Hybrid Variational/Ensemble Data Assimilation

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Outline

- Motivation of hybrid DA
- Elements of hybrid DA
- Preliminary results
- Alternative for updating perturbations
- Practice introduction

Why Hybrid?

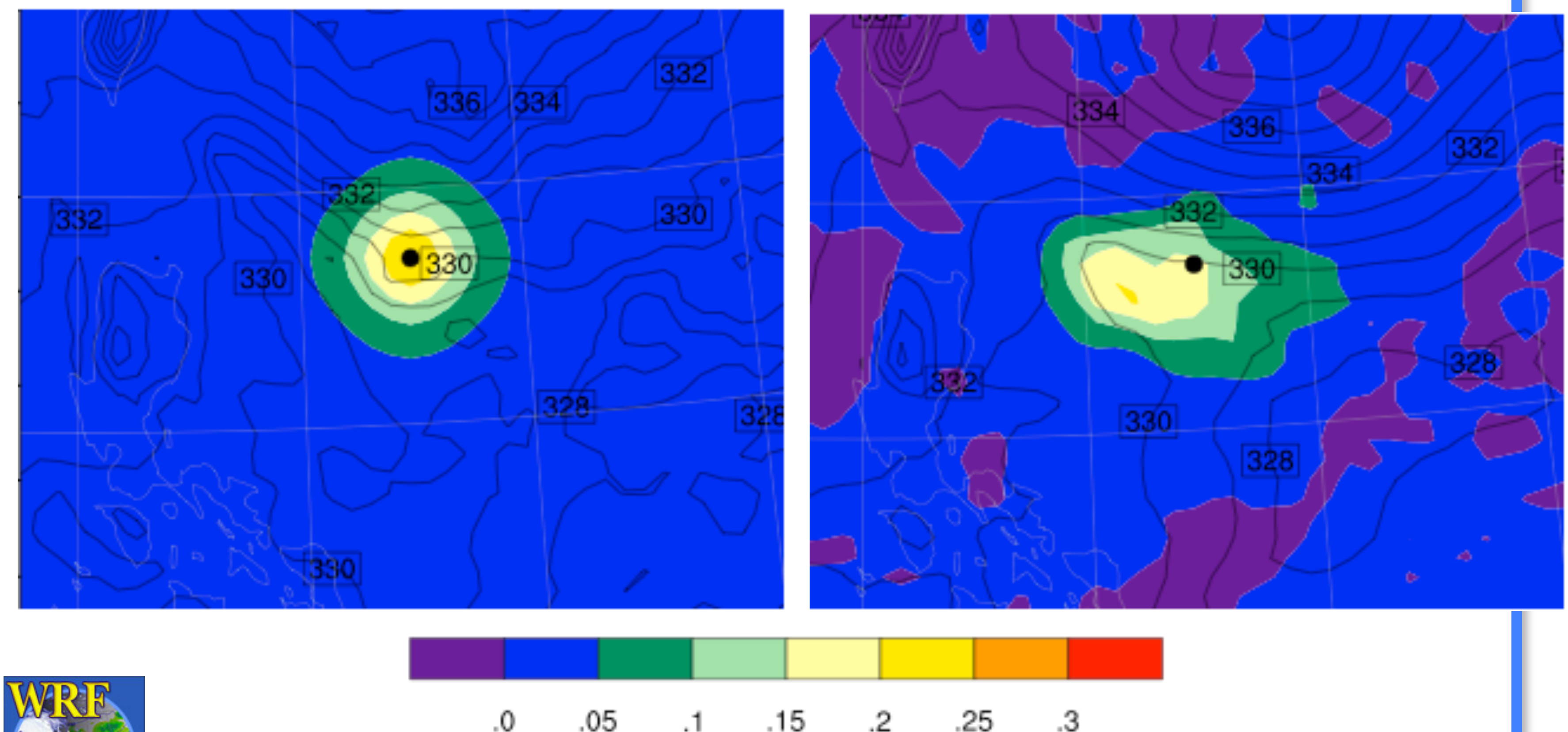
- Background errors are flow-dependent
  - 3D-Var uses static (“climate”) BE
  - 4D-Var implicitly uses flow-dependent information, but still starts from static BE
  - Hybrid: using flow-dependent information from ensemble in a variational DA system

- Hybrid can be robust for small size ensembles.

