Overview of WRF Data Assimilation

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WRFDA Tutorial - August 3-5, 2010

Acknowledgments and References

- WRF Tutorial Lectures (H. Huang and D. Barker)
- Data Assimilation concepts and methods
  (ECMWF Training Course, F. Bouttier and P. Courtier)
- Data Assimilation Research Testbed (DART) Tutorial
  (J. Anderson et al.)
- Analysis methods for numerical weather prediction
  (A.C. Lorenc, 1986, *Quart. J. R. Meteorol. Soc.*)
- Atmospheric Data Analysis
  (R. Daley, 1991, *Cambridge University Press, 457 pp.*)
- Atmospheric Modeling, Data Assimilation and Predictability
  (E. Kalnay, 2003, *Cambridge University Press, 341 pp.*)

Table of contents

1. Introduction
2. Simple Scalar Example
    - Kalman Filter equations
3. Modern Implementations
    - Sequential Algorithms
        - Ensemble Kalman Filter
        - 3D Variational (3DVar)
    - Smoothers
        - 4D Variational (4DVar)
4. WRFDA Overview

Motivation

- *A sufficiently accurate knowledge of the state of the atmosphere at the initial time.*
  (Today's weather)
- *A sufficiently accurate knowledge of the laws according to which one state of the atmosphere develops from another.*
  (Tomorrow's weather)

Vilhelm Bjerknes (1904)
(Peter Lynch)

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  (Today's weather)
- *A sufficiently accurate knowledge of the laws according to which one state of the atmosphere develops from another.*
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Vilhelm Bjerknes (1904)
(Peter Lynch)

Motivation

- Initial conditions for Numerical Weather Prediction (NWP)
- Calibration and validation
- Observing system design, monitoring and assessment
- Reanalysis
- Better understanding (Model errors, Data errors, Physical process interactions, *etc*)

From Empirical to Statistical methods

- Successive Correction Method (SCM, *Cressman 1959*)
  Each observation within a radius of influence L is given a weight w varying with the distance r to the model grid point:
  $w(r) = \frac{L^2 - r^2}{L^2 + r^2} (r \leq L)$
- Nudging
- Physical Initialization (PI), Latent Heat Nudging (LHN)

However...

- Relaxation functions are somewhat arbitrary
- **Good** forecast can be replaced by **bad** observations
- Noisy observations can create unphysical analysis

So...

Modern DA techniques are usually statistical

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What is the temperature in this room?

Notations

- x_t: "True" state
- x_o: Observation
- x_b: Background information
- $d = x_o - x_b$: Innovation or *Departure*

Hypotheses

- Observation and Background errors are uncorrelated, unbiased, normally distributed, with variance R and B *resp.*
- Analysis x_a is "optimal" in RMSE sense
- Linear Analysis: $x_a = \alpha x_o + \beta x_b = x_b + \alpha(x_o - x_b)$

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Best Linear Unbiased Estimate

The analysis value is $x_a = x_b + \alpha(x_o - x_b)$ and its error variance:

$$A = \overline{(x_a - x_t)(x_a - x_t)} = (1-\alpha)^2 B + \alpha^2 R$$

$$\frac{\partial A}{\partial \alpha} = 2\alpha(B+R) - 2B \qquad \frac{\partial A}{\partial \alpha} = 0 \;\Rightarrow\; \alpha = \frac{B}{B+R}$$

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Best Linear Unbiased Estimate (BLUE)

$x_a = x_b + K(x_o - x_b)$ with the definition of the *Kalman Gain*:

$$K = B(B+R)^{-1}$$

and the analysis error variance: $A^{-1} = B^{-1} + R^{-1}$

Statistically, the analysis is better than the observation ($A < R$) and the background ($A < B$)

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Variational Cost Function

This solution is equivalent to minimizing the cost function:

$$J(x) = \frac{1}{2}(x-x_b)^T B^{-1}(x-x_b) + \frac{1}{2}(x-x_o)^T R^{-1}(x-x_o) = \mathbf{J_b} + \mathbf{J_o}$$

Proof:

$$\nabla J = B^{-1}(x-x_b) + R^{-1}(x-x_o) = 0$$

$$\begin{aligned} \Rightarrow x_a &= x_b + \frac{B}{B+R}(x_o - x_b) \\ &= x_b + K(x_o - x_b) \end{aligned}$$

