WRF Guangzhou Tutorial

Recent Enhancements in the Numerics of the WRF Dynamical Core

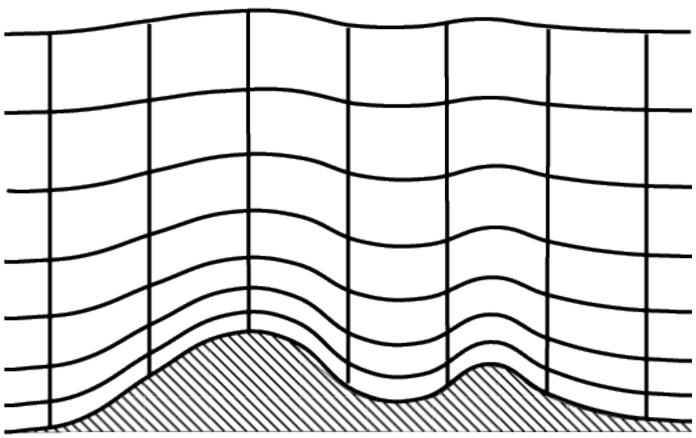
Hybrid Vertical Coordinate

Modifications for Moist LES Simulations

Joe Klemp

National Center for Atmospheric Research
Boulder, Colorado USA

WRF Terrain-Following Sigma (mass) Coordinate



Hydrostatic dry pressure: p_d

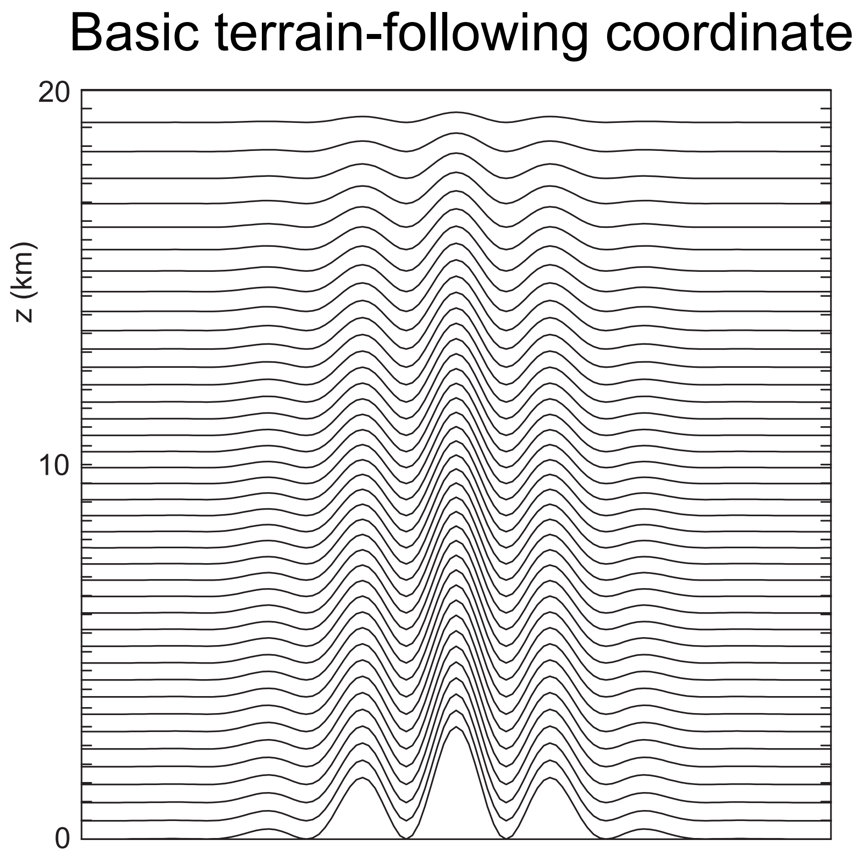
Vertical coordinate: $\eta = \dfrac{p_d - p_t}{p_s - p_t}$

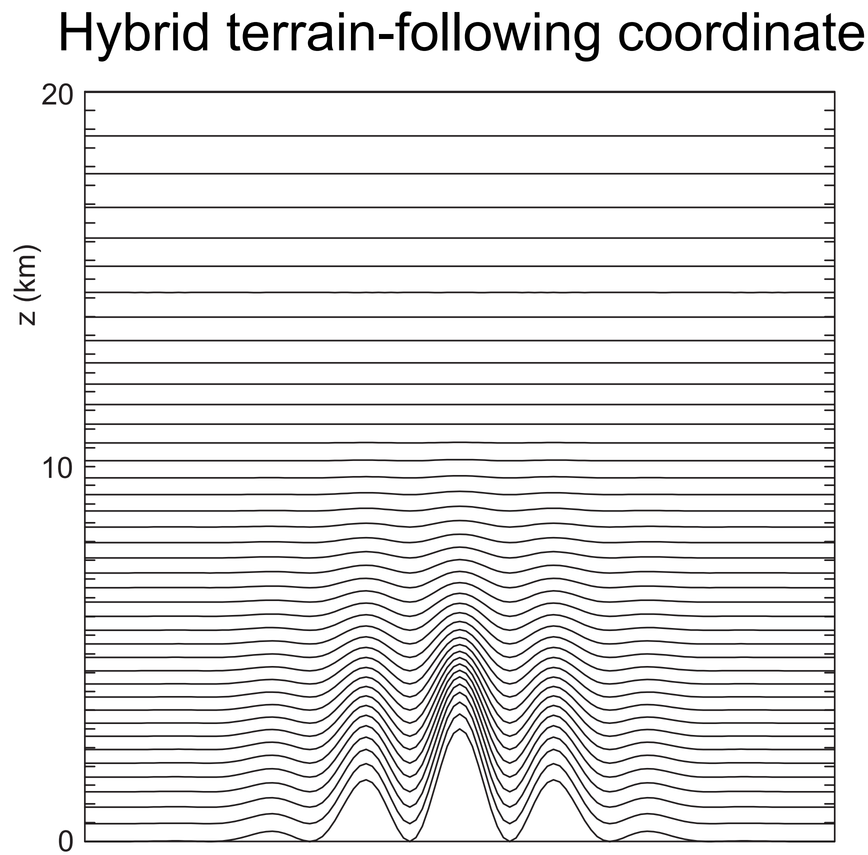
g (column mass per unit area): $\mu = p_s - p_t$

Layer mass per unit area: $\rho_d \Delta z = -\dfrac{1}{g}\Delta p_d = -\dfrac{\mu}{g}\Delta\eta$

Conserved prognostic variables: $\mu, \quad U = \mu u, \quad V = \mu v, \quad W = \mu w, \quad \Theta = \mu\theta$

WRF Hybrid Pressure Vertical Coordinate





WRF Hybrid Pressure Vertical Coordinate

Basic sigma coordinate

$$p_d = \eta(p_s - p_t) + p_t$$

$$\eta = \frac{p_p - p_t}{p_s - p_t}$$

Coordinate metric:

$$\mu_d(x,y,t) = \frac{\partial p_d}{\partial \eta} = \Delta p_c$$

Hybrid sigma coordinate

$$p_d = B(\eta)(p_s - p_t) + [\eta - B(\eta)](p_0 - p_t) + p_t$$

$B(\eta)$: Relative weighting between terrain-following and pure dry hydrostatic pressure coordinate:

$$\eta = \frac{p_d - p_t}{p_s - p_t} \quad \text{for } B(\eta) = \eta \quad \text{(basic sigma)}$$

$$\eta = \frac{p_d - p_t}{p_0 - p_t} \quad \text{for } B(\eta) = 0 \quad \text{(pure pressure)}$$

$$\mu_d(x,y,\eta,t) = \frac{\partial p_d}{\partial \eta} = B_\eta \Delta p_c + (1 - B_\eta)(p_0 - p_t)$$

$\Delta p_c = p_s - p_t$ ~ mass in each vertical column

WRF Hybrid Pressure Vertical Coordinate

Continuity equation for dry hydrostatic pressure:

$$\frac{\partial \mu_d}{\partial t} + (\boldsymbol{\nabla} \cdot \boldsymbol{V})_\eta = 0$$

Basic sigma coordinate

$$-\int_1^0 \frac{\partial \mu_d}{\partial t} d\eta =$$

$$\boxed{\begin{aligned} \frac{\partial \Delta p_c}{\partial t} &= \int_1^0 \boldsymbol{\nabla} \cdot \boldsymbol{V}_H d\eta \\ \Omega &= -\int_1^\eta \left(\frac{\partial \Delta p_c}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{V}_H \right) d\eta \end{aligned}}$$

