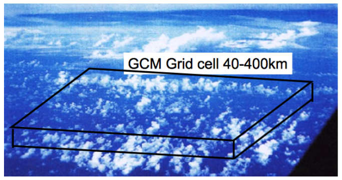
1) Concept…continued

Subgrid scale process

Any numerical model of the atmosphere must use a finite resolution in representing continuum certain physical & dynamical phenomena that are smaller than computational grid.

- Subgrid process (Energy perspective)





$\Delta x \to 0,$ the energy dissipation takes place by molecular viscosity

Objective of subgrid-scale parameterization

"To design the physical formulation of energy sink, withdrawing the equivalent amount of energy comparable to cascading energy down at the grid scale in an ideal situation."

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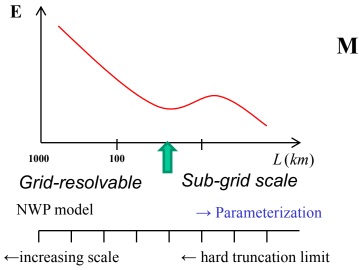
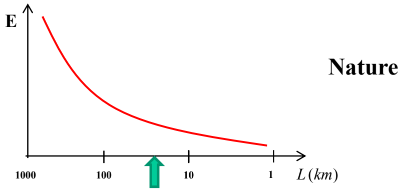
1) Concept…continued

※ Parameterization that are only somewhat smaller than the smallest resolved scales.



where truncation limit ; spectral gap

Unfortunately, there is no spectral gap



2) Subgrid scale process & Reynolds averaging

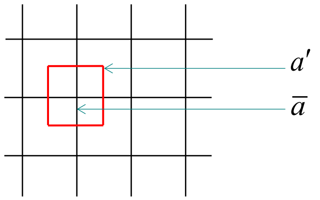
Consider prognostic water vapor equation

$$\frac{\partial \rho q}{\partial t} = -\frac{\partial \rho u q}{\partial x} - \frac{\partial \rho v q}{\partial y} - \frac{\partial \rho w q}{\partial z} + \rho E - \rho C \qquad \cdots(1)$$

In the real atmosphere,

$$u = \overline{u} + u', \qquad q = \overline{q} + q' \qquad \left(\begin{array}{l} ※\overline{a} : grid-resolvable \\ a' : subgrid\ scale\ perturbation \end{array} \right)$$

ρ' is neglected



Model predicts the domain-averaged quantity, $\overline{a}$

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2) Subgrid scale process & Reynolds averaging…continued

*** Rule of Reynolds average :** $\overline{q'} = 0,\ \ \overline{u'\overline{q}} = 0,\ \ \overline{\overline{u}\,\overline{q}} = \overline{u}\,\overline{q}$

then eq.(1) becomes

$$\frac{\partial \rho \overline{q}}{\partial t} = \underbrace{-\frac{\partial \rho \overline{u q}}{\partial x} - \frac{\partial \rho \overline{v q}}{\partial y} - \frac{\partial \rho \overline{w q}}{\partial z}}_{①} \underbrace{- \frac{\partial \rho \overline{u'q'}}{\partial x} - \frac{\partial \rho \overline{v'q'}}{\partial y} - \frac{\partial \rho \overline{w'q'}}{\partial z}}_{②} + \rho E - \rho C \quad \cdots(2)$$

① grid-resolvable advection (dynamical process)
② turbulent transport

*** How to parameterize the effect of turbulent transport**

a) $-\rho\overline{w'q'} = 0$: 0th order closure

b) $-\rho\overline{w'q'} = K\dfrac{\partial \overline{q}}{\partial z}$: 1st order closure (K-theory)

c) obtain a prognostic equation for $\overline{w'q'}$ from (1), (2)

$$\frac{\partial \rho w q}{\partial t} = -\frac{\partial \rho u w q}{\partial x} + \cdots$$

taking Reynolds averaging,

$$\frac{\partial \overline{\rho w'q'}}{\partial t} = \frac{\partial \overline{\rho w'w'q'}}{\partial z}$$

$$-\rho\overline{w'w'q'} = K'\frac{\partial \overline{\rho w'q'}}{\partial z} \qquad \text{: 2nd order closure}$$

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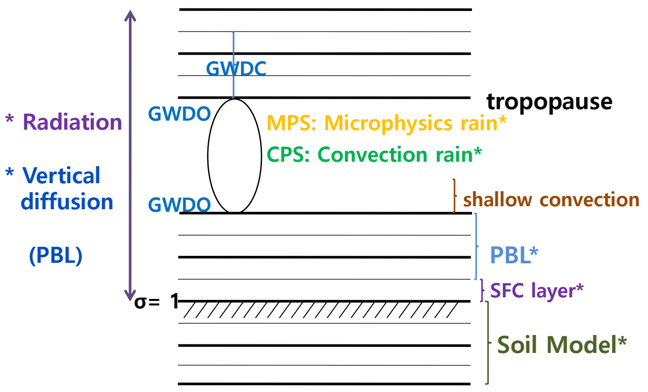
3) Schematic of physics algorithm : In modeled atmosphere : 7* (essential) ~10

$$\frac{d \ln \theta}{dt} = \frac{H}{c_p T}, \quad \frac{dq}{dt} = S, \quad \frac{d\vec{u}}{dt} = \nabla_z \vec{\tau}$$



* Radiation

* Vertical diffusion

(PBL)

GWDC

GWDO

GWDO

tropopause

MPS: Microphysics rain*

CPS: Convection rain*

shallow convection

PBL*

SFC layer*

σ= 1

Soil Model*

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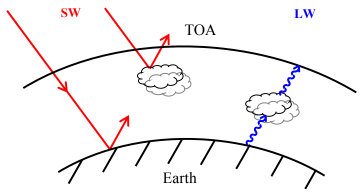
1. Radiation

1.1 Concept



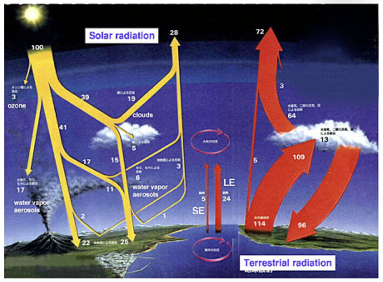
TOA (top of the atmos) : S = 1360 Wm^{-2}

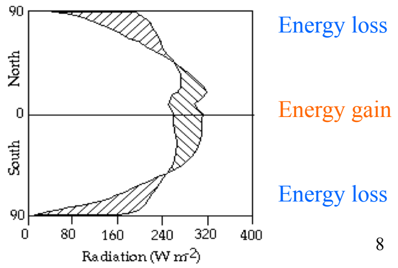
Mean Flux : $\frac{S}{4} = 340\ Wm^{-2}$ → Energy source for Earth

30% : reflected from the atmosphere clouds

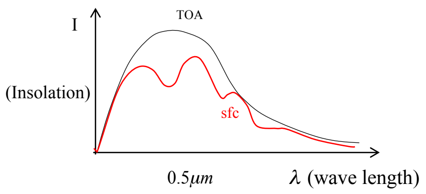
→ Back to space by terrestrial infrared radiation

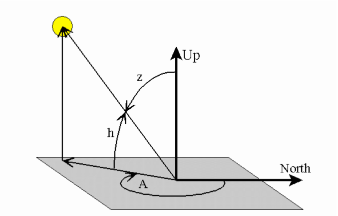
$\left\{ \begin{array}{l} \text{25\% : absorbed in the atmosphere} \\ \text{45\% : absorbed at the earth surface} \end{array} \right\}$



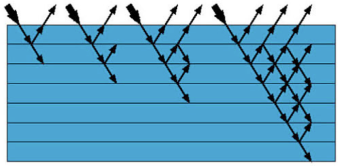


1.2 Solar radiative transfer





h = elevation angle, measured up from horizon
z = zenith angle, measured from vertical
A = Azimuth angle, measured clockwise from North



- At TOA,

$$F = S(\frac{dm}{d})^2 \cos\theta_0 \quad (\theta_0 : \text{Zenith angle})$$

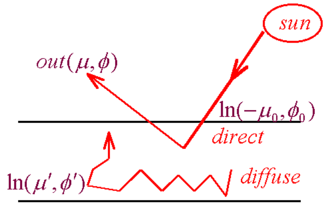
- Basic equations $\mu = \cos\theta$

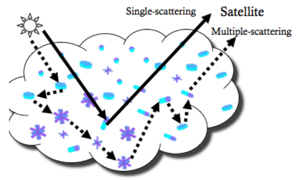
$$\mu\frac{dI(\tau,\mu,\phi)}{d\tau} = I(\tau,\mu,\phi) - J(\tau,\mu,\phi)$$

specific intensity — source function (absorption + scattering)

$$d\tau = -k_\nu \rho_a dz \qquad \tau(\text{optical depth}) = \int_z^{z_\infty} k_\nu(z')\rho_a(z')dz' = \int_0^p k_\nu(p')q(p')\frac{dp'}{g}$$

Absorption coeff.





