Development of the WRF Tangent Linear and Adjoint Models: Nonlinear and Linear Evolution of Initial Perturbations and Adjoint Sensitivity Analysis at High-Southern Latitudes

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1. Introduction

Development of the WRF tangent linear and adjoint models is an important step to set up the 4D-Var capability in the WRF variational data assimilation system (WRF-Var). During the past two years, significant efforts at NCAR have been devoted to this development. With the help of the automatic differentiation software TAF (Transformation of Algorithms in Fortran), the WRF tangent linear and adjoint models have been successfully coded, based on a simplified version of the Advanced Research WRF core (WRF-ARW).

Since the WRF-ARW is replacing the MM5 as the forecast model for AMPS (Antarctic Mesoscale Prediction System), the corresponding data assimilation component must be updated to obtain maximum benefits from the available observations in Antarctica. The WRF four dimensional variational data assimilation system (WRF 4D-Var) shares the same WRF-Var framework as WRF 3D-Var (Barker et al. 2004). It has the added benefits of using the model as a constraint; the implicit update of flow-dependent background field and the capability to assimilate data at the exact observation time.

As a feasibility study, we examined the nonlinear and linear evolutions of initial perturbations and performed sensitivity analysis of a cyclone on May 15, 2004 in Antarctica. WRF 4D-Var assimilation for the case will be conducted in the near future.

2. Development of the WRF tangent linear and Adjoint models

2.1 WRF simplified model

WRF is a sophisticated and fairly complicated model (the full nonlinear model will be referred to as WRF_NL, hereafter). We started with a basic, simplified version for its tangent linear and adjoint model development. This simplified nonlinear model (WRF_SN) excludes all the model physics and only simple diffusion (constant coefficient) is included.

The forecasts between WRF_NL and WRF_SN are compared. We carried out two experiments using the AMPS domain 1 configuration with the WRF SI initial conditions at 0000 UTC 15 May 2004 (Fig. 1). The 24-h forecasts of sea-level pressure (SLP) are shown in Fig. 2. It can be seen that the WRF_SN forecast captures the main features that the WRF_NL has produced. The major high/lows of the 24-h forecasts are located at almost the same locations in both experiments. The differences are their intensities. With model physics included, the WRF_NL predicted better forecasts compared with the observations. This indicates that WRF_SN can be used in the WRF 4D-Var system to obtain the main flow patterns. However, inclusion of model physics should be considered as a next step to obtain a more accurate 4D-Var analysis.

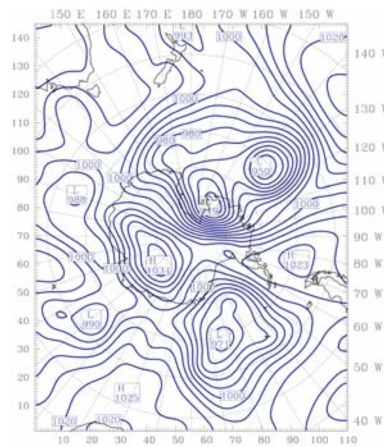

Fig. 1: Sea-level pressure distribution at 0000 UTC 15 May 2004 (interval: 5 hPa, AMPS domain 1)

2.2. Correctness verification of the WRF tangent linear and adjoint models

Based on the WRF_SN, TAF successfully generated its tangent linear and adjoint models. Since not all generated codes are correct, we had to conduct correctness verification. During the development, we kept close communications with the TAF developers. We reported any wrong behavior of TAF to FastOpt (http://www.FastOpt.com), which often results in bug fixes.

It must be pointed out that there are some subroutines that TAF incorrectly generated the tangent linear and adjoint codes. In these cases, we needed to carefully analyze the source code and manually correct the problem. We also found that TAF has difficulty in handling the WRF model integration scheme (3rd order Runge-Kutta large step and small acoustic steps). We made careful examinations of the integration schemes and developed a strategy to either store, or recalculate the basic state. To verify the correctness of the TAF generated codes, we developed the so-called tangent linear and adjoint check procedure following the method of Navon et al (1992).

