An Upper-Boundary Gravity-Wave Absorbing Layer for NWP Applications

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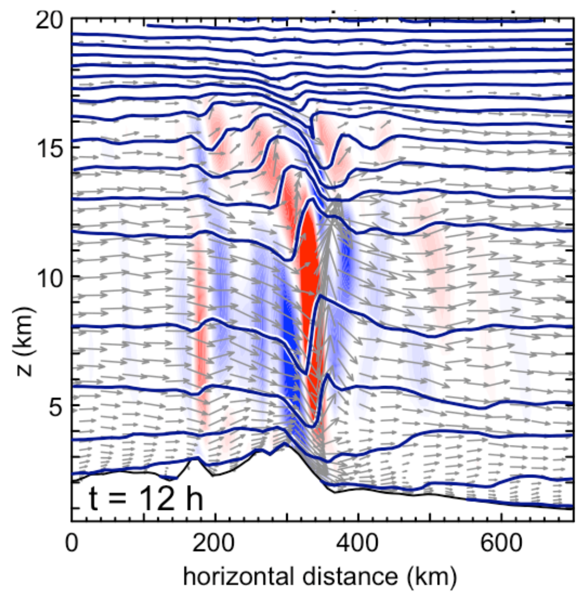


Preventing Artificial Reflection of Vertically Propagating Gravity-Wave Energy

- Radiation upper boundary conditions assume simplified conditions not well suited for real-data simulations.
- Rayleigh damping upper absorbing layer widely used for idealized simulations, but requires a known reference state.
- Horizontal diffusion absorbing layer often too weak due to linear stability constraints, and vertical diffusion alters the background environment.

Implicit Rayleigh Absorbing Layer in Split-Explicit Time Integration

Height-coordinate equations ($\mathbf{V}_H$, w, π, θ):

$$\mathbf{V}_H^{\tau+\Delta\tau} = \mathbf{V}_H^{\tau} - \Delta\tau\left(c_p\theta^t\boldsymbol{\nabla}_H\pi'^{\tau} - \boldsymbol{F}^t_{V_H}\right)$$

$$\pi_1 = \pi^{\tau} - \Delta\tau\left\{C^t\left[\boldsymbol{\nabla}\cdot\rho^t\theta^t\mathbf{V}_H^{\tau+\Delta\tau} + \tfrac{1}{2}\partial_z(\rho^t\theta^t w^{\tau})\right] - F^t_{\pi}\right\}$$

Vertically implicit

$$\left\{\begin{aligned} w^* &= w^{\tau} - \Delta\tau\left(c_p\theta^t\overline{\partial_z\pi'}^{\tau *} - g\frac{\overline{\theta'^{\tau}}}{\overline{\theta}} - F^t_w\right)\\ \pi^* &= \pi_1 - \tfrac{1}{2}\Delta\tau C^t\partial_z(\rho^t\theta^t w^*)\end{aligned}\right.$$

$$\boxed{w^{\tau+\Delta\tau} = w^* - R_w\Delta\tau\, w^{\tau+\Delta\tau}}$$

$$\theta'^{\tau+\Delta\tau} = \theta^{\tau} - \Delta\tau\left(\partial_z\theta^t\, w^{\tau+\Delta\tau} - F^t_{\theta}\right)$$

$$\pi'^{\tau+\Delta\tau} = \pi_1 - \tfrac{1}{2}\Delta\tau C^t\partial_z(\rho^t\theta^t w^{\tau+\Delta\tau}),$$

Effect of Applying Implicit Rayleigh Damping as an Adjustment Step

Combining the intermediate steps, the *w* equation becomes:

$$\frac{\partial w}{\partial \tau} + c_p \theta^t \partial_z \overline{\pi'}^{\tau} - g \frac{\theta'^{\tau}}{\overline{\theta}} + R_w w^{\tau+\Delta\tau} - \frac{\Delta\tau^2 c^{t2}}{4\rho^t \theta^t} \frac{\partial}{\partial z^2} \left\{ \rho^t \theta^t R_w w^{\tau+\Delta\tau} \right\} = F_w^t$$

Normal Rayleigh damping term that vanishes in hydrostatic limit

Additional damping term that remains effective in hydrostatic limit

Steady 2-D Linear Wave Equation Implicit Rayleigh Absorbing Layer

$$w(x,\tilde{z}) = \hat{w}(\tilde{z})e^{i\,k\,x} \qquad \tilde{z} = \frac{Nz}{U}$$

