Evaluation of Positive-Definite and Monotonic Limiters for Scalar Advection in ARW

Hailong Wang
CIRES, Univ. of Colorado, and NOAA/ESRL

Bill Skamarock
NCAR/MMM

Graham Feingold
NOAA/ESRL

Problem of RK3 Scalar Advection

- The 3rd order Runge-Kutta (RK3) scheme:
  - Not positive-definite
  - Not monotonic
  - Conservative if no clipping of negative values occurs

PD and Monotonic Limiters

- Positive-definite (PD) limiter:
  - Conservative but has overshoots
  - Is an option in current ARW;
  - Formulation: the V3.0 technical note.
- Monotonic (MO) limiter:
  - No new minima and maxima
  - Formulation is based on flux renormalization (Skamarock 2007; Zalesak 1979)

Formulation of MO limiter

- Scalar mass conservation equation

$$\frac{\partial \mu\phi}{\partial t} + \frac{\partial \mu u\phi}{\partial x} + \frac{\partial \mu v\phi}{\partial y} + \frac{\partial \mu\dot{\eta}\phi}{\partial \eta} = \mu S_\phi \qquad (1)$$

- The final step of RK3 integration

Centered flux divergence

$$(\mu\phi)^{t+\Delta t} = (\mu\phi)^t - \Delta t \left(\sum_i \boxed{\delta_{x_i} F_{x_i}^{**}} - \mu S_\phi^t \right) \qquad (2)$$

- In MO, (2) is replaced by

$$(\mu\phi)^{***} = (\mu\phi)^t + \Delta t \mu S_\phi^t \qquad (3a)$$

$$(\mu\phi)^{t+\Delta t} = (\mu\phi)^{***} - \Delta t \sum_i \boxed{\delta_{x_i} \left[F_{x_i}^{1^{***}} + R\left(F_{x_i}^{cor^{**}} \right) \right]} \qquad (3b)$$

Formulation of MO limiter

First-order upwind flux

$$(\mu\phi)^{***} = (\mu\phi)^{t} + \Delta t \mu S_{\phi}^{t} \qquad \text{(3a)}$$

$$(\mu\phi)^{t+\Delta t} = (\mu\phi)^{***} - \Delta t \sum_{i} \delta_{x_i} \left[F_{x_i}^{1^{***}} + R\left(F_{x_i}^{cor^{**}} \right) \right] \qquad \text{(3b)}$$

high-order flux correction: $F_{x_i}^{cor^{**}} = F_{x_i}^{**} - F_{x_i}^{1^{***}}$

Renormalized high-order flux correction

Renormalization Procedure:

1. Update the scalar mass using the first-order upwind flux
2. Estimate min and max updated mass
3. Use the min and max to renormalize the outgoing and incoming flux correction

(2) $\delta_{x_i} F_{x_i}^{**}$ $\longrightarrow$ (3) $\delta_{x_i} [r F_{x_i}^{**} + (1-r) F_{x_i}^{1^{***}}]$