T Analysis increments from a single T obs

1K difference, 1K error

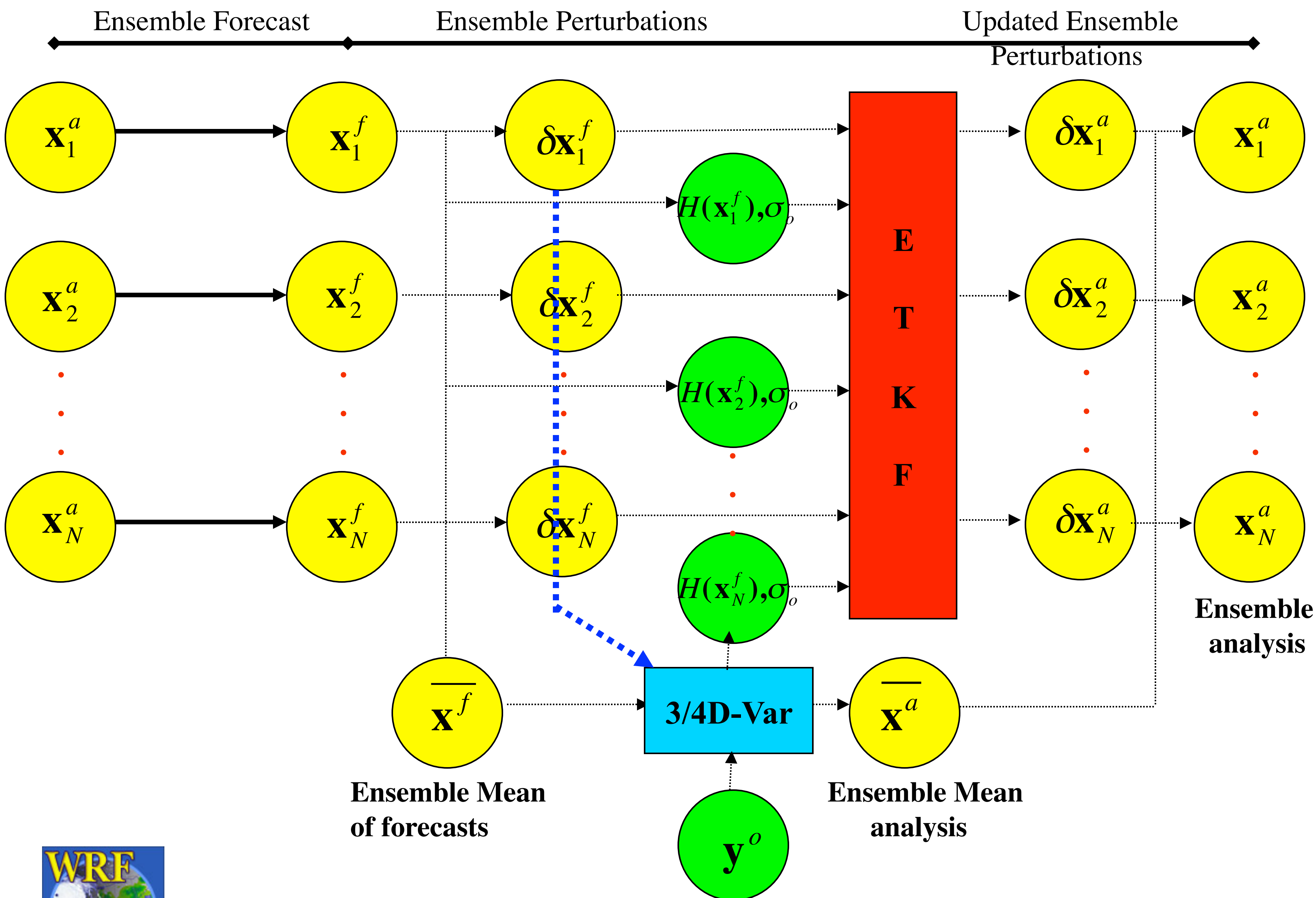
3DVAR | Hybrid (64 members)



