A new scale-aware 3DTKE scheme

- We have developed a scale-aware parameterization scheme based on the general form of the TKE equation in the WRF model to simulate 3D subgrid turbulent mixing;

- The scheme holds the promise of making the transition between the mesoscale and LES limits smooth, not only in the amount of subgrid mixing, but also in the parameterization formula.
Starting point of our development in WRF

\[ \overline{u_i'u_j'} = -K_{ij}^M \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) + \overline{u_i'u_j'}^{NL} \quad (i \neq j) \]

\[ \overline{u_i'\theta'} = -K_{ij}^H \frac{\partial \bar{\theta}}{\partial x_j} + \delta_{i3} \overline{u_i'\theta'}^{NL} \]

\[ K^M = Ce^{1/2} l \]

\[ \frac{\partial e}{\partial t} = \bar{u}_j \frac{\partial e}{\partial x_j} - u_i'u_j' \frac{\partial \bar{u}_i}{\partial x_j} + \frac{g}{\bar{\theta}_0} w'\theta' - \frac{\partial u_i'(e + p'/\rho_0)}{\partial x_i} - \varepsilon \]

Minimal requirements for extending the 3D TKE LES subgrid model to the mesoscale limit include the three key elements:

- The nonlocal fluxes;
- The mixing length scales;
- Horizontal diffusion parameterization.
Nonlocal heat flux

Conditional sampling based on LES data

\[
\overline{w' \theta'} = a \overline{w' \theta'}^c + (1-a) \overline{w' \theta'}^e + a(1-a)(w_c - w_e)(\theta_c - \theta_e)
\]

\[
\overline{w' \theta'}^{NL} = a(1-a)(w_c - w_e)(\theta_c - \theta_e)
\]

In mesoscale limit, the nonlocal heat flux gradually converges to a single profile (compare the profiles at resolutions 3km and 9km)

Vertical profile of SGS nonlocal heat flux for different resolutions normalized by surface heat flux.
Mixing length scale

In the LES limit, Deardorff’s length scale is applied:

\[
l_{LES} = \begin{cases} 
\min \left[ 0.76e^{1/2} \left| \frac{\partial \theta}{\partial z} \right|^{-1/2}, \Delta s \right] & \text{for } N^2 > 0 \\
\Delta s & \text{for } N^2 \leq 0
\end{cases}
\]

\[
\Delta s = (\Delta x \Delta y \Delta z)^{1/3}
\]

In the mesoscale limit, following MYNN Level-3 scheme, the vertical length scale is given as:

\[
\frac{1}{l_{MESO}} = \frac{1}{l_S} + \frac{1}{l_T} + \frac{1}{l_B}
\]

\(l_S\) is the length scale in the surface layer controlled by the effects of wall and stability, 
\(l_T\) the length scale depending on the turbulent structure of the PBL and 
\(l_B\) the length scale limited by the thermal stability.
Scale-aware transition between LES and mesoscale limit

\[
\overline{w'\theta'}^{\Delta x} = -K_H^{\Delta x} \frac{\partial \overline{\theta}}{\partial z} + \overline{w'\theta'}^{\Delta x, NL} = \overline{w'\theta'}^{\Delta x, L} + \overline{w'\theta'}^{\Delta x, NL}
\]

\[
\overline{w'\theta'}^{\Delta x, NL} = \overline{w'\theta'}^{NL} P_{NL}(\Delta x/z_i) \quad \overline{w'\theta'}^{\Delta x, L} = -K_H^{\Delta x} \frac{\partial \overline{\theta}}{\partial z}
\]

\[
K_H^{\Delta x} = C_{vertical} l_{\Delta x} e^{1/2}
\]

\[
l_{\Delta x} = P_L(\Delta x/z_i) l_{MESO} + \left[1 - P_L(\Delta x/z_i)\right] l_{LES}
\]
Horizontal diffusion

The scale-aware transition of horizontal diffusion:

\[ K_h = K_D + K_T \]

\[ = P_L(\Delta x / z_i)(C_s \Delta)^2 D / \sqrt{2} + \left[ 1 - P_L(\Delta x / z_i) \right] C_k l e^{1/2} \]

*\( K_D \) is the diffusivity based on the deformation (2D Smag).
\( K_T \) is the diffusivity based on the 1.5-order TKE.*
Improvements on numerics

• We have found that the new scale-aware 3DTKE scheme can be unstable when it is used in mesoscale simulations in which $dx, dy \gg dz$ (highly anisotropic grid).