Angular signature is mainly determined by single-scattering

$$J = J(\tau,\mu,\phi) = \frac{\tilde{\omega}}{4\pi}\int_0^{2\pi}\int_{-1}^{1} I(\tau,\mu',\phi')P(\mu,\phi;\mu',\phi')d\mu' d\phi \text{[diffuse (multiple) scattering]}$$

$$+\frac{\tilde{\omega}}{4\pi}F_0 P(\mu,\phi;-\mu_0,\phi_0)e^{-\frac{\tau}{\mu_0}}\text{[single(direct) scattering]}$$

P : Scattering phase function :redirects (μ',ϕ') (μ,ϕ)

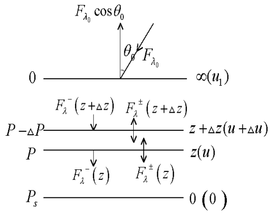
$\tilde{\omega} = \frac{\sigma_s}{\sigma_e}$: Scatteing albedo

scattering cross section/extinction(scattering + absorption) cross section

* remove ϕ dependency using $P(\cos\theta)$ function

* P, $\tilde{\omega}$, Albedo depend on λ, particle size & shape.

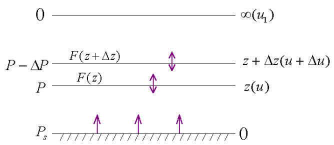
$$P(\cos\phi) = \sum_{l=0}^{N}\tilde{\omega}_l P_l(\cos\phi) \quad \text{: Legendre Polynomial}$$

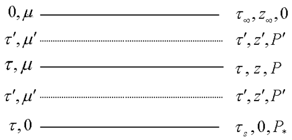


Radiative transfer equation solver. *simplification is needed*

→
- Discrete - ordinates method
- *Two - Stream* and Eddington's approximation
- Delta - function adjustment and similarity principle
- Delta - Four stream approximation

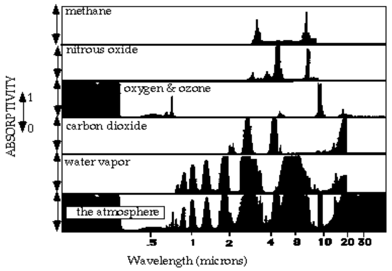
1.3 Terrestrial radiation

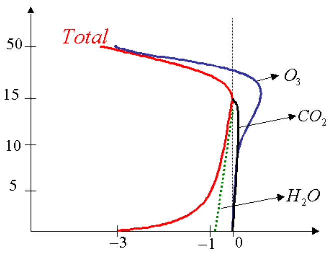




$$F(z) = F^{\uparrow}(z) - F^{\downarrow}(z)$$
$$\Delta F = F(z+\Delta z) - F(z)$$

$$\left.\frac{\partial T}{\partial t}\right|_{IR} = -\frac{1}{c_P\rho}\frac{\Delta F}{\Delta P} = -\frac{g}{c_P}\frac{\Delta F}{\Delta u}$$





In spectral bands (monochromatic)

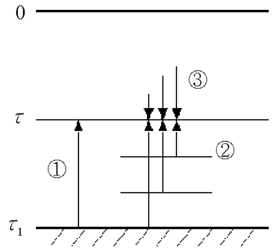
$$\uparrow \quad \mu\frac{dI_\nu(\tau,\mu)}{d\tau} = I_\nu(\tau,\mu) - B_\nu(T)$$

$$\downarrow \quad -\mu\frac{dI_\nu(\tau,-\mu)}{d\tau} = I_\nu(\tau,-\mu) - B_\nu(T)$$

B.C. $\begin{cases} SFC\ (\tau=\tau_1),\ I_\nu(\tau,\mu) = B_\nu(T_s) \\ TOP\ (\tau=0),\ I_\nu(0,-\mu) = 0 \end{cases}$

$$F^{\uparrow}(\tau) = 2\pi B_\nu(T_s)\int_0^1 e^{-\frac{(\tau_1-\tau)}{\mu}}\mu d\mu + 2\int_0^1\int_\tau^{\tau_1}\pi B_\nu[T(\tau')]e^{-\frac{(\tau'-\tau)}{\mu}}d\tau' d\mu$$

$$F^{\downarrow}(\tau) = 2\int_0^1\int_0^{\tau}\pi B_\nu[T(\tau')]e^{-\frac{(\tau-\tau')}{\mu}}d\tau' d\mu$$

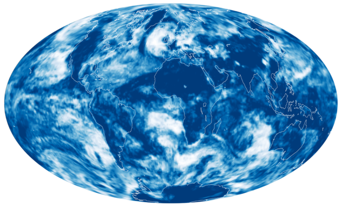


$$d\tau = -k_\nu \rho dz \quad \tau_1 = \int_0^{u_1} k_\nu du, \quad u_1 = \int_0^\infty \rho dz \text{ (path length)}$$

LW is time consuming !

1.4 Cloud fraction

i) Earlier method (Slingo)



$f = f_c$(convective cloud, CPS) $+ f_l$(large-scale cloud, MPS)

f_c : depends on precipitation, $p_{top},\ p_{bottom}$

f_l : depends on RH $= 1 - \left[\frac{1-RH}{1-RH_0}\right]^{0.5}$

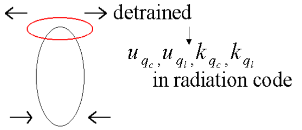
where RH_0 is the critical value of RH,

which is optimized based on observations

ii) Advanced method (Chou)

- inclusion of ice, liquid hydrometeors as prognostic water substances
- consistent treatment of water substance for both precipitation & radiative properties.

f_c : uses information of detrained water substances from sub-grid scale clouds in convective parameterization scheme (CPS)



detrained

$u_{q_c}, u_{q_l}, k_{q_c}, k_{q_l}$

in radiation code

f_l use the hydrometeor information from microphysics routine (MPS), $q_c, q_s, q_i, \cdots$

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iii) Radiation properties

$\tau_i^c = cwp[a_i + \frac{b_i}{r_{ei}}]\, f_{ice}$ (optical thickness)

$w_i^c = 1 - c_i - d_i r_{ei}$ (co-albedo)

$g_i^c = e_i - f_i r_{ei}$ (asymmetry factor)

$f_i^c = (g_i)^2$ (forward peak fraction)

a-f : coeff : depends upon band and k-

$\overline{\tau_c} = \sum_i \tau_i$ $\quad$ i : each gas

(The effective optical thickness for each spectral band)

- The long wave cloud emissivity (E_{cld})

$c'_f = E_{cld} c_f$

$E_{cld} = 1 - e^{-Dk_{abc} cwp}$

where $D = 1.66$: diffusivity factor

$k_{abc} = k_l(1 - f_{ice}) + k_i f_{ice}$: absorptivity coefficient

- For diagnostic microphysics scheme,

- cloud water scale height

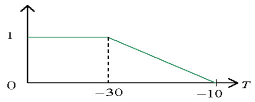
$h_1 = a \ln(1.0 + \frac{b}{g}\int_{P_T}^{P_S} q dp)$

- cloud droplet size,

$r_{ee} \begin{cases} = 10\mu m & \text{over ocean} \\ < 10\mu m & \text{over land} \end{cases}$: warm clouds

r_{ei}: $10\mu m$ (low) ~ $30\mu m$: (high) ice clouds

– ice fraction, f_{ice}



- For prognostic microphysics scheme,

: Broadband radiation : Radiative properties are explicitly computed from prognostic water substances

: Simplified radiation : Dudhia (1989)

α_p (absorption coefficient) $= \frac{1.66}{2000}(\frac{\pi N_0}{\rho_{rs}^3})^{\frac{1}{4}} \quad m^2g^{-1} = \begin{bmatrix} 2.34\times10^{-3} & m^2g^{-1} & \text{for snow} \\ 0.33\times10^{-3} & m^2g^{-1} & \text{for rain} \end{bmatrix}$

u_p (effective water path length) $= (\rho q_{rs})^{\frac{3}{4}} \Delta z \times 1000 \quad gm^{-2} \rightarrow \tau_p$ (transmission) $= \exp(-\alpha_p u_p)$

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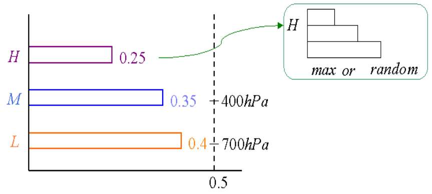
iv) Cloud overlapping



Maximum overlapping : 0.4

Minimum overlapping : 1.0

Random overlapping : $H + (1-H)M + \{1 - H - (1-H)M\}L = 0.6$

- Computation :

τ is scaled by A_c (cloud cover) at a given layer.

