Gravity waves in idealized MM5 simulations

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1 Introduction

The goal of this work is to determine how well gravity waves are represented in MM5. In particular, how much are they damped as they propagate and how should MM5 runs be set up to avoid unrealistic wave loss due to model damping?

To answer these questions, the two-dimensional gravity-wave problem that leads to the four-beam wave pattern, known as St. Andrew's Cross, is used. This pattern is produced when a stably stratified fluid at rest with constant Brunt Väisällä frequency is disturbed by a constant frequency oscillator. Gravity waves form and move away from the disturbance along four directions, forming an X shape, commonly called St. Andrew's Cross. This choice of problem is accompanied by a theory, is supported by physical experiments, and allows the MM5 wave propagation by the model's dynamic core to be distinguished from other model functions.

It is noted that the damping in the MM5 is strictly numerical, as real viscosity is not simulated by the dynamic core of the model. To explore the sensitivity of the MM5's numerical dissipation to changes in grid and wave parameters, a numerical analysis is done. The analysis uses the gravity-wave solution from the theory to evaluate loss of amplitude of a gravity wave subject to the discretization and time-stepping used in MM5.

The analysis is validated by numerical experiment for eight cases. For the MM5 simulation of St. Andrew's cross, moisture, Coriolis influences, and other effects that are important in the real atmosphere are eliminated, making model conditions close to those of theoretical solutions and numerical analysis. The numerical experiment results are also compared to the solution of Tabaei and Akylas (2003), which provides a gage for MM5's numerical damping in terms of theoretical viscous damping.

2 Numerical Analysis

For purposes of analyzing the numerical method of the MM5, the two-dimensional momentum and pressure equations are taken for a compressible, polytropic gas and linearized about a background state with no flow and constant Brunt Väisällä frequency and temperature. A damping term is added to the horizontal momentum equation to reflect artificial damping used in the MM5 (reference NCAR tech note and code). The relevant equations are

$$\rho_0 \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = D, \qquad (1)$$

$$\rho_0 \frac{\partial w}{\partial t} + \frac{\partial p}{\partial z} + \frac{g}{c_0^2} p = -\rho g + \frac{g}{c_0^2} p, \qquad (2)$$

$$\frac{\partial p}{\partial t} + \rho_0 c_0^2 \left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) - \rho_0 w g = 0, \qquad (3)$$

where u and w are the horizontal and vertical velocities, respectively, and p and ρ are the deviations from the background state pressure and density, respectively. The independent variables, x, z, and t are horizontal position, vertical position, and time, respectively. The subscript 0 refers to the background state, c is sound speed, and $D = -\Delta x^3 \partial^4 u / \partial x^4$ is the artificial dissipation term. With the addition of the continuity equation, the hydrostatic relation, and the perfect gas law, these equations describe the fluid of interest.

The approximate MM5 equations, (1) - (3), are discretized according to methods used in MM5. For example, in (1) a forward difference in time is used to make the replacement,

$$\frac{\partial u}{\partial t} \rightarrow (\mathrm{u}_{q,r}^{(n+1)} - \mathrm{u}_{q,r}^{(n)})/\Delta t, \qquad (4)$$

where u is horizontal velocity in the discrete system, and q, r and n refer to the position and time, respectively. The discrete counterpart to (1) - (3) is written for a gravity wave, for example the discrete horizontal velocity is,

$$\mathrm{u}_{q,r}^{(n)} \rightarrow U^n \exp i(k_x x_q + k_z z_r - \omega t^{(n)}). \qquad (5)$$

The discretized form of (1) - (3) with variables written in terms of their gravity-wave solutions are advanced in time following the MM5 scheme. That is, the horizontal momentum equation is explicitly advanced in time, providing the the updated value U^{n+1}. Then the vertical momentum and pressure equations are solved simultaneously for the unknowns, W^{n+1} and P^{n+1}, representing vertical velocity and pressure, respectively, at the next time step. This results in the ratios U^{n+1}/U^n

and W^{n+1}/W^n, which are used to compute the change in wave amplitude over one time step.

