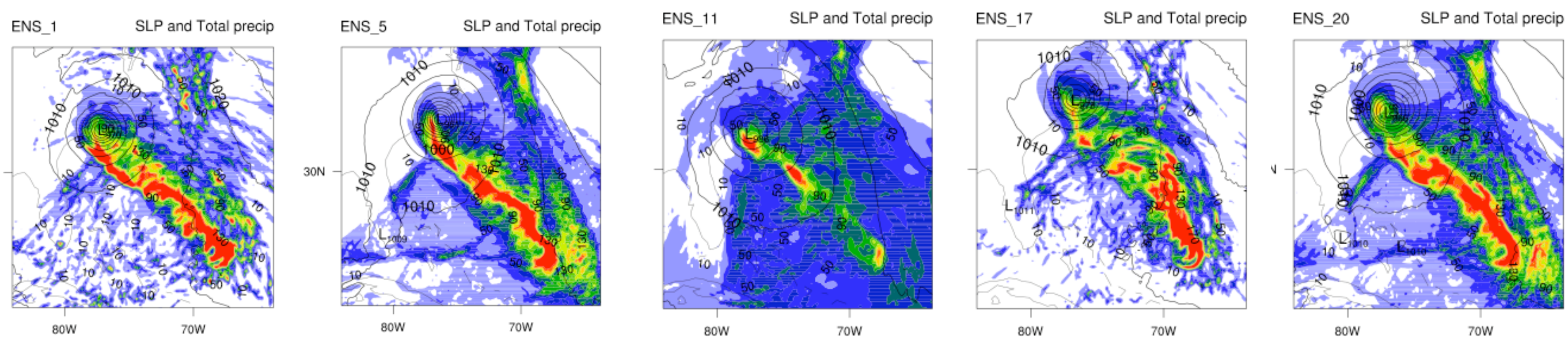
Ensemble Kalman Filters for WRF-ARW



Chris Snyder
MMM and IMAGe
National Center for Atmospheric Research

Preliminaries

Notation:

- $\mathbf{x}$ = model's state w.r.t. some discrete basis, e.g. grid-pt values
- $\mathbf{y} = \mathbf{Hx} + \varepsilon$ = vector of observations with random error ε
- Superscript *f* denotes forecast quantities, superscript *a* analysis, e.g. $\mathbf{x}^f$
- $\mathbf{P}^f = \mathrm{Cov}(\mathbf{x}^f)$ = forecast covariance matrix … a.k.a. **B** in Var

The Kalman Filter (KF)

Assume

- $\mathbf{x}^t \sim N(\overline{\mathbf{x}}^f, \mathbf{P}^f)$; Gaussian forecast errors
- $\epsilon \sim N(\mathbf{0}, \mathbf{R})$; Gaussian observation errors

KF analysis implements Bayes rule for Gaussians

- analysis equations:

$$\overline{\mathbf{x}}^a = \overline{\mathbf{x}}^f + \mathbf{K}(\mathbf{y} - \mathbf{H}\overline{\mathbf{x}}^f) \quad ; \quad \mathbf{P}^a = (\mathbf{I} - \mathbf{K}\mathbf{H})\mathbf{P}^f,$$

- Kalman gain

$$\mathbf{K} = \mathbf{P}^f\mathbf{H}^T(\mathbf{H}\mathbf{P}^f\mathbf{H}^T + \mathbf{R})^{-1}$$

Computationally difficult unless problem is small

- $\mathbf{P}^f$, $\mathbf{P}^a$ are $N_x \times N_x$, w/ $N_x = \dim \mathbf{x}$

Ensemble Kalman Filter (EnKF)

EnKF analysis step

- As in KF analysis step, but uses sample (ensemble) estimates for covariances
- e.g. one element of $\mathbf{P}^f\mathbf{H}^T$ is

$$\mathrm{Cov}(x^f, y^f) = N_e^{-1} \Sigma (x_i^f - \mathrm{mean}(x))(y_i^f - \mathrm{mean}(y^f))$$

  where $y^f = \mathbf{H}\mathbf{x}^f$ is the forecast, or prior, observation.
- Output of EnKF analysis step is ensemble of analyses