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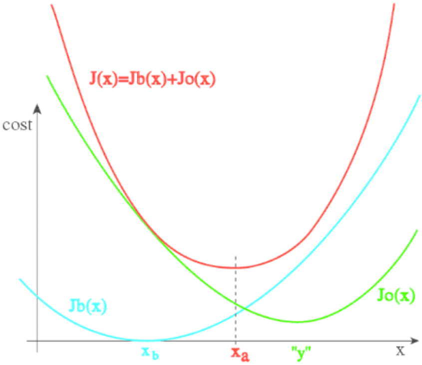
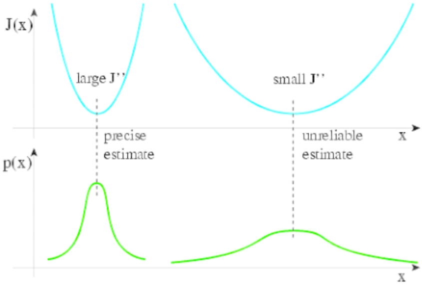
Analysis Accuracy



from Bouttier and Courtier 1999



Quality of the Analysis

The precision is defined by the convexity or **Hessian** $A = J''^{-1}$

Conditional Probabilities

According to Bayes Theorem, the joint pdf of x and x_o is:

$$P(x \wedge x_o) = P(x|x_o)P(x_o) = P(x_o|x)P(x)$$

Since $P(x_o) = 1$, $P(x|x_o) = P(x_o|x)P(x)$

We assumed the background and observation errors were Gaussian:
$P(x) = \lambda_b e^{[\frac{1}{2B}(x_b - x)^2]}$ and $P(x_o|x) = \lambda_o e^{[\frac{1}{2R}(x_o - x)^2]}$

$$\Rightarrow P(x|x_o) = \lambda_a e^{[\frac{1}{2R}(x_o - x)^2 + \frac{1}{2B}(x_b - x)^2]} = \lambda_a e^{-J(x)}$$

Maximum Likelihood

The minimum of the cost function J is also the estimator of x_t with the maximum likelihood

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Partial Conclusions

Under the aforementioned hypotheses, the BLUE:

- can be determined analytically through the Kalman gain K
- is also the minimum of a cost function $J = J_b + J_o$
- is optimal for minimum variance **and** maximum likelihood

Sequential Data Assimilation

Forecast model $M_{i \to i+1} = M$ from step i to $i+1$

$$x^t_{i+1} = M(x^t_i) + q_i$$

where q_i is the model error. As q_i is unknown and x^a_i is the best estimate of x^t_i, usually: $x^f_{i+1} = M(x^a_i)$

Forecast error

$$x^f_{i+1} - x^t_{i+1} = M(x^a_i) - M(x^t_i) - q_i \approx \mathbf{M}_i(x^a_i - x^t_i) - q_i$$

M is called the **Tangent-Linear** code of the non-linear model M

Sequential Data Assimilation

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Forecast error covariance matrix

$$P_{i+1}^f \approx \mathbf{M}_i \overline{(x_i^a - x_i^t)(x_i^a - x_i^t)^T} \mathbf{M}_i + \overline{q_i q_i^T} = \mathbf{M}_i P_i^a \mathbf{M}_i^T + Q_i$$

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Sequential Data Assimilation

We can use the forecast as background for the **BLUE** calculation

$$K_i = P_i^f (P_i^f + R)^{-1}$$

$$x_i^a = x_i^f + K(x_i^o - x_i^f)$$

$$(P_i^a)^{-1} = (P_i^f)^{-1} + R^{-1} \Rightarrow P_i^a = (I - K_i) P_i^f$$

Finally, we can distinguish the model space x from the observation space y and introduce an Observation Operator $H : x \mapsto y$, which is linearized: $H(x_i^a) - H(x_i^t) \approx \mathbf{H}(x_i^a - x_i^t)$

$$K_i = P_i^f \mathbf{H}_i^T (\mathbf{H}_i P_i^f \mathbf{H}_i^T + R)^{-1}$$

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The Extended Kalman Filter Algorithm

Analysis step i:

$$K_i = P_i^f \mathbf{H}_i^T [\mathbf{H}_i P_i^f \mathbf{H}_i^T + R]^{-1} \quad (1)$$

$$x_i^a = x_i^f + K_i [y^o - H x_i^f] \quad (2)$$

$$P_i^a = [I - K_i \mathbf{H}_i] P_i^f \quad (3)$$

Forecast step from i to $i+1$:

$$x_{i+1}^f = M(x_i^a) \quad (4)$$

$$P_{i+1}^f = \mathbf{M}_i P_i^a \mathbf{M}_i^T + Q_i \quad (5)$$

Hypotheses

- Gaussian distributions of errors
- $\mathbf{M}$: Linearization around non-linear Model M
- $\mathbf{H}$: Linearization around non-linear Observation Operator H