- Flux for each of $A_c, (1-A_c)$ → summation

- Issues : $A_c = 0$ or 1 🡐 in WRF versus partial cloudiness in GFS : WRF has partial cloudiness recently

- Interaction : Radiation $\xleftarrow{\text{reflect } SW, LW}$ Cloud

Radiation $\xrightarrow[\text{enhance or reduce cloud activity}]{}$ Cloud

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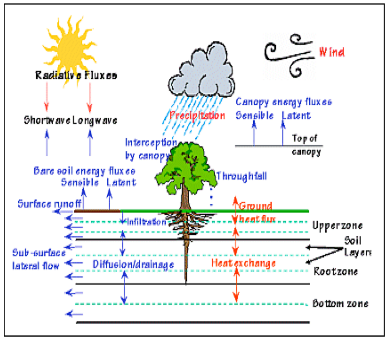
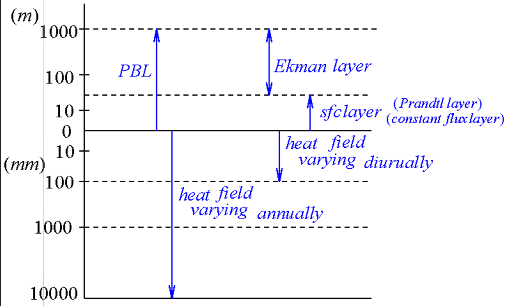
2. Land-surface processes

2.1 Concept : Surface layer + soil model





Atmospheric surface layer : the lowest part of the atmospheric boundary layer (typically about a tenth of the height of the BL) where mechanical (shear) generation of turbulence exceeds buoyant generation or consumption. Turbulent fluxes and stress are nearly constant with height.

➔ **In atmospheric models**, it is defined the height of the lowest model level.

2.2 Surface layer parameterization

Surface layer schemes calculate friction velocities and exchange coefficients that enable the calculation of surface heat and moisture fluxes by the land-surface models. These fluxes provide a lower boundary condition for the vertical transport done in the PBL Schemes.

Over water surfaces, the surface fluxes and surface diagnostic fields are computed in the surface layer scheme itself. Sea surface temperature can be predicted by the surface energy budget and mixed layer mixing

1) Bulk method (before 1990, MM4)

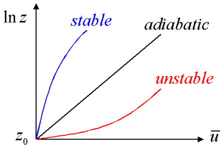
$$H_0 = \rho C_p C_H |\vec{V}_a| \Delta T$$
$$E_0 = \rho L C_H |\vec{V}_a| \Delta q M_a$$
$$\vec{\tau}_0 = \rho C_D |V_a| \vec{V}_a$$

C_D, C_H : *prescribed*
= 0.01 over land, = 0.001 over water

2) Monin-Obukov similarity

$$\frac{k_z}{u_*}\frac{\partial u}{\partial z} = \phi_m(z/L), \quad \frac{k_z}{u_*}\frac{\partial \theta}{\partial z} = \phi_t(z/L)$$

Integrate, $F_m = \int_{z_0}^{h_s} \frac{dz}{z}\phi_m dz = \ln(\frac{h_s}{z_0}) - \psi_m(h_s, z_0, L)$



$$\vec{u} = C_u[\ln\frac{z}{z_0} + \Phi]$$

Φ : curving factor

※ **Profile function** : ϕ_m and ϕ_t

Dyer and Hicks formula for similarity

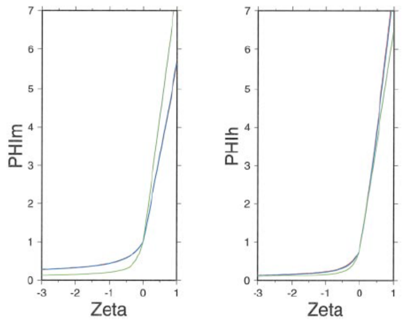
- unstable (L < 0)

$$\phi_m = (1 - 16\frac{0.1h}{L})^{-\frac{1}{4}} \quad \text{for } u, v$$
$$\phi_t = (1 - 16\frac{0.1h}{L})^{-\frac{1}{2}} \quad \text{for } \theta, q$$

- stable (L > 0)

$$\phi_m = \phi_t = (1 + 5\frac{0.1h}{L})$$

where $L = u_*^2\bar{\theta}/(kg\theta_*) = -\frac{\rho C_p \theta_0 u_*^3}{kgH_0}$



Kantha (2003)

$$\frac{h_s}{L} = \frac{\phi_m^2(hs/L)}{\phi_t(hs/L)} Ri = \zeta \text{ (Zeta=hs/L)}$$

where $Ri = \frac{g}{\bar{\theta}}\frac{\partial \theta}{\partial z}(\frac{\partial u}{\partial z})^{-2}$

※ **Useful relation** :

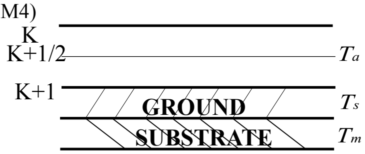
Given the F_m, F_H $C_D = k^2 / F_m{}^2, C_Q = C_H = k^2/(F_m F_t)$ $u_* = kU/F_m$

$$\tau_0 = \rho k_m \frac{du}{dz} = -\overline{u'w'} = \rho C_d U^2$$
$$H_0 = -\rho C_p k_h \frac{d\theta}{dz} = \rho C_p \overline{\theta' w'} = -\rho C_p C_H U \Delta\theta$$
$$E_0 = -\rho L \overline{q' w'} = -\rho L C_q U \Delta q$$

2.3 Soil model

1) **Slab model :** force-restore method (before 1990 : MM4)

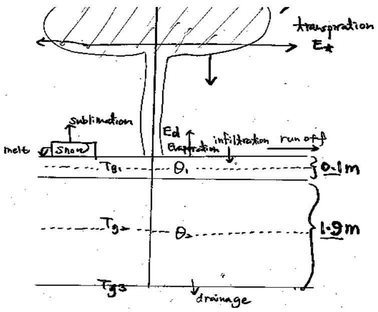
$$\frac{\partial T_s}{\partial t} = \lambda_T(R_n - LE - H) - \frac{2\pi}{\tau}(T_s - T_a)$$
$$\frac{\partial T_s}{\partial t} = \frac{1}{\tau}(T_s - T_m) \quad \textbf{: } \boldsymbol{T_m}\textbf{, daily mean}$$



2) **Multi-layer model** : OSU method

$$\frac{\partial T_s}{\partial t} = \lambda_T(R_n - LE - H) \quad \textbf{: surface T}$$
$$(\rho C)_i \frac{\partial T_g}{\partial t} = \frac{\partial}{\partial z}(\lambda T_g \frac{\partial T_g}{\partial z}) \quad \textbf{: soil T}$$
$$\frac{\partial \Theta}{\partial t} = \frac{\partial}{\partial z}(D\frac{\partial \Theta}{\partial z}) + K\frac{\partial \Theta}{\partial z} + F_\Theta \quad \textbf{: soil moisture}$$

- NOAH, SIB, PLACE, VIC, CLM, etc

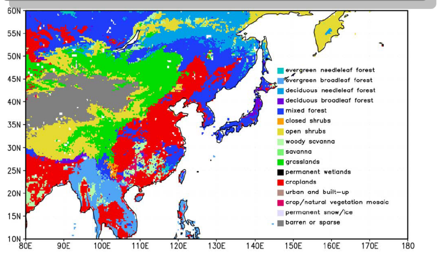


RA analysis: 2-layer soil mode l

2.4 Vegetation type → z_0, Albedo

Vegetation types

MODIS (satellite)