EnKF forecast step

- Each member integrated forward with full nonlinear model
- Monte-Carlo generalization of KF forecast step

Relation of Var and KF

Analysis equations

- Variational: compute $\mathbf{x}^a$ as minimizer of
$$J(\mathbf{x}) = (\mathbf{x} - \mathbf{x}^f)(\mathbf{P}^f)^{-1}(\mathbf{x} - \mathbf{x}^f)^T + (\mathbf{y}^o - \mathbf{H}\mathbf{x})\mathbf{R}^{-1}(\mathbf{y}^o - \mathbf{H}\mathbf{x})^T$$
- Kalman filter,
$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}(\mathbf{y}^o - \mathbf{H}\mathbf{x}^f), \quad \mathbf{K} = \mathbf{P}^f\mathbf{H}^T(\mathbf{H}\mathbf{P}^f\mathbf{H}^T + \mathbf{R})^{-1}$$

These are equivalent

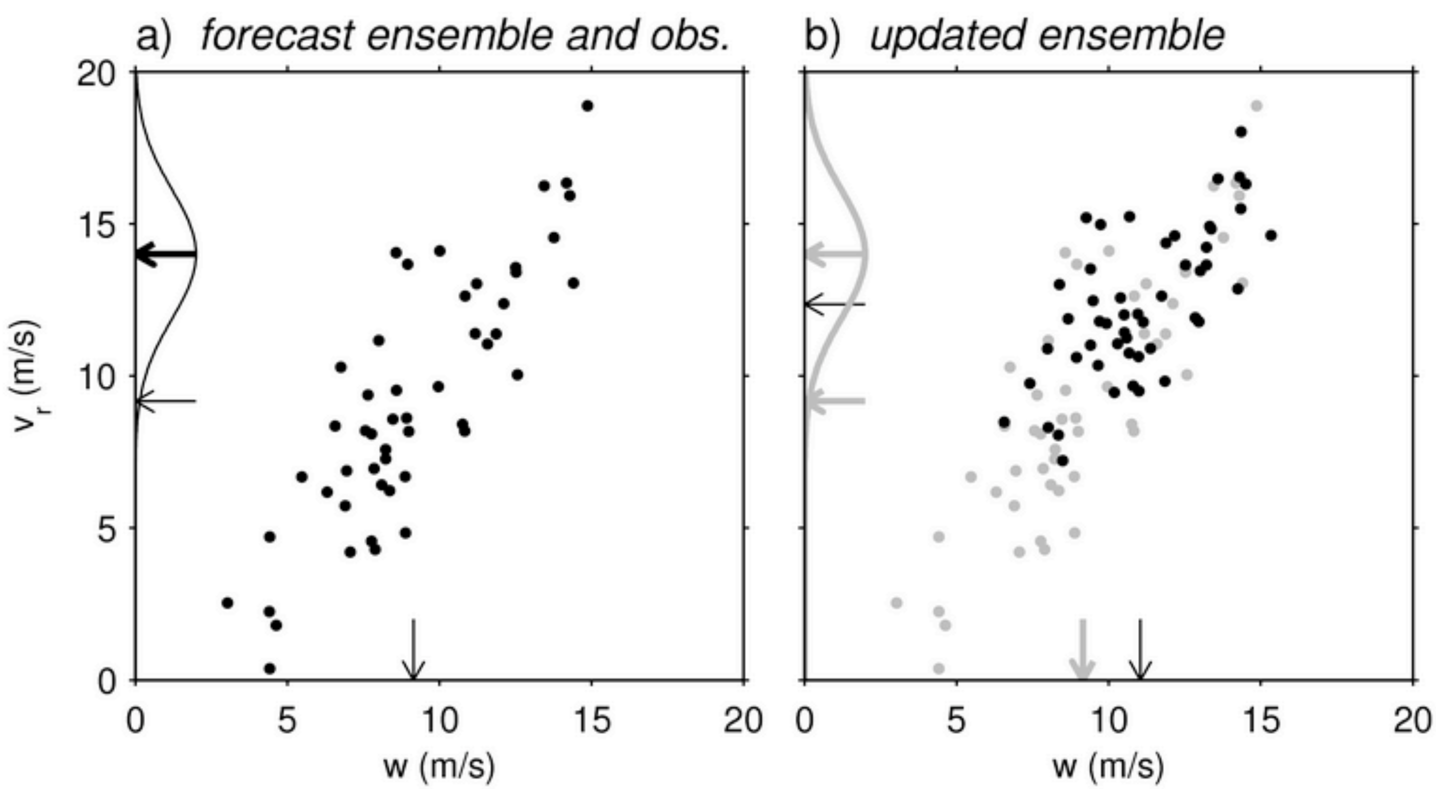
… as long as $\mathbf{P}^f$ and $\mathbf{R}$ are the same in both systems

