

No Ri-critical turbulence-closure for stably stratified geophysical flows

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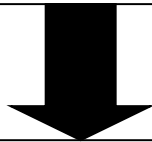
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Atmospheric turbulence and planetary boundary layers (PBLs)

Physics

Revised paradigm for stratified turbulence: self-control and self-organisation



Revised turbulence-energetics, turbulence-closure and PBL theory and modelling



Geo-sciences

PBLs link atmosphere, hydrosphere, lithosphere and cryosphere within weather & climate systems



Improved “linking algorithms” in weather & climate models



**Progress in understanding and modelling
weather & climate systems**

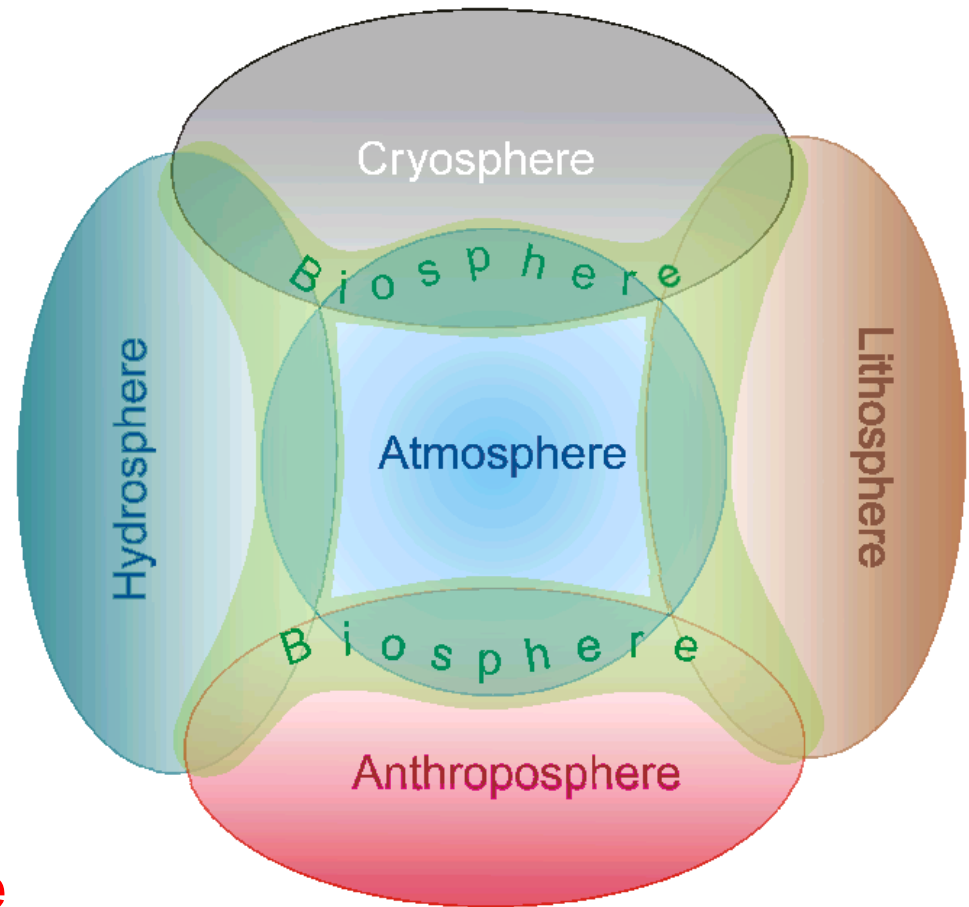


Geospheres in climate system

Turbulence performs vertical transports of energy, matter and momentum in the air and water

Atmosphere, hydrosphere, lithosphere and cryosphere are coupled through turbulent planetary boundary layers PBLs (dark green lenses)

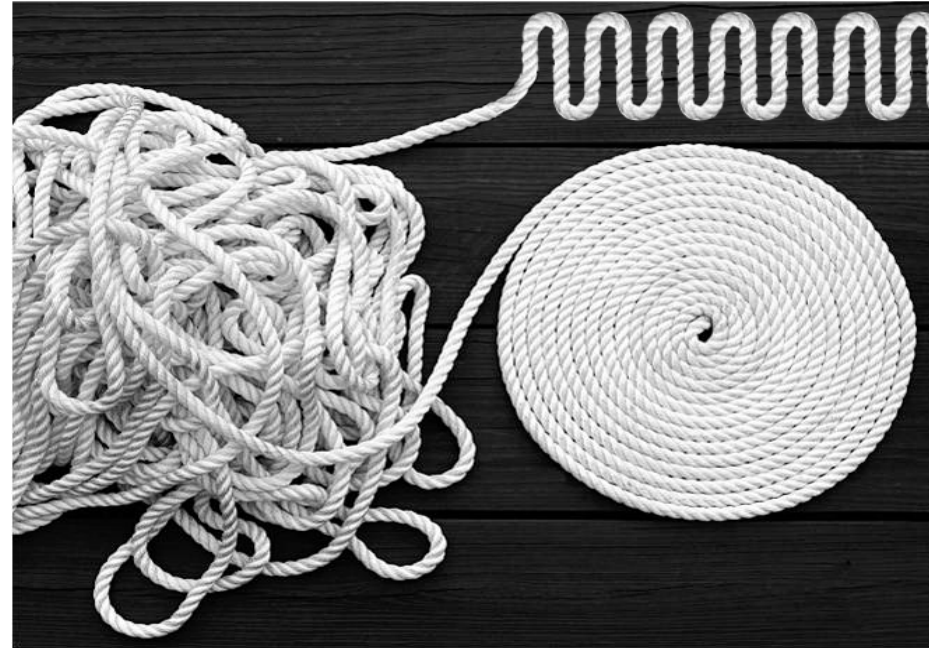
PBLs include 90% biosphere and the entire anthroposphere



Changing the paradigm

TRADITIONAL:

- **Fluid flow (implied neutral) = mean flow (regular) + turbulence (chaotic) with forward energy cascade from larger to smaller eddies towards viscous dissipation**



REVISED: **Geophysical fluid flow (stable/unstable) =**

- mean flow +
- usual turbulence with forward cascade towards dissipation +
- **anarchy turbulence (inverse energy transfer)** from smaller to larger eddies (e.g., merging plumes in turbulent convection)
- towards **large organised structures** (secondary circulations)



Role of planetary boundary layers (PBLs): TRADITIONAL VIEW



**Surface fluxes between
AIR
and**

WATER (or LAND)

**fully characterise interaction between
ATMOSPHERE and OCEAN/LAND**

**Monin-Obukhov similarity theory (1954)
(conventional framework for determining
surface fluxes in operational models)
disregards non-local features of
convective and long-lived stable PBLs**



Role of PBLs: MODERN VIEW



Because of very stable stratification in the atmosphere and ocean beyond PBLs and convective zones, the density increments at the PBL outer boundaries prevent the entities delivered by the surface fluxes (or emissions) to efficiently penetrate from the PBL into the free atmosphere or deep ocean.

Hence the PBL heights and the fluxes due to entrainment at the PBL outer boundary essentially control local weather including extreme weather events

- heat waves associated with convection,
- strong stable stratification triggering air pollution, etc.

This modern view (relevant also to hydrosphere) requires accurate modelling of the

- PBL height (depth) and
- turbulent entrainment at the PBL outer boundary



Very shallow boundary layer separated from the free atmosphere by **capping inversion**



PBL height visualised by smoke blanket (Johan The Ghost, Wikipedia)

Capping inversion restricts the PBL-free flow exchange



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Different types of PBL

Classification by **the sign of the surface buoyancy flux** B_s

Stable $B_s < 0$

Neutral $B_s = 0$

Unstable (convective) $B_s > 0$

disregards **free-flow Brunt-Väisälä frequency** N (at $z > h$).