Wave equation within absorbing layer:

$$\frac{\partial}{\partial \tilde{z}^2}\left\{(1 - F^2 - i\tfrac{1}{4}\alpha^2\beta_w)\hat{w}\right\} + [1 - K^2(1 - i\beta_w)]\hat{w} = 0$$

where: $K = \dfrac{kU}{N}, \qquad F = \dfrac{f}{kU}, \qquad \alpha = kc\Delta\tau, \qquad \beta_w = \dfrac{R_w}{kU},$

Wave equation below absorbing layer:

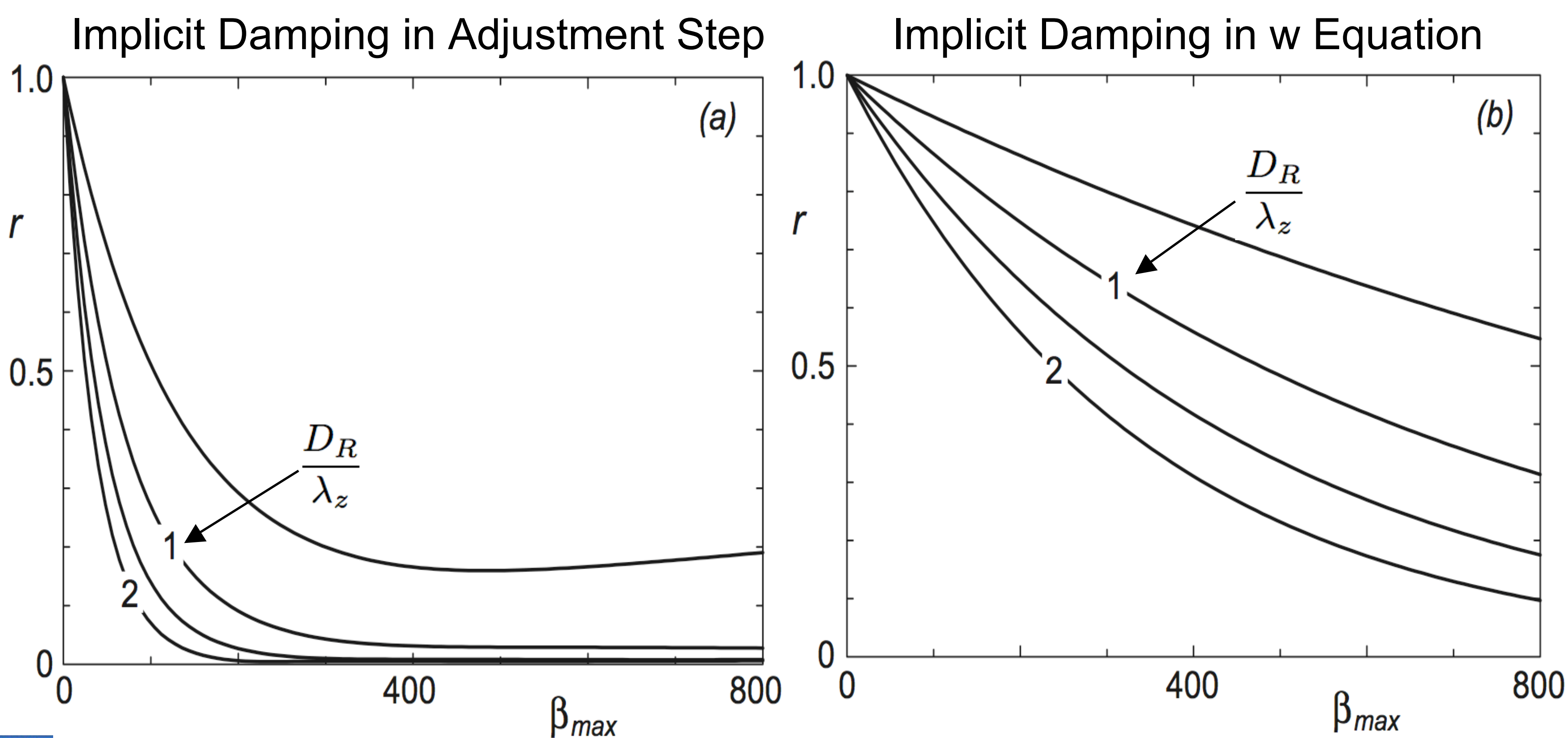
$$\hat{w}_l(\tilde{z}) = C_1 e^{i\Lambda_z(\tilde{z}-\tilde{z}_d)} + C_2 e^{-i\Lambda_z(\tilde{z}-\tilde{z}_d)}, \quad \Lambda_z = \left(\frac{1-K^2}{1-F^2}\right)^{\frac{1}{2}} = N\lambda_z/U$$