EnKF Analysis/Update

Example: update w given v_r observation

- calculate $v_r^f = \mathbf{Hx}^f$ for each member and $d = \mathsf{Var}(v_r^f) + \mathbf{R}$
- update w via

$$w^a = w^f + (\mathsf{Cov}(w^f, v_r^f)/d)(v_r^o - v_r^f + \epsilon), \quad \epsilon \sim N(0, \mathbf{R})$$



Snyder and Zhang 2003 *MWR*

Flavors of EnKF

ETKF

- $\mathbf{P}^f$ is sample covariance from ensemble
- Analysis increments lie in ensemble subspace
- Computationally cheap--reduces to N_x x N_x matrices
- Useful for EF but not for DA: In Var "hybrid" system, ETKF updates ensemble deviations but **not** ensemble mean

"Localized" EnKF

- Cov(y,x) assumed to decrease to zero at sufficient distances
- Reduces computations and allows increments outside ensemble subspace
- $\exists$ approximate equivalence with α-CV option in Var--different way of solving same equations
- Numerous variants; DART provides several with interfaces for WRF

Data Assimilation Research Testbed (DART)

DART is general software for ensemble filtering:

- Assimilation scheme(s) are independent of model
- Interfaces exist for numerous models: WRF (including global and single column), CAM (spectral and FV), MOM, ROSE, others
- See http://www.image.ucar.edu/DAReS/DART/

Parallelization

- Forecasts parallelized at script level as separate jobs; also across processors, if allowed by OS
- Analysis has generic parallelization, independent of model and grid structure

WRF/DART

Consists of:

- Interfaces between WRF and DART (e.g. translate state vector, compute distances, …)
- Observation operators
- Scripts to generate IC ensemble, generate LBC ensemble, advance WRF

Easy to add fields to state vector (e.g. tracers, chem species)

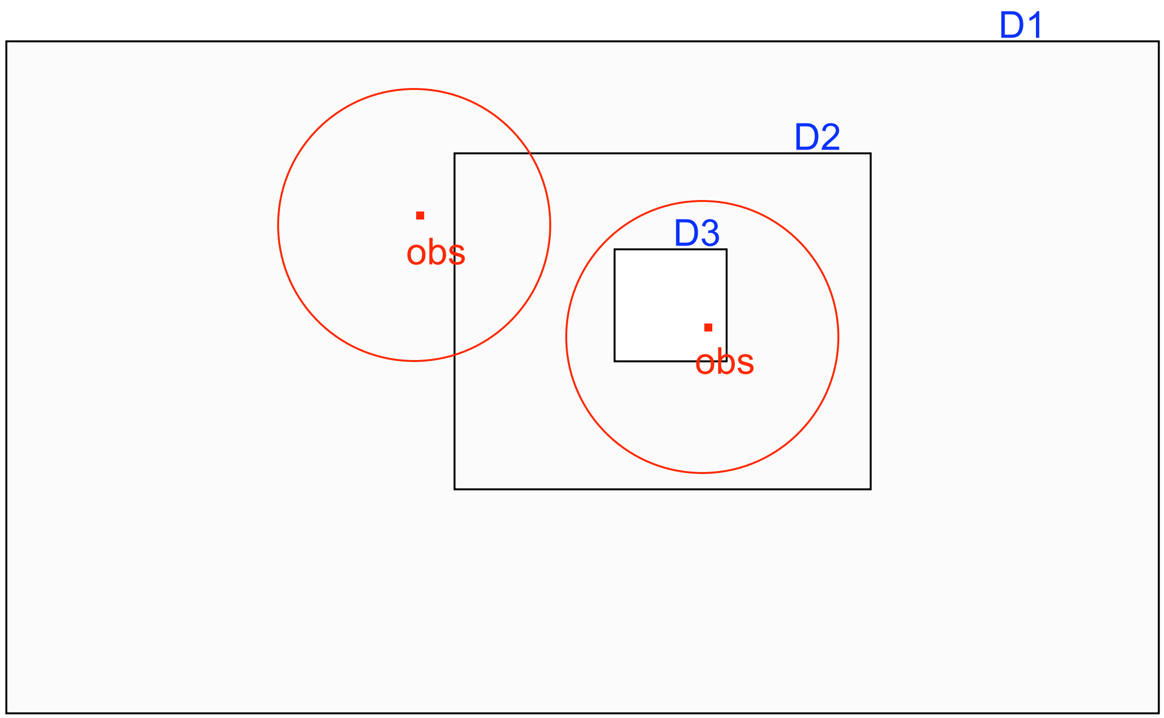
- Plan to add namelist control of fields in state vector