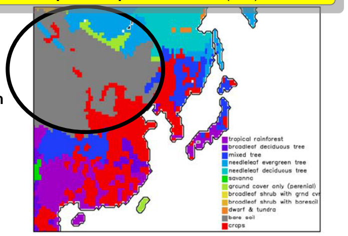


13 type data set
1 degree

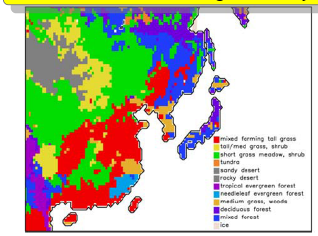
12 type data set
20 min

The Simple Biosphere model (SiB)

quite broad
desert region

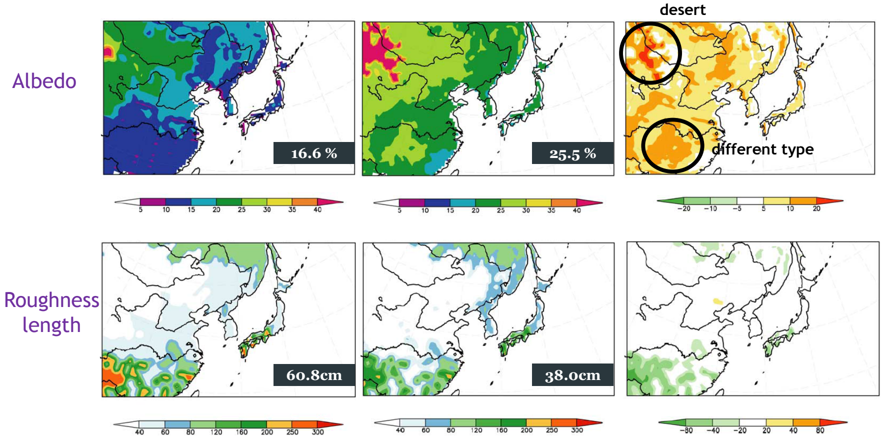


United States Geological Survey's (USGS)



Albedo and roughness length (z_0)

Sib **USGS** **USGS-Sib**



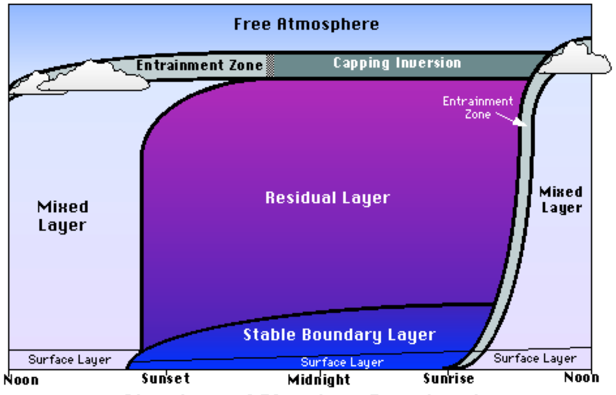
Much uncertainties in calibration/optimization !!!

---

3. Vertical diffusion (PBL)

3.1 Concept

- computes the parameterized effects of vertical turbulent eddy diffusion of momentum, water vapor and sensible heat fluxes



Structure of Planetary Boundary Layer

adapted from Stull, 1988.

3.2 Planetary Boundary Layer Structure : schematic

Daytime profiles

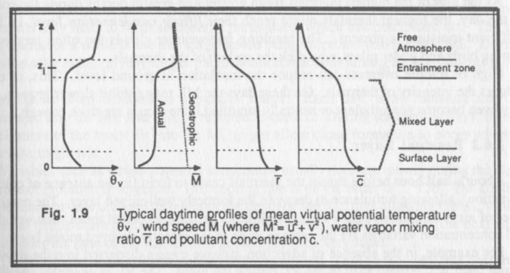


Fig. 1.9 Typical daytime profiles of mean virtual potential temperature $\bar{\theta}_v$, wind speed $\bar{M}$ (where $\bar{M}^2=\bar{u}^2+\bar{v}^2$), water vapor mixing ratio $\bar{r}$, and pollutant concentration $\bar{c}$.

Daytime flux profiles

Nighttime flux profiles

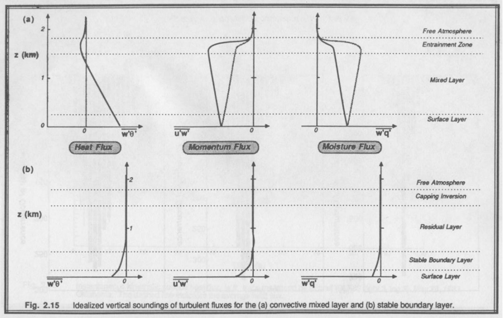


Fig. 2.15 Idealized vertical soundings of turbulent fluxes for the (a) convective mixed layer and (b) stable boundary layer.

Stull (1988)

3.3 Classifications : how to determine, k_c

i) Local diffusion (Louis 1979)

$$\frac{\partial c}{\partial t}=\frac{\partial}{\partial z}(-\overline{wc})=\frac{\partial}{\partial z}(k_c\frac{\partial c}{\partial z}) \quad k_c: \text{ diffusivity, } \quad k_m,k_t=l^2 f_{m,t}(Ri)\left|\frac{\partial U}{\partial z}\right|$$

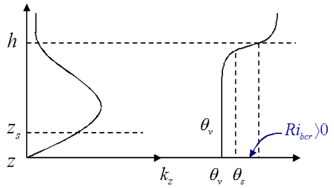
Local Richardson number

ii) Nonlocal diffusion (Troen and Mahrt 1986)

$$\frac{\partial c}{\partial t}=\frac{\partial}{\partial z}(-\overline{wc})=\frac{\partial}{\partial z}(k_c(\frac{\partial c}{\partial z}-\gamma_c))$$

$$k_{zm}=kw_s z(1-\frac{z}{h})^p,\ h=R_{ibcr}\frac{\theta_m}{g}\frac{U^2(h)}{(\theta_v(h)-\theta_s)}$$

$$\theta_s=\theta_{va}+\theta_T\ (=b\frac{\overline{(\theta_v'w')_0}}{w_s}),\ w_s=u_*\phi_m^{-1}$$



iii) Eddy mass-flux diffusion (Siebesma et al. 2007)

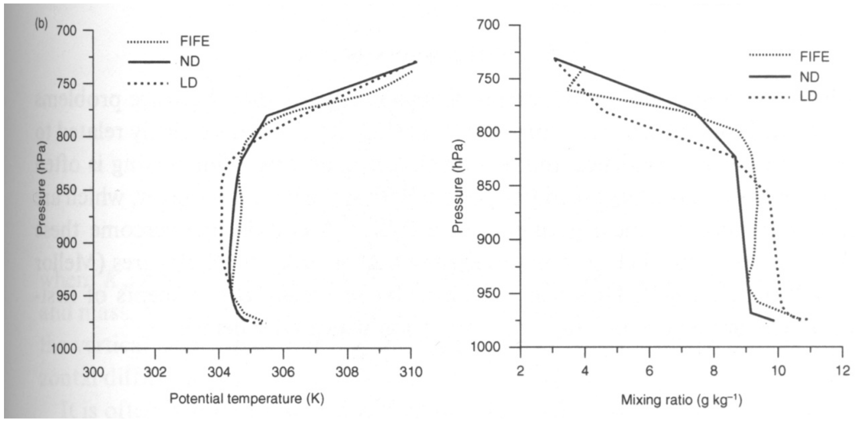
$$\frac{\partial c}{\partial t}=\frac{\partial}{\partial z}(-\overline{wc})=-\frac{\partial}{\partial z}[\underbrace{-k_c\frac{\partial\bar{c}}{\partial z}}_{\text{small eddies}}+\underbrace{M(c_u-\bar{c})}_{\text{strong updrafts}}]$$

iv) TKE (Turbulent Kinetic Energy) diffusion (Mellor and Yamada 1982)

TKE equation : $$\frac{\partial\overline{u_iu_j}}{\partial t}+u_j\frac{\partial\overline{u_iu_j}}{\partial x_j}=-\frac{\partial}{\partial x_k}[\overline{u_iu_ju_k}+\frac{1}{\rho}\ \cdots]$$

$$\overline{u_iu_j} \Longrightarrow k_z = \text{ fn (TKE)}$$

3.4 Local versus nonlocal



Local scheme (LD) typically produces unstable mixed layer in order to transport heat upward. But, observation (FIFE) shows nearneutral or slightly stable BL in the upper part of BL

Hong and Pan (1996)