Solve by matching p and w at bottom of absorbing layer:

$$\boxed{r = \left|\frac{C_2}{C_1}\right| = \left|\frac{\Lambda_z\hat{w}(\tilde{z}_d) + i\partial_{\tilde{z}}\hat{w}(\tilde{z}_d)}{\Lambda_z\hat{w}(\tilde{z}_d) - i\partial_{\tilde{z}}\hat{w}(\tilde{z}_d)}\right|}$$

Reflection Coefficient for Implicit Rayleigh Absorbing Layer

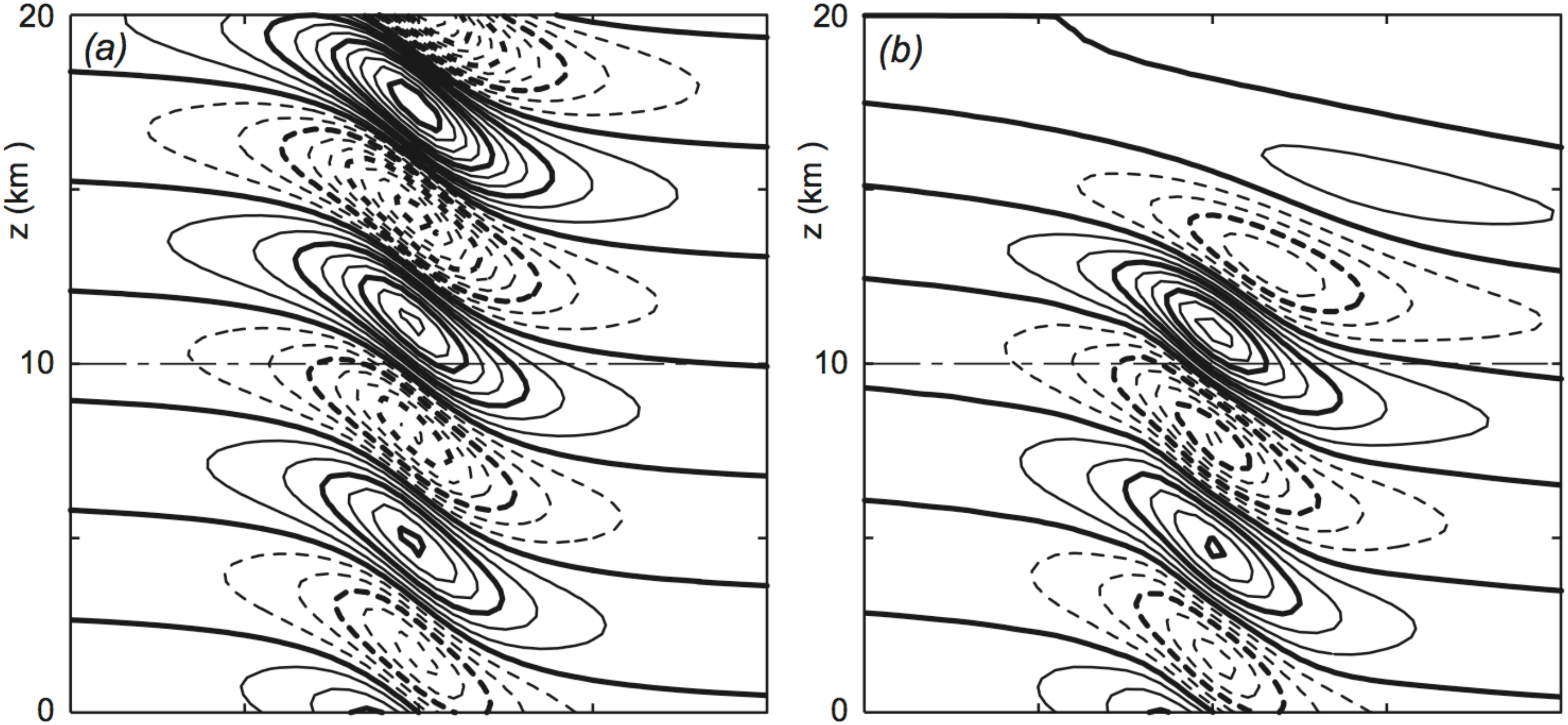
$$\beta_w(\tilde{z}) = \beta_{max} \sin^2\left(\frac{\pi}{2}\frac{\tilde{z} - \tilde{z}_d}{\tilde{z}_t - \tilde{z}_d}\right)$$