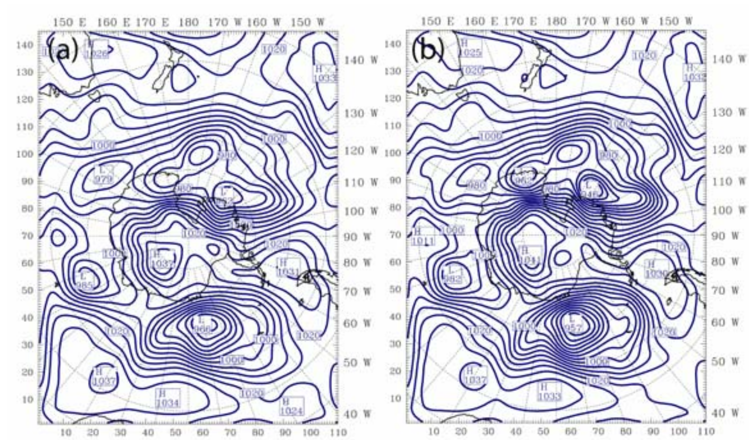

Fig. 2: 24-h forecasts of SLP (interval: 5 hPa) by (a) WRF_NL and (b) WRF_SN

Let *F(X)* denote a nonlinear subroutine (function), and *g_F(X, g_X)* and *a_F(X, a_X)* denote its tangent linear and adjoint subroutines (functions), respectively. The correctness of the tangent linear *g_F(X, g_X)* code can be tested against its nonlinear *F(X)* code using the Taylor-Lagrange formula:

$$\lim_{g\_x \to 0} \frac{F(X + g\_X) - F(X)}{g\_X^T \cdot g\_F(X, g\_X)} = 1 \qquad (1)$$

where *X* is the input vector and *g_X* is the perturbation on the input vector.

For each subroutine (function), the adjoint code *a_F(X, a_X)* is tested against the tangent linear code *g_F(X, g_X)* using the adjoint relation. Let *g_Y=g_F(X, g_X)* and *a_Y=a_F(X, g_Y*), the relation is:

$$< g\_Y, g\_Y > = < a\_Y, g\_X >. \qquad (2)$$

If the tangent linear and adjoint codes are formulated correctly, the above two relations should be held up to the machine accuracy. If the subroutine of the generated code fails the test, the testing will be performed from top to lower blocks (loops) until the source of error is identified.

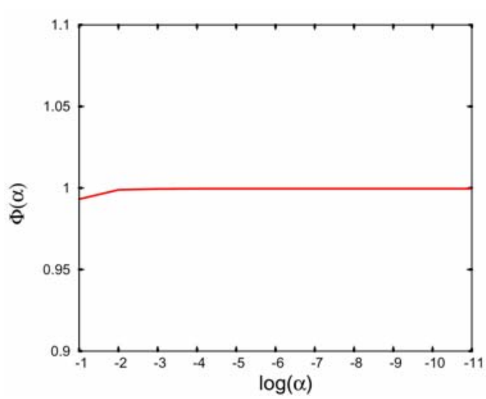

Fig. 3: Variation of $\phi(\alpha) = \frac{F(X+\alpha X) - F(X)}{\alpha X^T \cdot g\_F(X, \alpha X)}$ with respect to log(α)

For the selected Antarctica case (Fig. 1), we performed the tangent linear and adjoint check. The initial and boundary conditions produced by WRF SI are used as basic state input, the perturbations are assigned the value of its basic state multiplied by a scaling coefficient α (10^{-n}, n=1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11). The WRF integration subroutine ***solve_em***, and its tangent linear and adjoint passed the correctness check. Figure 3 shows the tangent linear testing results. For values of α that are small but not too close to the machine zero, $\Phi(\alpha)$ should be close to 1. The testing is performed on NCAR IBM machine with 64 bits precision. For the scaling coefficient α smaller than 10^{-2}, a unit value of $\Phi(\alpha)$ is obtained (Fig. 3). The result shown in Figure 3 verifies that the tangent linear code of ***solve_em*** subroutine is correct and can be safely used in the WRF 4D-Var system.