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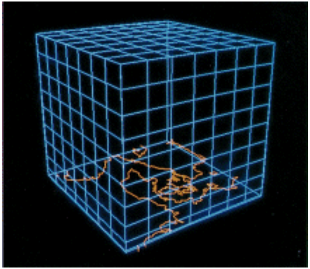
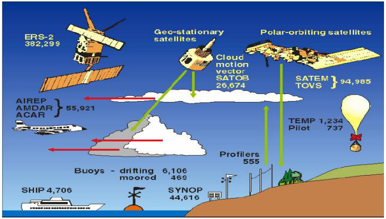
From scalar to vector: dimensions

$x \to \mathbf{x}$
Number of grid points $\approx 10^7$
Dimension of P^f, $P^a \approx 10^7 \times 10^7$



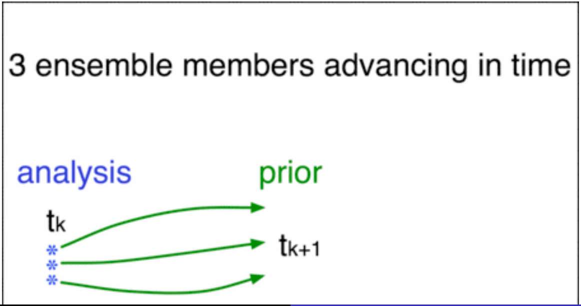


$y^o \to \mathbf{y^o}$
Number of observations $\approx 10^6$
Dimension of $R \approx 10^6 \times 10^6$

Ensemble Kalman Filter (EnKF)

Hypotheses

- Monte Carlo approximation to pdfs
- Gaussian distributions used for computing update
- Localization in space: for each model grid point, only a few observations are used to compute the analysis increment.



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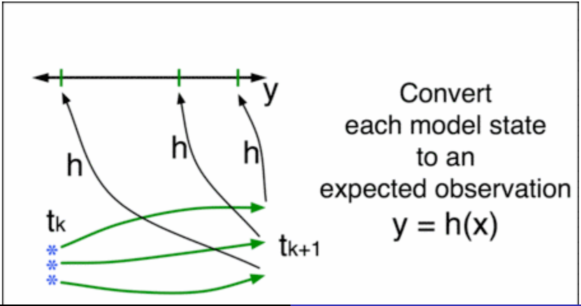

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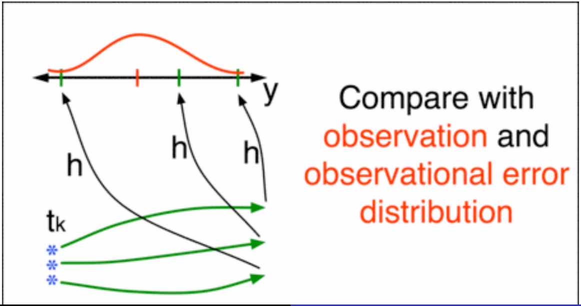

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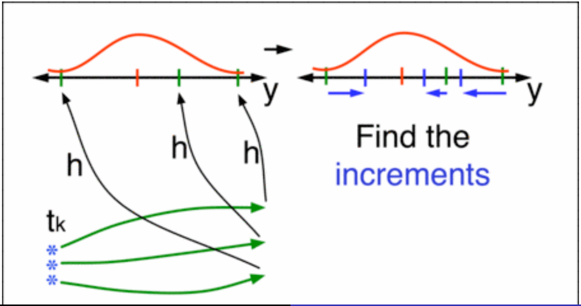

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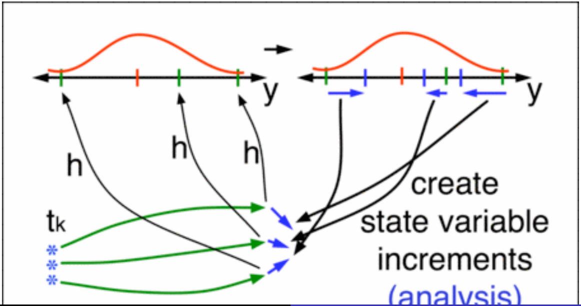