$$K = F = \alpha = 0.1$$

Linear Mountain-Wave Simulation with Implicit Rayleigh Absorbing Layer

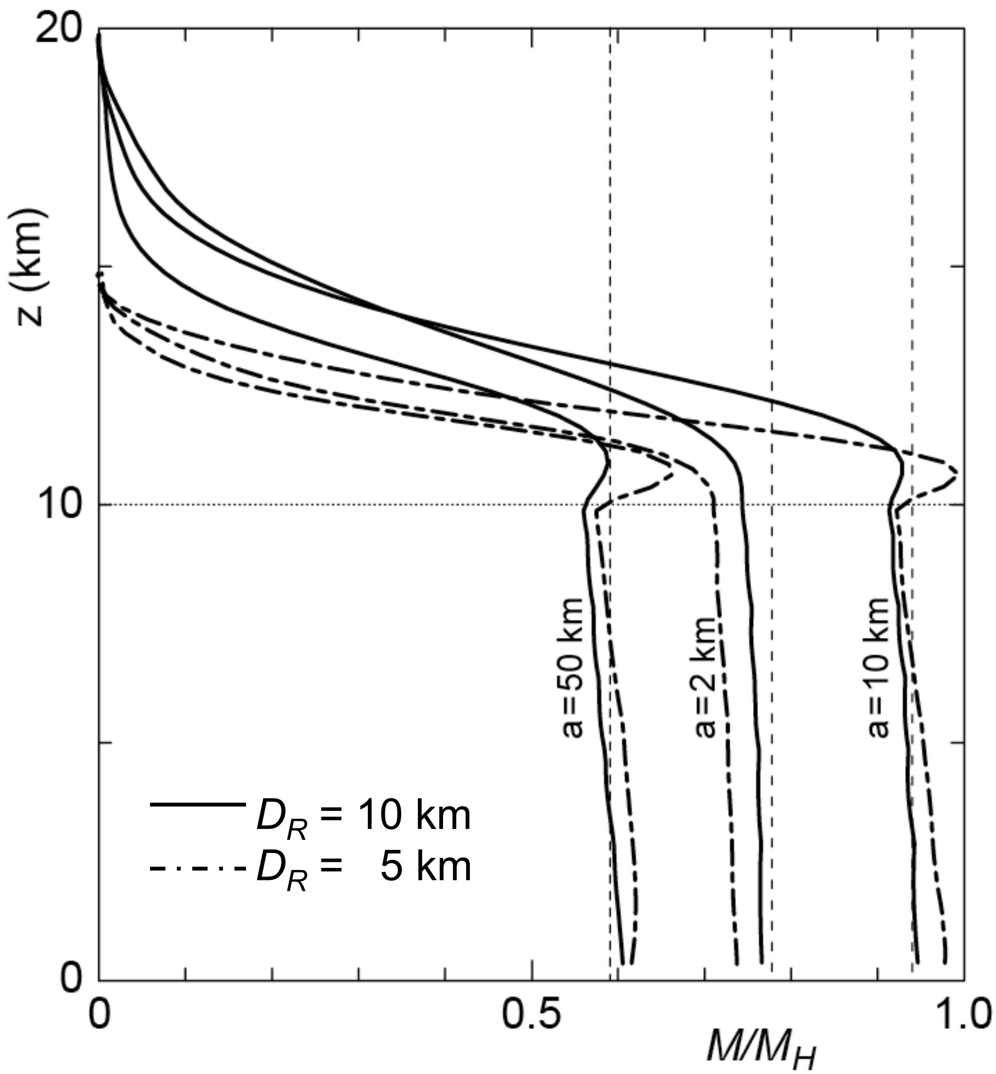
Vertical Velocity for 10 m Bell Mountain with *a* = 10 km half-width