Recover μ_d from: $\mu_d = \Delta p_c(x, y)$

Hybrid sigma coordinate

$$-\int_1^0 \frac{\partial \mu_d}{\partial t} d\eta = -\int_1^0 B_\eta \frac{\partial \Delta p_c}{\partial t} d\eta$$

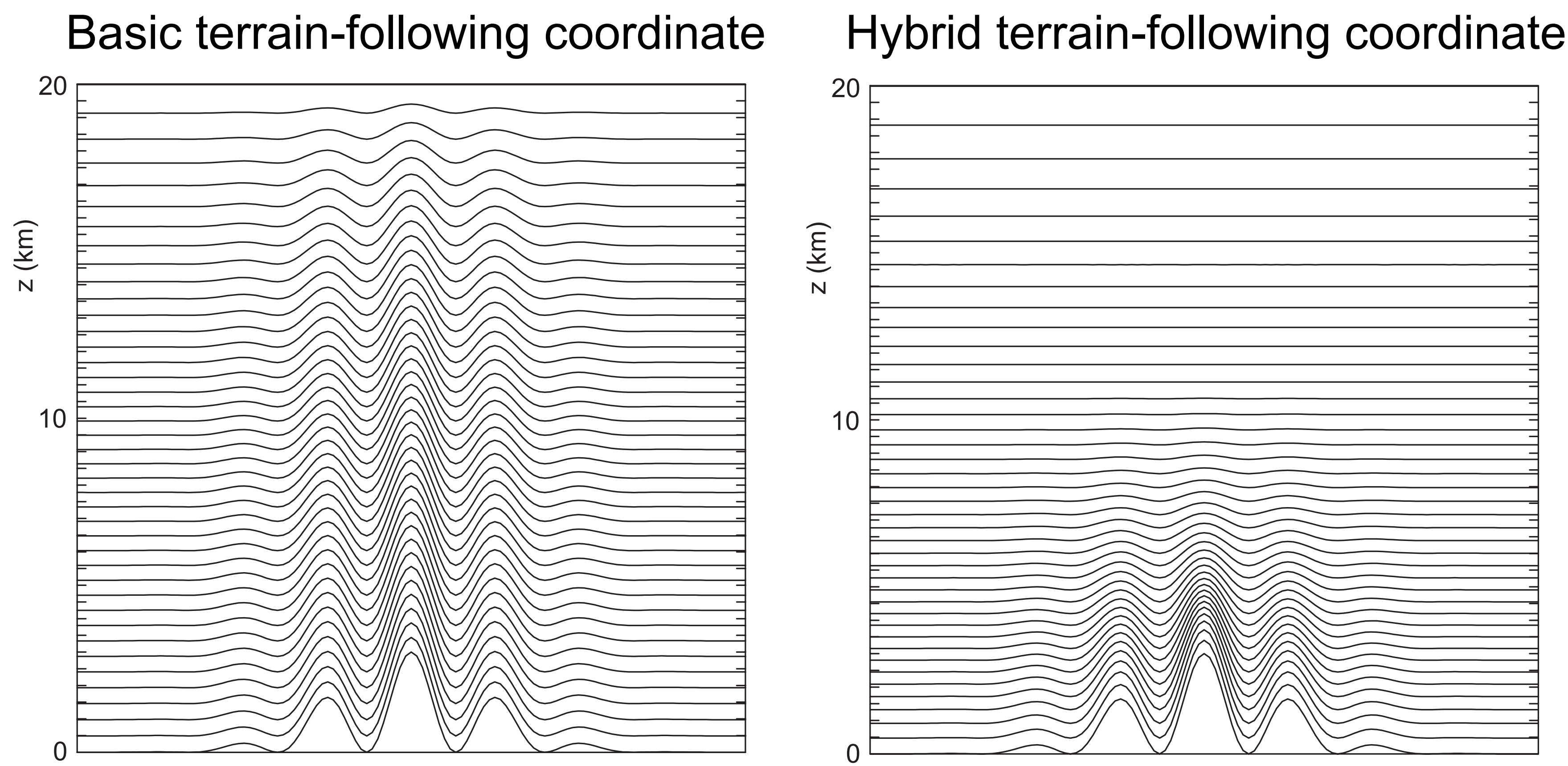
$$\boxed{\begin{aligned} \frac{\partial \Delta p_c}{\partial t} &= \int_1^0 \boldsymbol{\nabla} \cdot \boldsymbol{V}_H d\eta \\ \Omega &= -\int_1^\eta \left(B_\eta \frac{\partial \Delta p_c}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{V}_H \right) d\eta \end{aligned}}$$

$$\mu_d = C_1(\eta)\Delta p_c(x, y) + C_2(\eta)$$

$$\begin{aligned} C_1(\eta) &= B_\eta \\ C_2(\eta) &= (1 - B_\eta)(p_0 - p_t) \end{aligned}$$

(C_1, C_2 depend only on η)