We account for N and distinguish

Stable

nocturnal stable (NS) $N = 0$

long-lived stable (LS) $N > 0$

Neutral

truly neutral (TN) $N = 0$

conventionally neutral (CN) $N > 0$

Unstable shear-free (convective cells) in two-layer fluid $N = 0$

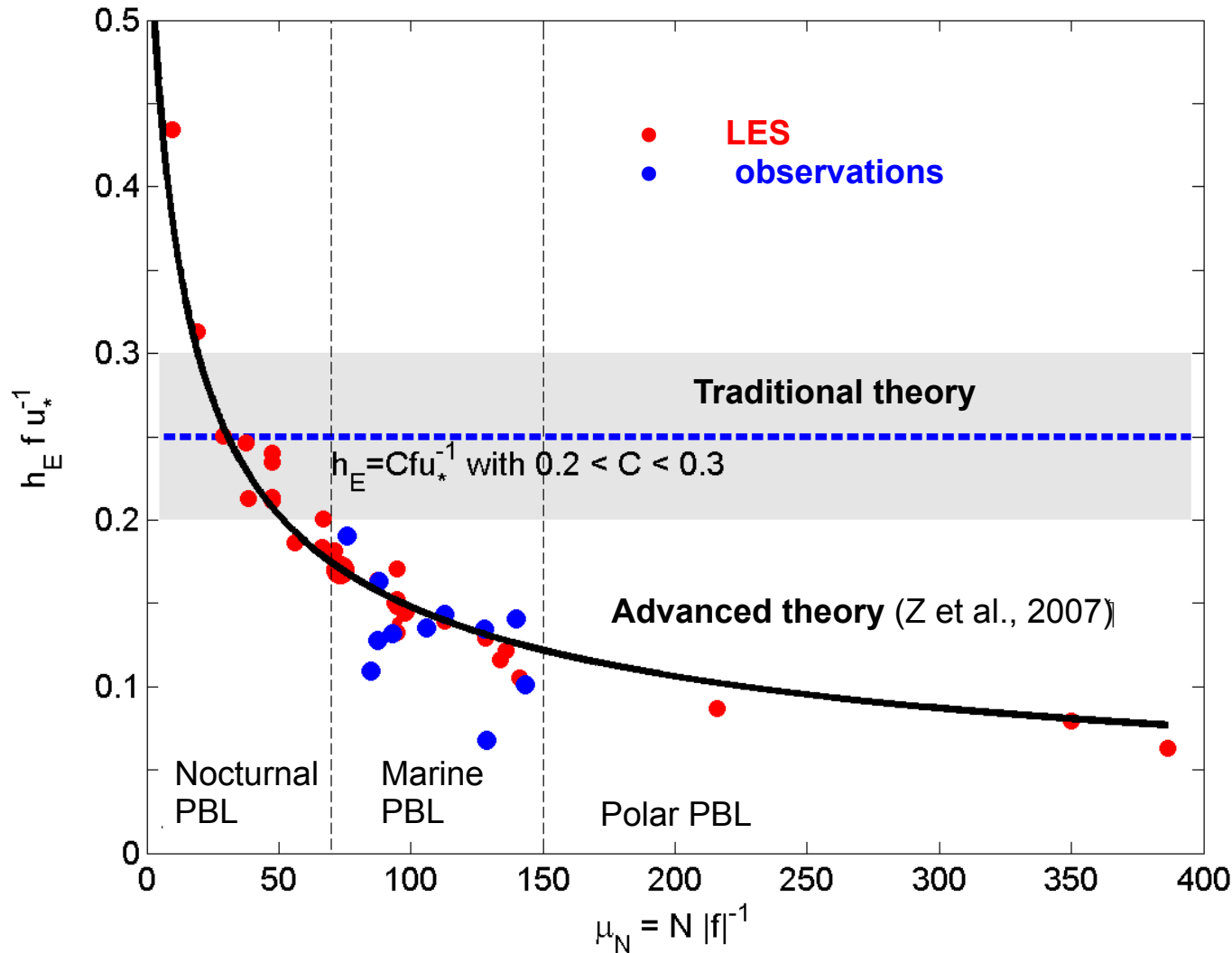
in stratified fluid $N > 0$

Unstable sheared (convective rolls) in two-layer fluid $N = 0$

in stratified fluid $N > 0$



PBL shallowing due to free-flow stability



Dashed line –
traditional TN
PBL model

Heavy curve –
CN PBL model

Red points –
LES

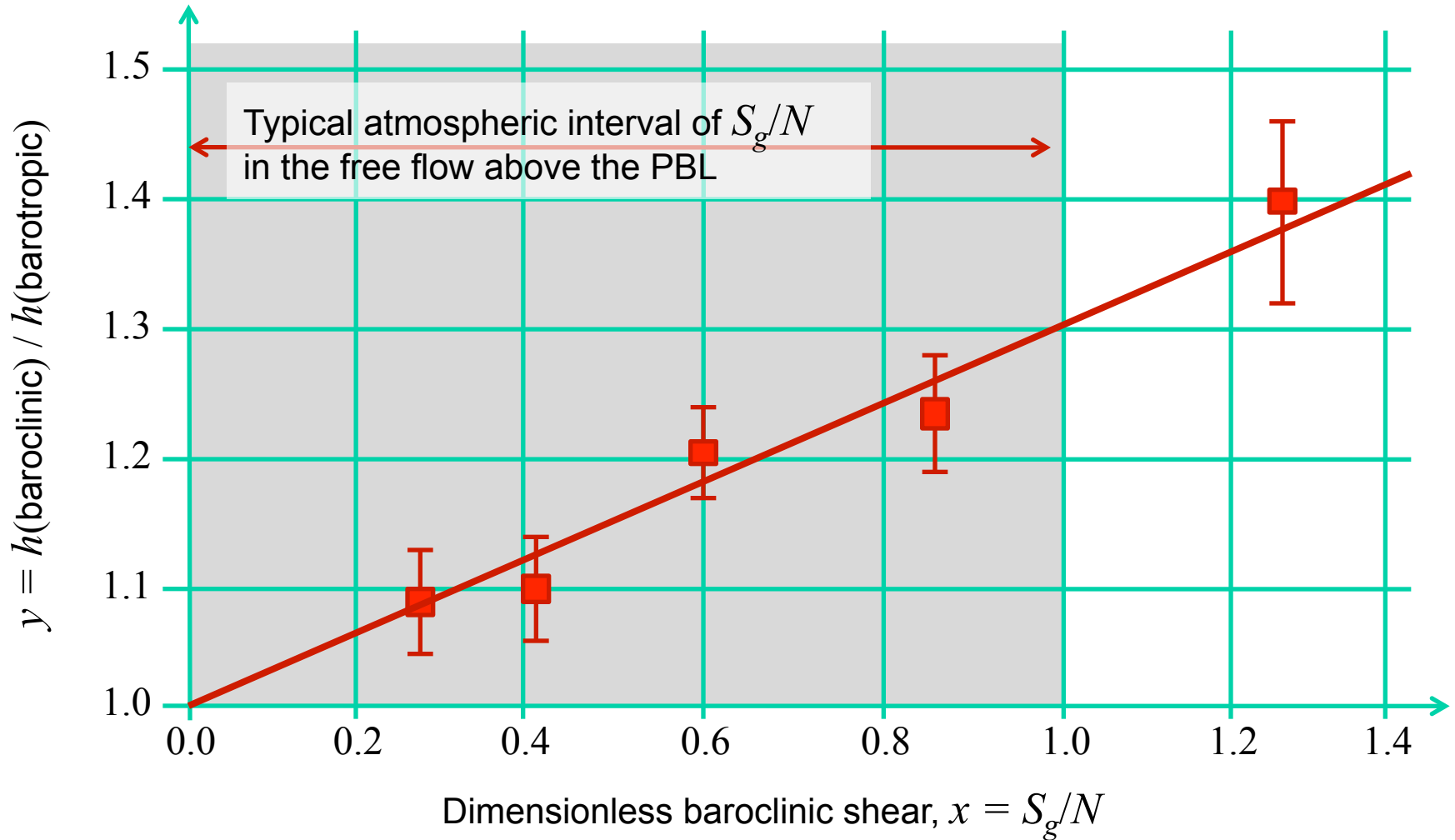
Blue points –
atmospheric
data

The effect of free-flow Brunt-Väisälä frequency N on the equilibrium CN PBL height h_E



PBL deepening due to baroclinic shear

Theoretical model $y = (1 + 0.67x)^{1/2}$ against LES (LESDATABASE64, NERSC)



Turbulence-closure theory milestones

Boussinesq 1877 Turbulent transfer is similar to molecular transfer but stronger

→ down-gradient transport → K -theory (eddy viscosity, conductivity, diffusivity)

Richardson (1920, 1922) role of stratification (Ri), the forward energy cascade

Prandtl (1930s) mixing length $l \sim z$, velocity scale $u_T \sim l dU/dz$, viscosity $K \sim l u_T$

Kolmogorov (1941) quantified the cascade, closure as a problem of energetics:

- budget equation for turbulent kinetic energy (TKE)
- TKE dissipation rate expressed through the turbulent-dissipation length scale

$u_T \sim (K \epsilon T)^{1/2}$, $K \sim l_\epsilon u_T$ **underlies further developments through 20th century**

Obukhov (1946) TKE-closure extended to stratified flows by adding the buoyancy flux in TKE equation, introduced Obukhov length scale L , but kept $l \sim z$

Monin & Obukhov (1954) M-O similarity theory (MOST) for the atmospheric surface layer → z/L

Mellor & Yamada (1974) hierarchy of K -closures → turbulence cut-off problem

Turbulence cut-off problem: Ri and Re

Buoyancy $b = (g/\rho_0)\rho \approx (g/T_0)d\Theta/dz$ (g – gravity acceleration, ρ – density)

Velocity shear $S = dU/dz$ (U – velocity, z – height)

$$Ri = \frac{db/dz}{(dU/dz)^2}$$

Richardson number characterises static stability:

the higher Ri (or z/L), the stronger suppression of turbulence

Key question What happens with turbulence at large Ri ?

Historical answer At Ri exceeding critical value ($Ri_{\text{critical}} < 1$) turbulence degenerates, and the flow becomes laminar (Richardson, 1920; Taylor, 1931; Prandtl, 1930, 1942; Chandrasekhar, 1961;...)

Observations in nature and numerical (LES, DNS) experiments

GEOPHYSICAL (very high Re) turbulence is maintained up to $Ri > 10^2$

Lab experiments at with low Re Flow becomes laminar at supercritical Ri



Mainstream in turbulence closure theory

Prandtl-1930's followed the Boussinesq idea of down-gradient transport (K -theory), determined $K \sim lu_T$, and expressed turbulent velocity u_T heuristically through the mixing length l and velocity gradient

Kolmogorov-1942 (**for neutrall stratification**) followed Prandtl concept of eddy viscosity $K_M \sim lu_T$; determined $u_T = (\text{TKE})^{1/2}$ through the turbulent kinetic energy (TKE) budget equation with dissipation rate $\varepsilon \sim (\text{TKE})/t_T \sim (\text{TKE})^{3/2}/l$

Obukhov-1946 and then the entire turbulence community extended Kolmogorov's closure to **stratified flows** **keeping it untouched**, except for inclusion of the buoyancy flux in the TKE equation

This approach **overlooked turbulent potential energy** TPE and its interaction with TKE and **employed Prandtl's relation** $K \sim lu_T$ to both K_M and eddy conductivity K_H , **which caused unrealistic cut off turbulence in supercritically stable stratification**

Mellor and Yamada (1974) developed **corrections preventing the turbulence cut-off**



Energy- & flux-budget (EFB) closure (2007-13)

Budget equations for major statistical moments

Turbulent kinetic energy (TKE) E_K

Turbulent potential energy (TPE) E_P

Vertical flux of temperature $F_z = \langle \theta w \rangle$ [or flux of buoyancy $(g/T)F_z$]

Vertical flux of momentum $\tau_{iz} = \langle u_i w \rangle$ ($i = 1, 2$)

New equation for the dissipation time scale $t_T = E_K / \varepsilon_K = l(E_K)^{-1/2}$

Accounting for TPE, vertical heat flux (killed TKE in Kolmogorov-type closures)
drops out from the equation for total turbulent energy (TTE = TKE + TPE)

Heat-flux equation restricts F_z through counter-gradient heat transfer and yields
self-preservation of turbulence and no Ri -critical in the energetic sense

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EFB disclosed two different regimes of stably stratified turbulence

"Strong-mixing turbulence" in boundary layer flows with $K_M \sim K_H$ at $Ri < Ri_c$

"Wave-like turbulence" in free atmosphere with $Pr_T = K_M / K_H \sim 4 Ri$ at $Ri \gg Ri_c$

New vision: PBL height separates strong-mixing and wave-like regimes

Conventional theories (e.g. Monin-Obukhov) overlook wave-like regime



Turbulent potential energy – analogy to Lorenz (1955) available potential energy

Buoyancy fluctuation proportional to displacement of fluid particle

$$b' = \frac{g}{\rho_0} \rho' = \frac{g}{\rho_0} \frac{\partial \langle \rho \rangle}{\partial z} z' = N^2 z'$$

Potential energy proportional to **squared** buoyancy/temperature

$$\begin{aligned} (E_P)' &= \frac{1}{z'} \int_z^{z+z'} b' z dz \\ &= \frac{1}{2} \frac{(b')^2}{N^2} = \frac{1}{2} \left(\frac{\beta}{N} \right)^2 (\theta')^2 = \left(\frac{\beta}{N} \right)^2 (E_\theta)' \end{aligned}$$



Turbulent energy budgets

Kinetic energy $E_K = \frac{1}{2} \langle u_i u_i \rangle$

$$\varepsilon_K = \frac{E_K}{t_T}$$

$$\frac{DE_K}{Dt} + \frac{\partial \Phi_K}{\partial z} = \tau S + \beta F_z - \varepsilon_K$$

Potential energy $E_P = \frac{1}{2} \left(\frac{\beta}{N} \right)^2 \langle \theta^2 \rangle$

$$\varepsilon_P = \frac{E_P}{C_P t_T}$$

$$\frac{DE_P}{Dt} + \frac{\partial \Phi_P}{\partial z} = -\beta F_z - \varepsilon_P$$

Total energy $E = E_K + E_P$

$$\frac{DE}{Dt} + \frac{\partial (\Phi_K + \Phi_P)}{\partial z} = \tau S - (\varepsilon_K + \varepsilon_P)$$

Buoyancy flux βF_z drops out from the turbulent total energy budget



Budget equation for the vertical turbulent flux of momentum $\tau_{i3} = \langle u_i w \rangle$

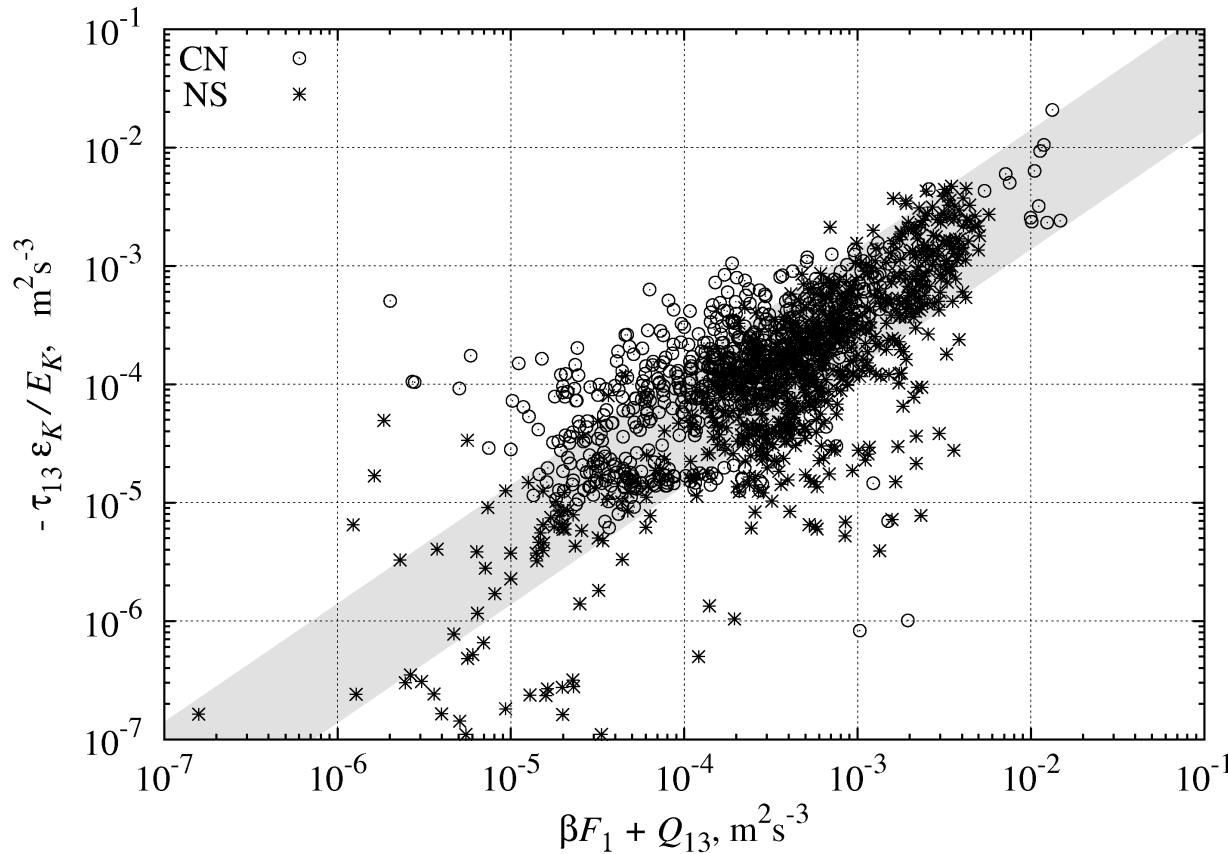
$$\frac{D\tau_{i3}}{Dt} + \frac{\partial}{\partial z} \Phi_i^{(\tau)} = -2E_z \frac{\partial U_i}{\partial z} - \varepsilon_{i3(\text{eff})}^{(\tau)}$$