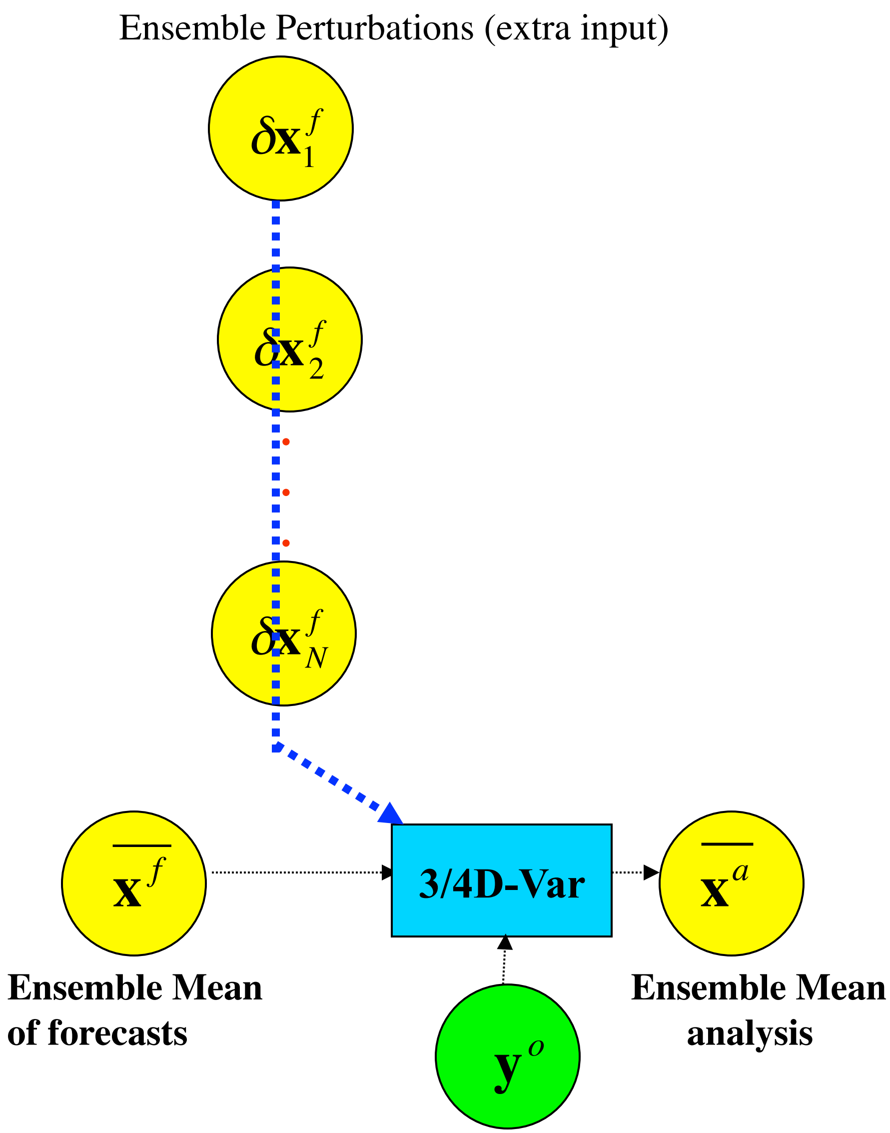
Elements of Hybrid DA

- Ensemble forecasts: *WRF-ensemble forecasts*
- Ensemble Transform Kalman Filter (ETKF):
    - Update forecast/background ensemble perturbations to analysis ensemble perturbations
- A Variational DA to update ensemble mean.

Cycling WRF/WRFDA/ETKF System (Hybrid DA)



Hybrid DA: Variational Part



Hybrid DA formulation

- Ensemble covariance is included in the 3DVAR cost function through augmentation of control variables.

$$J(\mathbf{x}_1', \boldsymbol{\alpha}) = \beta_1 J_1 + \beta_2 J_e + J_o$$

$$= \beta_1 \frac{1}{2} \mathbf{x}_1'^T \mathbf{B}^{-1} \mathbf{x}_1' + \underline{\beta_2 \frac{1}{2} \boldsymbol{\alpha}^T \mathbf{C}^{-1} \boldsymbol{\alpha}} + \frac{1}{2} (\mathbf{y}^{o'} - \mathbf{H}\mathbf{x}')^T \mathbf{R}^{-1} (\mathbf{y}^{o'} - \mathbf{H}\mathbf{x}')$$