Linear Analytic Solution

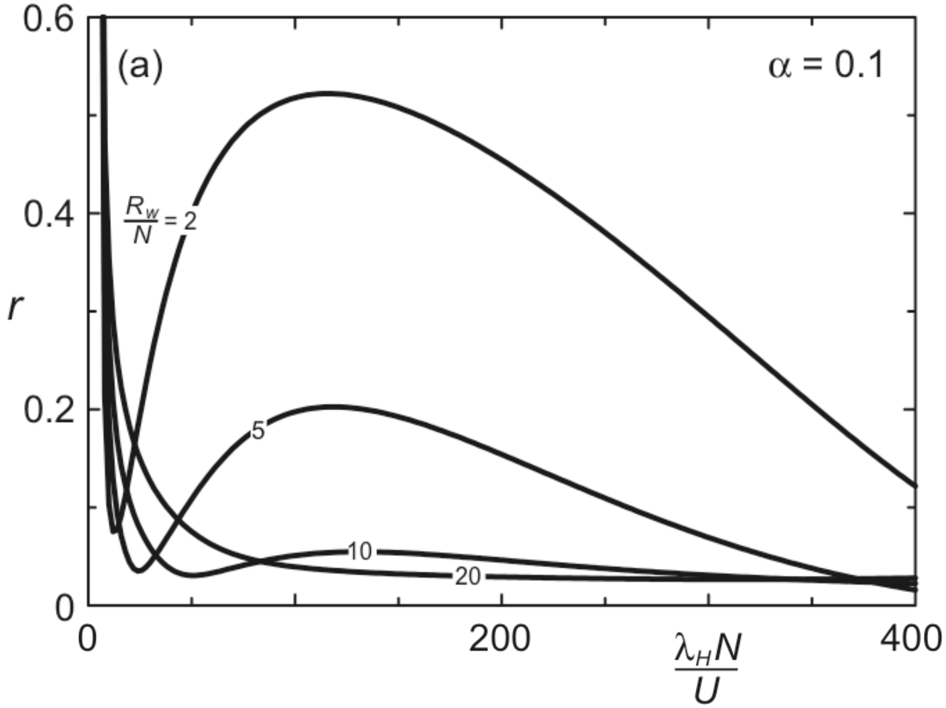
Numerical Simulation at *Ut*/*a*=108

Vertical Momentum-Flux Profiles

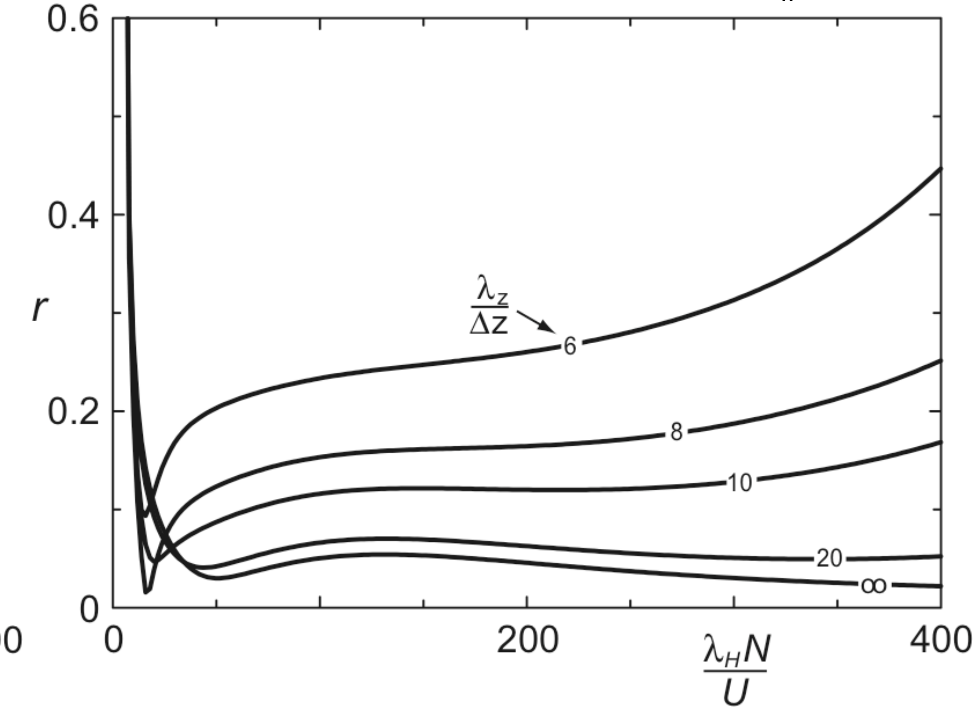


Reflection Coefficient for $D_R/\lambda_z=1$