Effective dissipation

$$\varepsilon_{i3(\text{eff})}^{(\tau)} \equiv 2\nu \left\langle \frac{\partial u_i}{\partial x_k} \frac{\partial w}{\partial x_k} \right\rangle + \beta F_i - \frac{1}{\rho_0} \left\langle p \left(\frac{\partial u_i}{\partial z} \frac{\partial w}{\partial x_i} \right) \right\rangle \sim \frac{\tau_{i3}}{t_T}$$



LES verification of Kolmogorov closure for effective dissipation of the turbulent flux of momentum $\varepsilon_{i3(\text{eff})}^{(\tau)}$



LES incapable of modelling viscous terms (presumably negligible in strongly stable stratification)



Budget equation for the vertical turbulent flux of potential temperature $F_z = \langle \theta w \rangle$

$$\frac{DF_z}{Dt} + \frac{\partial}{\partial z} \Phi_z^{(F)} = C_\theta \beta \langle \theta^2 \rangle - 2E_z \frac{\partial \Theta}{\partial z} - \frac{F_z}{C_F t_T}$$

(1) We have shown that pressure term combines with mean-squared potential-temperature-fluctuation term:

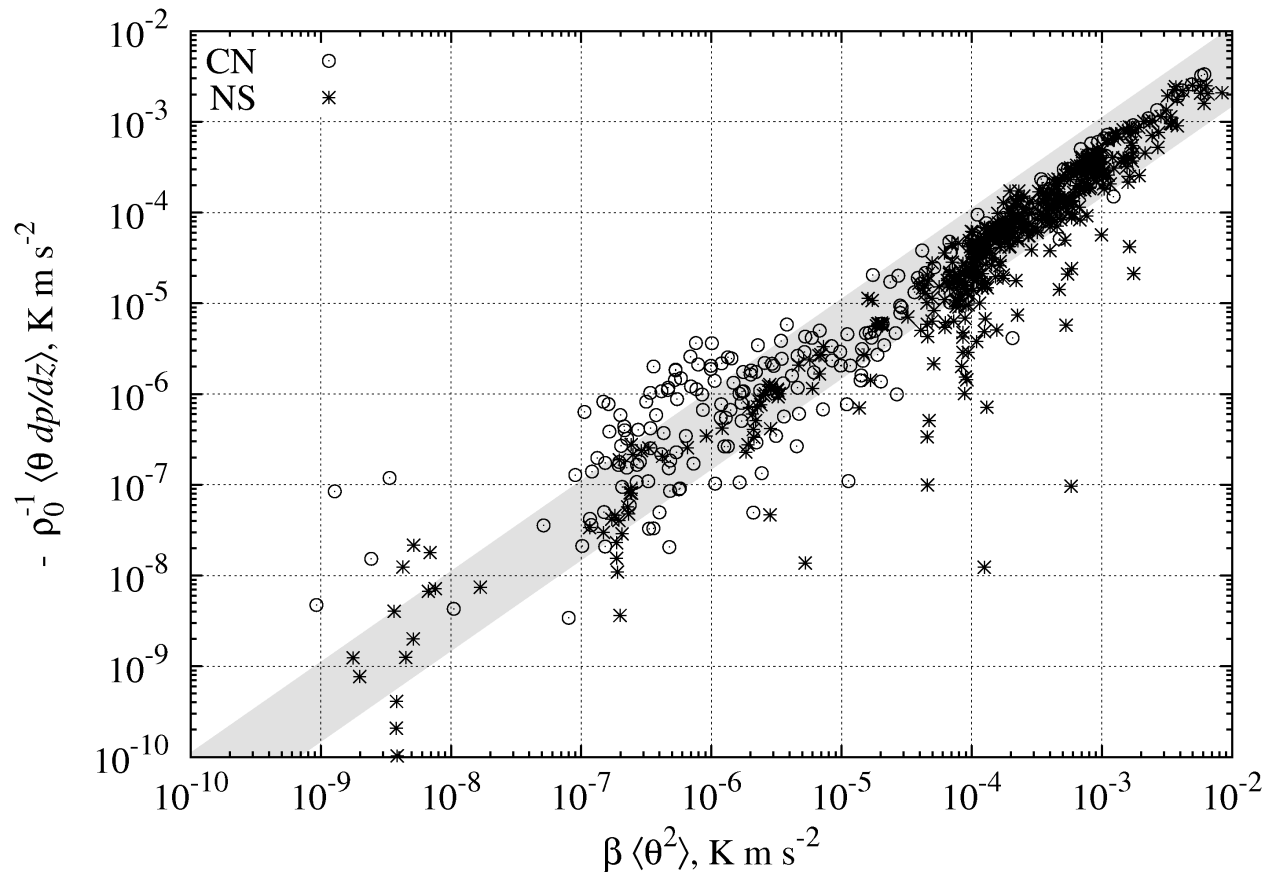
$$\frac{1}{\rho_0} \left\langle \theta \frac{\partial p}{\partial z} \right\rangle \sim \beta \langle \theta^2 \rangle$$

(2) On r.h.s. of the equation, 1st term (generation of positive heat flux) counteracts to 2nd term (generation of negative heat flux) and yields

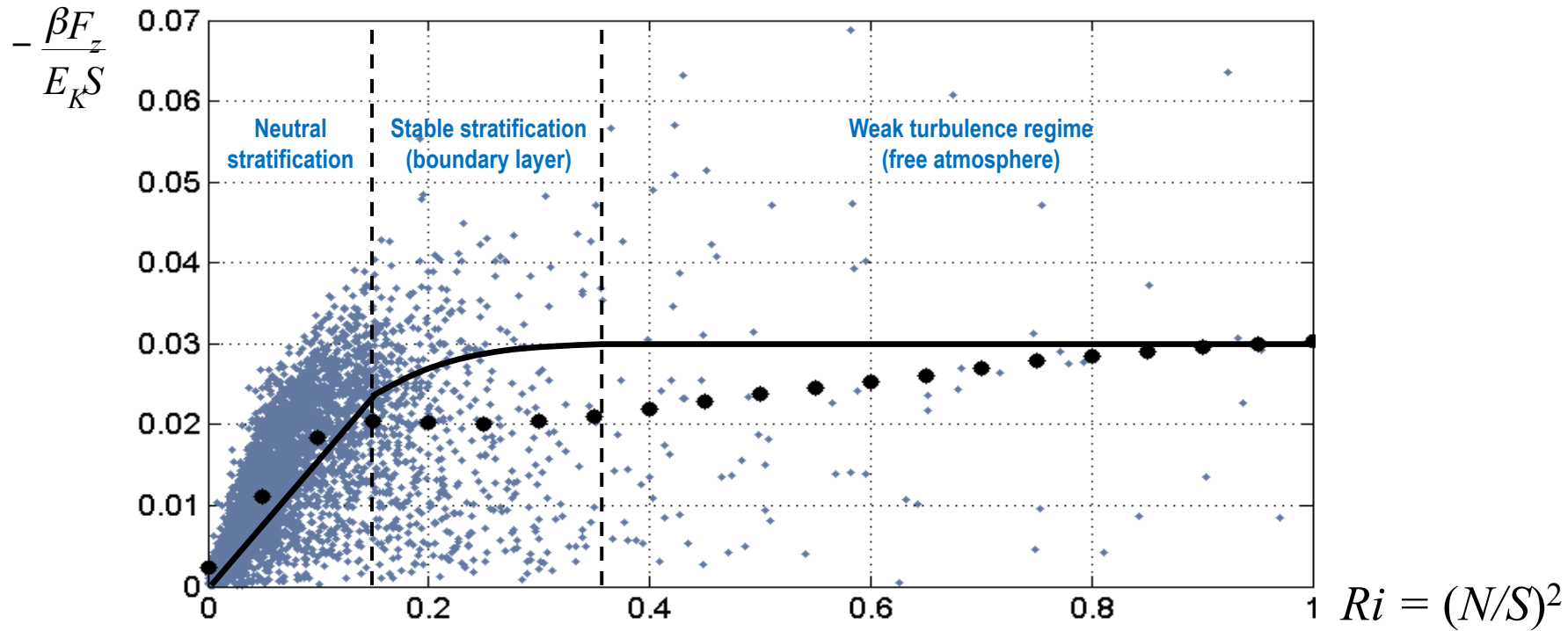
self-control of turbulence in very stable stratification



LES verification of our parameterization of the pressure term $\rho_0^{-1} \langle \theta \partial p / \partial z \rangle \sim \beta \langle \theta^2 \rangle$



Ri -dependence of the buoyancy flux $B = \beta F_z$



Data (Sheba) and theory disprove very concept of eddy-conductivity $B = K_H N^2$

Almost neutral stratification ($0 < z/L < 0.5$) **MOST OK**

$$B \sim E_K^{1/2} z N^2, \quad K_H \sim E_K^{1/2} z$$

“z-less stratification” ($0.5 < z/L \ll 10$) **MOST OK**

$$B \sim E_K N, \quad K_H \sim E_K / N$$

Very stable stratification ($z/L \gg 10$) **MOST fails**

$$B \sim E_K S, \quad K_H \sim E_K S / N^2$$



Turbulent dissipation time and length scales

By definition, time scale $t_T \equiv E_K / \varepsilon_K$ and length scale $l \equiv E_K^{1/2} t_T$

The steady-state TKE budget $\tau S + \beta F_z \equiv \tau S (1 - Ri_f) = \varepsilon_K \equiv - \frac{E_K}{t_{TE}}$

Flux Ri number $Ri_f \equiv \frac{-\beta F_z}{\tau S} = \frac{\tau^{1/2}}{SL}$ Obukhov length $L = \frac{\tau^{3/2}}{-\beta F_z}$ $Ri_f \rightarrow R_\infty < 1$

Shear: neutral $S = \frac{\tau^{1/2}}{kz}$, extreme stable (TKE) $S \rightarrow \frac{-\beta F_z}{R_\infty \tau} = \frac{\tau^{1/2}}{R_\infty L}$

Interpolation yields **empirical law** valid in any stratification $S = \frac{\tau^{1/2}}{kz} \left(1 + \frac{k}{R_\infty} \frac{z}{L} \right)$ $k / R_\infty = 1.6$

Combining **this law** with the TKE equation yields

$$t_{TE} = \frac{kz}{E_K^{1/2} + C_\Omega \Omega z} \left(\frac{E_K}{\tau} \right)^{3/2} \frac{1 - Ri_f / R_\infty}{1 - Ri_f}$$

where kz plays the role of a “**master length scale**”



Relaxation equation for dissipation time scale

Evolution of t_T is controlled by

tendency towards equilibrium

$$t_T \rightarrow t_{TE}$$

counteracted by **distortion** due to non-stationary processes and heterogeneity causing mean-flow and turbulent transports.