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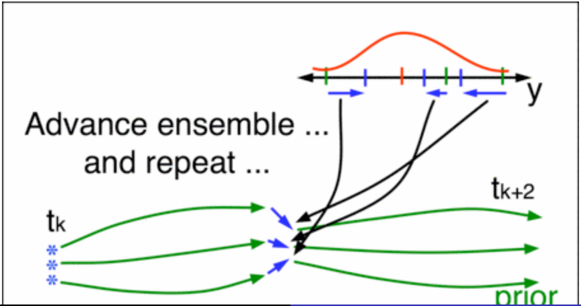

Ensemble Kalman Filter (EnKF)

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- Monte Carlo approximation to pdfs
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Advantages

- Easy to implement and provides estimate of Analysis Accuracy
- H and M need not be linearized

Drawbacks

Localization avoids degeneracy from under-sampling and reduces spurious noise, but it affects model internal balance

3D Variational Data Assimilation (3DVar)

Hypotheses

Avoid calculating K by solving the equivalent minimization problem defined by the cost function:
$J(x) = \frac{1}{2}(x - x_b)^T B^{-1}(x - x_b) + \frac{1}{2}(y^o - H(x))^T R^{-1}(y^o - H(x))$

$$\nabla J(x) = B^{-1}(x - x_b) - \mathbf{H}^T R^{-1}[y - H(x)]$$

$\mathbf{H}^T$ is called the **Adjoint** of the linearized observation operator

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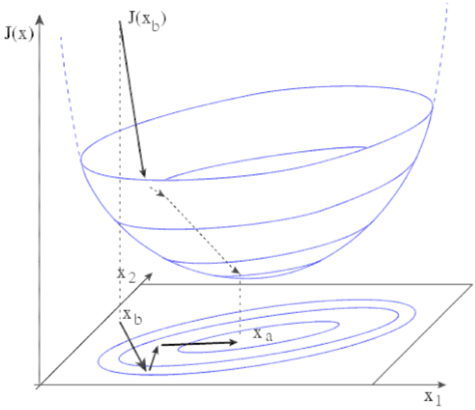


from Bouttier and Courtier 1999

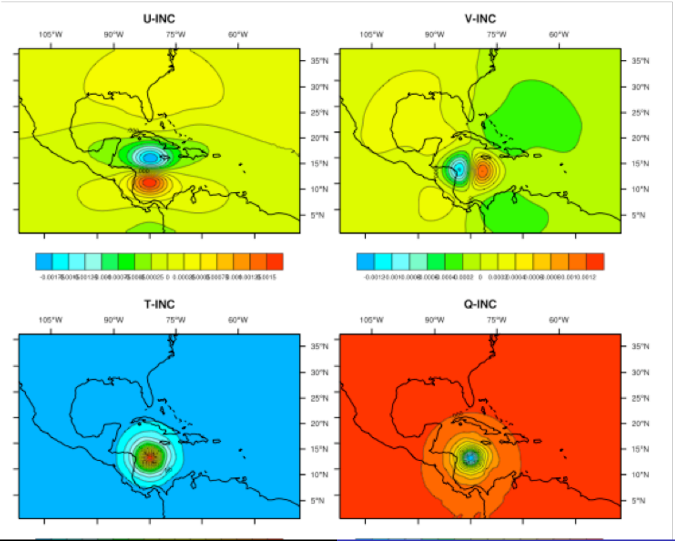
Minimization Algorithm

- Iterative minimizer
  $\rightarrow$ several simulations
- Steepest Descent, Quasi-Newton, Conjugate Gradient, *etc*

Preconditioning

- Improve Condition Nb
- Faster convergence

Single Observation Experiment



3D Variational Data Assimilation (3DVar)

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Advantages

- Easy to use with complex observation operators
- Can add external weak or *penalty* constraints J_c

Drawbacks

- Sub-optimal for strongly non-linear observation operators
- All observations are assumed to be instantaneous

3D Variational Data Assimilation (3DVar)

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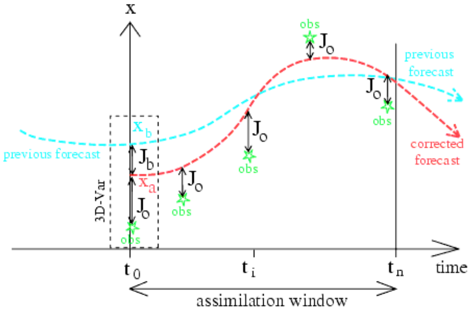
4D Variational Data Assimilation (4DVar)

Hypotheses

- Generalization of 3DVar for observations distributed in time
- Analysis variable x defined at the **beginning** of time window
- Find model trajectory minimizing the distance to observations