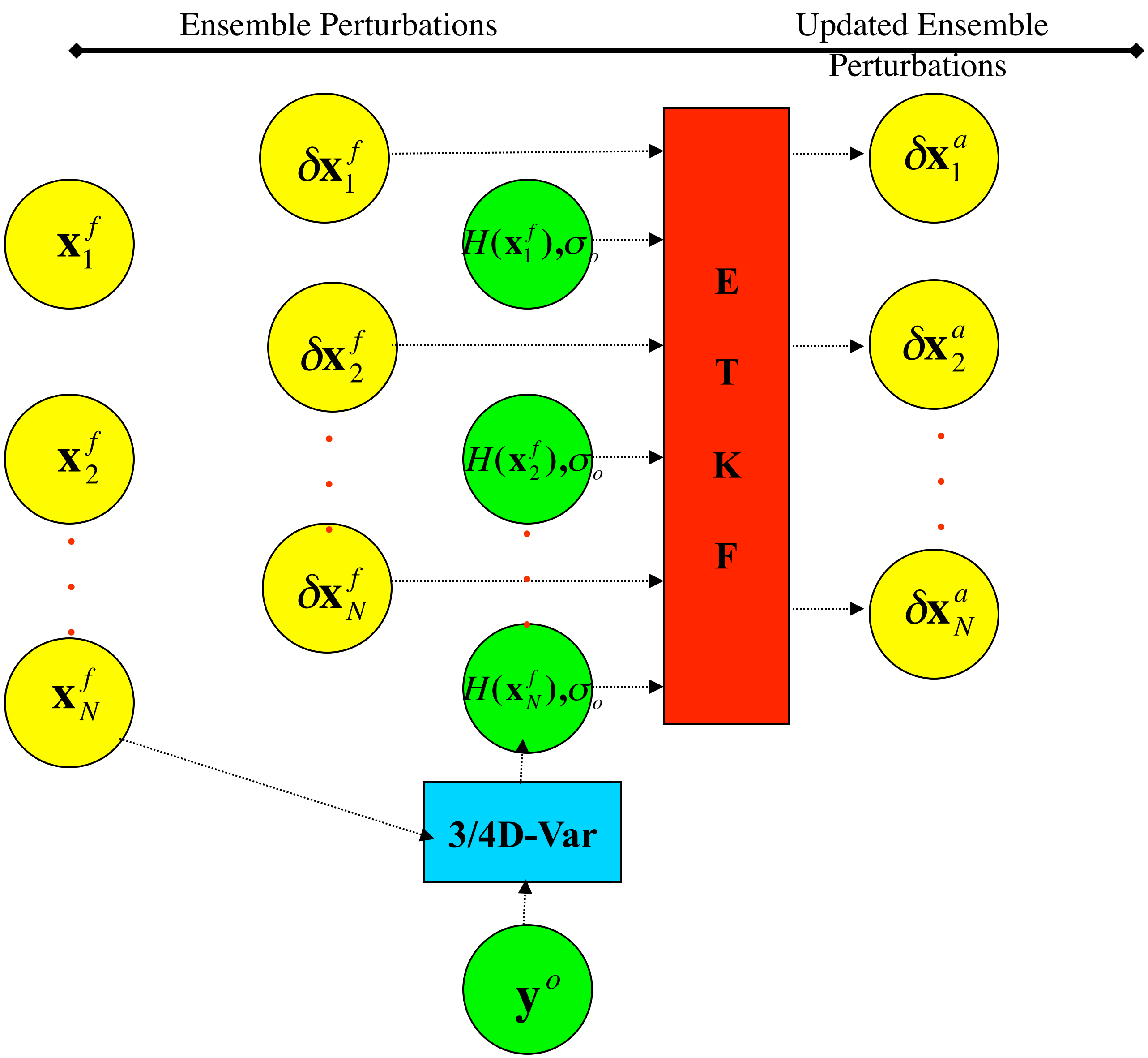
Extra term associated with extended control variable

$$\mathbf{x}' = \mathbf{x}_1' + \underline{\sum_{k=1}^{K} (\boldsymbol{\alpha}_k \circ \mathbf{x}_k^e)}$$

Extra increment associated with ensemble

$\mathbf{B}$ 3DVAR static covariance; $\mathbf{R}$ observation error covariance; K ensemble size;
$\mathbf{C}$ correlation matrix for ensemble covariance localization; $\mathbf{x}_k^e$ kth ensemble perturbation;
$\mathbf{x}_1'$ 3DVAR increment; $\mathbf{x}'$ total (hybrid) increment; $\mathbf{y}^{o'}$ innovation vector;
$\mathbf{H}$ linearized observation operator; β_1 weighting coefficient for static covariance;
β_2 weighting coefficient for ensemble covariance; $\boldsymbol{\alpha}$ extended control variable.