A few external users (5-10) so far

Nested Grids in WRF/DART

Perform analysis across multiple nests simultaneously

- Innovations calculated w.r.t. finest availble grid
- All grid points within localization radius updated



Var/DART

DART algorithm

- First, calculate “observation priors:” $\mathbf{H}(\mathbf{x}^f)$ for each member
- Then solve analysis equations

Possible to use Var for $\mathbf{H}(\mathbf{x}^f)$, DART for rest of analysis

- Same interface as between Var and ETKF: $\mathbf{H}(\mathbf{x}^f)$ are written by Var to gts_omb_oma files, then read by DART
- Allows EnKF within existing WRF/Var framework, and use of Var observation operators with DART
- Under development

Some Applications

Radar assimilation for convective scales

- Altug Aksoy (NOAA/HRD) and David Dowell (NCAR)

Assimilation of surface observations

- Dowell
- Also have single-column version of WRF/DART from Josh Hacker (NCAR)

Tropical cyclones

- Ryan Torn (SUNY-Albany), Yongsheng Chen (York), Hui Liu (NCAR)

GPS occultation observations

- Liu

References

Bengtsson T., C. Snyder, and D. Nychka, 2003: Toward a nonlinear ensemble filter for high-dimensional systems. *J. Geophys. Res.*, **62**(D24), 8775-8785.

Dowell, D., F. Zhang, L. Wicker, C. Snyder and N. A. Crook, 2004: Wind and thermodynamic retrievals in the 17 May 1981 Arcadia, Oklahoma supercell: Ensemble Kalman filter experiments. *Mon. Wea. Rev.*, **132**, 1982-2005.

Snyder, C. and F. Zhang, 2003: Assimilation of simulated Doppler radar observations with an ensemble Kalman filter. *Mon. Wea. Rev.*, **131**, 1663-1677.

Torn, R. D., G. J. Hakim, and C. Snyder, 2006: Boundary conditions for limited-area ensemble Kalman filters. *Mon. Wea. Rev.*, **134**, 2490-2502.

Hacker, J. P., and C. Snyder, 2005: Ensemble Kalman filter assimilation of fixed screen-height observations in a parameterized PBL. *Mon. Wea. Rev.*, **133**, 3260-3275.

Caya, A., J. Sun and C. Snyder, 2005: A comparison between the 4D-Var and the ensemble Kalman filter techniques for radar data assimilation. *Mon. Wea. Rev.*, **133**, 3081-3094.

Chen, Y., and C. Snyder, 2007: Assimilating vortex position with an ensemble Kalman filter. *Mon. Wea. Rev.*, **135**, 1828-1845.

Anderson, J. L., 2007: An adaptive covariance inflation error correction algorithm for ensemble filters. *Tellus A*, **59**, 210-224.

Snyder, C. T. Bengtsson, P. Bickel and J. L. Anderson, 2008: Obstacles to high-dimensional particle filtering. *Mon. Wea. Rev.*, accepted.

Aksoy, A., D. Dowell and C. Snyder, 2008: A multi-case comparative assessment of the ensemble Kalman filter for assimilation of radar observations. Part I: Storm-scale analyses. *Mon. Wea. Rev.*, accepted.

http://www.mmm.ucar.edu/people/snyder/papers/