WRF Hybrid Pressure Vertical Coordinate



WRF Hybrid Pressure Vertical Coordinate

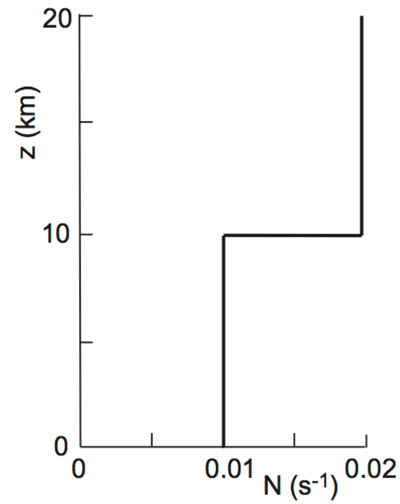
$$H = 2000 \text{ m}$$

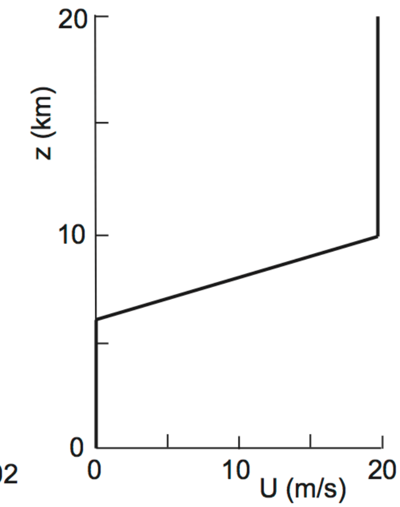
$$U_m = 20 \text{ m/s}$$

$$\Delta x = 1000 \text{ m}$$

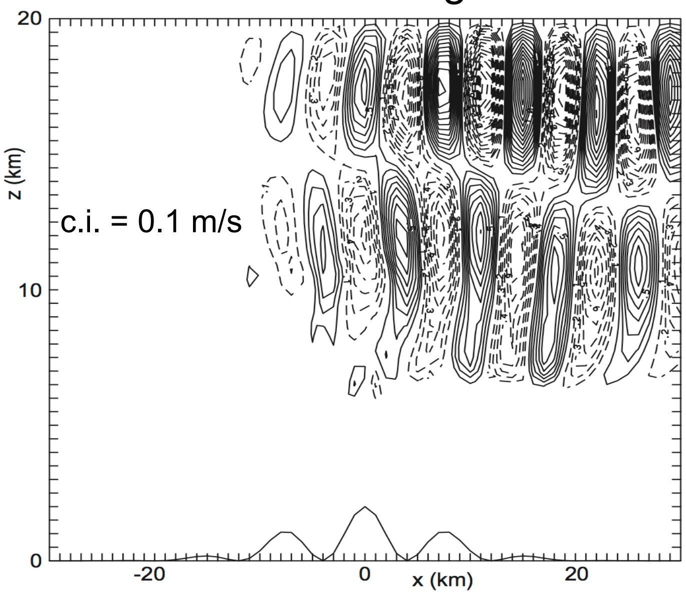
$$\Delta z = 500 \text{ m}$$

$$\Delta t = 12 \text{ s, ns} = 6$$

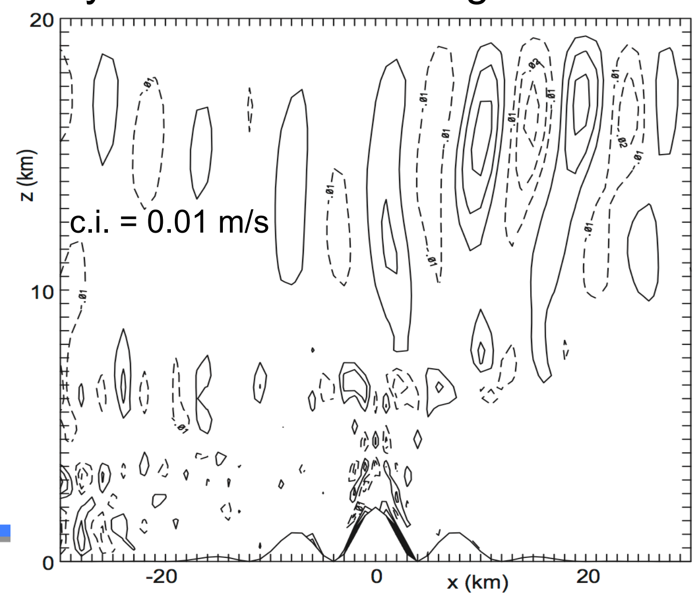




Basic terrain-following coordinate



Hybrid terrain-following coordinate

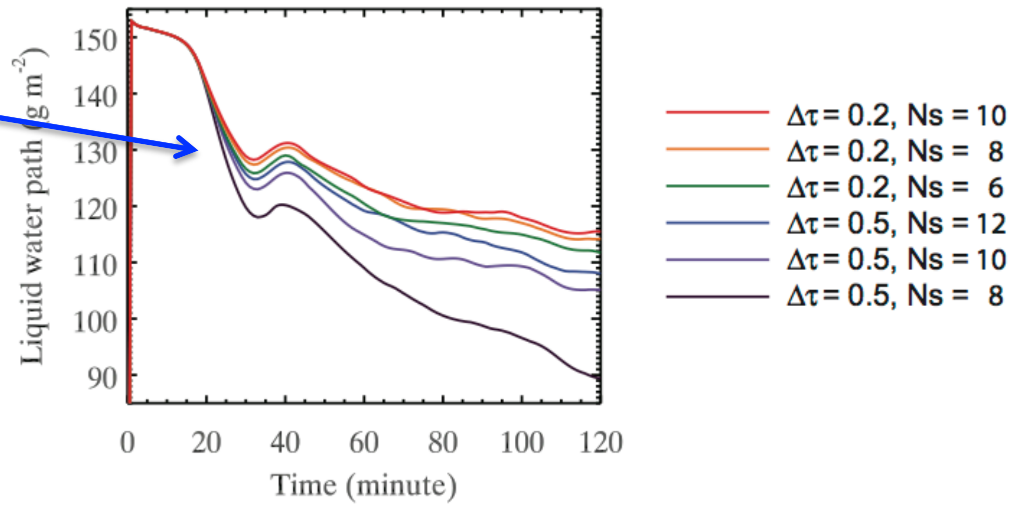


Improvement of WRF Dynamics for LES Applications

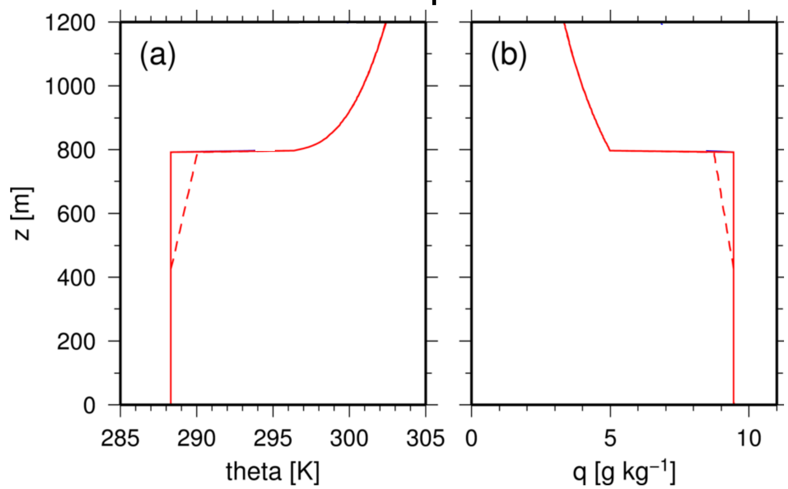
Strong Sensitivity to the Acoustic Time Step
in WRF LES Simulations of Low Clouds

Yamaguchi and Feingold (JAMES, 2012)

$\Delta x = 35$ m, $\Delta z \simeq 5$ m

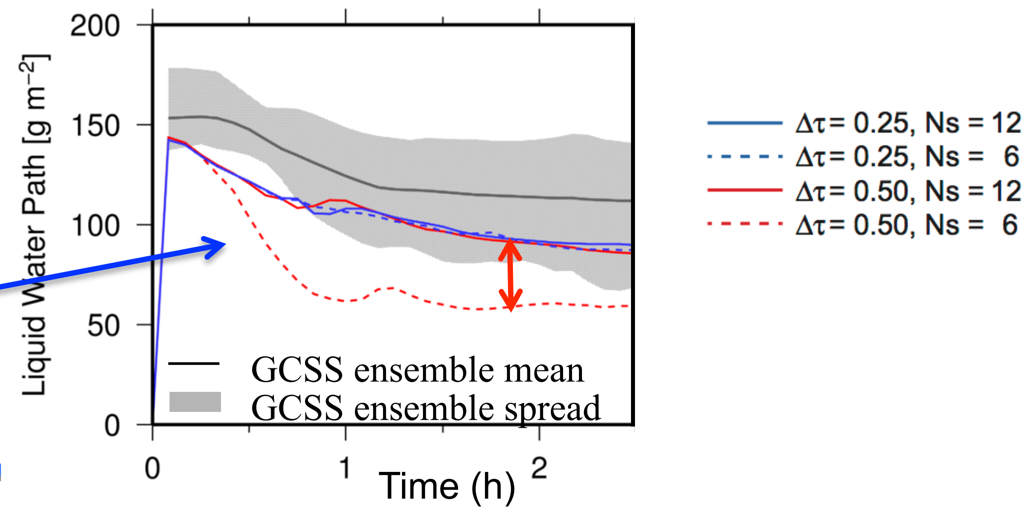


Initial profiles



Xiao et al. (JAMES, 2015)

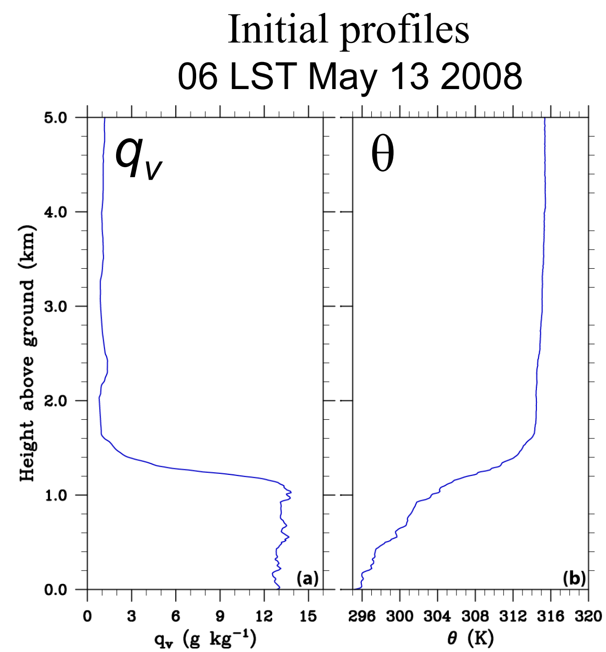
$\Delta x = 50$ m, $\Delta z \simeq 5$ m