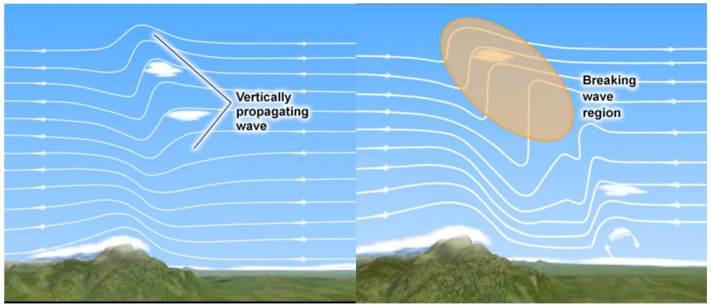
---

4. Gravity Wave Drag

- GWDO : GWD induced by sub-grid scale orography
- GWDC : GWD induced by precipitating deep convection

4.1 Concept

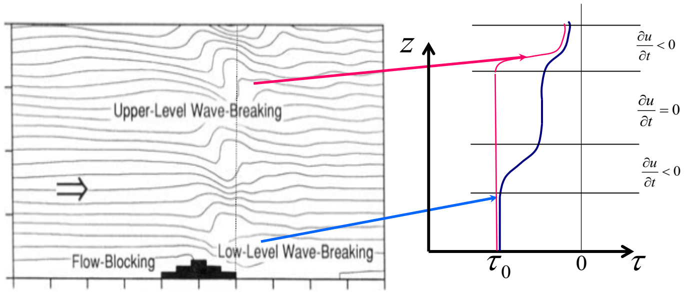
This scheme (GWDO) includes the effect of mountain induced gravity wave drag from sub-grid scale orography including convective breaking, shear breaking and the presence of critical levels. Effects are strong in the presence of strong vertical wind shear and thermally stable layer.



* In smoothed model orography, momentum stress near mountain cannot be generated

---

4.2 Enhanced lower tropospheric gravity wave drag (Kim and Arakawa 1995)



$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial \tau_x}{\partial z}$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial \tau_y}{\partial z}$$

Stress at reference level : $$\tau_0 = -E\frac{1}{\Delta x}\frac{\rho_0 U_0^3}{N_0}\frac{Fr^2}{Fr^2 + 0.5/OC},\ U_0 = \frac{1}{h}\int_{k=1}^{k=kpbl} Udz$$

Reference level (KA95) : Max (2, KPBL)

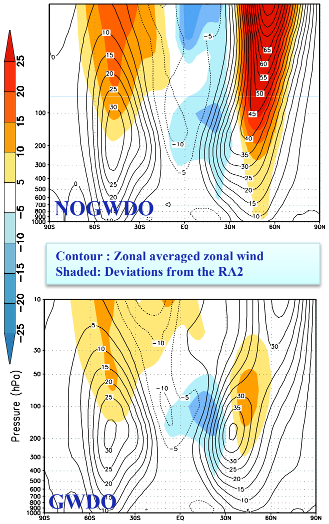
Earlier method : the conventional Ri number-based wave- breaking mechanism using the saturation hypothesis, which works mainly in the upper atmosphere

Advanced: the higher-moment orographic statistics-based wave- breaking mechanism using half-theory (Scorer parameter ~ BVF**2 / U**2) and half-empiricism obtained from mesoscale mountain wave simulations, which breaks in the lower atmosphere as well as upper layer, together with flow blocking

---

4.3 Impact of GWDO

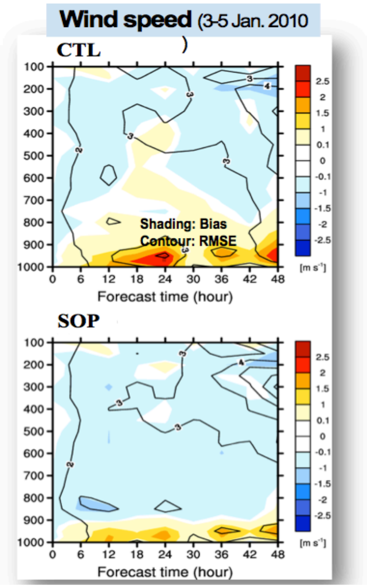
Zonal-averaged zonal wind (96/97 DJF)



Contour : Zonal averaged zonal wind
Shaded: Deviations from the RA2

➔ Improves the upper level jets (Kim and Hong, 2009)

Wind speed (3-5 Jan. 2010)



Flow blocking ➔ Improves the low-level winds (Choi and Hong, 2015)

4.4 Non-orographic gravity wave drag (GWD)

Conventional GWD parameterizations

- Simple assumptions on GW sources with either a spatiotemporally uniform distribution (e.g., Warner and McIntyre, 2001 in Unified Model; Scinocca, 2003 in IFS)

Source-based GWD parameterizations

(e.g., Choi et al., 2018 in KIM (Korean Integrated Model, Hong et al. 2010) ; Richter et al., 2010 in CESM)

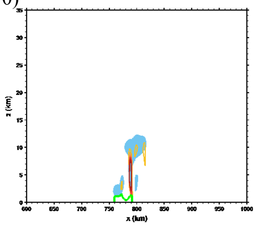
Convective GWD parameterizations

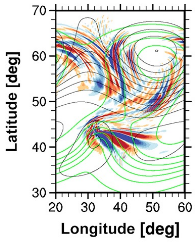
- Chun and Baik (1998, 2002): first to analytically formulate the GW momentum flux spectrum for convective GWs
- Beres (2004), Song and Chun (2005): non-stationary GW parts together with spatiotemporal variations in the convective source
- Song and Chun (2008): ray-based approach
- Choi and Chun (2011): moving speed of the convective source and wave-propagation direction, were determined

Jet/Front GWD parameterizations

- Not yet explored sufficiently
- Charron and Manzini (2002), Richter et al. (2010): introduced a frontogenesis function to diagnose the generation of frontal GWs
- de la Cámara and Lott (2015): based on the theoretical results of GW generation by potential vorticity anomalies

Convective GWs





Jet/front induced GWs

4.5 Improvement by source-based non-orographic GWD (Choi et al., 2018)

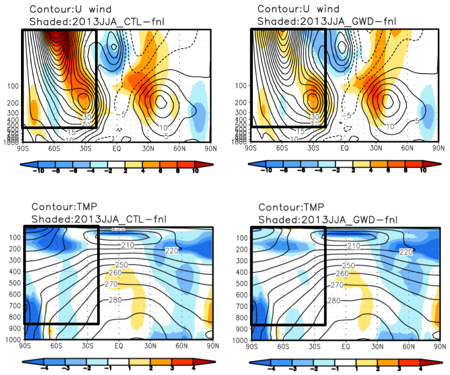
Seasonal simulation

(2013JJA)

CTL **NEW**

Zonal-mean zonal wind

Zonal-mean temperature



↑ Shading: Bias against the analysis

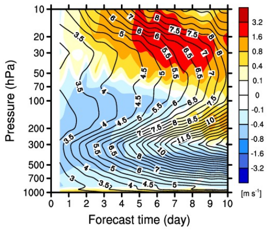
This study is the first to investigate the effects of both the convective and frontal GWD schemes, at both the seasonal and medium-range time scales

Medium-range forecast

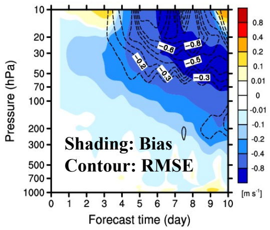
(January 2016)

Wind speed

CTL



NEW-CTL

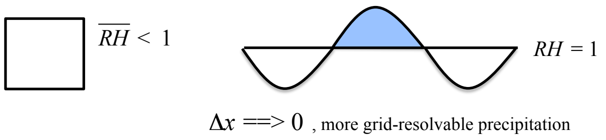


Precipitation Processes

Concept : precipitation algorithms (CPS and MPS)

- In real atmosphere, dynamical motion -> RH > 1 => clouds form -> produces rain
- In modeled atmosphere, RH < 1

But generate clouds by sub-grid scale motion → requires parameterized process
(it is often related to the deep convection processes)
Deep convection : 2~10 km



Δx ==> 0 , more grid-resolvable precipitation

Thus, we need the cumulus parameterization scheme to account for releasing conditional instability due to subgrid scale motion

- Grid-resolvable (Microphysics scheme : MPS) : Supersaturation → clouds
- Subgrid scale (Cumulus parameterization scheme : CPS) : CAPE removal → clouds