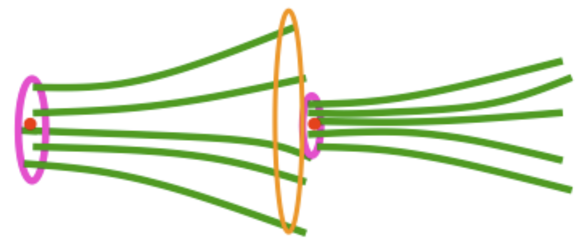
Hybrid DA: Ensemble Part



ETKF formulation

- The ETKF (Bishop et al. 2001) finds the transformation matrix **T** to update forecast/background perturbations to analysis perturbations

$$\delta \mathbf{x}^a = \delta \mathbf{x}^f \mathbf{T}$$



- So that the analysis error covariance

$$\mathbf{P}^a = \frac{1}{N-1}(\delta \mathbf{X}^a)(\delta \mathbf{X}^a)^{\mathrm{T}}$$

$$\mathbf{T} = \mathbf{C}(\mathbf{\Gamma} + \mathbf{I})^{-1/2}\mathbf{C}^{\mathrm{T}}$$

- Where **C** contains eigenvectors and **Γ** eigenvalues of a NxN (N is ensemble size) matrix

$$[\mathbf{H}(\delta \mathbf{x}^{\mathrm{f}})]^{\mathrm{T}}\mathbf{R}^{-1}[\mathbf{H}(\delta \mathbf{x}^{\mathrm{f}})]/(N-1)$$

$$\mathbf{H}(\delta \mathbf{x}_k^f) = H(\mathbf{x}_k^f) - \overline{H(\mathbf{x}_k^f)}$$

Inflation factor in ETKF

- Analysis error variance is usually underestimated from ETKF due to sampling error, i.e., analysis perturbations' spead is too small.
  - So perturbations need to be inflated by a factor ∏

$$\mathbf{T} = \mathbf{\Pi C}(\mathbf{\Gamma} + \mathbf{I})^{-1/2}\mathbf{C}^T$$

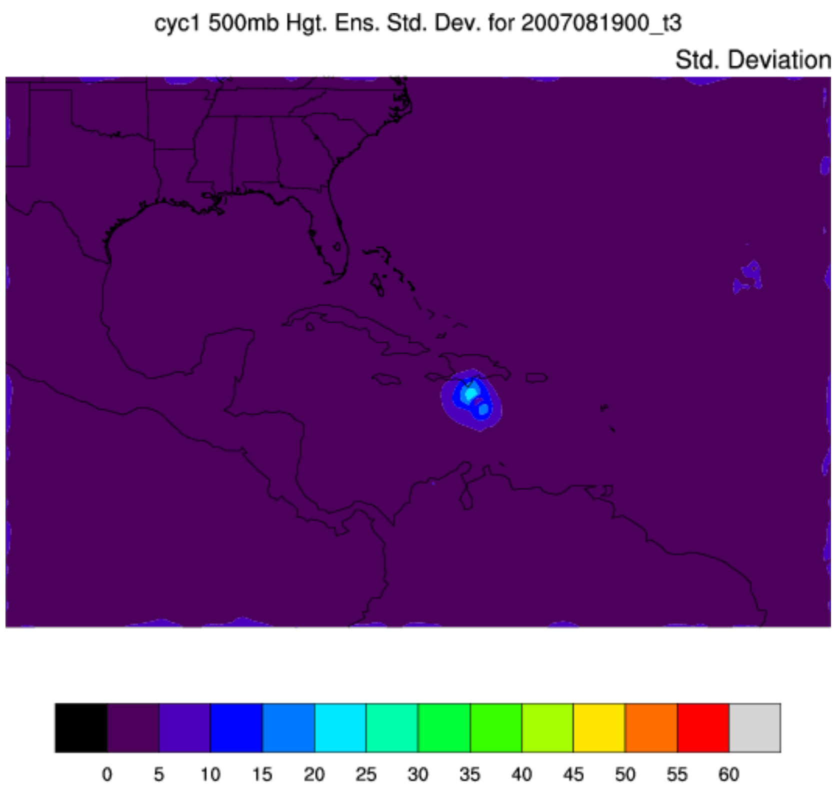
  - Inflation factor is estimated from innovation vector (Wang and Bishop, 2003).