DYCOMS II RF02 nighttime marine stratocumulus case

DOE ARM Southern Great Plains Statocumulus Case

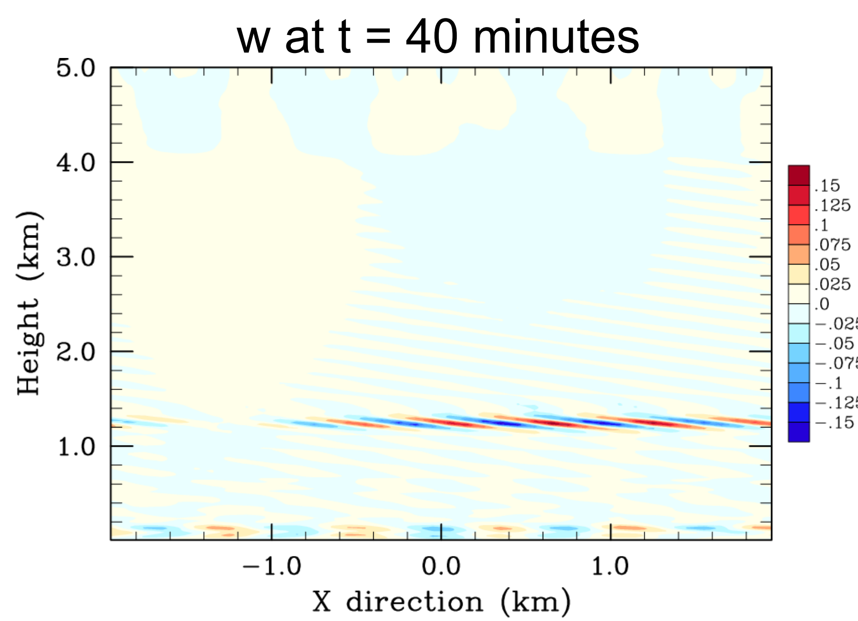
- Small time step sensitivity and numerical noise also occurs in WRF LES simulations for this case.
- Noise is present during first 2 hours of the simulation before cloud forms.



Stripped-down dycore test shows noise

- 2-D domain, 4 km (X) by 5 km (Z)
- dx = 100 m, dz = 20 m
- dt = 0.5 s, N_{ns} = 12
- No microphysics; No radiation; No surface forcing (only initial perturbation); No SGS mixing scheme; No Coriolis force

Xiao et al. (JAMES, 2015)



Acoustic Step Time Integration Using $\Theta = \mu_d \theta$

Vertical coordinate: $\eta = \dfrac{p_d - p_t}{\mu_d} \qquad \mu_d = p_{ds} - p_t$

$U^{\tau+\Delta\tau}$

$$\frac{\partial U}{\partial t} + \left(\mu_d \alpha \frac{\partial p}{\partial x} + \frac{\alpha}{\alpha_d}\frac{\partial p}{\partial \eta}\frac{\partial \phi}{\partial x}\right)^{\tau} = R_U^t$$

$\mu_d^{\tau+\Delta\tau}, \ \Omega^{\tau+\Delta\tau}$

$$\frac{\partial \mu_d}{\partial t} + \frac{\partial U}{\partial x}^{\tau+\Delta\tau} + \frac{\partial \Omega}{\partial \eta}^{\tau+\Delta\tau} = 0$$

$\Theta^{\tau+\Delta\tau}$

$$\frac{\partial \Theta}{\partial t} + \left(\frac{\partial U\theta^t}{\partial x} + \frac{\partial \Omega\theta^t}{\partial \eta}\right)^{\tau+\Delta\tau} = R_\Theta^t$$

$W^{\tau+\Delta\tau}, \ \phi^{\tau+\Delta\tau}$

$$\begin{cases} \dfrac{\partial W}{\partial t} + g\overline{\left(\mu_d - \dfrac{\alpha}{\alpha_d}\dfrac{\partial p}{\partial \eta}\right)}^{\tau} = R_W^t \\ \mu_d^t \dfrac{\partial \phi}{\partial t} + U^{\tau+\Delta\tau}\dfrac{\partial \phi}{\partial x}^t + \Omega^{\tau+\Delta\tau}\dfrac{\partial \phi}{\partial \eta}^t - g\overline{W}^{\tau} = R_\phi^t \end{cases}$$

$\alpha_d^{\tau+\Delta\tau}$

$$\alpha_d^{\tau+\Delta\tau} = -\frac{1}{\mu_d^{\tau+\Delta\tau}}\frac{\partial \phi}{\partial \eta}^{\tau+\Delta\tau}$$

$p^{\tau+\Delta\tau}$

$$p^{\tau+\Delta\tau} = p_0\left(\frac{R_d \Theta^{\tau+\Delta\tau}[1 + (R_v/R_d)q_v^t]}{p_0 \mu_d^{\tau+\Delta\tau}\alpha_d^{\tau+\Delta\tau}}\right)^{\gamma}$$

Acoustic Step Time Integration Using $\Theta_m = \mu_d \theta \left(1 + \frac{R_v}{R_d} q_v\right)$

Vertical coordinate: $\eta = \frac{p_d - p_t}{\mu_d} \qquad \mu_d = p_{ds} - p_t$

$U^{\tau+\Delta\tau}$ $\qquad \frac{\partial U}{\partial t} + \left(\mu_d \alpha \frac{\partial p}{\partial x} + \frac{\alpha}{\alpha_d} \frac{\partial p}{\partial \eta} \frac{\partial \phi}{\partial x}\right)^{\tau} = R_U^t$

$\mu_d^{\tau+\Delta\tau},\ \Omega^{\tau+\Delta\tau}$ $\qquad \frac{\partial \mu_d}{\partial t} + \frac{\partial U}{\partial x}^{\tau+\Delta\tau} + \frac{\partial \Omega}{\partial \eta}^{\tau+\Delta\tau} = 0$

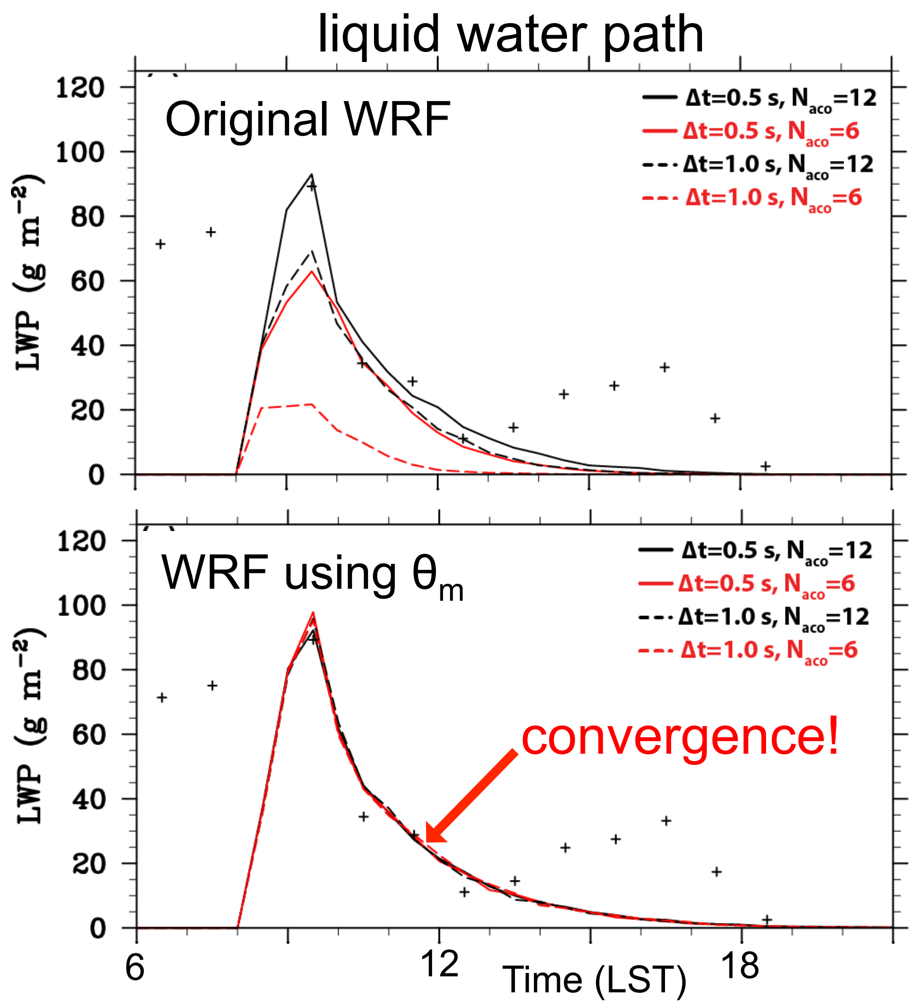
$\Theta_m^{\tau+\Delta\tau}$ $\qquad \frac{\partial \Theta_m}{\partial t} + \left(\frac{\partial U \theta_m^t}{\partial x} + \frac{\partial \Omega \theta_m^t}{\partial \eta}\right)^{\tau+\Delta\tau} = R_{\Theta_m}^t$