Sensitivity Tests

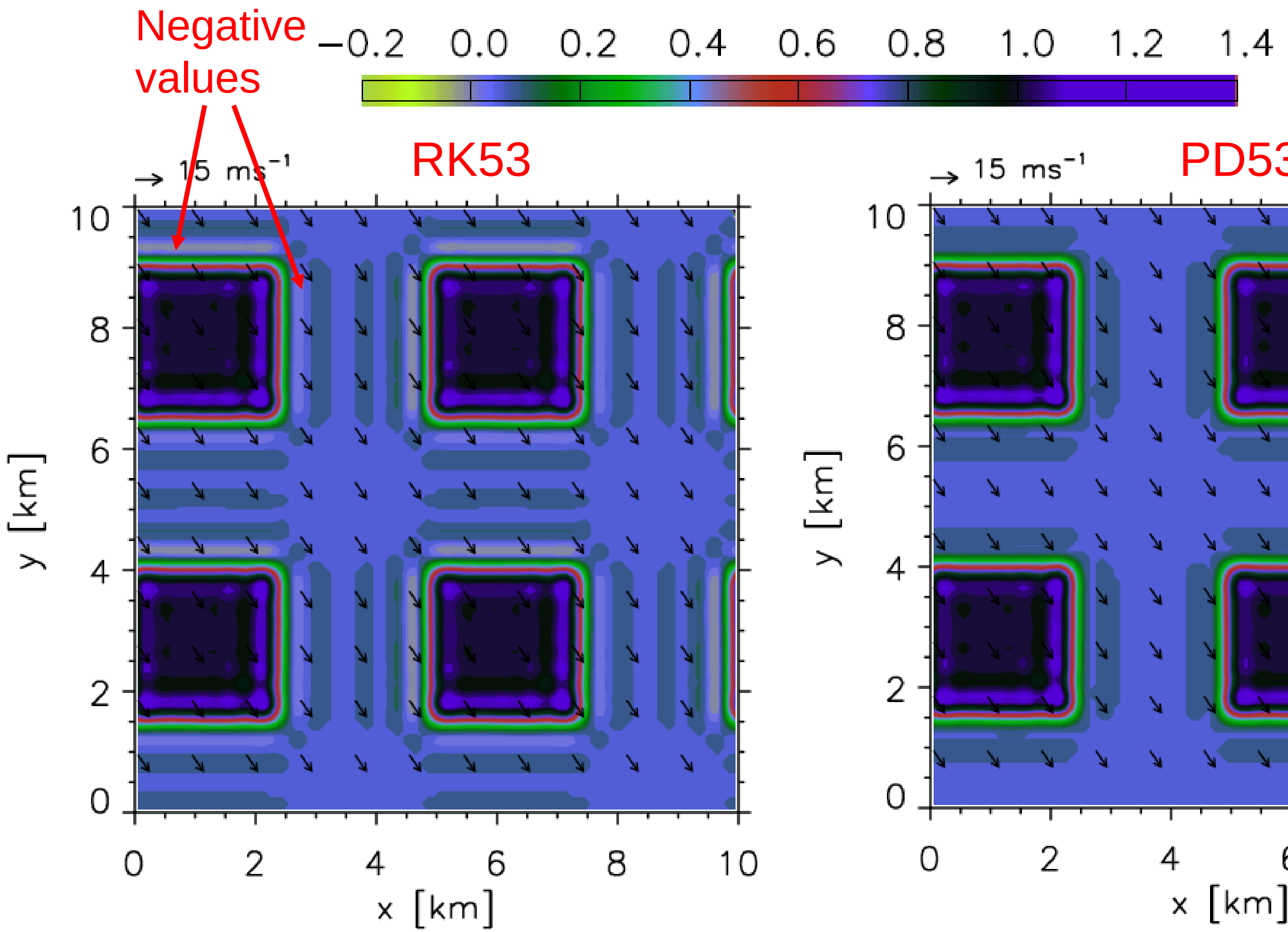
- Passive tracers:
  - Released in 4 cubes (25x25x30 grids);
  - Initial concentration of 1;
- Cloud condensation nuclei (CCN), cloud droplets, and rain drops:
  - New two-moment warm-rain microphysics (Feingold et al. 1998);
  - Equations for CCN, cloud droplet and rain drop;
  - Initial CCN concentration is 100 cm^{-3}.

Test Simulations

- Setup:
  - 100x100X50 grids; dx=dy=100 m; dz~30 m;
  - dt=1 s; U~10 ms^{-1}; Cr~0.1;
  - TKE closure for turbulence and mixing;
  - Periodic B.C. in both x and y;
- Basic RK3 with or without PD/MO limiter;
- 5^{th} (3^{rd}) order accuracy of horizontal (vertical) advection vs. 6^{th} (4^{th}) order;