• To make the model stable, I have used the implicit method instead of original explicit method to solve the TKE equation and model diffusion equations.

$$\frac{\partial e}{\partial t} + U_j \frac{\partial e}{\partial x_j} = - (u_i u'_j) \frac{\partial U_i}{\partial x_j} + \delta_{i3} \frac{g}{\theta_v} u'_i \theta'_v - \frac{1}{\rho} \frac{\partial (p' u'_i)}{\partial x_i} - \frac{\partial (u'_j e)}{\partial x_j} - \varepsilon$$

$$e_{k}^{n+1} - e_{k}^{n} = \Delta t \left( D_h - A_h + P \right)_{k}^{n} + \Delta t \left( D_v - A_v - \frac{c_e e^{1/2}}{l} e \right)_{k}^{n+1}$$
A real case of fair weather

From: 2016.08.29.08
To: 2016.08.30.08

D01: 3km
D02: 1km
D03: 500m

3km: GFS analysis
1km: ndown from 3km output
500m: ndown from 1km output

<table>
<thead>
<tr>
<th>Res</th>
<th>nx*ny</th>
<th>dt</th>
</tr>
</thead>
<tbody>
<tr>
<td>3km</td>
<td>793*853</td>
<td>15s</td>
</tr>
<tr>
<td>1km</td>
<td>805*805</td>
<td>5s</td>
</tr>
<tr>
<td>500m</td>
<td>1001*1001</td>
<td>2s</td>
</tr>
</tbody>
</table>
250-m resolution visible MODIS-Terra image at 02:40 UTC 29 Aug 2016
Vertical profiles of simulated potential temperature at 0500 UTC 29 Aug 2016 for the Station Baoshan (31.40°N, 121.45°E)
Operational NWP system in SMS (Shanghai Meteorological Service)

**SMS-WARMS (WRF-ADAS Realtime Modeling System):** 9km resolution, WRF3.5.1+ADAS 5.3.3, 72h prediction

**SMS-WARR (WRF-ADAS Rapid Refresh System):** 3km resolution, cooling starting at 02 am (local time), doing data assimilation every hour, and making 12-hour prediction, boundary condition from 9km system
Precipitation event in Meiyu front on 9 June 2017
24h precipitation forecast in SMS-WARMS with 9-km resolution
The new scheme does not deteriorate the distribution and intensity of precipitation.
Threat Score of precipitation

12h precipitation forecast with 3km WARR

24h and 48h precipitation forecast forecast with 9km WARMS

INIT: 17060900

Precipitation (mm)

INIT: 2017060900

Precipitation (mm)

3DTKE, YSU, SH, MYJ, BouLac
**km_opt in WRF namelist**

km_opt selects method to compute $K$

- **1**: constant (khdif and kvdif used)
- **2**: 1.5-order TKE prediction (Deardorff’s model, usually used in LES, not recommended for $dx > 2$ km, not appropriate for meso)
- **3**: 3D Smagorinsky (usually used in LES)
- **4**: 2D Smagorinsky (for horizontal diffusion only, this option is most often used with a PBL scheme)
- **5**: New scale-aware 3DTKE scheme (SMS-3DTKE). This option extends original Deardorff’s model ($km_{opt}=2$) to the mesoscale, and can be used in LES, mesoscale and the gray zone in between. In the horizontal diffusion, the new scheme blends 2D Smag ($km_{opt}=4$) and TKE-based $K$ ($km_{opt}=2$). In mesoscale limit, horizontal diffusion is recovered to 2D Smag ($km_{opt}=4$). The new scheme can replace option 2 and convectional PBL schemes. In LES limit, option 5 is recovered to option 2. Thus, option 5 unifies all diffusion effects into one framework.

bl_pbl_physics = 0
Thank you for your attention!