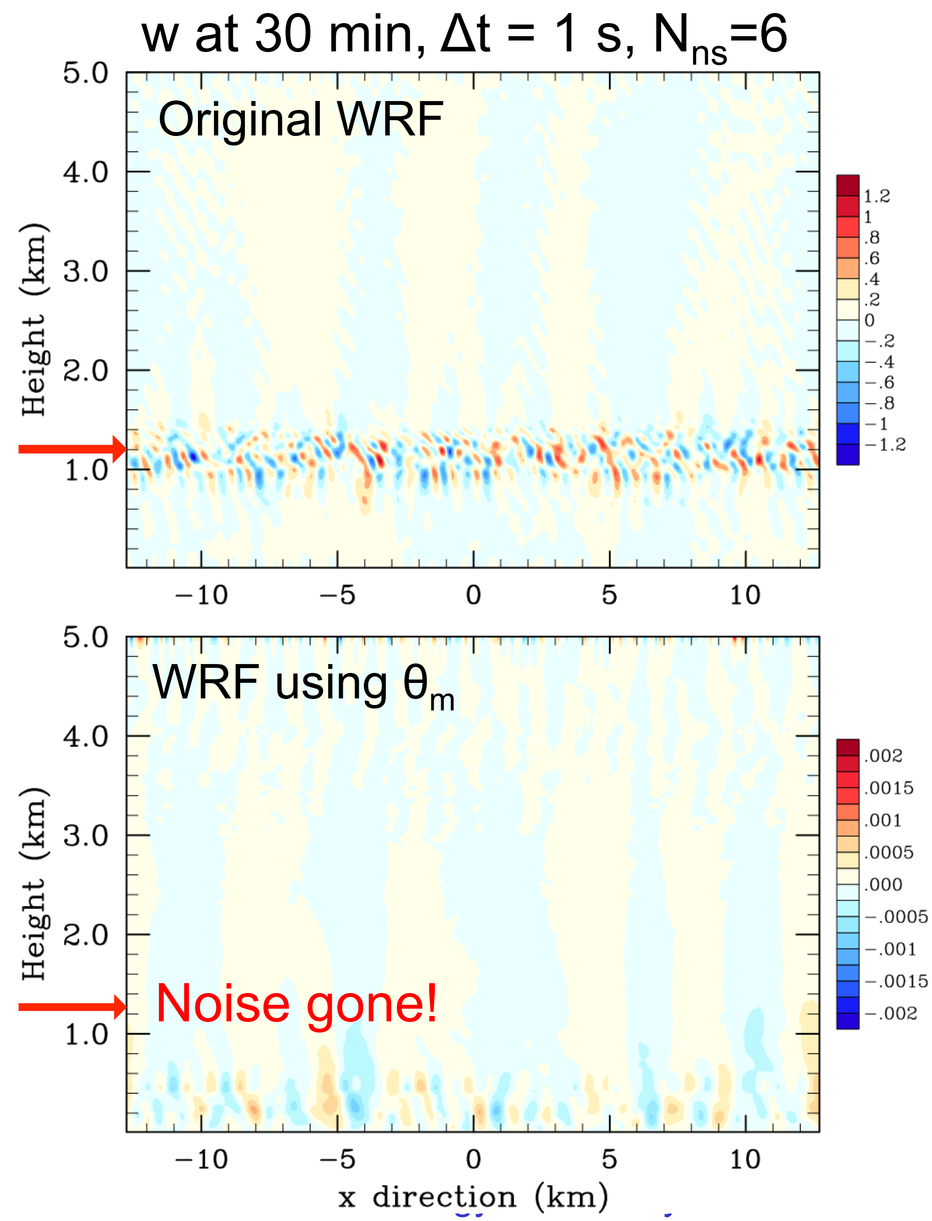
$W^{\tau+\Delta\tau},\ \phi^{\tau+\Delta\tau}$ $\qquad \begin{cases} \frac{\partial W}{\partial t} + g\overline{\left(\mu_d - \frac{\alpha}{\alpha_d}\frac{\partial p}{\partial \eta}\right)}^{\tau} = R_W^t \\ \mu_d^t \frac{\partial \phi}{\partial t} + U^{\tau+\Delta\tau} \frac{\partial \phi}{\partial x}^t + \Omega^{\tau+\Delta\tau} \frac{\partial \phi}{\partial \eta}^t - g\overline{W}^{\tau} = R_\phi^t \end{cases}$

$\alpha_d^{\tau+\Delta\tau}$ $\qquad \alpha_d^{\tau+\Delta\tau} = -\frac{1}{\mu_d^{\tau+\Delta\tau}} \frac{\partial \phi}{\partial \eta}^{\tau+\Delta\tau}$

$p^{\tau+\Delta\tau}$ $\qquad p^{\tau+\Delta\tau} = p_0 \left(\frac{R_d \Theta_m^{\tau+\Delta\tau}}{p_0 \mu_d^{\tau+\Delta\tau} \alpha_d^{\tau+\Delta\tau}}\right)^{\gamma}$

DOE ARM Southern Great Plains Stratocumulus Case





3-D domain 25.6×25.6×5 km^3,dx=dy=100 m, dz=20 m, Morrison 2-mom. microphysics, RRTMG radiation, Deardorff 1.5-order TKE

Xiao et al. (JAMES, 2015)