This **counteraction** is described by **RELAXATION EQUATION**

$$\frac{Dt_T}{Dt} - \frac{\partial}{\partial z} K_T \frac{\partial t_T}{\partial z} = -C_R \left(\frac{t_T}{t_{TE}} - 1 \right)$$

$C_R \sim 1$ relaxation constant (differs for increasing/decreasing regimes)

K_T is the vertical turbulent exchange coefficient ($\sim$ to eddy viscosity)



Major results

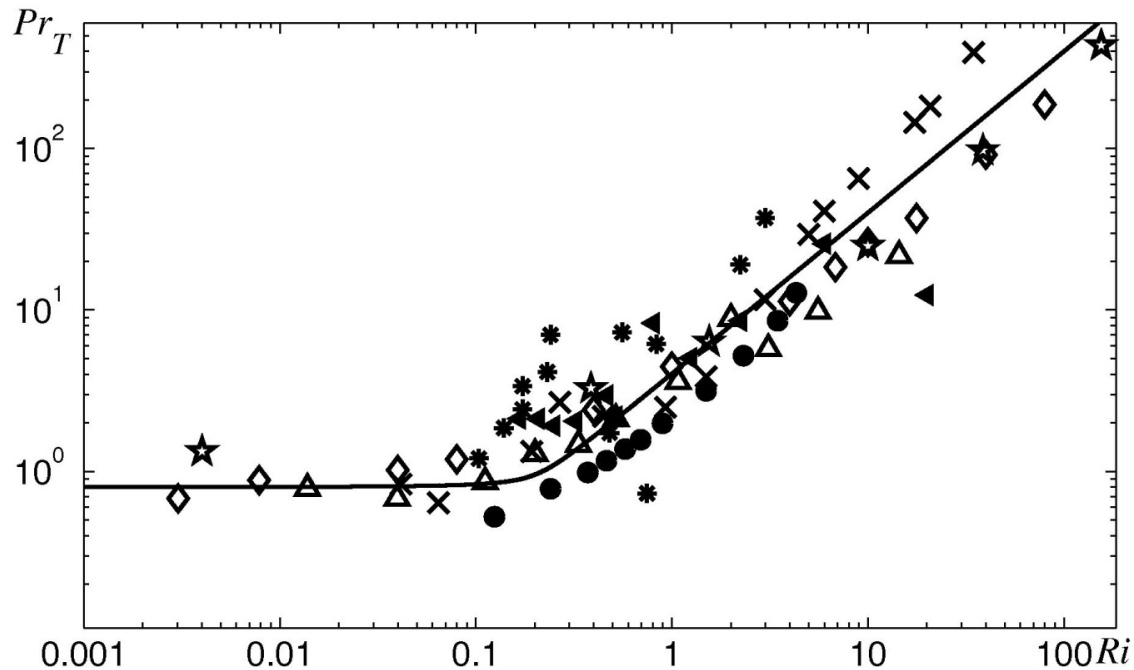
- CONCEPT of turbulent potential energy analogous to Lorenz's **available potential energy**; both $\sim$ **squared** density (O & T, 1986)
- CONCEPT of self-control: **down-gradient buoyancy flux $\rightarrow$ TPE $\rightarrow$ compensating counter-gradient flux / TPE converts back into TKE**
- Geophysical (high- Re) flows **remain turbulent at supercritical Ri** .
“Critical” $Ri_c \sim 0.25$ demarcates two different turbulent regimes:
 - known strong turbulence with $K_M \sim K_H$ at $Ri < Ri_c$ typical of PBLs
 - new weak turbulence with $Pr_T = K_M / K_H \sim 4Ri$ at $Ri \gg Ri_c$ in free flow
- Hierarchy of closure models of different complexity – for use in research and operational modelling (incl. new dissipation time scale)
- Revision of Monin-Obukhov surface-layer similarity theory (MOST)
- Experiments confirm EFB theory up to $Ri \sim 10^3$ (free atmosphere/ocean)



Examples of empirical verification



Turbulent Prandtl number $Pr_T = K_M/K_H$ versus Ri

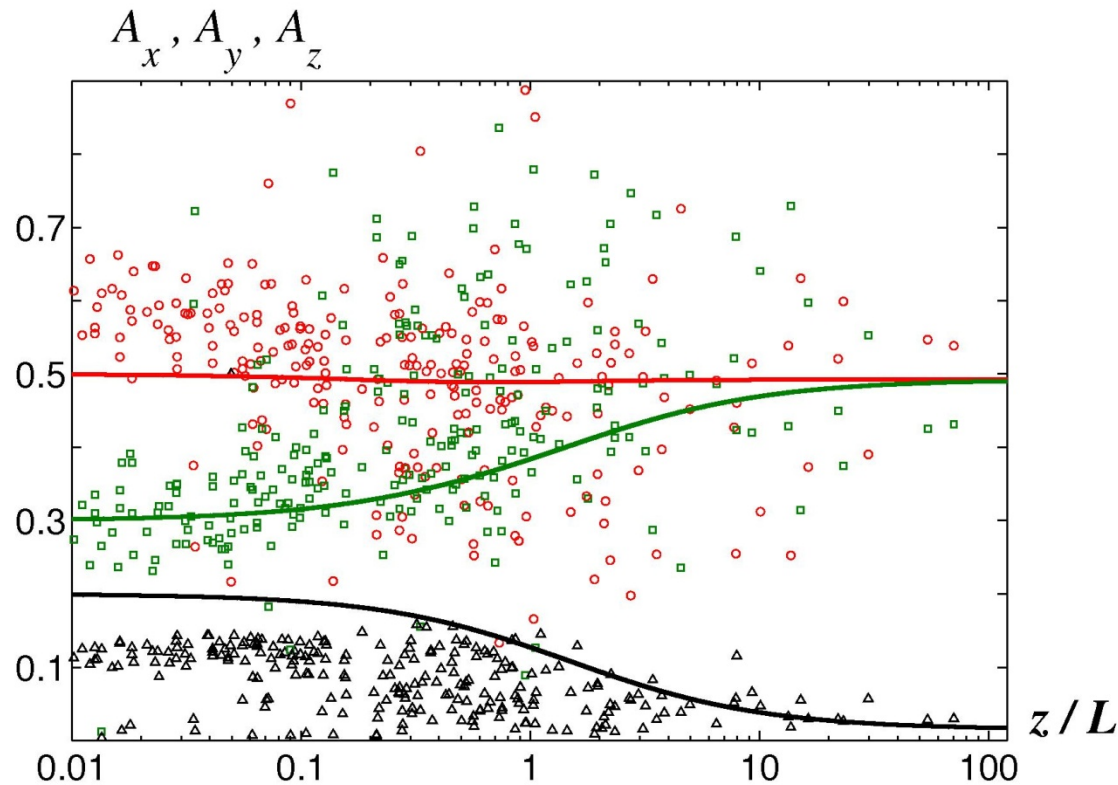


Atmospheric data: $\blacktriangleleft$ (Kondo et al., 1978), $*$ (Bertin et al., 1997); laboratory experiments: $\times$ (Rehmann & Koseff, 2004), $\diamond$ (Ohya, 2001), $\bullet$ (Strang & Fernando, 2001); DNS: $\star$ (Stretch et al., 2001); and LES: $\triangleleft$ (Eau, 2009). The curve shows our EFB theory. The “strong” turbulence ($Pr_T \approx 0.8$) and the “weak” turbulence ($Pr_T \sim 4 Ri$) match at $Ri \sim 0.25$.

MOST assumes $Pr_T = \text{constant}$ (Reynolds Analogy)
Prior closures $\rightarrow$ heuristic corrections to R-analogy



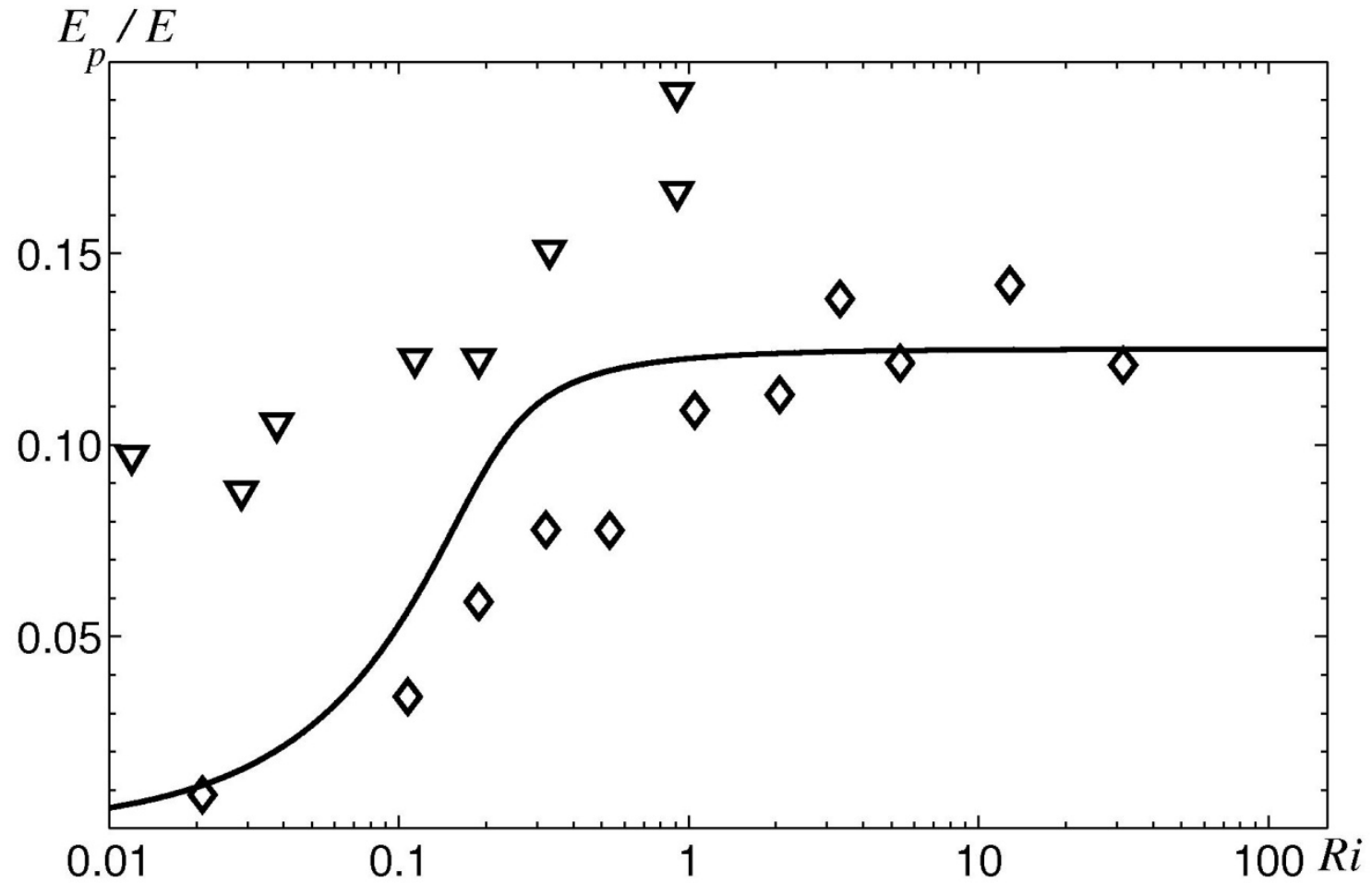
Longitudinal A_x , transverse A_y & vertical A_z TKE shares vs. z/L



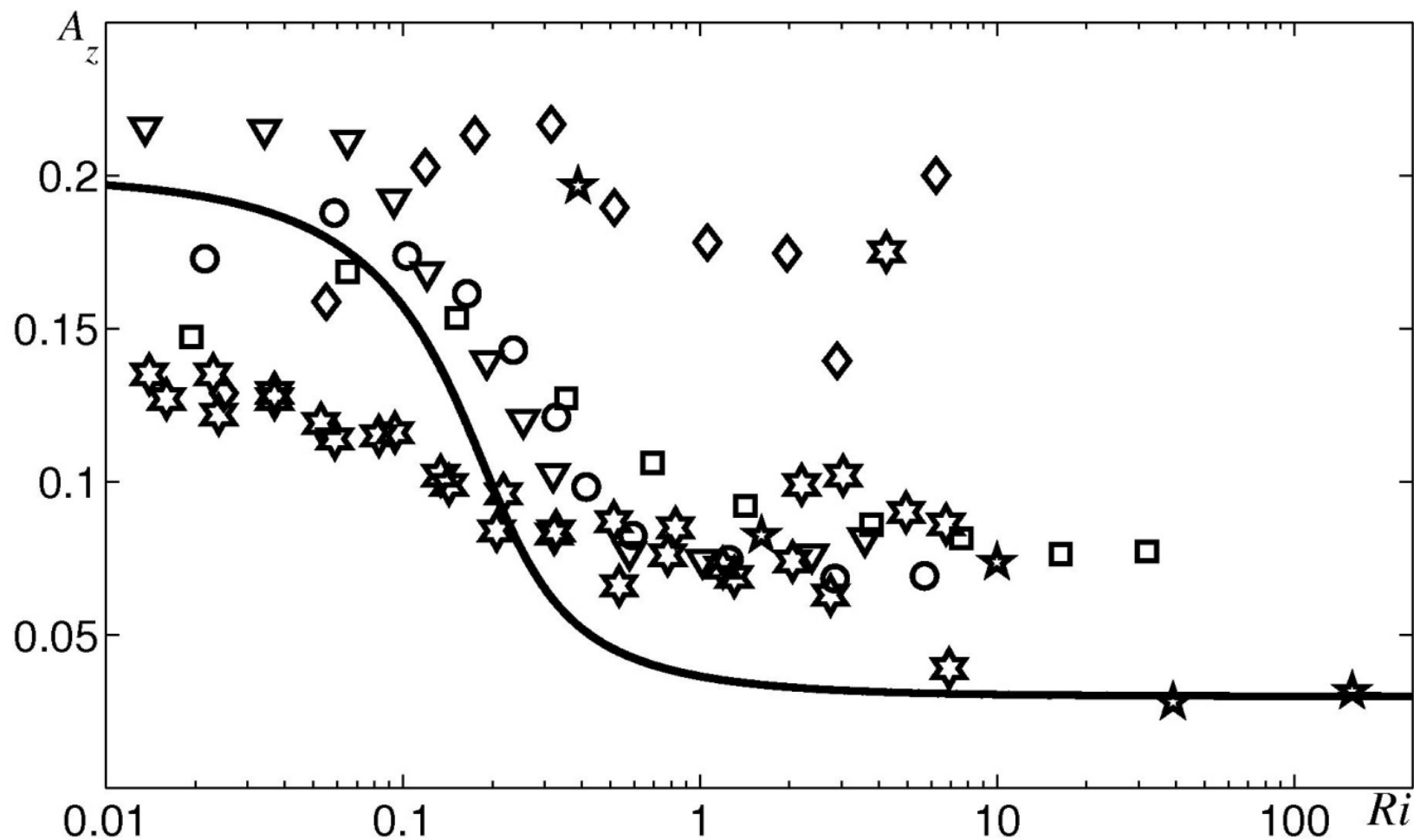
Experimental data from Kalmykian expedition 2007 of the Institute of Atmospheric Physics (Moscow). Theoretical curves are plotted after the EFB theory. The **traditional “return-to-isotropy” model overlook the stability dependence of A_y** clearly seen in the Figure. The strongest stability, $z/L = 100$, corresponds to $Ri = 8$.