Dependence on R_w with good vertical resolution

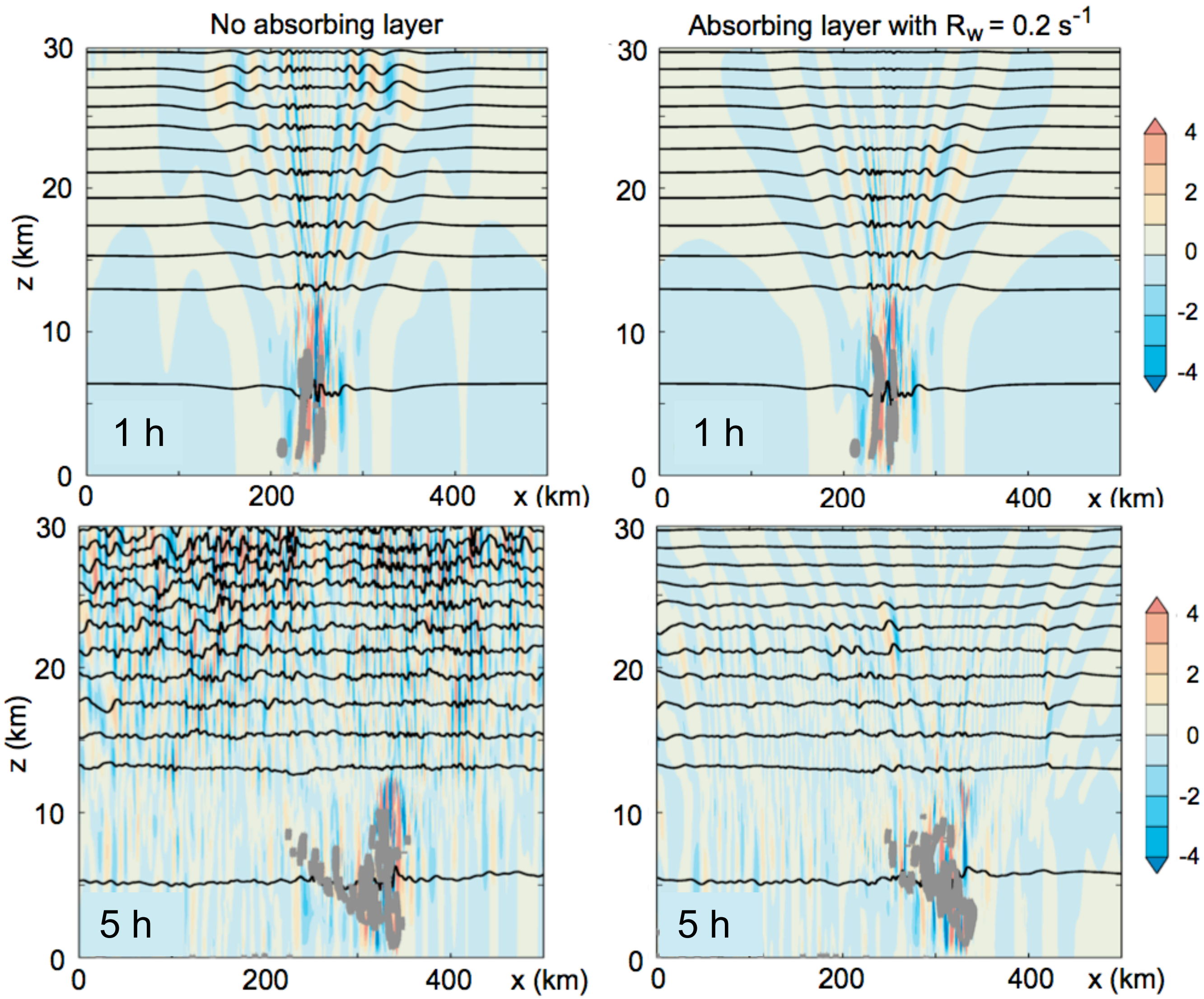


Dependence on vertical resolution for $R_w/N=10$



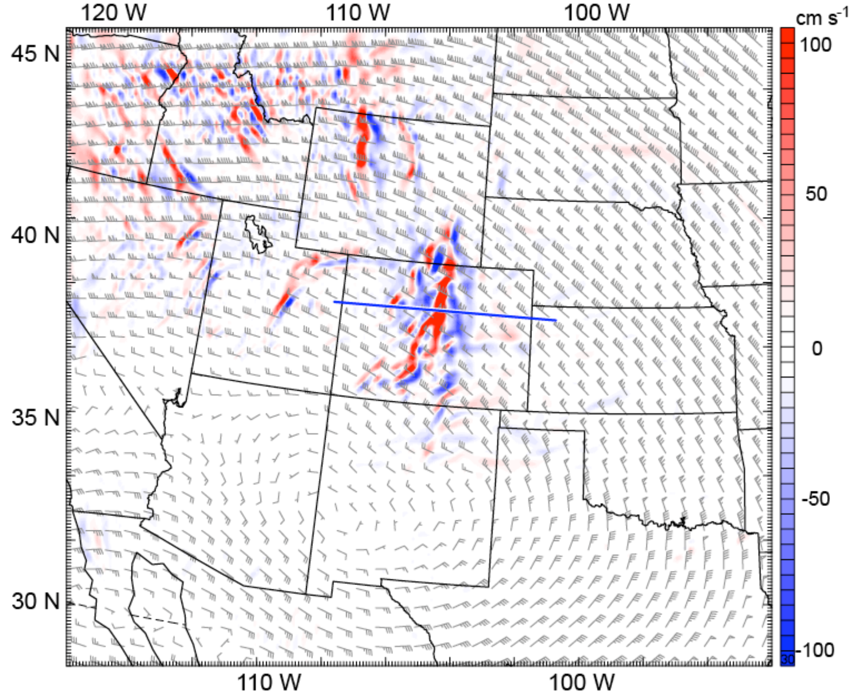
Default Value of damping coefficient in WRF/ARW V3.0 is $R_w = 0.2\ s^{-1}$