Idealized 2-D WRF Moist ABL Simulation, $\Delta x = 100$ m

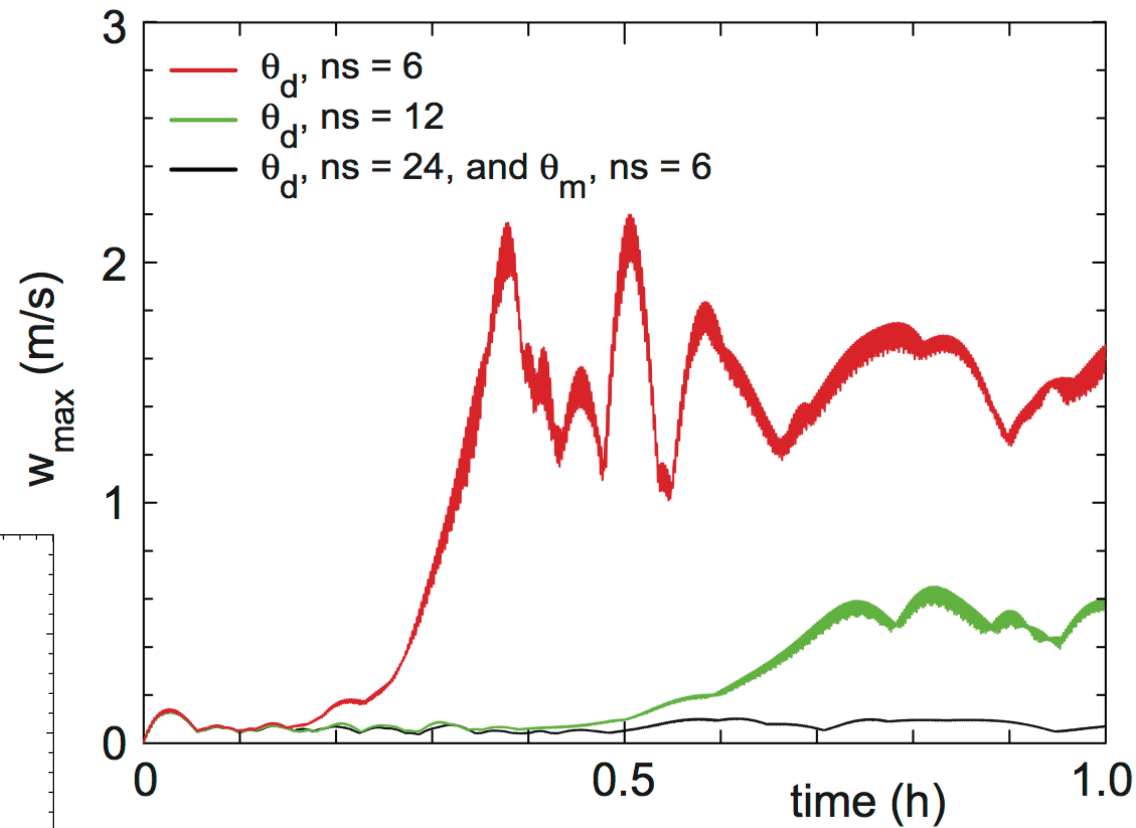
$U_m = 12$ m/s

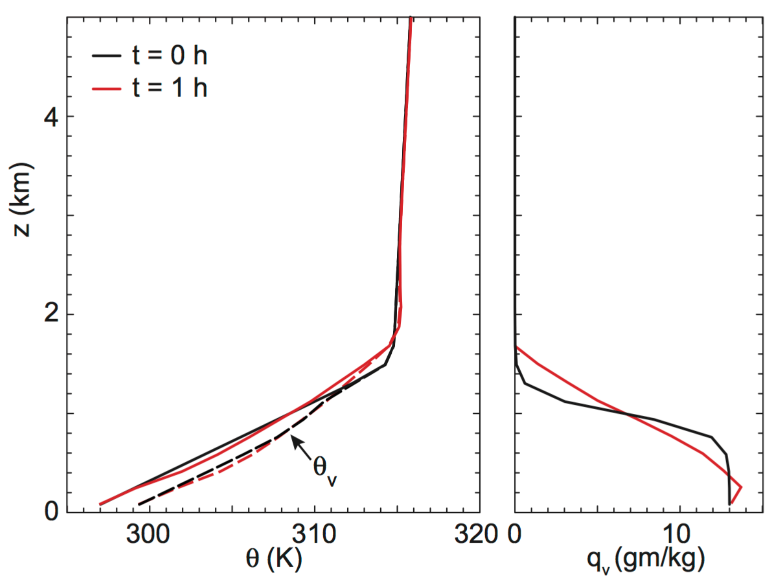
$\Delta x = 100$ m

$\Delta z = 200$ m

$\Delta t = 1$ s, ns = 6

No moist physics
No explicit diffusion





Using θ_m, the sounding profile at 1 h is unchanged from the initial sounding.