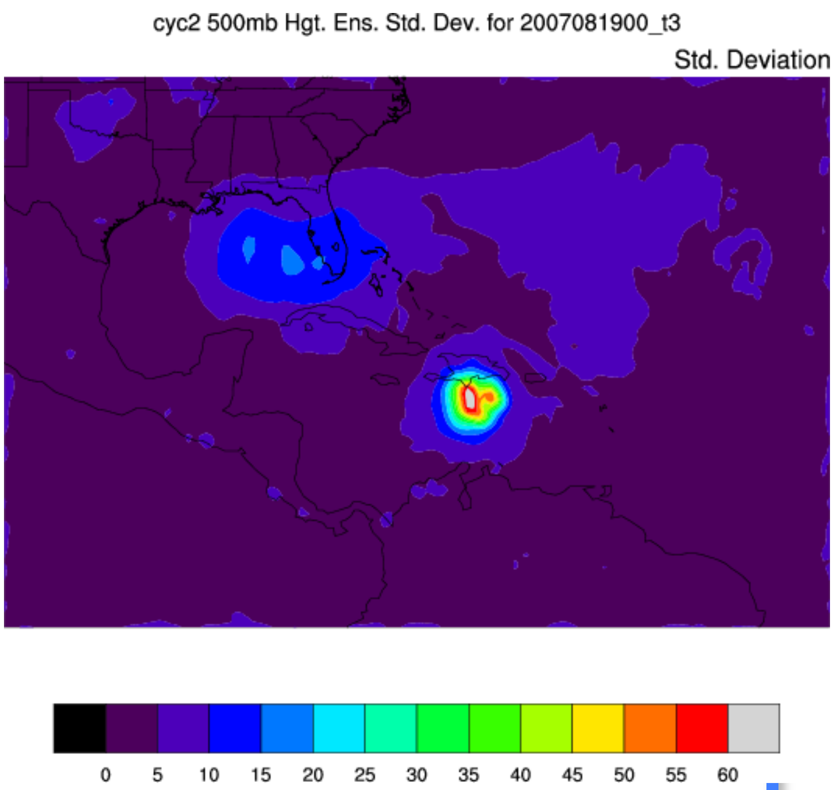
Preliminary results

- Ensemble size: 10
- Test Period: 15th August - 15th September 2007
- Cycle frequency: 3 hours
- Observations: GTS conventional observations
- Deterministic ICs/BCs: Down-scaled GFS forecasts
- Ensemble ICs/BCs: Produced by adding spatially correlated Gaussian noise to GFS forecasts.
- Horizontal resolution: 45km
- Number of vertical levels: 57
- Model top: 50 hPa

Ensemble spread: 500 hPa height (m) std. dev.