The adjoint code of ***solve_em*** also passed the adjoint check. In the adjoint relation, the left-hand side (lhs) involves only the tangent linear code, while the right-hand side (rhs) involves also the adjoint code. If lhs and rhs have the same value with machine accuracy, the adjoint code is correct compared with the tangent linear code. Using NCAR IBM machine with 64 bits precision, lhs and rhs for the May 15, 2004 Antarctica case are 0.2168395953702E+12 and 0.2168395953705E+12, respectively. This indicates that the adjoint code is right (For 64 bites machine, 12 digital accuracy is expected).

3. Nonlinear and linear evolutions of the initial perturbations at high-southern latitudes

In this section, we investigate the linearity of the initial error evolution and verify the validity of the WRF tangent linear model. This test is very useful for the setup of the 4D-Var assimilation windows and estimates of the forecast errors from the initial conditions.

The method used in this study is simple. First, we construct initial perturbations using the WRF 3D-Var increments (Fig.4) which are produced by assimilating 8 radiosonde soundings in Antarctica at 0000 UTC 15 May 2004. Two parallel nonlinear forecasts are conducted, one without the initial perturbations, and the other with the increments added to the initial conditions. The forecast differences of the two parallel runs are analyzed to evaluate the nonlinear evolution of the initial perturbations (Fig. 5a, b and c). Meanwhile, we applied the initial perturbations (Fig. 4) to the WRF tangent linear model, and the results of the WRF tangent linear model integration is used to analyze the linear evolution of the initial perturbations (Fig, 5d, e and f). Fig. 5 shows the comparison results of the nonlinear and linear evolutions of the initial perturbations at 24-h forecast.

The 24-h tangent linear solutions displayed in Fig. 5d, e and f are very similar to the difference fields of the two nonlinear integrations (fig. 5a, b and c). The pattern similarity between the linear and nonlinear solutions is greater than the similarity in amplitudes. The amplitude of the forecasted perturbations from the tangent linear model is slightly larger than that from the difference of the two nonlinear runs.

These results undoubtedly show that the initial perturbation evolution is well represented by the WRF tangent linear solutions up to at least 24 hours for the 90-km resolution domain at high-southern latitudes. In fact, we performed another set of experiments with a mid-latitude cyclone case, and the conclusion is also valid. When the model resolution is reduced, the validity of the WRF tangent linear approximation to represent the initial data uncertainties is also reduced. It must be noted that the basic state of the WRF tangent linear is updated every time step from the WRF_SN in this study. Additional examinations on the basic state update using WRF_NL and different update frequency are under way. All these studies can certainly provide useful information for the WRF 4D-Var setup, especially for the 4D-Var assimilation time windows configuration.