Idealized 2-D WRF Moist ABL Simulation, Δx = 1 km

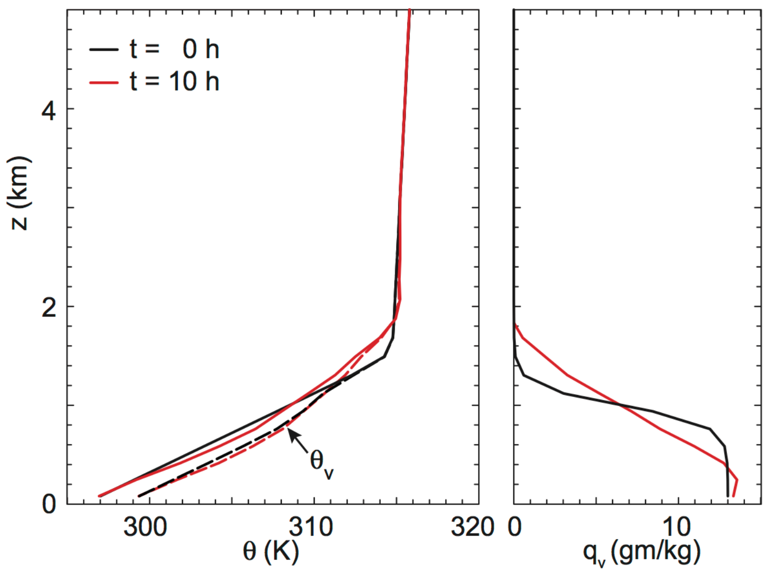
$U_m = 12$ m/s

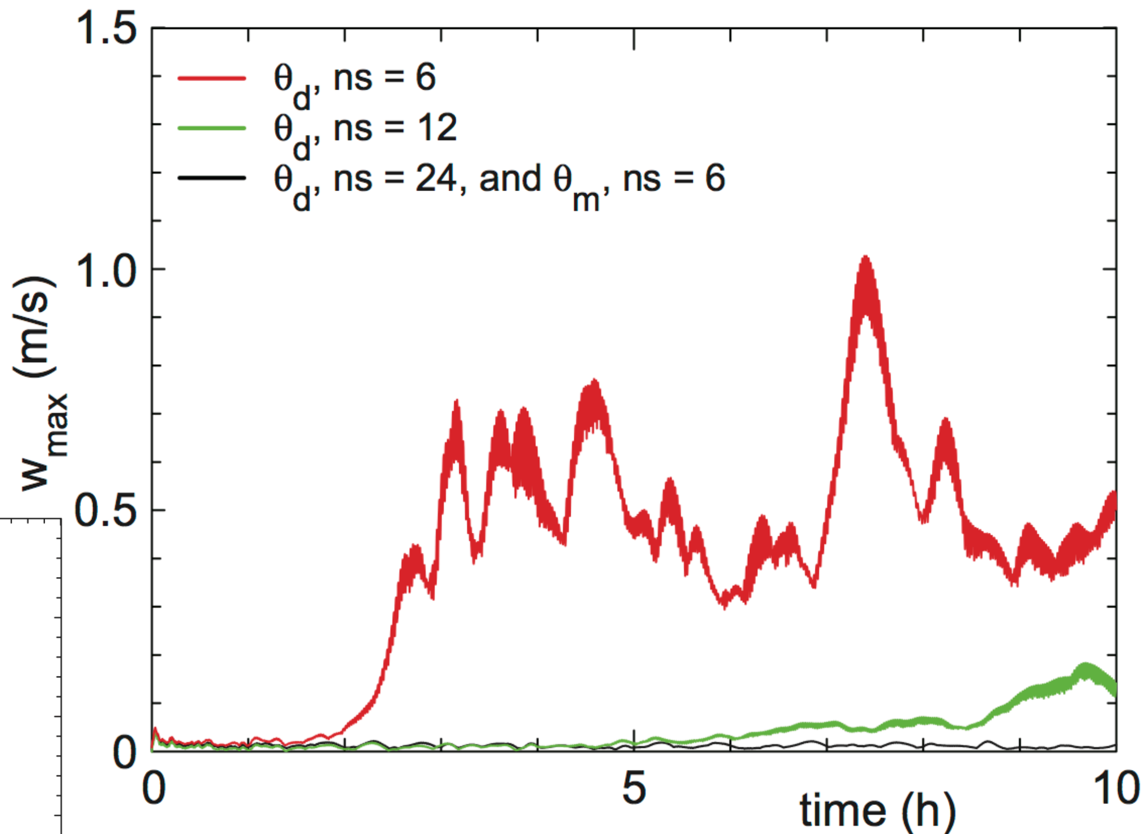
$\Delta x = 1000$ m

$\Delta z = 200$ m

$\Delta t = 10$ s, ns = 6

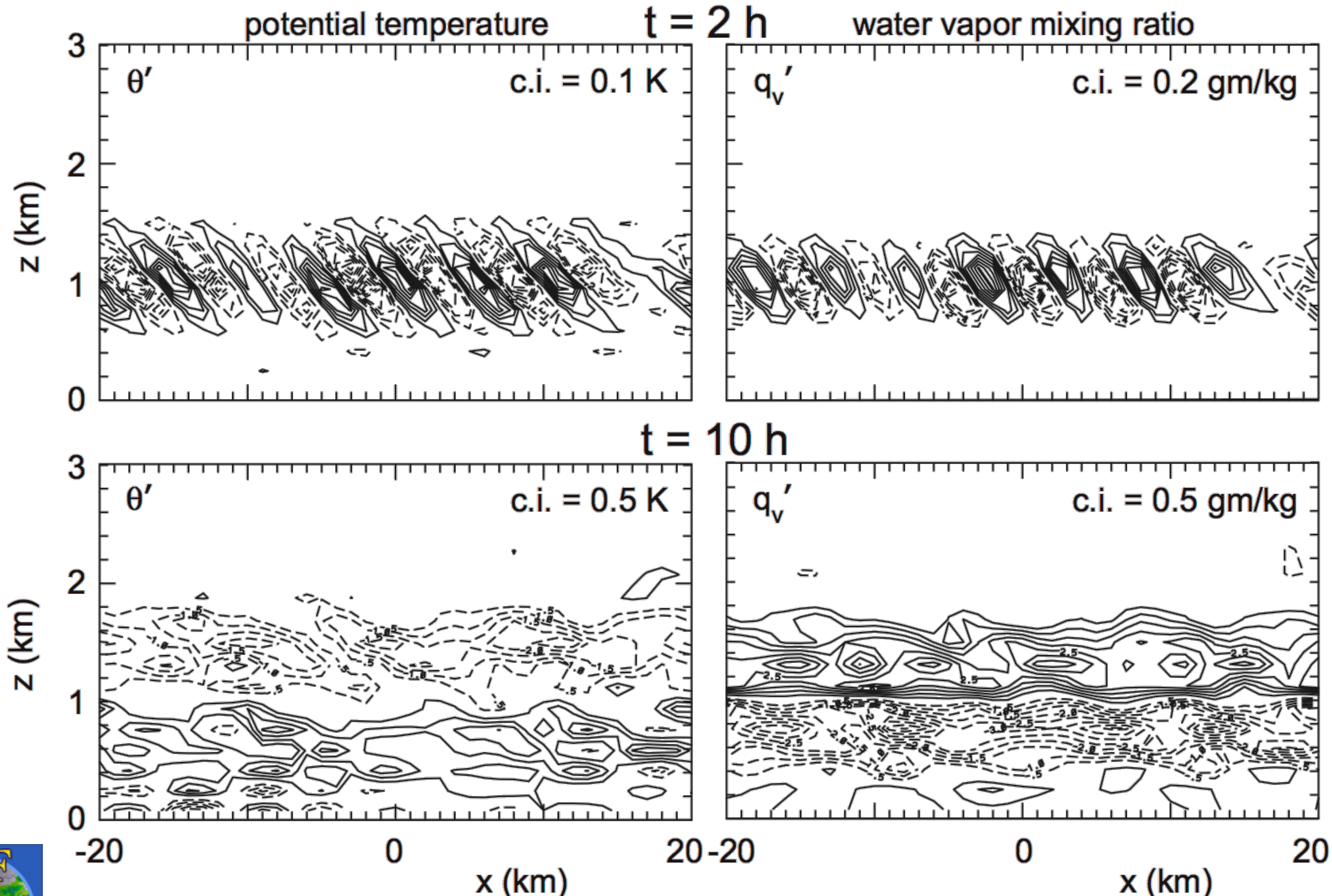
No moist physics
No explicit diffusion





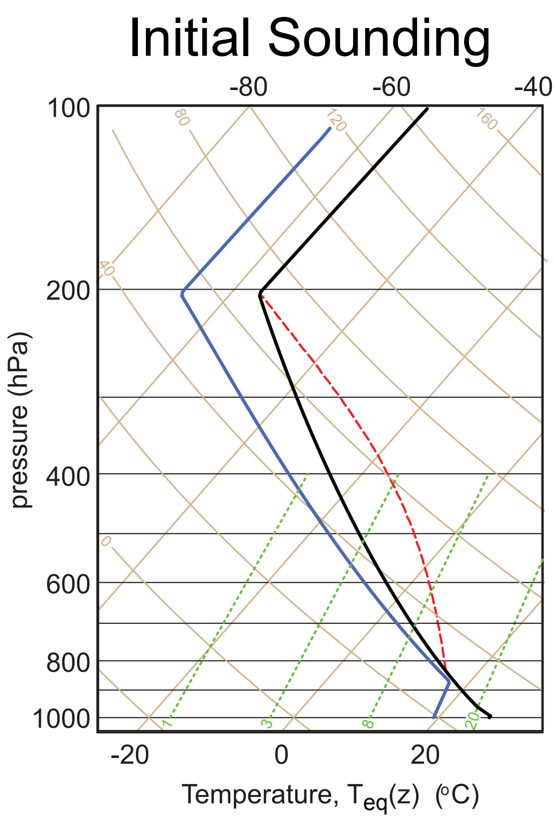
Using θ_m, the sounding profile at 10 h is unchanged from the initial sounding.

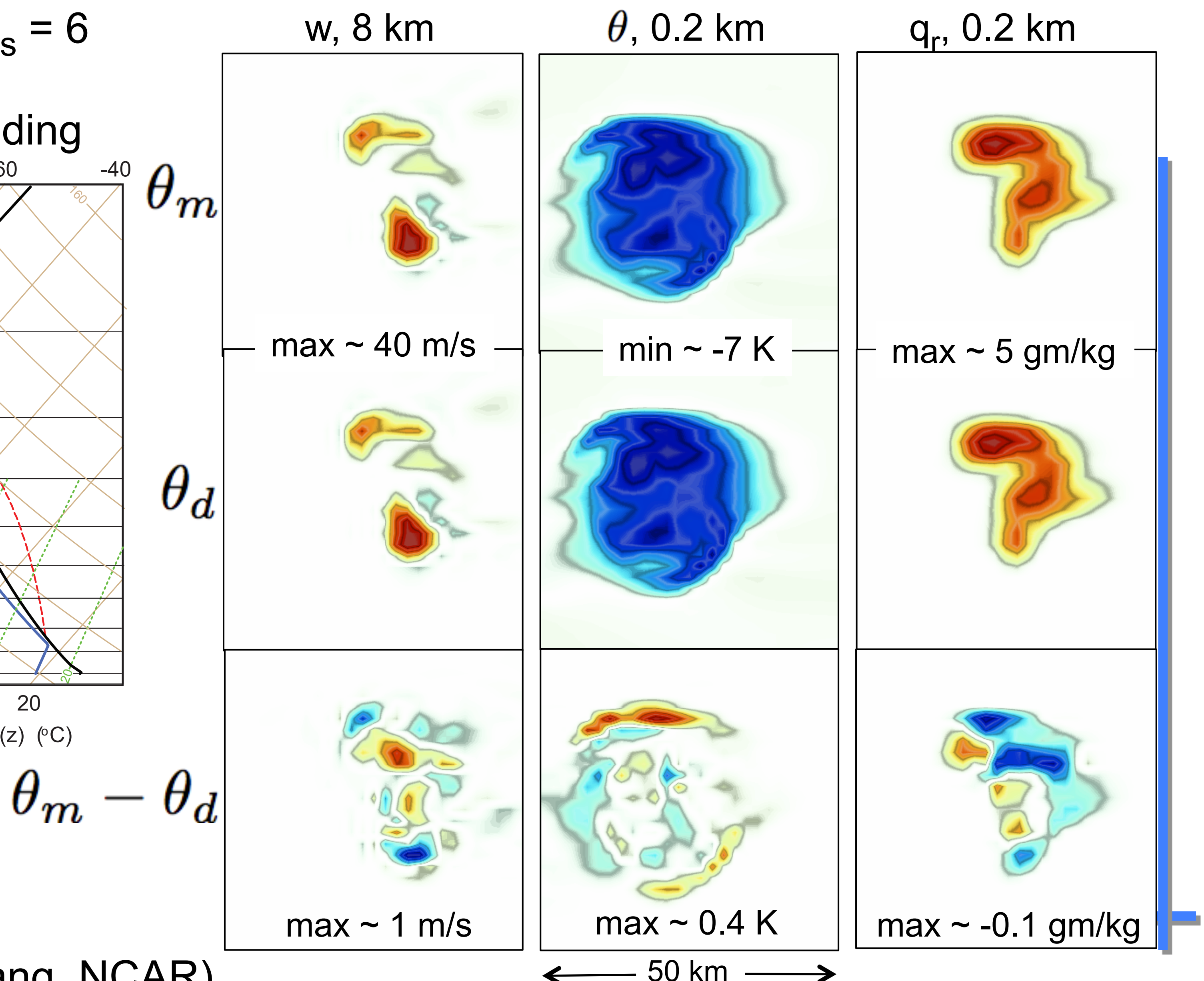
Idealized Test Case, 2-D WRF Prototype, Δx = 1 km