4D Variational Data Assimilation (4DVar)

Hypotheses

- Generalization of 3DVar for observations distributed in time
- Analysis variable x defined at the **beginning** of time window
- Find model trajectory minimizing the distance to observations

The Cost Function becomes:

$$J(x) = \frac{1}{2}(x-x_b)^T B^{-1}(x-x_b)+\frac{1}{2}(y^o-HM(x))^T R^{-1}(y^o-HM(x))$$

$$\nabla J(x) = B^{-1}(x - x_b) - \mathbf{M}^T \mathbf{H}^T R^{-1}[y - HM(x)]$$

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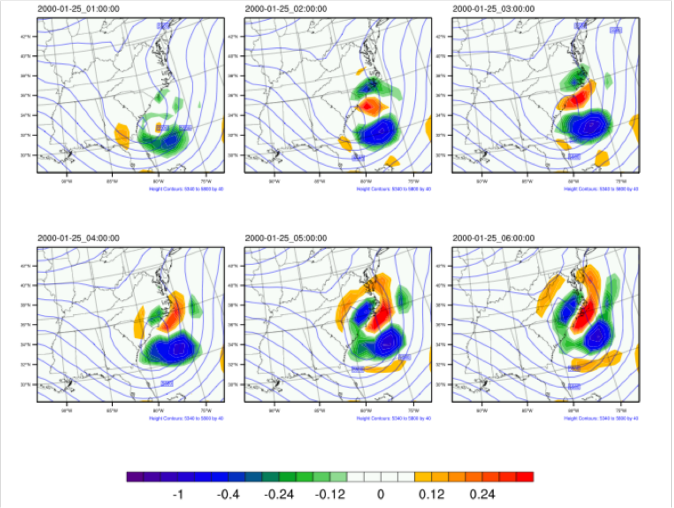

4D Variational Data Assimilation (4DVar)

Hypotheses

- Generalization of 3DVar for observations distributed in time
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- Find model trajectory minimizing the distance to observations

Advantages

Model internal balance is more prone to be respected

Drawbacks

- The development and maintenance of the Adjoint model $\mathbf{M}^T$ can be cumbersome
- Limitation of the "perfect model" assumption

4D Variational Data Assimilation (4DVar)

Hypotheses

- Generalization of 3DVar for observations distributed in time
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- Find model trajectory minimizing the distance to observations

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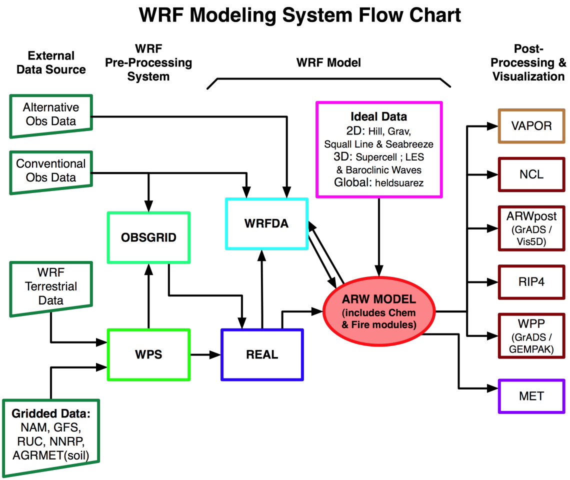
WRF Data Assimilation (WRFDA)



WRF Data Assimilation (WRFDA)

Community WRF DA System

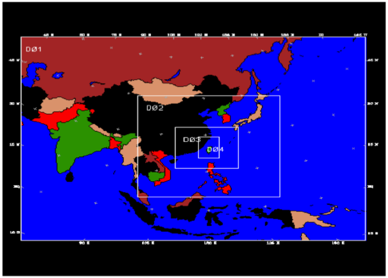
- Regional/Global
- Research/Operations
- Deterministic/Probabilistic