The terms treated as fast-changing, or acoustic, in the MM5 appear on the left hand side of (1) - (3), and are updated on small time steps, while the slowly-changing terms which appear on the right hand side, are updated on large time steps. In the analysis a large time step equal to four small time steps is taken as a typical value. Once the gravity wave completes its combination of small and large time steps, the updated ratios for U and W are combined to get the ratio for the resultant speed. This ratio, called A_a, represents the impact of numerical damping on the local fluid speed associated with a gravity wave.

Values of A_a are shown in Figure 1 for a range of grid and wave parameters. The values are less than 1 indicating decreasing, rather than growing magnitudes for fluid speed. Values of A_a increase as grid cells per wave, or wave resolution, increases in either the horizontal or the vertical directions. However, the surface of A_a values is nearly two-dimensional, indicating greater sensitivity to changes in horizontal parameters than vertical ones.

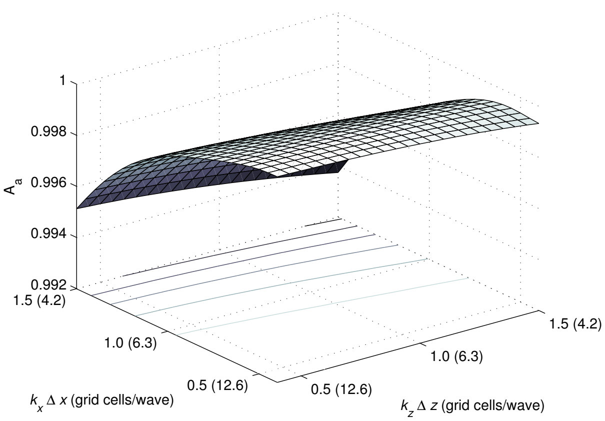

Figure 1. Beam amplitude predicted by analysis for a range of horizontal and vertical parameters.

3 Numerical Experiment

The MM5 is implemented for an idealized atmosphere for eight sets of grid and wave parameters specified in Tables 1 and 2. In all cases waves are represented by at least four grid cells, both horizontally and vertically. Simulations are in a single domain without moisture, Coriolis effects, a planetary boundary layer, or tropopause. The terrain is uniform and the background state of the atmosphere is given by an ideal gas in hydrostatic balance with no flow and a temperature profile of the standard atmosphere (Holton 1992), resulting in a nearly uniform Brunt Väisälä frequency of $0.012 s^{-1}$.

Wave	k_x (m^{-1})	k_z (m^{-1})	ω (s^{-1})
Short	0.00048	$\pm$0.0033	0.00175
Long	0.00024	$\pm$0.0033	0.000873

Table 1. Wave parameters

Case	Δx (m)	Δz (m)	Wave
1	3000	434	Short
2	3000	217	Short
3	2500	434	Short
4	2700	217	Short
5	3000	434	Long
6	3000	217	Long
7	2500	434	Long
8	2700	217	Long

Table 2 MM5 grid spacing and waves

The background state is maintained at all boundaries except at the left hand boundary where a perturbation satisfying the gravity-wave solution is specified. The perturbation is only applied over a limited vertical segment approximately equal to one vertical wavelength. In order to simulate the two-dimensional problem in a three-dimensional atmosphere, the disturbance is applied uniformly across the extraneous dimension (the N-S or y dimension in MM5 notation), while the other three lateral boundaries retain their no-flow conditions.

The resulting flow contains an upward-traveling beam and a downward-traveling beam, forming the right half of the X shape in St. Andrew's Cross. An example of a beam developing is shown in Figure 2. Beam-center velocities in upper beams are measured from MM5 data and combined with the known background density to obtain beam-center amplitudes used in dissipation evaluations.