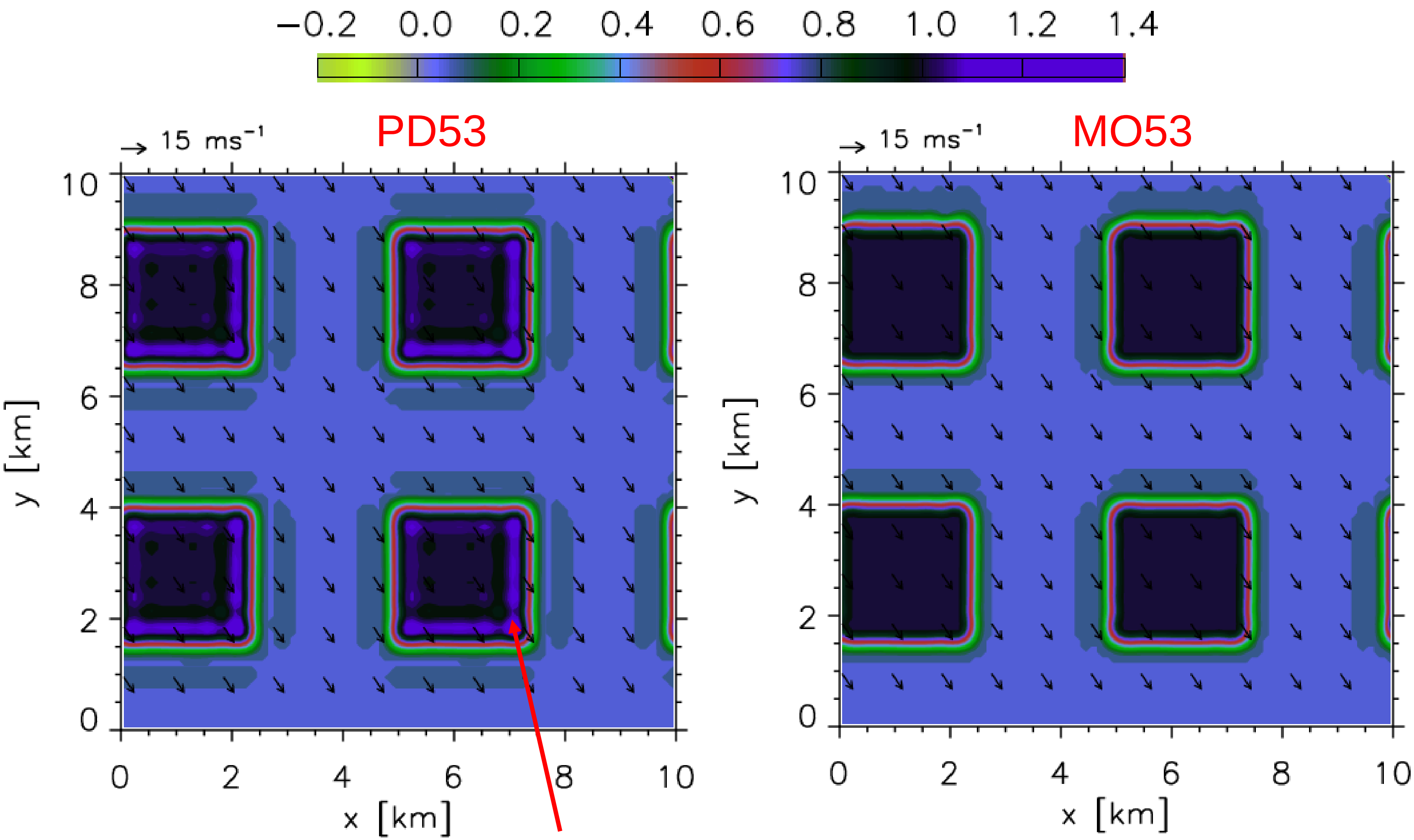
RK53, PD53, MO53 RK64, PD64, MO64

Results: tracer concentrations