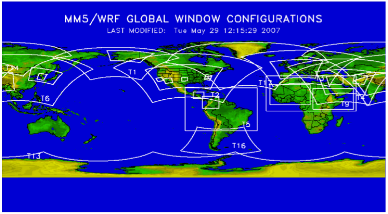
Algorithms

- 3DVar, 4DVar (Regional)
- Ensemble (ETKF/EnKF)
- Hybrid Var/Ens

Model: WRF

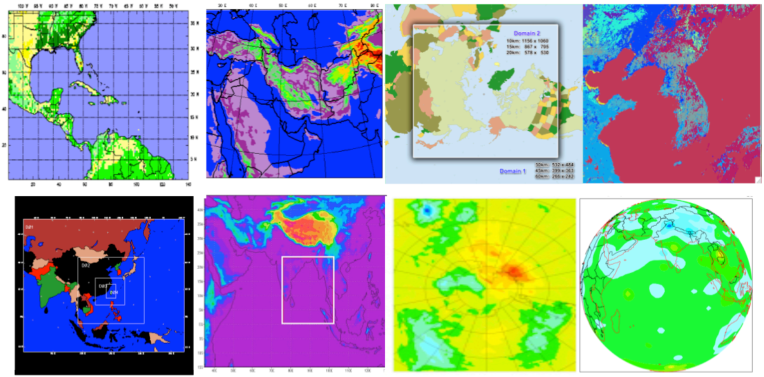
ARW, NMM





WRFDA Program

- NCAR Staff: 20FTE, 10 projects
- Ext. collaborators (AFWA, KMA, CWB, BMB): 10 FTE
- Community Users: 40



WRFDA Observations

Conventional

- Surface (SYNOP, METAR, SHIP, BUOY)
- Upper Air (TEMP, PIBAL, AIREP, ACARS, TAMDAR)

Bogus

- Tropical Cyclone Bogus
- Global Bogus

WRFDA Observations

Remotely Sensed Retrievals

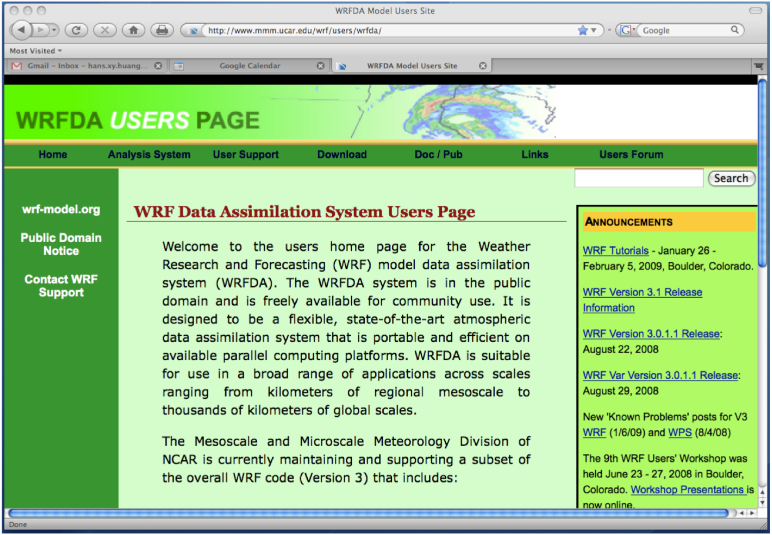
- Atmospheric Motion Vectors (from GEOs and Polar)
- SATEM Thickness
- Ground-based GPS TPW/Zenith Total Delay
- SSM/I oceanic surface wind speed and TPW
- Scatterometer oceanic surface winds
- Wind Profiler
- Radar Radial Velocities and Reflectivities
- Satellite Temperature, humidity, thickness profiles
- GPS Refractivity (COSMIC)

WRFDA Observations

Satellite Radiances (RTTOV or CRTM Radiative Transfer)

- HIRS (from NOAA-16, 17, 18 and METOP-2)
- AMSU-A (from NOAA-15, 16, 18, EOS-Aqua and METOP-2)
- AMSU-B (from NOAA-15, 16, 17)
- MHS (from NOAA-18 and METOP-2)
- AIRS (from EOS-Aqua)
- SSMIS (from DMSP-16)

www.mmm.ucar.edu/wrf/users/wrfda



Conclusions

- Observations y^o
- Background x_b
- Observation Operator H
- Innovations $y^o - H(x_b)$

- Observation Error R
- Bkg/Ana Error P^f, P^a
- Tangent-Linear $\mathbf{H}$, $\mathbf{M}$
- Adjoint $\mathbf{H^T}$, $\mathbf{M^T}$

(Extended) Kalman Filter (quasi-)linear statistical algorithm

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Simplifications for practical implementation

- Ensemble methods: EnKF
- Variational methods: 3DVar, 4DVar

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Conclusions

Warning

WRFDA should NOT be used as a *black box*

- Processing of Observations (Quality Control, Bias Correction)
- Modeling of Background and Observation error covariances
- Accounting for Model errors and Non-Linearities

Thank you for your attention...