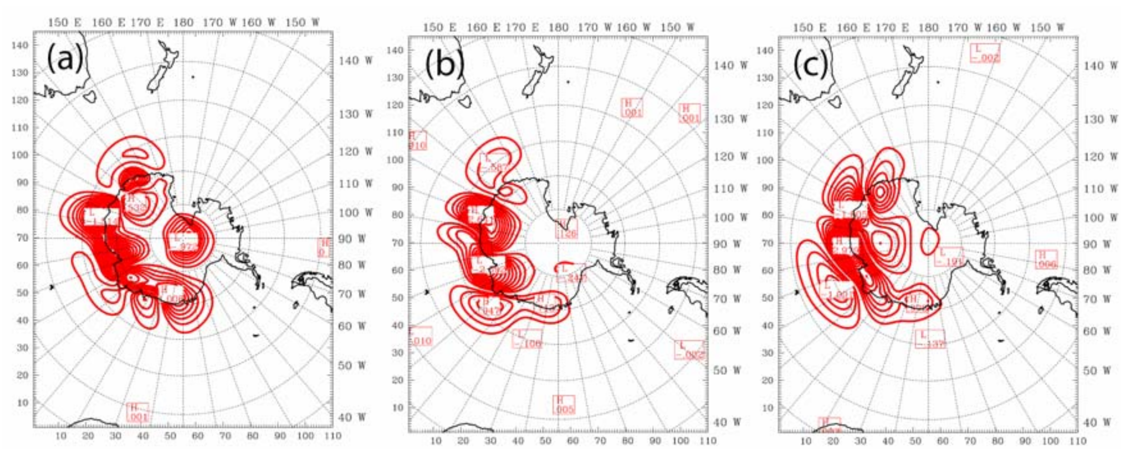

Fig. 4: Initial perturbations at 500 hPa (WRF 3D-Var increments by assimilating 8 radiosonde station data) at 0000 UTC 15 May 2004: (a) $\delta\theta$, (b) δu, and (c) δv

4. Adjoint sensitivity analysis

The adjoint technique is a computationally efficient method to determine the sensitivity of a forecast with response to model initial conditions (Errico and Vukicevic 1992; Xiao et al. 2002). It can be used to locate high-sensitivity regions where small perturbations can have relatively large effects on forecast features.

In sensitivity analysis, the model output of interest is usually referred to as the system's response (Zou et al. 1993). Sensitivity is a measure of the effect of changes in input variables on a selected model response. For the Antarctic cyclone, we consider a functional response $R(X)$ of the form

$$R(X) = -\sum_{i,j} \mu(i,j) \,, \qquad (3)$$

where μ **(x,y)** represents the mass within the column in the model domain at (x,y) at a specific forecast time, and the summation includes the model grid points centered at the surface low at that time. Using the WRF adjoint model integration, the gradient of the defined response (adjoint sensitivity) can be calculated.

The sensitivity of the response $R(X)$ to variations in the initial conditions X is defined by

$$VR(X, \Delta X) = \hat{X}^T \Delta X \,, \qquad (4)$$

where $\hat{X}$ is obtained by integrating the adjoint model backward with forcing at the time of interest. The adjoint sensitivity is then examined for the initial conditions as determined by a backward integration of the adjoint model over a specific time period. With the analysis, we can study the sensitivity to each variable of the initial conditions. If an initial condition in the positively sensitive area is increased, the predicted cyclone intensity will be increased (stronger). It is vise versa for the negatively sensitive area. This is true assuming the linear approximation is valid. The adjoint sensitivity analysis for the Antarctic cyclone will be shown in our presentation.