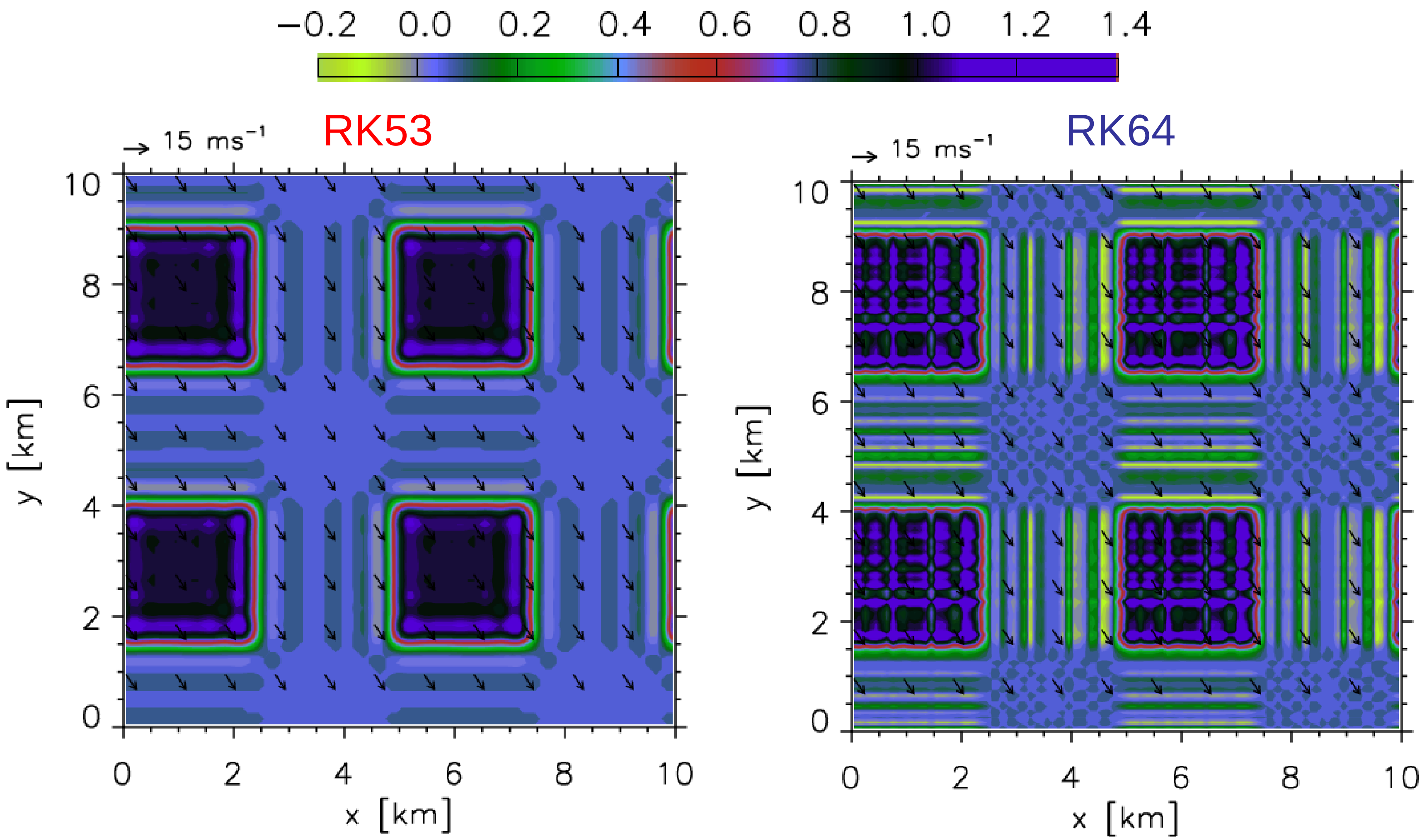
Cross section at 750m,10 min into the simulation