The share of turbulent potential energy $E_p / (E_p + E_K)$

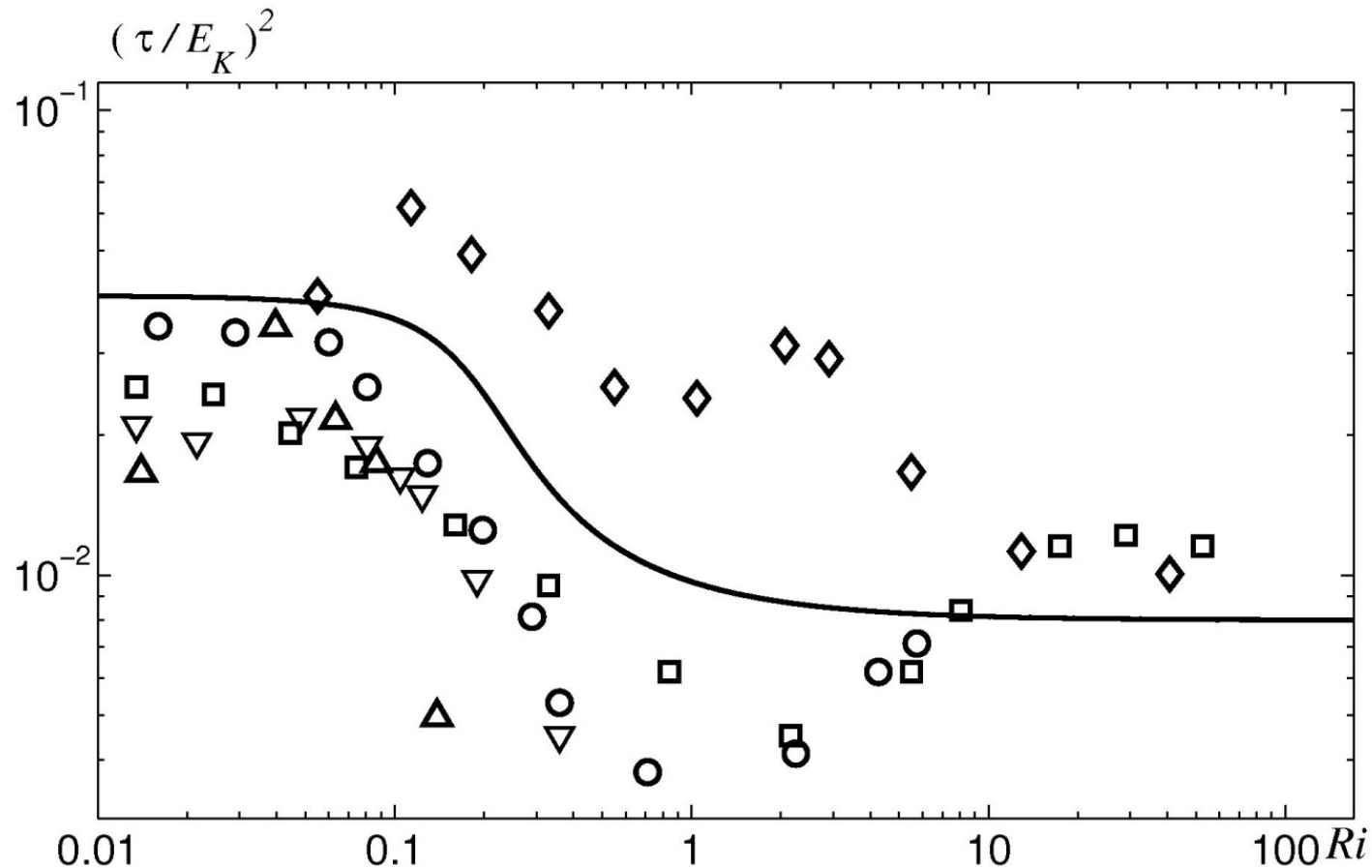


The share of the energy of the vertical velocity E_z / E_K



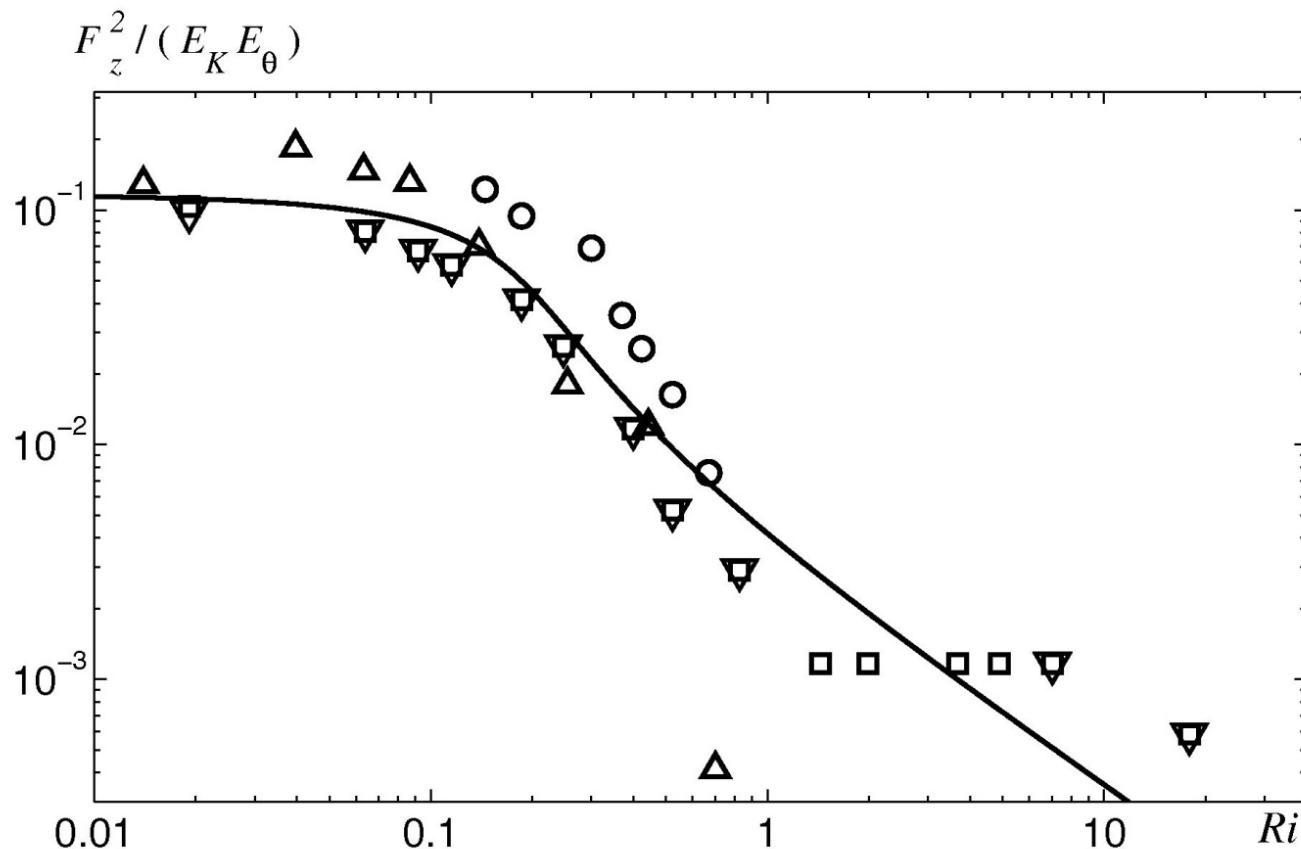
Dimensionless vertical flux of momentum: two plateaus
corresponding to the *strong* and *weak* turbulence regime
traditional closures and **MOST** assumes $\tau/E_K = \text{constant}$

s

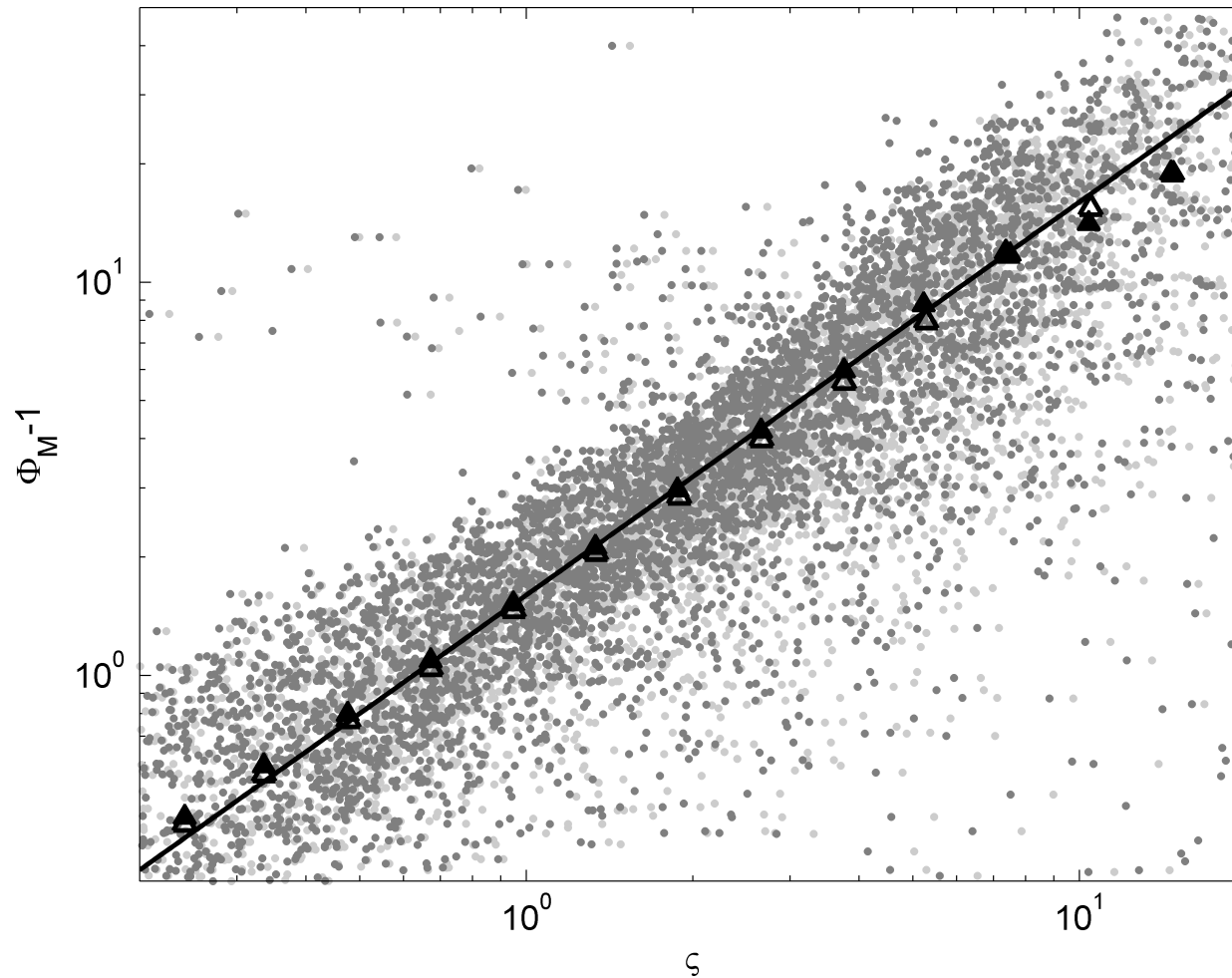


Dimensionless heat flux: practically constant in *strong* turbulence and sharply decreases in *weak* turbulence