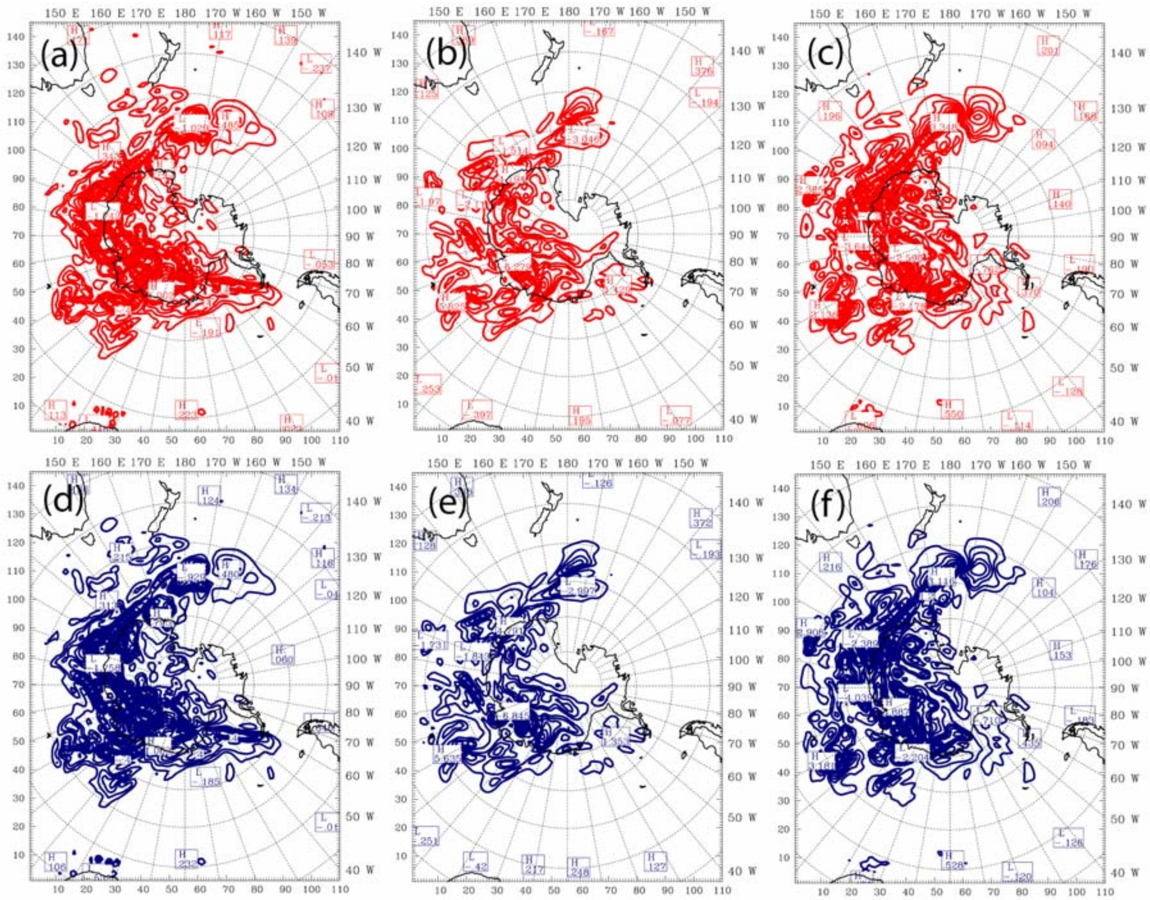

Fig. 5: The difference of (a) potential temperature, (b) u component and (c) v component of wind between two parallel WRF_SN runs and the increment of (d) potential temperature, (e) u component and (f) v component of wind by WRF tangent linear model integration at 500 hPa at 0000 UTC 16 May 2004 (24-h forecast)

5. Summary and conclusions

The WRF tangent linear and adjoint models are crucial components of the WRF 4D-Var system. The up-to-date technical achievements toward this goal are summarized as follows:

- We performed simplifications of the WRF model, to facilitate the development of the WRF tangent linear and adjoint models. All the physics are excluded in the WRF_SN except the horizontal diffusion. The numerical forecasts of the May 15, 2004 Antarctic cyclone case show that the major feature predicted by WRF_NL can be captured by the WRF_SN.
- The WRF tangent linear and adjoint models have been successfully produced by the automatic differentiation software TAF. The generated codes are revised and bug-fixed. The tangent linear and adjoint correctness are verified.
- The study of the nonlinear and linear evolution of the initial perturbations for the Antarctic cyclone indicates that the linearity of the developed WRF tangent linear model is valid for at least 24 hours for the AMPS domain 1. This indicates that major portion of the forecasted perturbations can be described by the tangent linear model solutions for at least 24 hours with the 90 km model resolution.
- With the developed WRF adjoint model, the sensitivity can be examined to study the most sensitive region and variables to a specific forecast aspect. The WRF 4D-Var will be built with the generated tangent linear and adjoint codes soon. We will show more detailed sensitivity analysis of the Antarctic cyclone during the workshop presentation.

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