Results: tracer concentrations

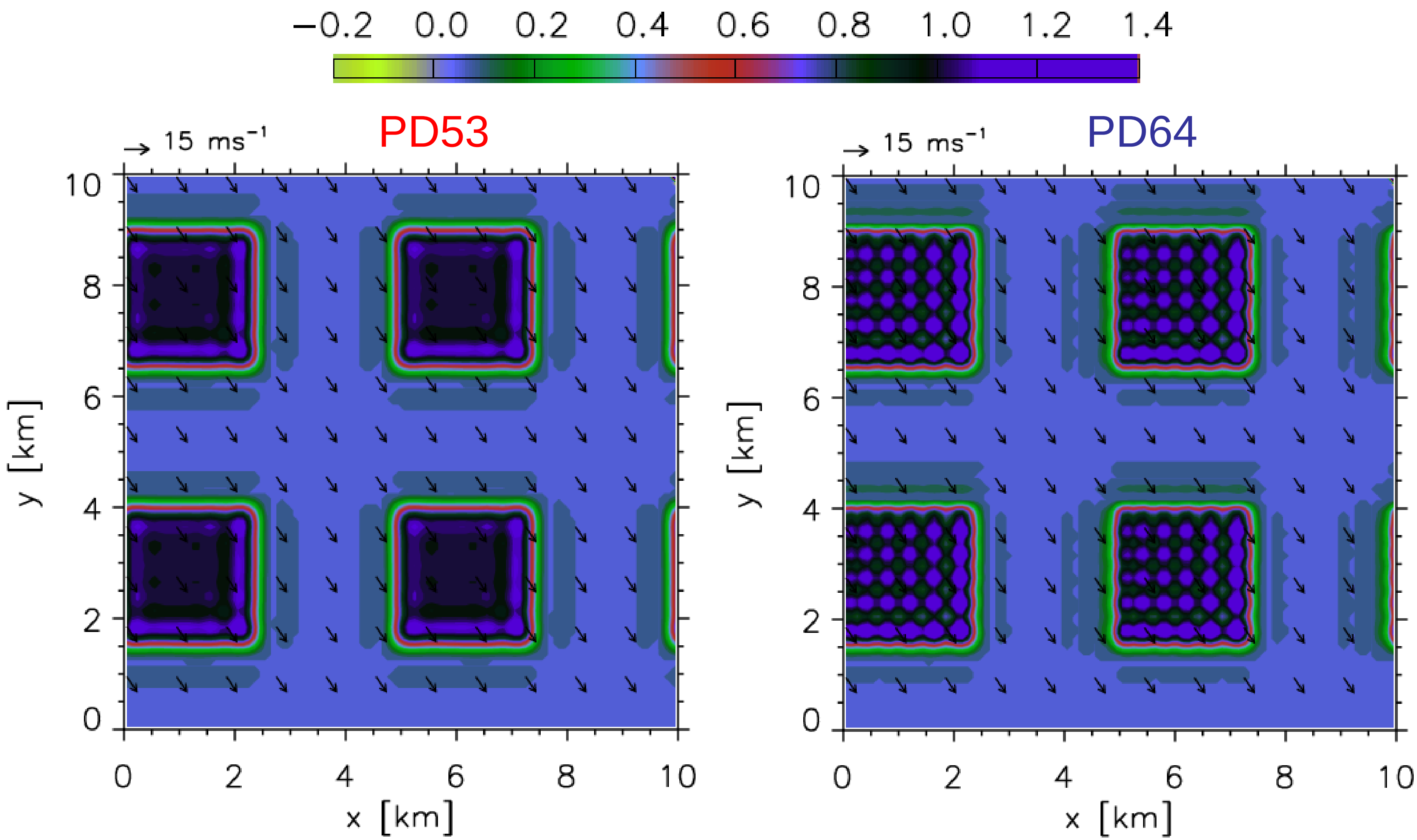


Up to 30% overshoots

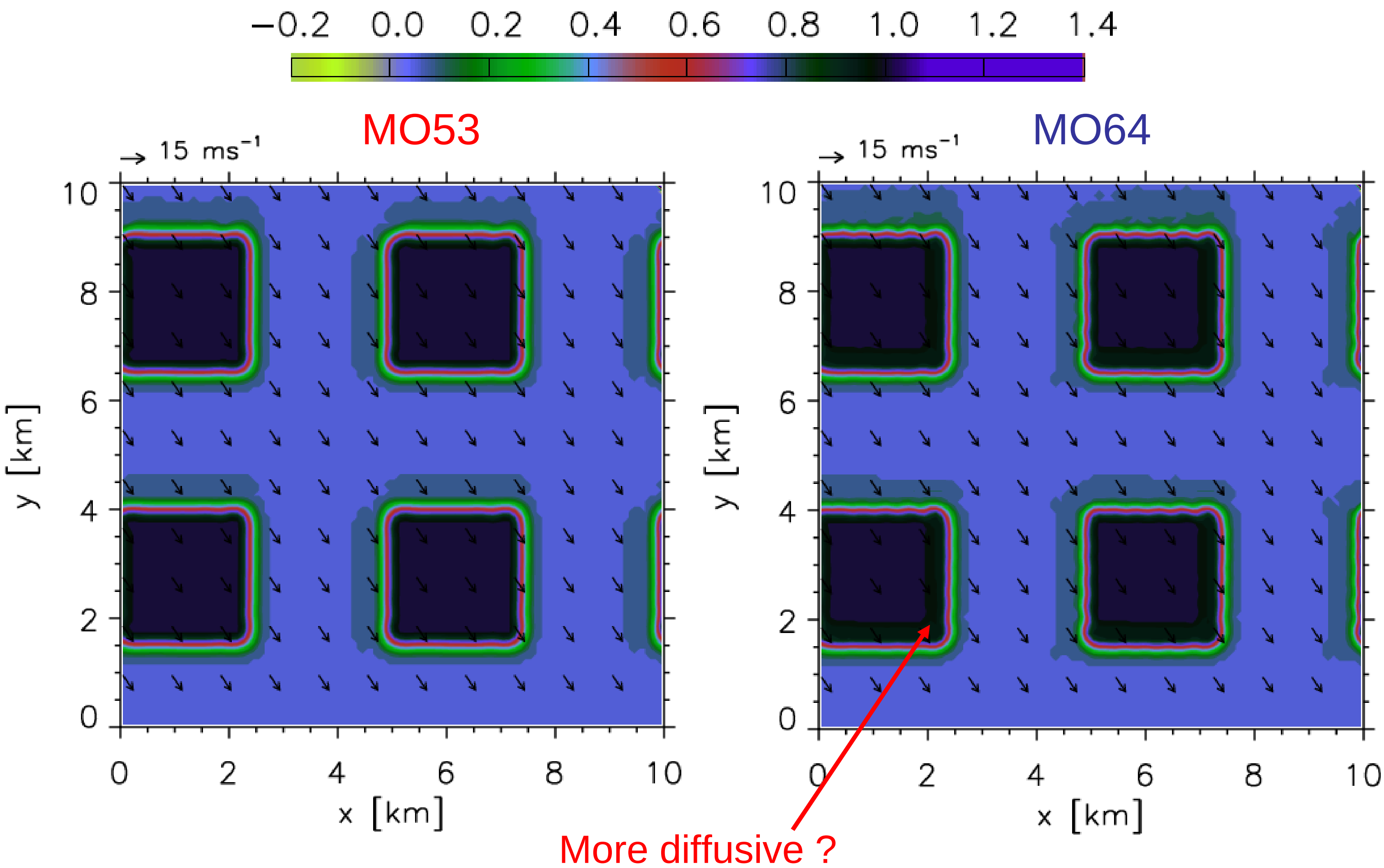
Results: tracer concentrations



Results: tracer concentrations

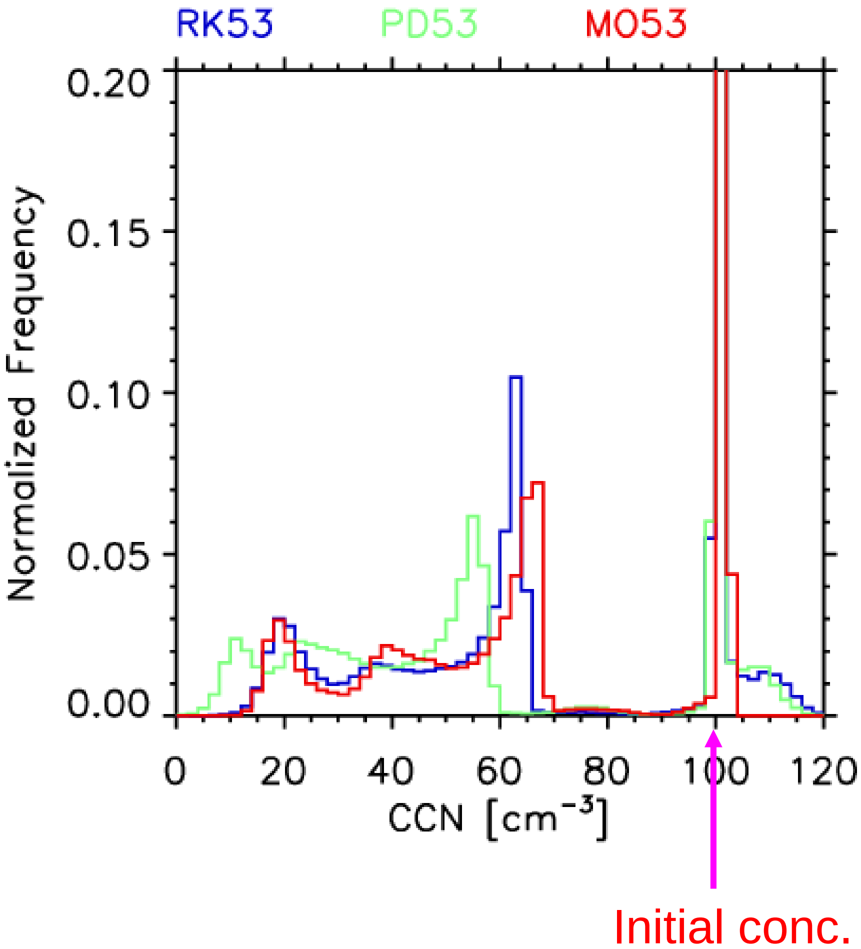


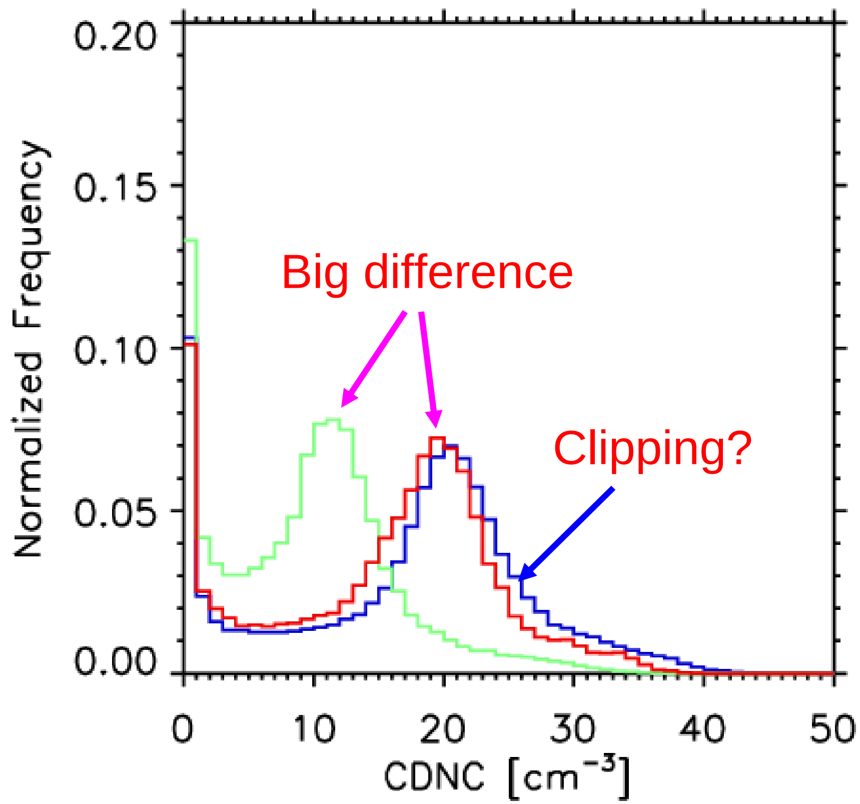
Results: tracer concentrations