Free atmosphere diffusion

The length scale at a given level is determined as a function of stability profile of the adjacent levels. The algorithm relies on the computation of the maximum vertical displacement allowed, for a parcel of air having the mean kinetic energy of the level as initial TKE.

\[
\int_{z}^{z+l_{up}} \frac{g}{\theta_0} \left( \theta(z') - \theta(z) \right) dz' = e(z)
\]

\[
\int_{z-l_{down}}^{z} \frac{g}{\theta_0} \left( \theta(z) - \theta(z') \right) dz' = e(z)
\]

\[l = \min(l_{up}, l_{down})\]

The BouLac mixing length is blended with MYNN length scale in a transition layer above PBL.

*Bougeault-Lacarrere (1989)*

The calculation of TKE in new scheme is not only confined in PBL.
The differences of 3DTKE compared to conventional PBL schemes

\[
\frac{\partial e}{\partial t} + \overline{U_j \frac{\partial e}{\partial x_j}} = -(u_i'u_j') \frac{\partial \overline{U_i}}{\partial x_j} + \delta_{ij} \frac{g}{\theta_v} u_i' \theta'_v - \frac{1}{\rho} \frac{\partial (p' u_i')}{\partial x_i} - \frac{\partial (u'_j e)}{\partial x_j} - \varepsilon
\]

\[
-(u_i'u_j') \frac{\partial \overline{U_i}}{\partial x_j} = -u''^2 \frac{\partial \bar{u}}{\partial x} - u'' v' \frac{\partial \bar{u}}{\partial y} - \frac{u' w'}{\partial z} - v'' \frac{\partial \bar{v}}{\partial x} - v'' u' \frac{\partial \bar{v}}{\partial y} - \frac{v' w'}{\partial z} - w'' \frac{\partial \bar{w}}{\partial x} - w'' v' \frac{\partial \bar{w}}{\partial y} - \frac{w' w'}{\partial z} \]

1. The 3DTKE scheme uses the complete form of TKE prognostic equation without any approximations;
2. The advection of TKE is calculated in the model dynamics framework, which insures the numerical consistency between subgrid mixing and model dynamics.
The differences of 3DTKE compared to conventional PBL schemes

\[
\frac{\partial \overline{u}}{\partial t} = -\overline{u} \frac{\partial \overline{u}}{\partial x} - \overline{v} \frac{\partial \overline{u}}{\partial y} - \overline{w} \frac{\partial \overline{u}}{\partial z} - \frac{1}{\rho} \frac{\partial p}{\partial x} - f \overline{v} - \frac{\partial u'u'}{\partial x} - \frac{\partial u'v'}{\partial y} - \frac{\partial u'w'}{\partial z}
\]

\[
\frac{\partial \overline{v}}{\partial t} = -\overline{u} \frac{\partial \overline{v}}{\partial x} - \overline{v} \frac{\partial \overline{v}}{\partial y} - \overline{w} \frac{\partial \overline{v}}{\partial z} - \frac{1}{\rho} \frac{\partial p}{\partial y} + f \overline{v} - \frac{\partial v'u'}{\partial x} - \frac{\partial v'v'}{\partial y} - \frac{\partial v'w'}{\partial z}
\]

\[
\frac{\partial \overline{w}}{\partial t} = -\overline{u} \frac{\partial \overline{w}}{\partial x} - \overline{v} \frac{\partial \overline{w}}{\partial y} - \overline{w} \frac{\partial \overline{w}}{\partial z} - \frac{1}{\rho} \frac{\partial p}{\partial z} + g - \frac{\partial w'u'}{\partial x} - \frac{\partial w'v'}{\partial y} - \frac{\partial w'w'}{\partial z}
\]

3. The 3DTKE scheme combines the horizontal and vertical subgrid turbulent mixing into a single energetically consistent framework;

4. The 3DTKE scheme includes the tendency from subgrid mixing in vertical velocity equation. In hydrostatic model, there is no prognostic equation of vertical velocity.