---

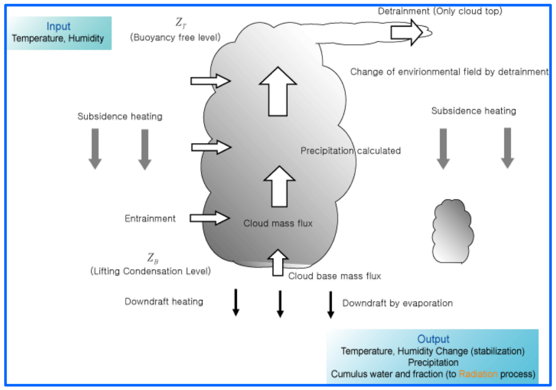
5. Cumulus parameterization scheme

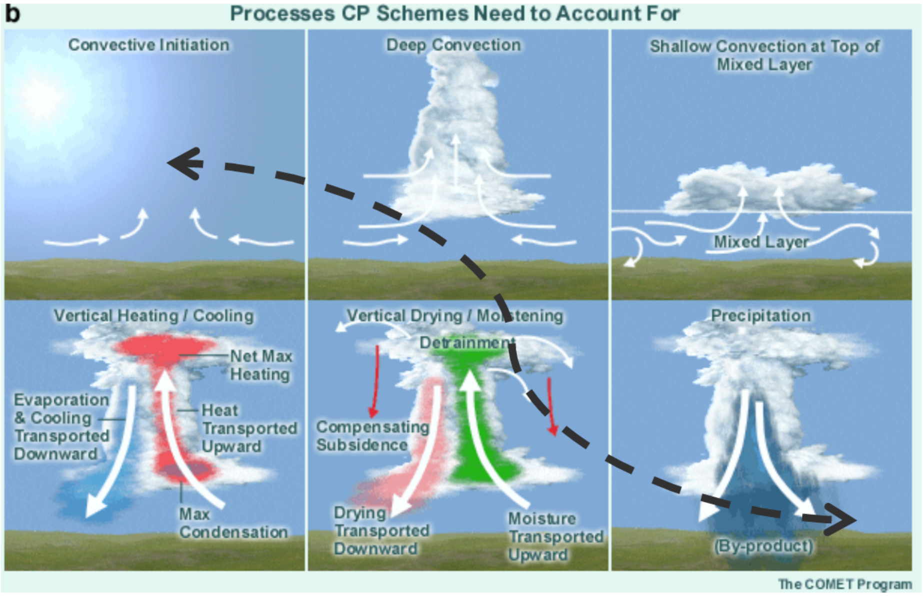
5.1 Concept

- represents deep precipitating convection and feedback to large-scale
- must formulate the collective effects of subgridscale clouds in terms of the prognostic variable in grid scale

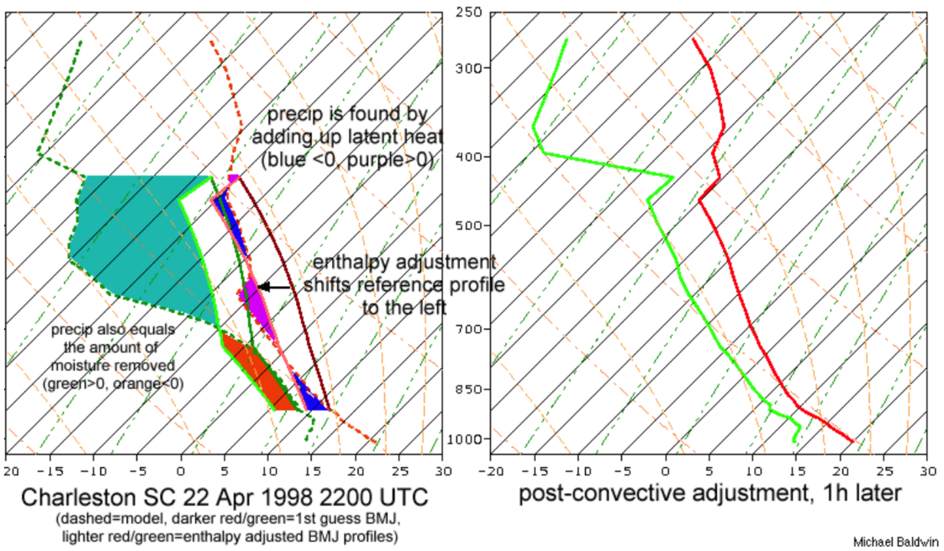
Parameterized convection
Deep convection
Subgridscale precipitation
Implicit precipitation

Jul 25, 2010, Seoul





CPS does not consider the detailed evolution of convection !



Charleston SC 22 Apr 1998 2200 UTC
(dashed=model, darker red/green=1st guess BMJ, lighter red/green=enthalpy adjusted BMJ profiles)

post-convective adjustment, 1h later

CPS consider changes in profile before and after convection

5.2 Kuo scheme (1965)

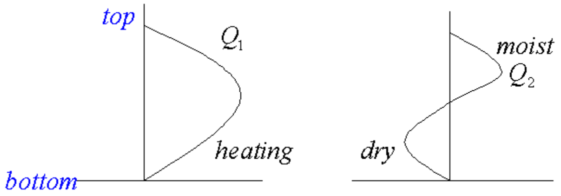
: Cloud is formed in proportional to column-integrated moisture convergence

$$M_t = -\frac{1}{g}\int_0^{P_s} \nabla\cdot(vq)dp + F_{g_s}$$

- Heating and moistening profiles (prescribed)

$$\frac{d\theta}{dt} = \frac{1}{\pi}\left[gL(1-b)M_t Q_1 + Q_r\right] \qquad \frac{dq}{dt} = -g(1-b)M_t Q_2$$

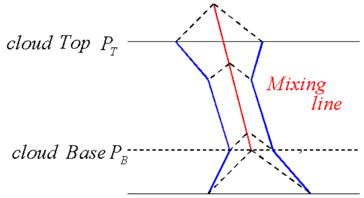
$$\int_0^{P_s} Q_1 dp = \int_0^{P_s} Q_2 dp = 1$$

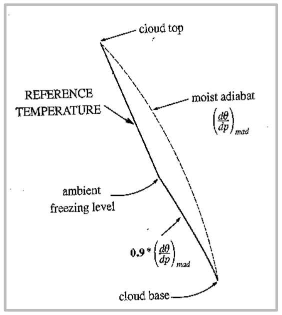


- Kuo type scheme has been widely used before 1990

5.3 Betts-Miller scheme (1986)

: adjust toward reference profiles that are based on observational evidence of convective equilibrium





Reference profile to be adjusted

$$\theta_R'(P) = \bar{\theta}(P_B) + \beta\ M_\theta(P - P_B)$$

$$\frac{\partial q}{\partial p} = \beta\left(\frac{\partial q}{\partial p_*}\right)_M \quad \beta = \frac{\partial p^*}{\partial p}$$

$$M_\theta = 0.85\left(\frac{\partial\theta^*}{\partial P^*}\right)_M$$

Energy Constraints : $\int_{P_B}^{P_{T+1}} C_P\left(T_R - \bar{T}\right)dp = 0$

- Convective tendencies & Precipitation

$$\left(\frac{\partial\bar{T}}{\partial t}\right)_{Cu} = \frac{T_R - \bar{T}}{\tau}$$

$$\left(\frac{\partial\bar{q}}{\partial t}\right)_{Cu} = \frac{q_R - \bar{q}}{\tau}$$

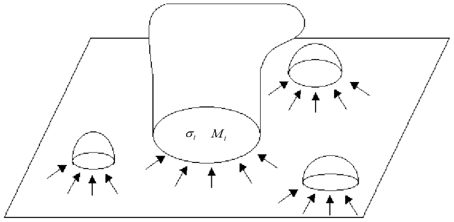
$$\Pr ecip = \int_{P_0}^{P_T}\left(\frac{q_R - \bar{q}}{\tau}\right)\frac{dP}{g}$$

5.4 Mass-flux schemes : Arakawa-Schubert (1974)

i) Concept

-- Mass flux approach, cloud ensemble, quasi-equilibrium
-- Theoretical frame work for CPS

- Area is large enough so that cloud ensemble can be a statistical entity
- Area is small enough so that cloud environment is approximately uniform horizontally



M_i : vertical mass flux through ith cloud

σ_i : fractional area covered by ith cloud

$M_c \equiv \sum_i M_i$: total vertical mass flux

environment

$\rho M = M_c + \tilde{M}$

: net mass flux/unit large-scale horizontal area

---

ii) Quasi-equilibrium : cloud forcing ~ large-scale adjustment

: CPS computes the warming (cooling) in the grid box due to adiabatic descent (ascent), rather than computing latent heat release in cloud models

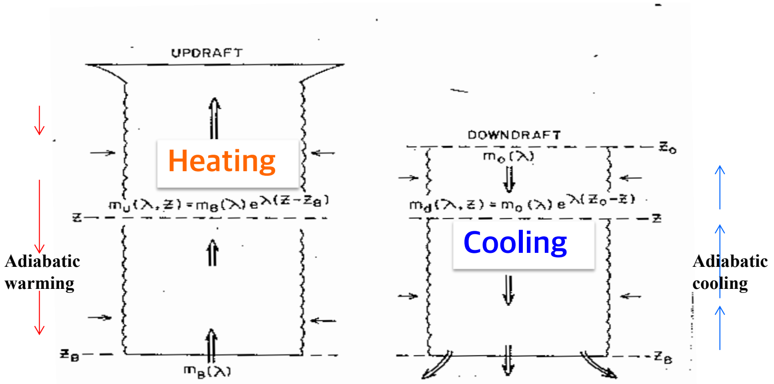


FIG. 4.10. Model for updraft and downdraft of cloud type λ (from Johnson 1976).