WRF t+3 valid at 2007081900

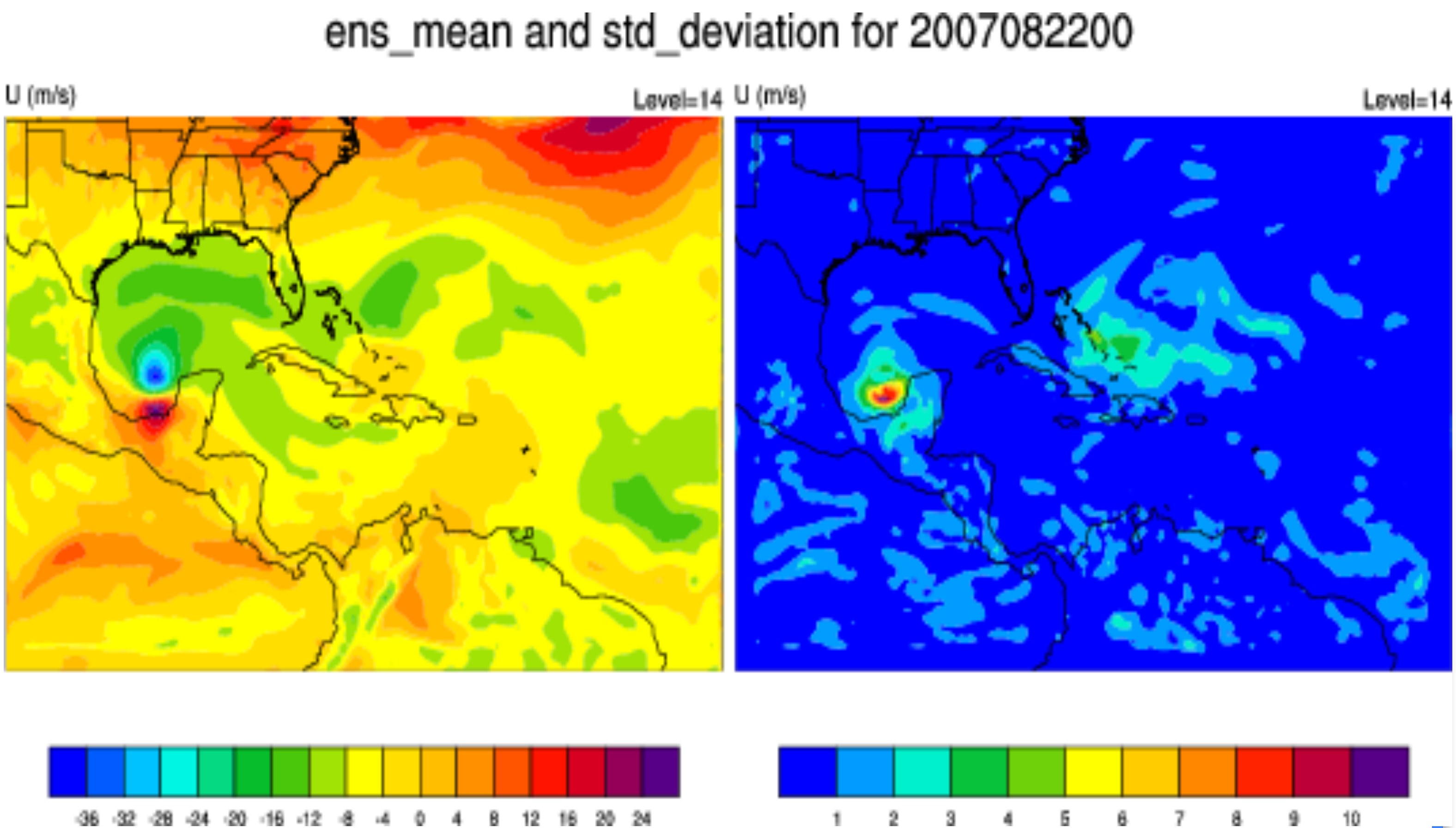




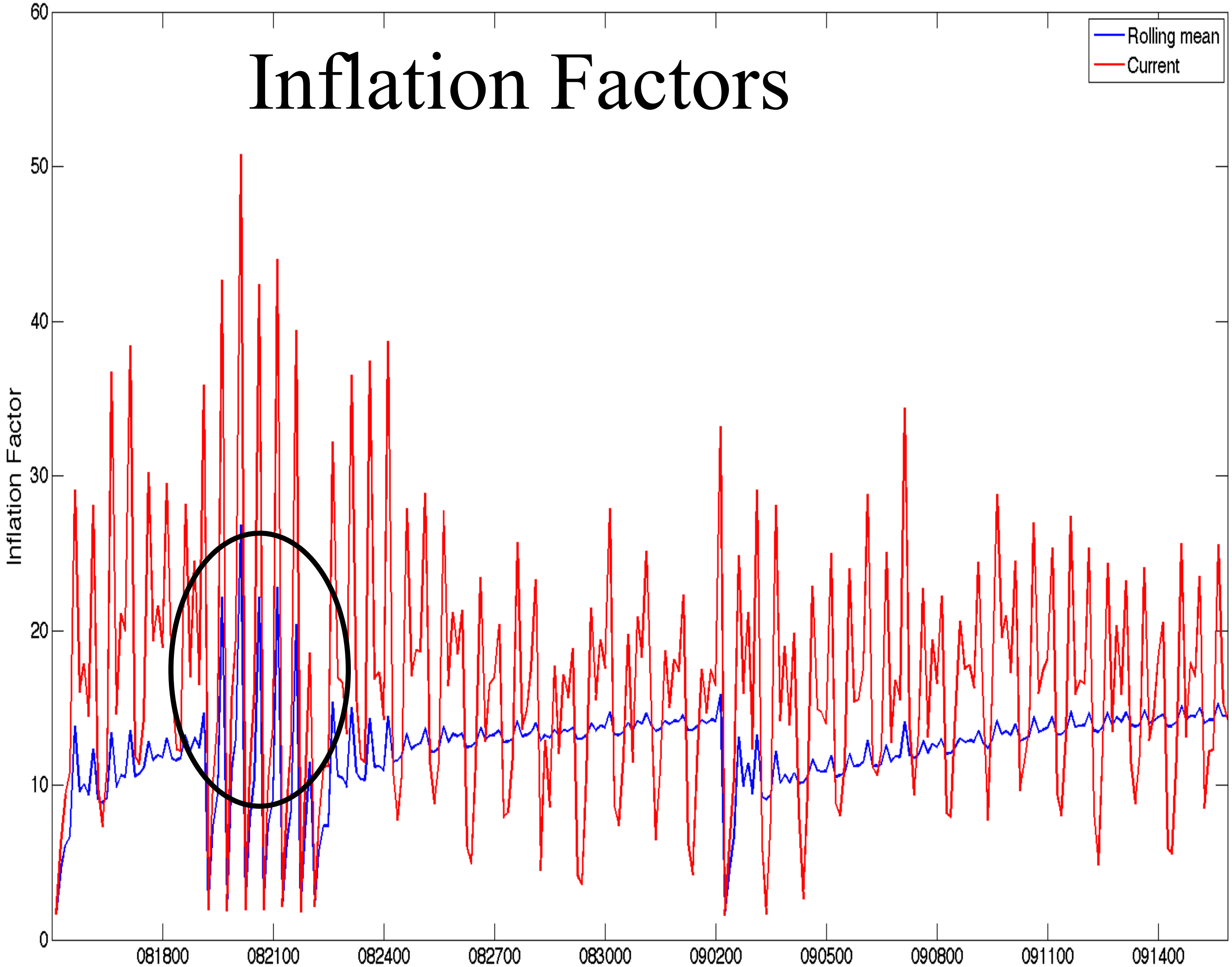