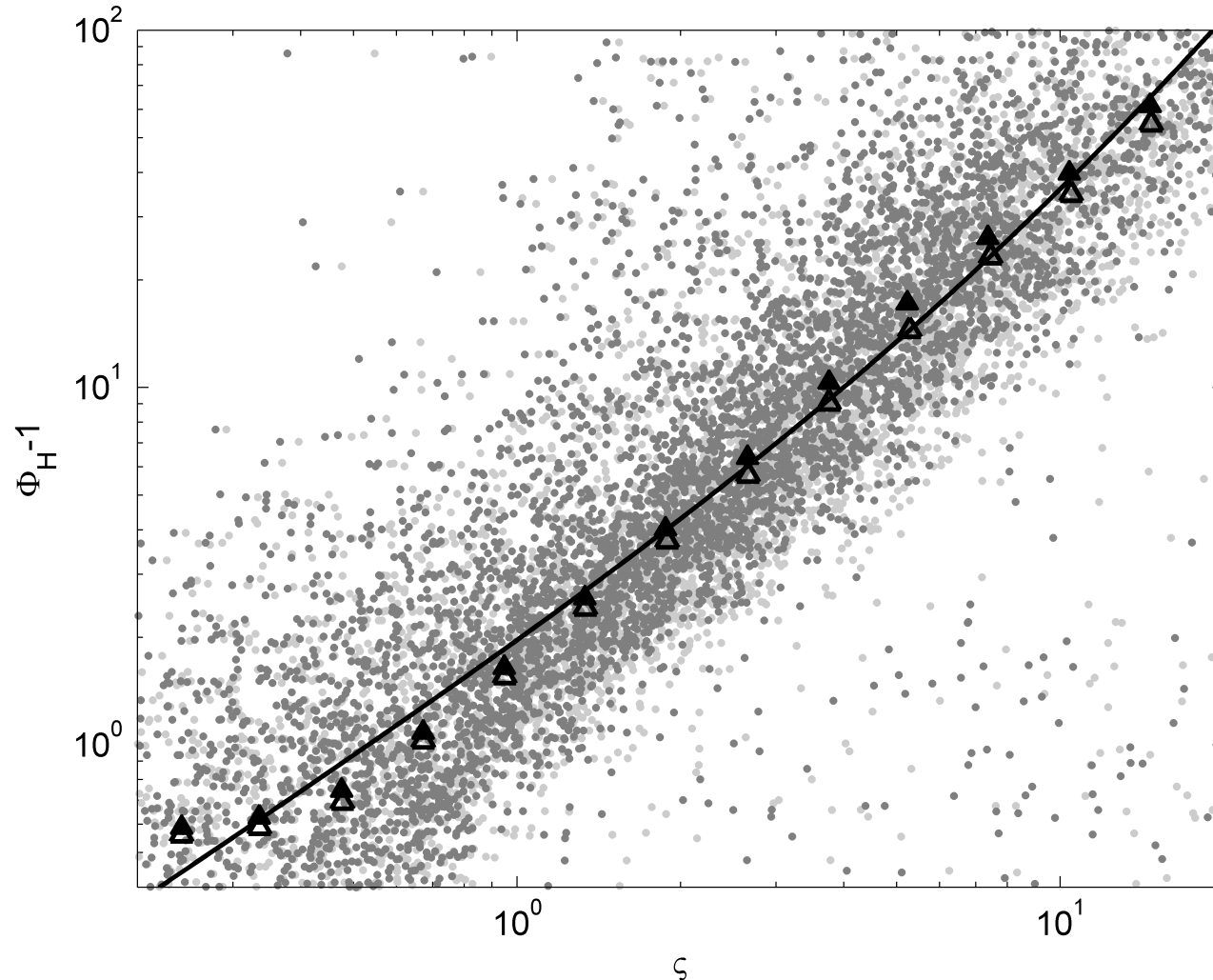
MOST assumes $F_z / (E_K E_\theta)^{1/2} = \text{constant}$
traditional closures overlooked this dependence



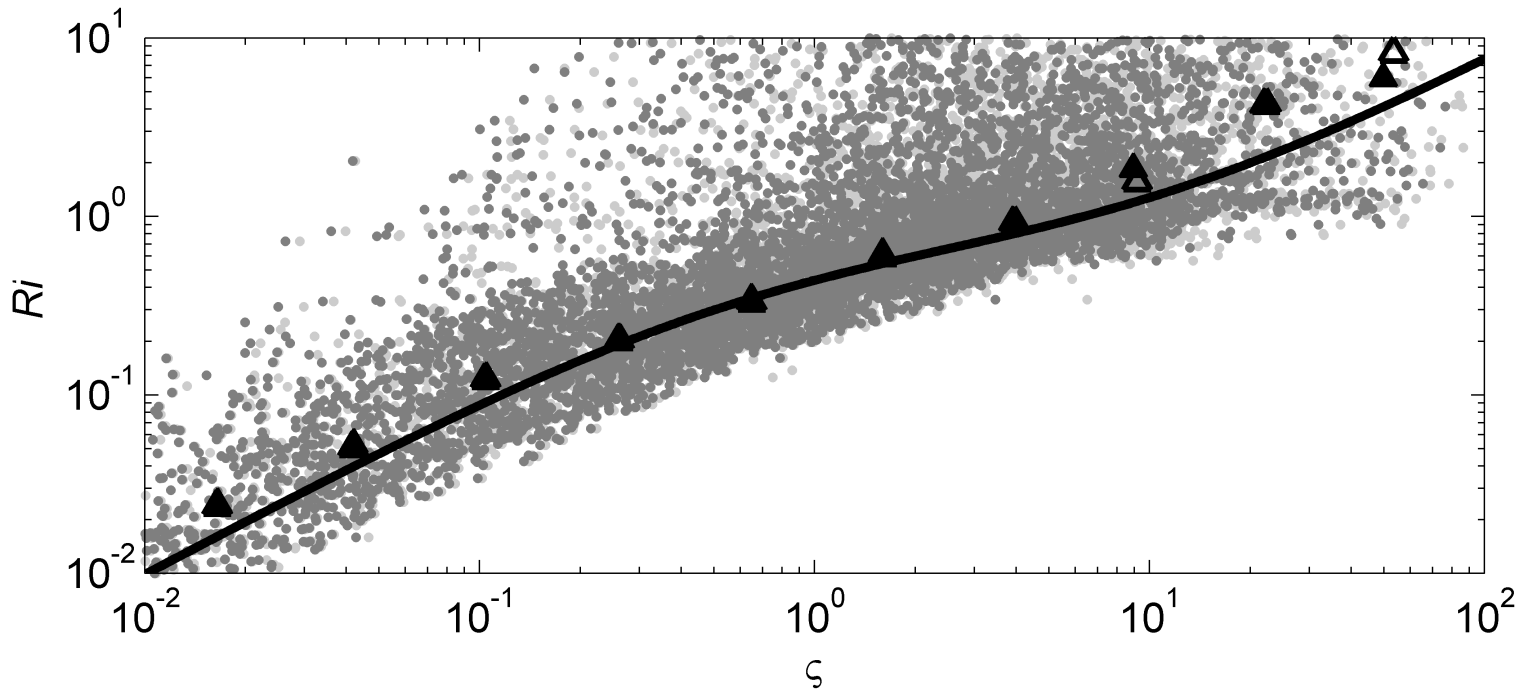
**Dimensionless velocity gradient $\Phi_M = (kz / u_*) / (\partial U / \partial z)$
versus $\zeta = z/L$ after LES (dots) and the EFB model (curve)**
MOST OK



**Dimensionless temperature gradient $\Phi_H = (-k_T z \tau^{1/2} / F_z)(\partial \Theta / \partial z)$
versus $\zeta = z/L$ after LES (dots) and the EFB model (curve)**
MOST and conventional closure models fail



Richardson number, Ri , versus $\zeta = z/L$ after LES (dots) and the EFB model (curve)



General closure model: energy & flux equations

Kinetic energy $\frac{DE_K}{Dt} - \frac{\partial}{\partial z} K_E \frac{\partial E_K}{\partial z} = -\tau_{i3} \frac{\partial U_i}{\partial z} + \beta F_z - \frac{E_K}{t_T}$

Potential energy $\frac{DE_P}{Dt} - \frac{\partial}{\partial z} K_E \frac{\partial E_P}{\partial z} = -\beta F_z - \frac{E_P}{C_P t_T}$

Momentum flux $\frac{D\tau_{i3}}{Dt} - \frac{\partial}{\partial z} K_{FM} \frac{\partial \tau_{i3}}{\partial z} = -2E_z \frac{\partial U_i}{\partial z} - \frac{\tau_{i3}}{C_\tau t_T}$

Θ -flux $\frac{DF_z}{Dt} - \frac{\partial}{\partial z} K_{FH} \frac{\partial F_z}{\partial z} = -2(E_z - C_\theta E_P) \frac{\partial \Theta}{\partial z} - \frac{F_z}{C_F t_T}$

Turbulent exchange coefficients for energies and fluxes are taken proportional to the eddy viscosity

$$K_E / C_E = K_{FM} / C_{FM} = K_{FH} / C_{FH} = K_T / C_T = E_z t_T$$



General closure model:

Vertical TKE

To characterise stability we use, instead of Ri_f , the energy-ratio $\Pi = E_P / E_K$ {in the steady-state $\Pi = C_P Ri_f / (1 - Ri_f)$ } and employ our steady-state solution to express E_z / E_K and E_K / τ as universal functions of Π determined from our prognostic equations

Dissipation time scale

Similarly, we express the equilibrium time scale t_{TE} through Π

$$t_{TE} = \frac{kz}{E_K^{1/2} + C_\Omega \Omega z} \left(\frac{E_K}{\tau} \right)^{3/2} \left(1 - \frac{\Pi}{\Pi_\infty} \right)$$

and determine t_T after our relaxation equation

$$\frac{Dt_T}{Dt} - \frac{\partial}{\partial z} K_T \frac{\partial t_T}{\partial z} = -C_R \left(\frac{t_T}{t_{TE}} - 1 \right)$$



Optimal meteorological EFB closure model

For operational modelling

we recommend model based on 3 prognostic equations for:

- the the two turbulent energies E_K and E_P
- and the dissipation time scale t_T
- in combination with diagnostic eddy viscosity & eddy conductivity

Advantages

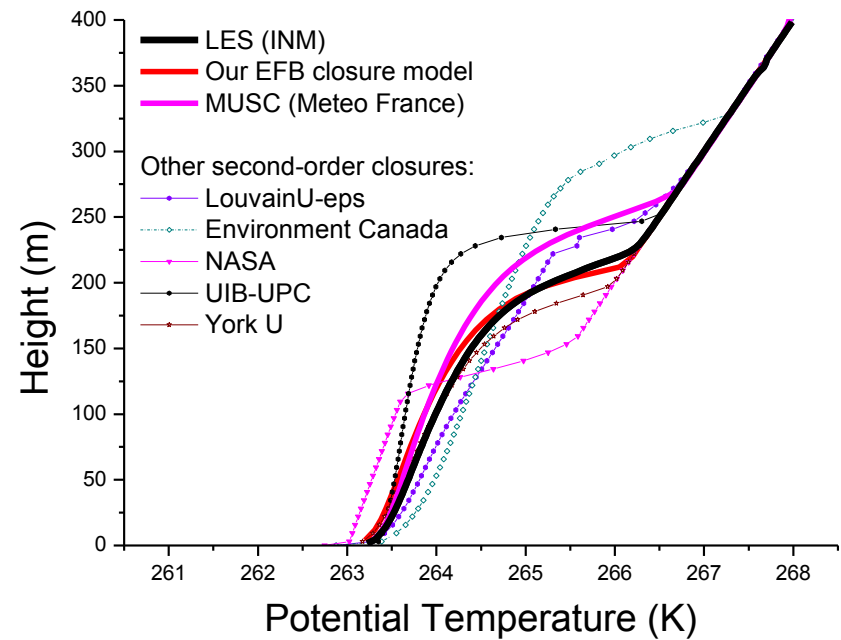
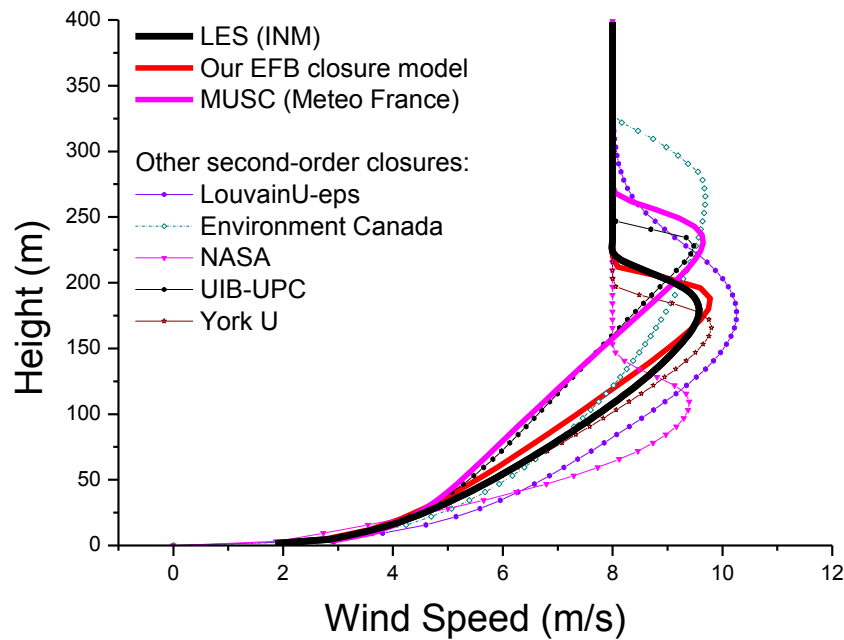
- consistent energetics with no Ri-critical
- advanced concept of the turbulent dissipation time scale
- “energy stratification parameter” preventing artificial extremes
- essential anisotropy of turbulence
- generally non-gradient and non-local turbulent transports