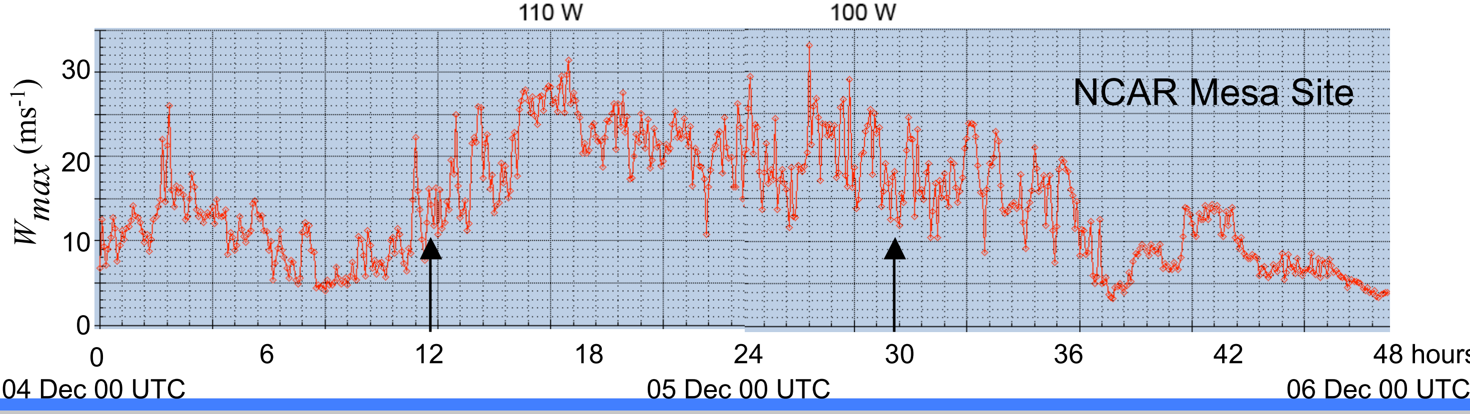
Idealized Squall-Line Simulation



WRF Forecast over Colorado Front Range

Model Initialized 04 Dec 2007 00 UTC





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