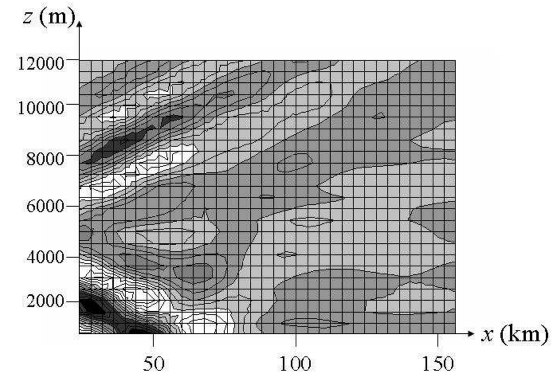

Figure 2. Example of MM5 gravity-wave response.

4 Theory

The nonlinear, two-dimensional, viscous Boussinesq beam problem was solved by Tabaei and Akylas (2003) for a stably stratified fluid with constant Brunt Väisällä frequency. The solution prescribes the loss in wave amplitude with distance from the disturbance, along the center of the beam. That amplitude result is interpretted here for a compressible fluid, in which the background density factor, $\sqrt{\rho_0}$, constitutes the leading correction to Boussinesq results (Lighthill 1978). This gives the beam-center amplitude,

$$V\sqrt{\rho_0} \propto \frac{1}{|\xi|^{2/3}}, \tag{6}$$

where V is the fluid speed and ξ is a beam-following coordinate. This amplitude prediction is used to gage whether MM5 beam amplitudes are diminished by MM5 damping more or less than they would be by molecular viscosity.

The analytic solutions also provide a dispersion relation between the wave frequency and wave numbers. A calculation using this relation, provides the group velocity, which pertains to the movement of wave energy.

5 Results

In order to show that numerical dissipation analysis describes MM5 performance, the two are compared. First, the results of the numerical experiment are quantified in terms of amplification factor. The experimental amplification factor is based on fluid speeds observed along a beam, $V(\xi)$. Values of $V(\xi)$ at a fixed time are interpretted as equivalent local speeds at different times. For example, losses over the period Δt, when waves travel from ξ_0 to ξ_1, are given by the amplification factor

$$(A_e)^N = \frac{V(\xi_1)\sqrt{\rho_0(\xi_1)/\rho_0(\xi_0)}}{V(\xi_0)}, \tag{7}$$

where $N = (\xi_1 - \xi_0)/(C_g \Delta t)$ is the number of large time steps and C_g is the group velocity from theory. In this formulation the speed at ξ_1 is transformed to the equivalent speed at ξ_0 using the result that $V\sqrt{\rho_0}$ is conserved along a compressible, inviscid beam. An average value for A_e using all available points along the beam is used to represent each of the eight numerical cases.

Numerical dissipation is typically quantified in terms of the order r in $A = 1 - Cf^r$, where C is a constant, and f depends on grid and wave parameters (Strikwerda 1989). In the current problem f is not readily determined and therefore the quantity $\ln(1 - A)$, which varies like $r \ln f$ is used for both the experiment and analysis results. For the experiment a provisional dissipation is $\ln(1 - A_e)$. This is adjusted by the average difference,

$$\langle \log(1 - A_e) - \log(1 - A_a) \rangle \tag{8}$$

over all eight cases, to eliminate any constant difference and allow a direct comparison.

The experimental dissipation is compared to the analysis dissipation for the eight simulated beams in Figure 3, where labels indicate case numbers from Table 2. The diagonal line indicates agreement between analysis and experiment, with the amount of dissipation increasing toward the top right of the diagonal. Markers are close to the line, indicating the analysis is an approximate predictor of numerical dissipation. This agreement validates the analysis as a predictor of dissipation of gravity waves by the MM5 dynamic core.

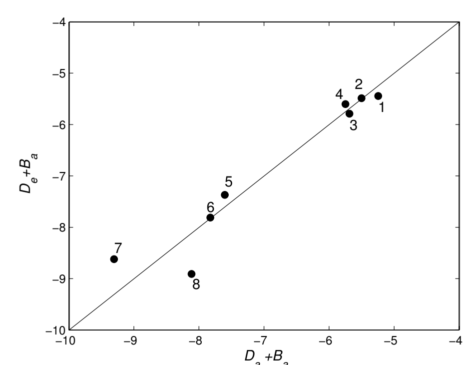

Figure 3. Comparison of dissipation values predicted by analysis and observed in experiment for eight numerical experiment cases.