EFB compared to case study GABLS-1



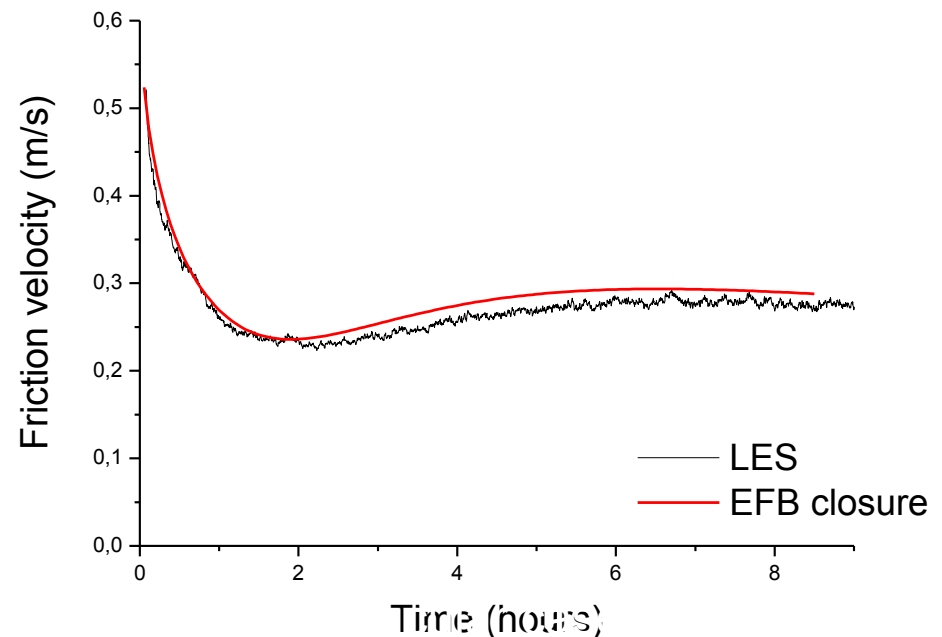
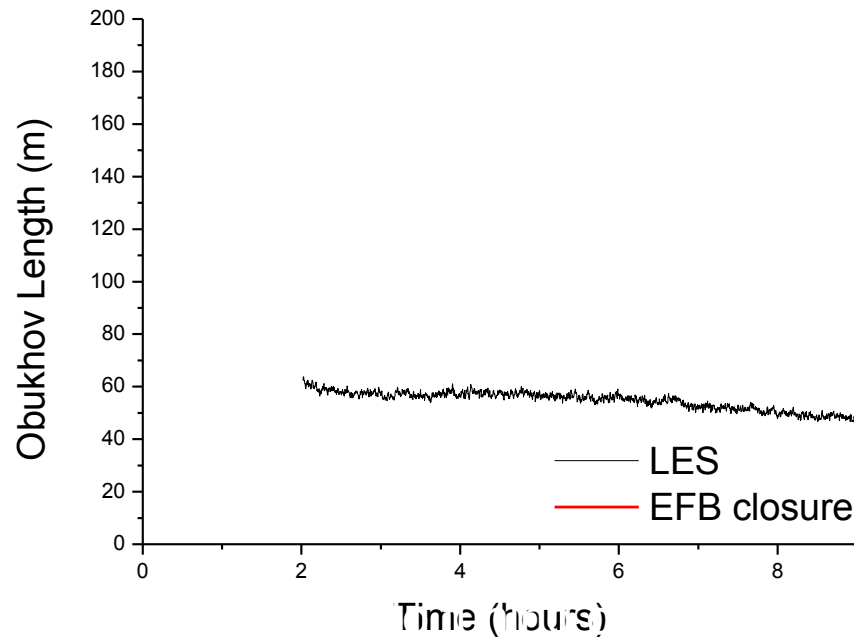
Comparison with GABLS1 (Holtslag et al, 2003) Nocturnal Stable PBL



EFB-closure profiles of the wind speed and potential temperature
compared with the GABLS1 LES