Idealized Supercell Simulation, dx = 2 km, t = 1 h

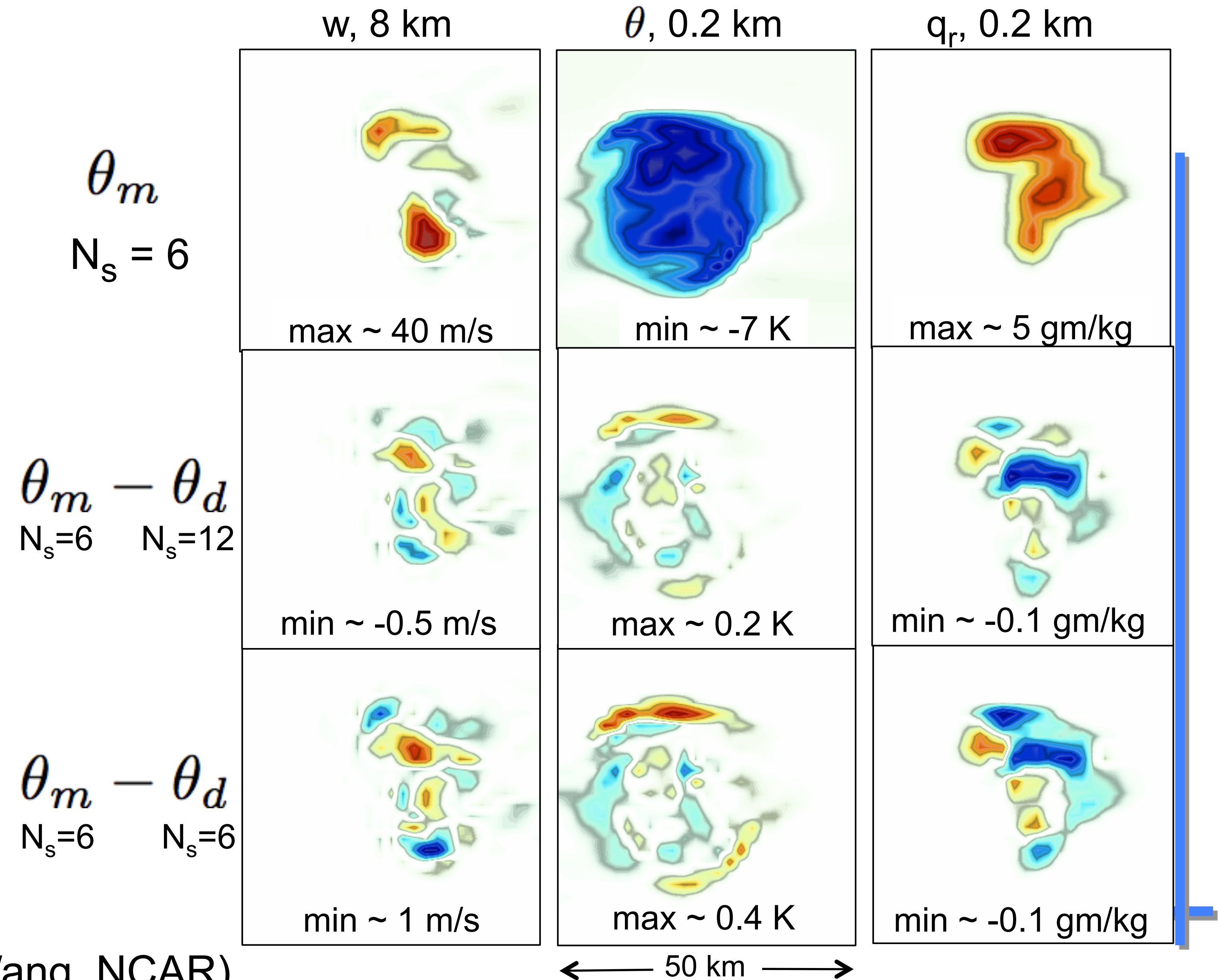
δt = 12 s, N_s = 6





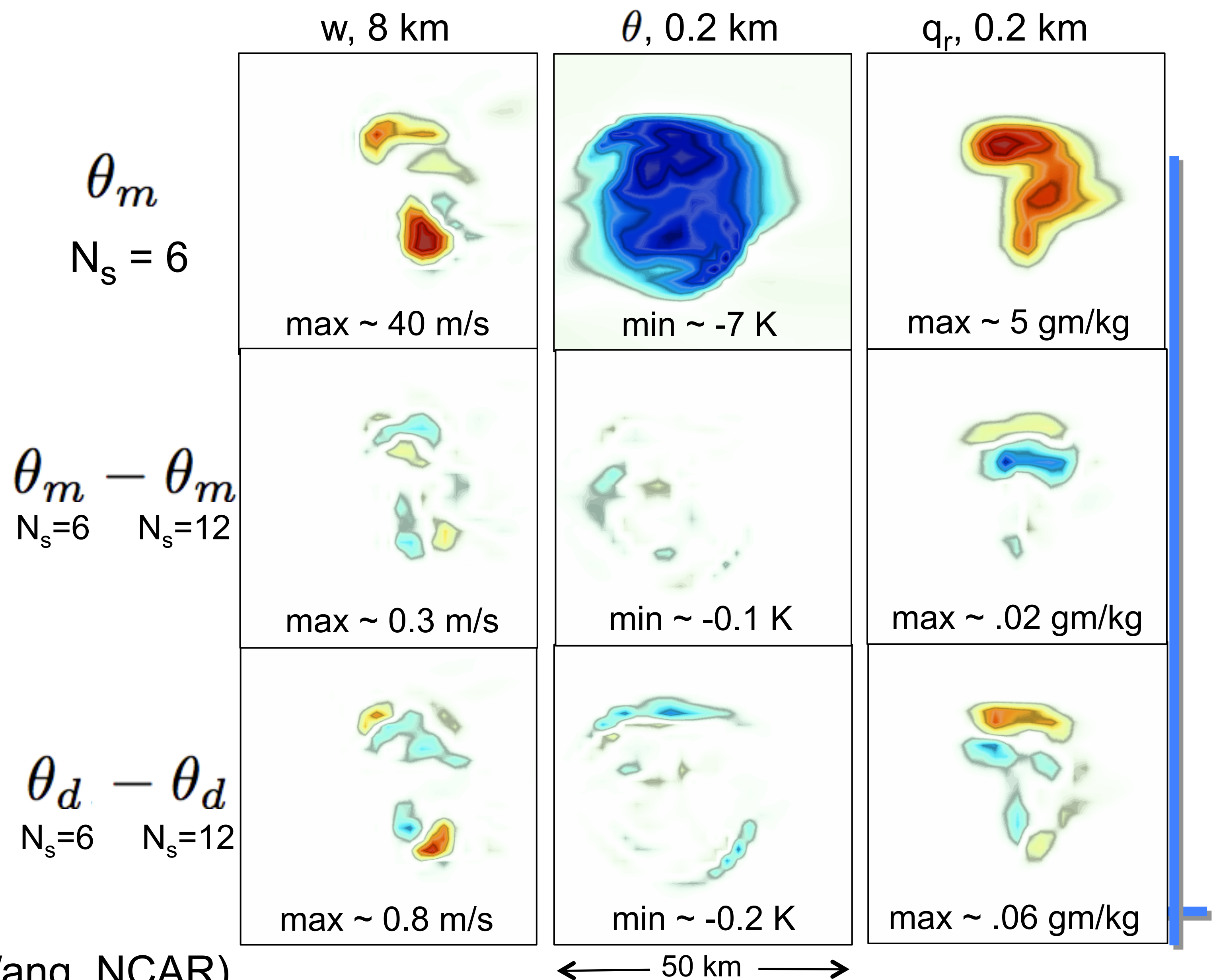
(Wei Wang, NCAR)

Idealized Supercell Simulation, dx = 2 km, t = 1 h



(Wei Wang, NCAR)

Idealized Supercell Simulation, dx = 2 km, t = 1 h



(Wei Wang, NCAR)

Summary

- In WRF LESs of low clouds, significant sensitivity of simulated cloud properties to the acoustic time step and the growth of numerical noise is found to occur at low levels in the vicinity of strong vertical gradients of water vapor.
- The sensitivity appears to be caused by the neglect of temporal variations in water vapor in computing the pressure during the acoustic time steps in the WRF time-split numerical integration scheme.
- This artificial behavior has been removed from WRF by solving for a moist potential temperature θ_m as a prognostic variable on the acoustic time steps (instead of θ), which removes the explicit appearance of q_v on the small time steps.
- Moist θ_m as a prognostic was variable released in WRF version 3.7
  - Set option use_theta_m = 1 (default = 0)
  - Include patch from WRF Web page “WRF Model Version 3.7: Known Problems and Fixes”
- The potential impact of θ_m in other WRF applications needs to be examined.
- A hybrid sigma coordinate in WRF should provide more accurate numerics, particularly in computing horizontal pressure gradients. (Hopefully available in WRF next year).