---

iii) Energy budget equations

Large-scale flux across grid box

Exchange of S between environment and clouds

$$\frac{\partial}{\partial t}\rho(1-\sigma_c)\tilde{s} = -\vec{\nabla}\cdot\overline{(\rho\tilde{V}S)} - \frac{\partial}{\partial Z}(\tilde{M}\tilde{S}) - \sum_i\left(\frac{\partial M_i}{\partial Z} + \rho\frac{\partial\sigma_i}{\partial t}\right)S_{i_b} - LE + \tilde{Q}_R \quad \text{: Environment MSE}$$

$$\frac{\partial}{\partial t}\rho\sum\sigma_i s_i = -\frac{\partial}{\partial z}\left(\sum_i M_i s_i\right) + \sum_i\left(\frac{\partial M_i}{\partial z} + \rho\frac{\partial\sigma_i}{\partial t}\right)S_{ib} + \sum_x\left(LC_i + Q_{Ri}\right) \quad \text{: in-cloud MSE}$$

$S_i : C_pT + gz$ (dry static energy) of ith cloud

$S_{ib} : C_PT + gz$ of the air entraining into or detraining from the ith cloud

C_i : condensation in the ith cloud

E : evaporation of liquid water in the environment

Q_r : Radiative heating

- Entrainment : $\frac{\partial M_i}{\partial z} + \rho\frac{\partial\sigma_i}{\partial t} > 0, \quad S_{ib} = \tilde{S}$

- Detrainment : $\frac{\partial M_i}{\partial z} + \rho\frac{\partial\sigma_i}{\partial t} < 0, \quad S_{ib} = S_i$

---

iv) Approximation

- Assume $\sigma_c \ll 1$, $\bar{s} \simeq \tilde{s}$ (grid-mean, grid-resolvable = environment)

$$\frac{\partial}{\partial t}\rho\bar{s} = -\nabla\cdot(\rho\bar{v}\bar{s}) - \frac{\partial}{\partial z}(\rho\bar{w}\bar{s}) - \overline{\nabla\cdot(\rho v\bar{s} - \rho\bar{v}\bar{s})}$$

$$+M_c\frac{\partial\bar{s}}{\partial z} - \sum_{\boxed{dc}}\left(\frac{\partial M_i}{\partial z} + \rho\frac{\partial\sigma_i}{\partial t}\right)\underline{(\delta_i - \bar{s})} - LE + \widetilde{\theta_R}$$

Detraining clouds

detrainiment, entrainment

Adiabatic warming due to hypothetical subsidence between the clouds

$$\frac{\partial}{\partial t}\rho\bar{q} = -\nabla\cdot(\rho\bar{v}\bar{q}) - \frac{\partial}{\partial z}(\rho\bar{w}\bar{q}) - \overline{\nabla\cdot(\rho\bar{v}q - \rho\bar{v}\bar{q})}$$

$$\underline{+M_c\frac{\partial\bar{q}}{\partial z}} - \sum_{dc}\left(\frac{\partial M_i}{\partial z} + \rho\frac{\partial\sigma_i}{\partial t}\right)(q_i - \bar{q}) - E$$

- Spectral cloud ensemble :

$$M_c(z) = \underline{\int_0^{\lambda_{\max}} m(z,\lambda)d\lambda}$$

Sub-ensemble

mass flux of between λ and $d\lambda+\lambda$

$$= \int_0^{\lambda_{\max}} \underline{m_B(\lambda)}\eta(z,\lambda)d\lambda$$

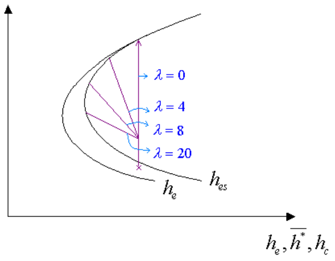
Mass flux at cloud base

$$\eta(z,\lambda) \equiv \frac{m(z,\lambda)}{m_B(\lambda)}$$; normalized subensemble mass flux

v) Closure

$$\frac{\partial m(z,\lambda)}{\partial z} = \mu(z,\lambda)\eta(z,\lambda)$$

$$\eta(z,\lambda) = e^{\lambda(z-zB)}$$; mass flux profile



- Cloud work function : measure of buoyancy

$$A(\lambda) = \int_{z_B}^{z_D(\lambda)} \eta(z,\lambda) g \frac{T_c(z,\lambda) - \overline{T}(z)}{\overline{T}} dz$$

- Q-G equilibrium

$$\frac{dA(\lambda)}{dt} = \underbrace{\left.\frac{dA(\lambda)}{dt}\right|_{LS}}_{} + \underbrace{\left.\frac{dA(\lambda)}{dt}\right|_{C}}_{} \simeq 0$$

Large-scale forcing
>0 : destabilized

Adjustment
<0 : stabilization

Kernel : Cloud scheme kinetic energy

$$K_{ij} = \frac{A_i' - A_i}{(m_B \Delta t)} \qquad \sum_j^{i_{\max}} K_{ij}(m_B \Delta t)_j + F_i = 0 \quad => m_B$$

$$\longrightarrow \text{compute } \frac{\partial \overline{s}}{\partial t}, \frac{\partial \overline{q}}{\partial t} \text{ with } \eta, m_B$$

5.5 Other schemes

*** Arakawa-Schubert type mass flux schemes**

Grell scheme (1993) : Updraft/downdraft couplet without lateral mixing to find the deepest cloud
Simplified AS (SAS, Han and Pan 2011): revised cloud physics from the Grell
Relaxed AS (RAS, Moorthi and Suarez 1992): linearized profile function

*** Other mass flux schemes : Low-level control convective schemes** (Stensrud 2007)

Kain and Fritsch (2004) : CAPE based sophisticated convective plume model
Emanuel (1991) : Stochastic mixing cloud model
Tiedtke (1989) : Large-scale moisture convergence (KUO) based mass flux (ECMWF IFS model)
Gregory-Rowntree (1990): Parcel buoyancy based turbulence in cloud model (UK model)

---

6. Shallow Convection

6.1 Concept

more vigorous vertical mixing of q and T above the mixed layer top. With the enhanced vertical eddy transport between LCL and inversion level, this process does not allow the excess moisture trapped near the surface in synoptically inactive regions (non-precipitating convection).

➔ Cooling and moistening above LCL and heating and warming below.

6.2 Classification

- Moist adjustment type : Betts and Miller (1993), Lock et al. (2000), Tiedtke (1983)
- Mass flux type : Kain (2004), Park and Bretherton (2009), Han and Pan (2011)

Tiedtke (1983)

$$\frac{\partial T}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K\left[\frac{\partial T}{\partial z} + \Gamma\right]\right)$$

$$\frac{\partial q}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K \frac{\partial q}{\partial z}\right)$$

Han and Pan (2011)

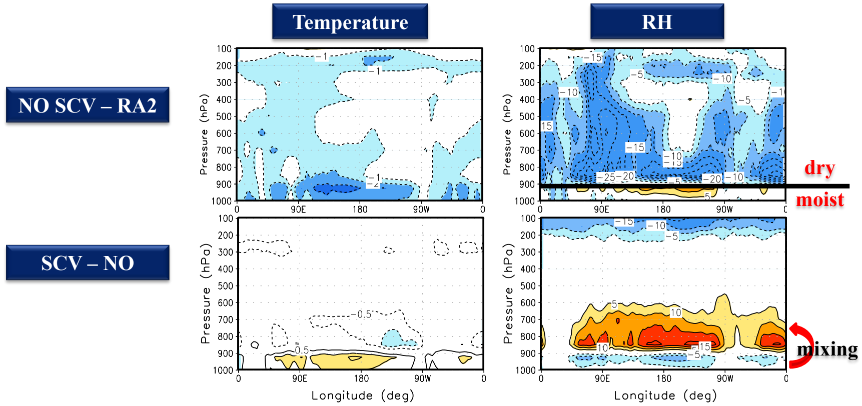
$$\frac{1}{\eta}\frac{\partial \eta}{\partial z} = \varepsilon - \delta$$

$$\frac{\partial(\eta s)}{\partial z} = (\varepsilon\overline{s} - \delta s)\eta$$

$$\frac{\partial[\eta(q_v + q_l)]}{\partial z} = \eta[\varepsilon\overline{q}_v - \delta(q_v + q_l) - r]$$

6.3 Impact of the shallow convection scheme (SCV)

: JJA 1996 simulation in a GCM



* SCV plays crucial role over the oceans.