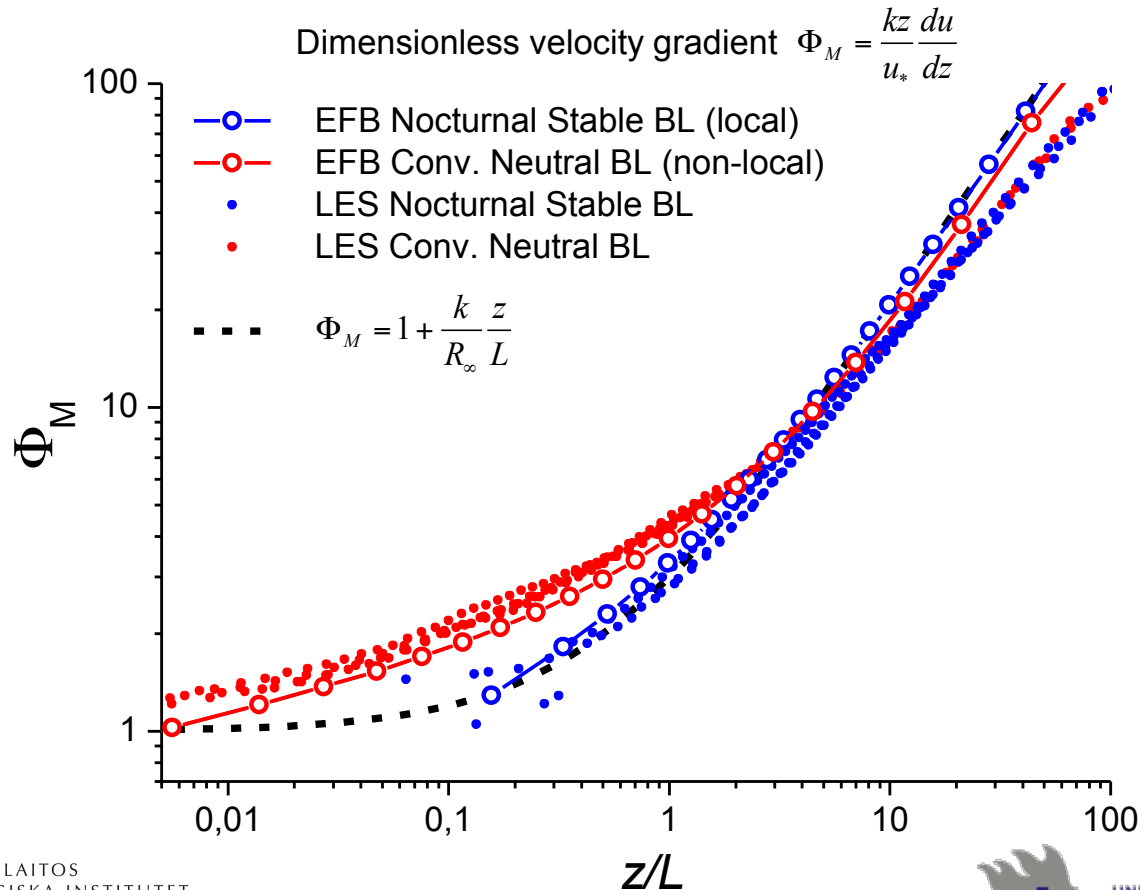


Temporal development of the Obukhov length-scale and friction velocity

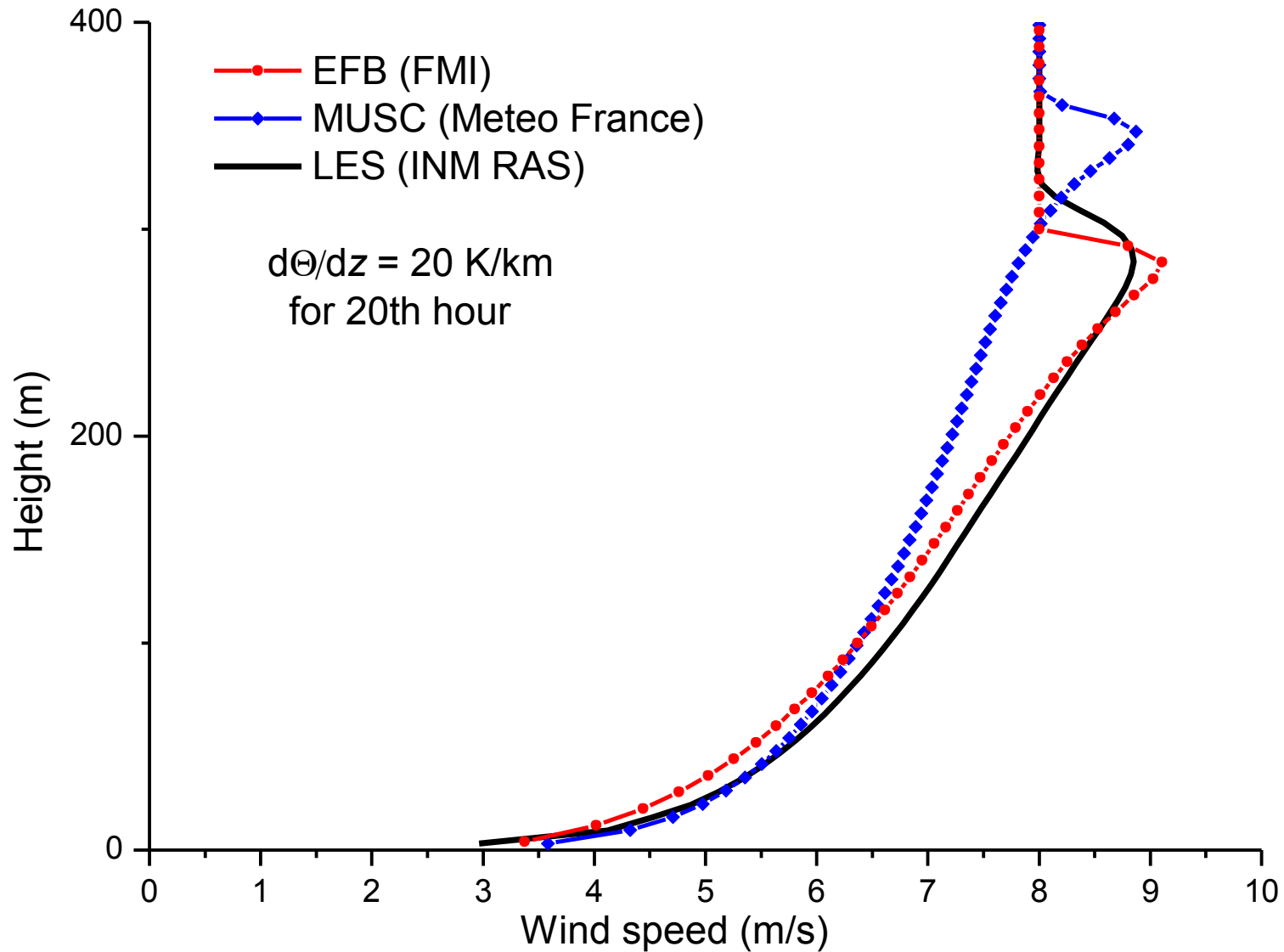


Relaxation equation for the dissipation time scale

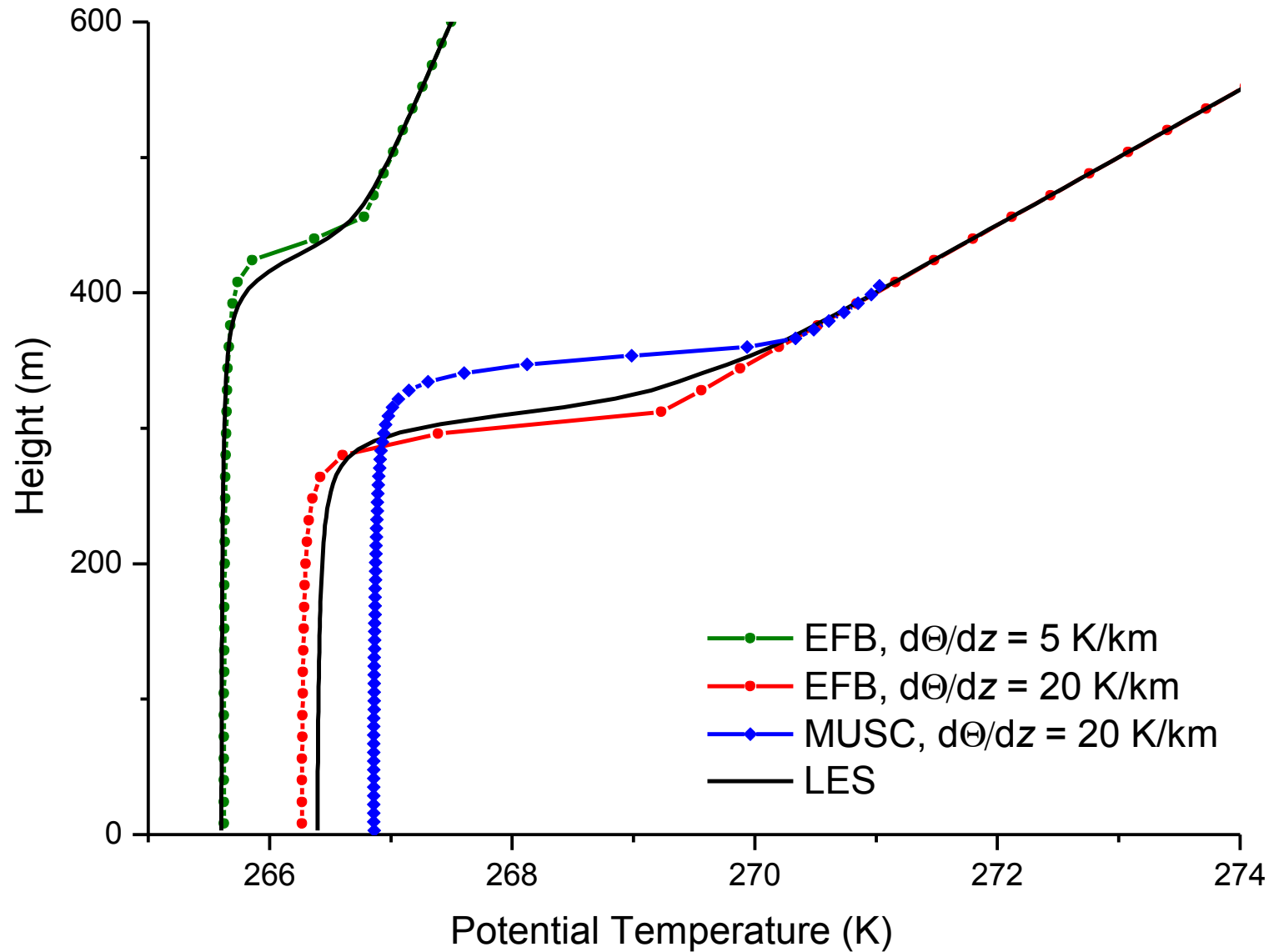
$$\frac{Dt_T}{Dt} - \frac{\partial}{\partial z} K_T \frac{\partial t_T}{\partial z} = -C_R \left(\frac{t_T}{t_{TE}} - 1 \right), \quad K_T = C_T E_z t_T, \quad C_T = 4, \quad C_R = 0.5$$



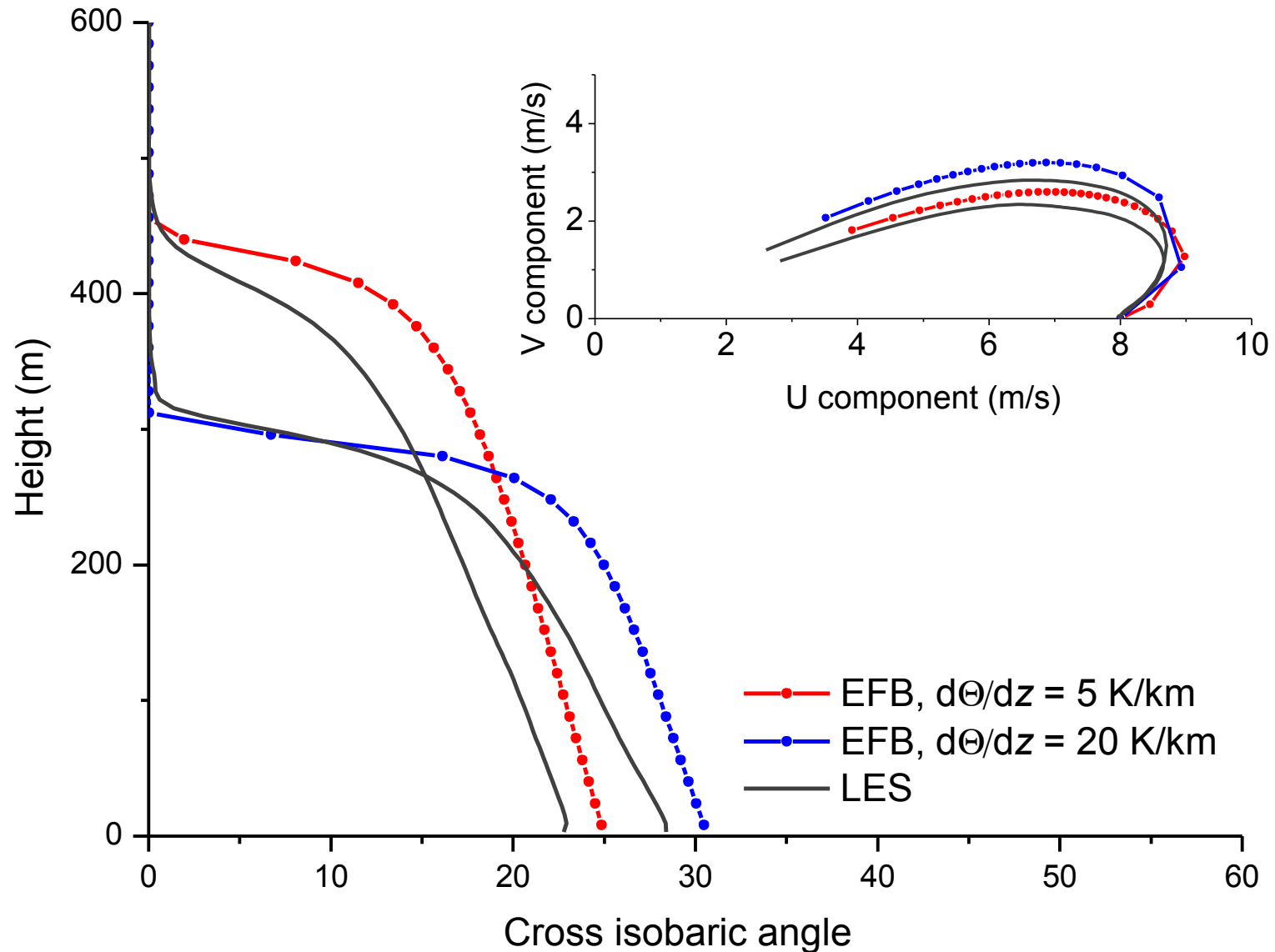
Vertical profiles of the wind speed in Conventionally Neutral BL



Vertical profiles of potential temperature in Conventionally Neutral BL



Cross isobaric angle and wind hodograph in Conventionally Neutral BL



Conclusions

- **TKE budget equation is INSUFFICIENT**

E_K and E_P are equally important $\rightarrow E = E_K + E_P$

- There is **no Ri_c in the energetic sense**; experimental data confirm this theoretical conclusion up to $Ri \sim 10^3$
- $Ri \sim 0.2-0.3$ (hydrodynamic instability limit) **separates regimes of “strong” and “weak” turbulence**
- Newly discovered “**weak turbulence regime**” is **typical of free atmosphere and deep ocean**, wherein it determines turbulent transport of the energy and momentum and diffusion of passive scalars
- The EFB closure provides advanced tools for research and modelling applications



Conclusions

- EFB turbulence closure → new vision and modelling of geophysical stably stratified turbulence
- **No Ri_c in the energetic sense:** experimental data confirm this conclusion up to $Ri \sim 10^3$
- **Instead:** $Ri \sim 0.2-0.3$ (hydrodynamic instability limit) separates regimes of “strong” and “weak” turbulence → the boundary between PBL and free atmosphere → **another view at the PBL height**
- MOS is applicable to the “strong” turbulence regime typical of boundary layer flows but inapplicable to **“weak turbulence” typical of free atmosphere / ocean**



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Major results

- CONCEPT of turbulent potential energy analogous to Lorenz's **available potential energy**; both $\sim$ **squared** density (O & T, 1986)
- CONCEPT of self-control: **down-gradient buoyancy flux $\rightarrow$ TPE $\rightarrow$ compensating counter-gradient flux / TPE converts back into TKE**
- Geophysical (high- Re) flows **remain turbulent at supercritical Ri** .
“Critical” $Ri_c \sim 0.25$ demarcates two different turbulent regimes:
 - known strong-mixing turbulence $K_M \sim K_H$ at $Ri < Ri_c$ typical of PBLs
 - new wave-like turbulence $Pr_T = K_M / K_H \sim 4Ri$ at $Ri \gg Ri_c$ free flows
- Hierarchy of closure models of different complexity – for use in research and operational modelling
- Observations in atmosphere, LES and DNS confirm EFB theory up to $Ri \sim 10^3$ (free atmosphere/ocean)



PBL height and air pollution



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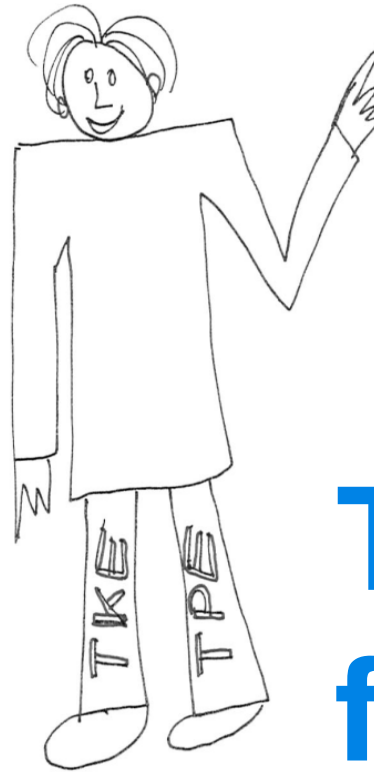
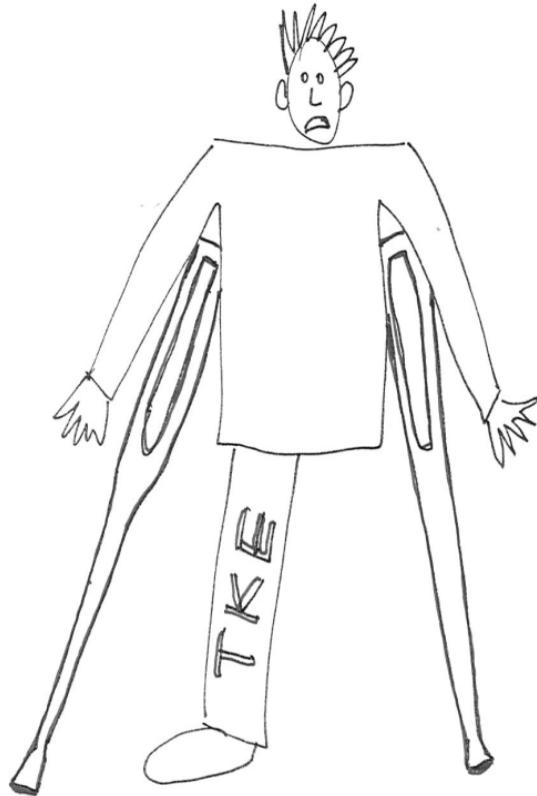


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Turbulence does not degenerate up to very strong stratification

From only TKE



to TKE, TPE and
self-control of
the buoyancy flux

Thank you for your attention