For comparison to viscous theory, MM5 beam amplitudes as a function of the beam-following coordinate ξ are fit to the function,

$$V(\xi)\sqrt{\rho_0(\xi)} = V(\xi_0)\sqrt{\rho_0(\xi_0)}\left(\frac{\xi}{\xi_0}\right)^{-F}, \qquad (9)$$

to find the exponent F. The value of F is compared to the the value 2/3 from (6) to indicate the MM5 level of damping relative to theoretical, viscous damping. An example of amplitude along a beam is shown in Figure 4 for case 3 (from Table 2), with markers to indicate MM5 values, a solid curve to show the best-fit function for those values, and a dashed line to show theory. In this example, MM5 amplitude decreased faster than the theoretical prediction for a beam in a viscous fluid. The oscillations in the MM5 values are largely due to the beam center following a staircase pattern through the MM5 grid.

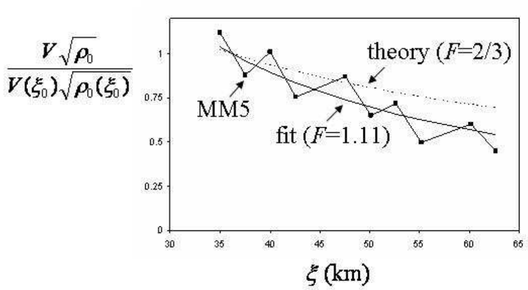

Figure 4 Example of beam amplitude from MM5, best fit curve, and theory.

A summary of F values for all eight cases is shown in Figure 5. This shows that for cases 1 - 4, MM5 damping is greater than theoretical viscous damping, while for the remainder it is less. Variations in damping are shown with horizontal parameters, and not vertical parameters, because the analysis shows the dominance of horizontal parameters in numerical damping levels.

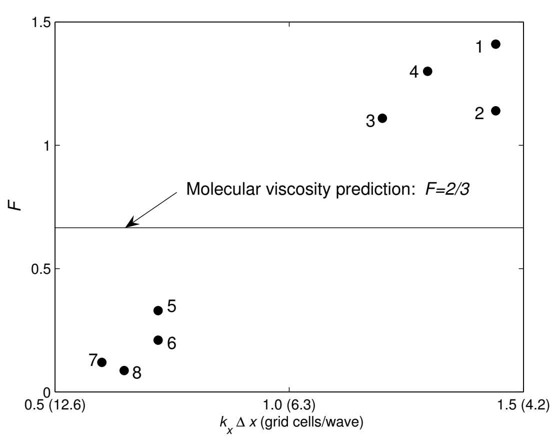

Figure 5. Beam amplitude loss in terms of F for the eight numerical experiment cases.

6 Conclusions

MM5 gravity wave beams are compared to numerical analysis and theory in terms of damping of gravity waves. Both numerical analysis and MM5 results show refining the resolution of the wave in either the horizontal or vertical, decreases numerical dissipation. The sensitivity of the damping to the refinement is greater in the horizontal than the vertical, reflecting the influence of the horizontally based artificial dissipation.

Due to the relative sensitivity to horizontal spacing, the horizontal spacing alone can be used to approximately predict numerical dissipation. The amount of numerical dissipation for a wave represented by seven horizontal grid cells is approximately equivalent to the dissipation predicted by theory accounting for molecular viscosity. Numerical dissipation is greater or less than this amount if the the wave is resolved by fewer or more horizontal cells, respectively.

These results could be used to better understand MM5 results and as a consideration in selecting grids. This approach establishes a way to perform idealized numerical experiments that can be applied to other models for model investigation or to compare different models on the basis of their core calculations as opposed to their parameterizations or boundary conditions.

7 Acknowledgments

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