7. Microphysics scheme

large-scale precipitation
grid-resolvable scale precipitation
explicit moisture scheme
cloud scheme
non-convective precipitation scheme

7.1 Concept

- Remove supersaturation after deep and shallow convection, and feedback to large-scale

7.2 Classification : according to the complexity in microphysics

i) Diagnostic : condensation, evaporation of falling precipitation

ii) Bulk microphysics : hydrometeors with size distribution in inverse-exponential function
- Single moment : predict mixing ratios of hydrometeors
- Double moment : + number concentrations
- Triple moment : + reflectivity

iii) Bin microphysics : divides the particle distribution into a number of finite size or mass categories.

7.3 Precipitate size distributions

Marshall and Palmer(1948) : exponential law
Heymsfield and Platt (1984) : Power law

$$N_R(D_R) = aD_R^{\ b}$$

The rain and snow particles are assumed to follow the size distribution derived by Marshall and Palmer(1948), and Gunn and Marshall(1958), respectively. The size distributions for both rain and snow are formulated according to an inverse-exponential distribution and its formula for rain can be expressed by

$$N_R(D_R) = N_{0R}\exp(-\lambda_R D_R)$$

for rain, where N_{0R} is the intercept parameter of the rain distributions. Slope parameter is

$$\lambda_R = \left(\frac{\pi\rho_w N_{0R}}{\rho q_R}\right)^{1/4}$$

Due to the size distribution in exponential manner (integration of precip for whole size results in constant), we can apply the bulk property microphysics terms.

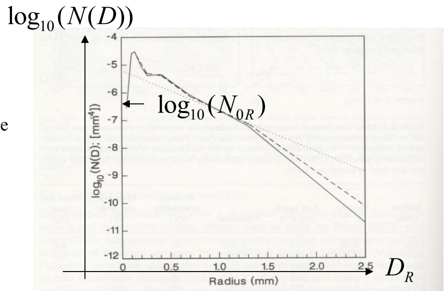


Fig. 9.21. Evolution of a drop spectrum in a subsaturated rainshaft including the effects of coalescence and breakup. The spectrum at the top of the rainshaft is Marshall–Palmer with a rainfall rate of 110 mm h^{-1}. (······), 0 km; (- - -), 1 km; (–), 2 km; R = 110 mm h^{-1}; t = 24 min. From Tzivion et al. (1989). *Journal of Atmospheric Sciences*, **46**, 21. American Meteorological Society. Reproduced with permission

$$\Gamma(x) = \int_0^\infty t^{x-1}\exp(-t)dt$$

$$\int_0^\infty D_R^{\ 4-1}\exp(-\lambda_R D_R)dD_R$$

$$= \Gamma(4)/\lambda_R^{\ 4}$$

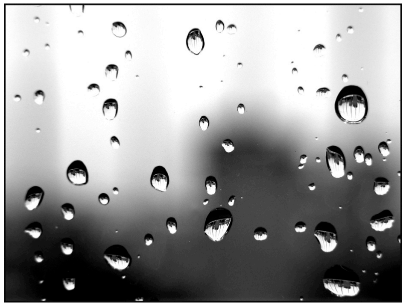
7.4 Bulk Method : 1-Moment versus 2-Moment

Mixing Ratio
(1-moment/ 2-moment scheme)

$$\left(\int\frac{dM(D_R)}{dt}dN_{DR}\right)/\rho = \frac{dq}{dt}(kgkg^{-1}s^{-1})$$

Number concentration
(2-moment scheme)

$$\left(\int\frac{d\Pr ob(D_R)}{dt}dN_{DR}\right) = \frac{dN}{dt}(m^{-3}s^{-1})$$



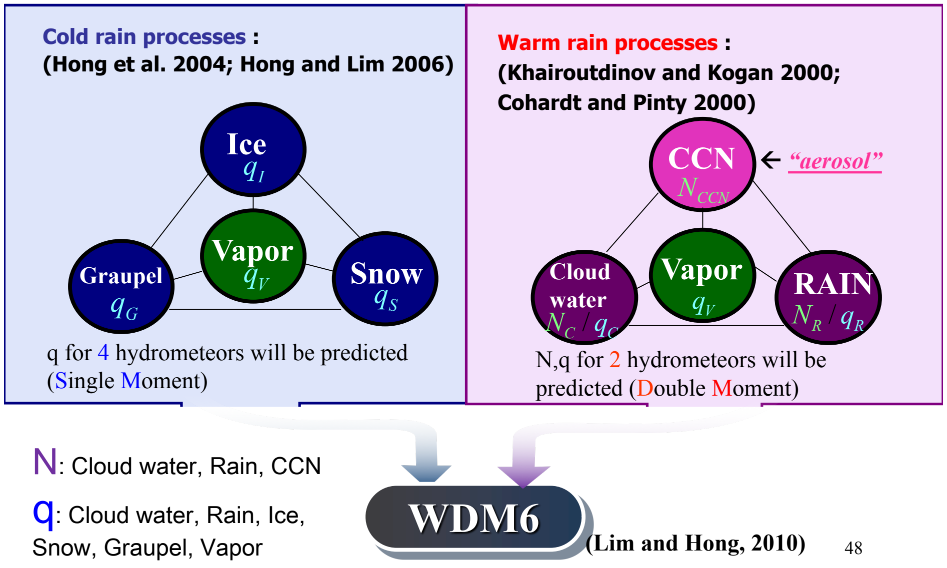
Single moment scheme

$$dN_{DR} = N_{0R}\exp(-\lambda_R D_R)dD_R$$

Double moment scheme

$$dN_{DR} = N_R\lambda_R^2(N_R)D_R\exp(-\lambda_R D_R)dD_R$$

7.5 Bulk Method : 1-Moment (WSM) versus 2-Moment (WDM)



Cold rain processes :
(Hong et al. 2004; Hong and Lim 2006)

q for 4 hydrometeors will be predicted (Single Moment)

Warm rain processes :
(Khairoutdinov and Kogan 2000; Cohardt and Pinty 2000)

N,q for 2 hydrometeors will be predicted (Double Moment)

N: Cloud water, Rain, CCN

q: Cloud water, Rain, Ice, Snow, Graupel, Vapor

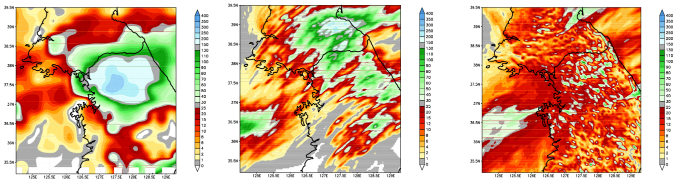
(Lim and Hong, 2010)

Resolution Dependency

Cut-off horizontal grid length: current status in research communities

- PBL : ~50 m (Mirocha, 2008 WRF workshop)
- GWDO : ~ 3 or 1 km (hydrostatic approximation)
- GWDC: ~ 3 or 1 km (go with CP)
- Cumulus parameterization : ~ 3 or 1 km (cloud resolving scale)

CPS gray-zone issue : A heavy rainfall simulated by WRF at 3 km



OBS **No CPS** **With CPS**

Gray-zone : partly resolved and partly parameterized (Hong and Dudhia 2012)

- CPS (1 km~10 km) : Gerard, Grell and Freitas, Arakawa and Ming, Pan and Han, Kwon and Hong
- PBL (100 m~1 km) : Honnert, Boutle, Shin and Hong
- Other processes such as shallow convection may also consider gray-zone

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Thanks for your attention !

Modeling is to understand what is happening in nature !