Results: CCN and drops

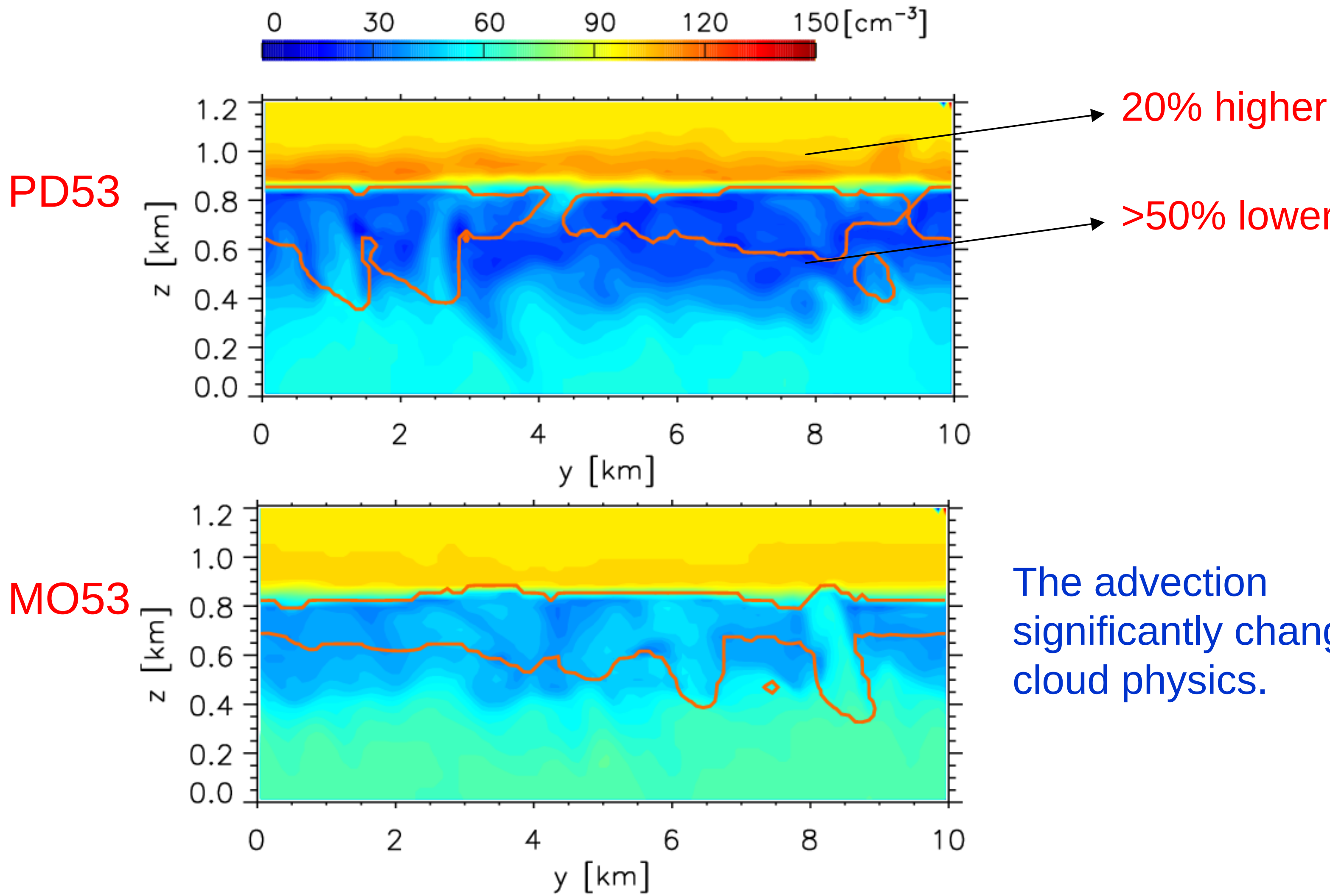
At the end of the 4th simulation hour





Results: CCN + drops

Cross section of total particle # conc. at the end of the 4th simulation hour



The advection significantly changed cloud physics.

Summary

- PD limiter removes negative values produced by the basic RK3, but has significant overshoots;
- Monotonic limiter avoids both undershoots and overshoots, and is much less dispersive than PD;
- PD (MO) is 7% (8%) more expensive than RK3;
- 6^{th} (4^{th}) order approximation of horizontal (vertical) advection is more dispersive than 5^{th} (3^{rd}) order one;
- Numerical advection errors have substantial impacts on cloud physics.

We acknowledge support from NOAA's Climate Goal. Hailong Wang is supported by the CIRES Post-doctoral Fellowship. We thank Hongli Jiang, Adrian Hill, and Mary Barth for thoughtful discussions.