Modest inflations factors used

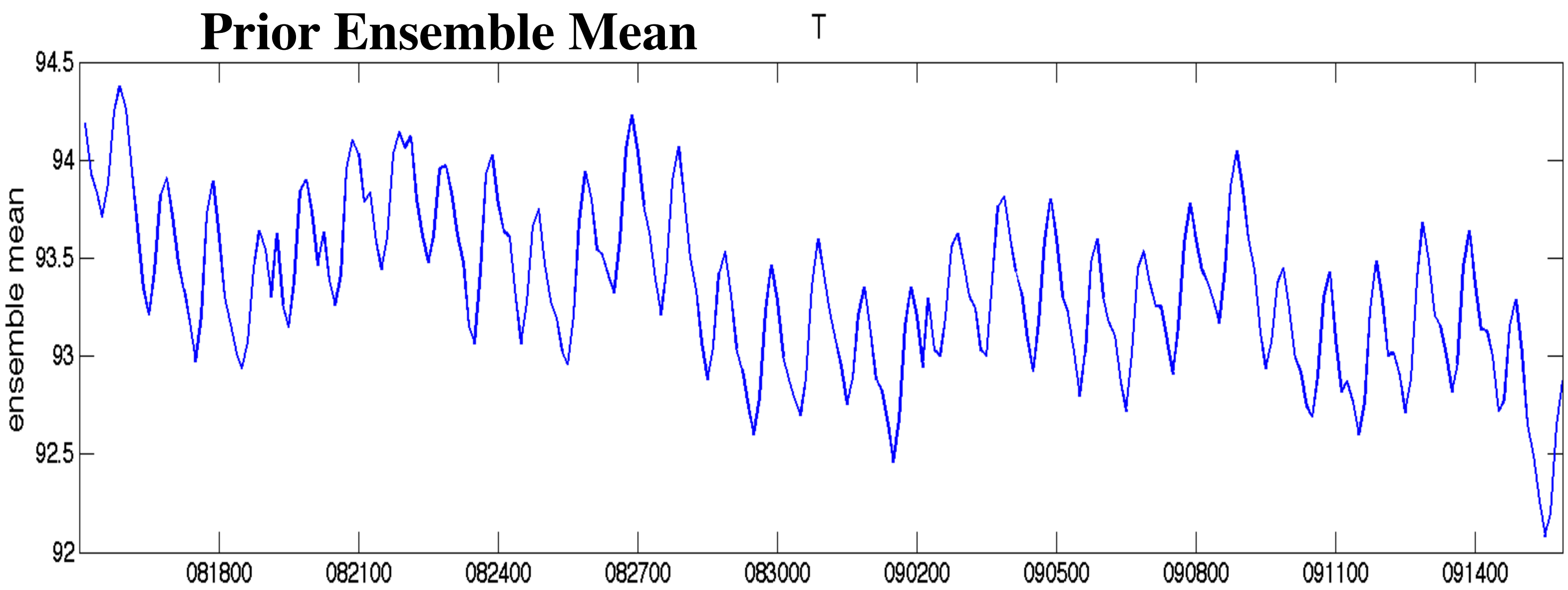
Higher inflations factors used

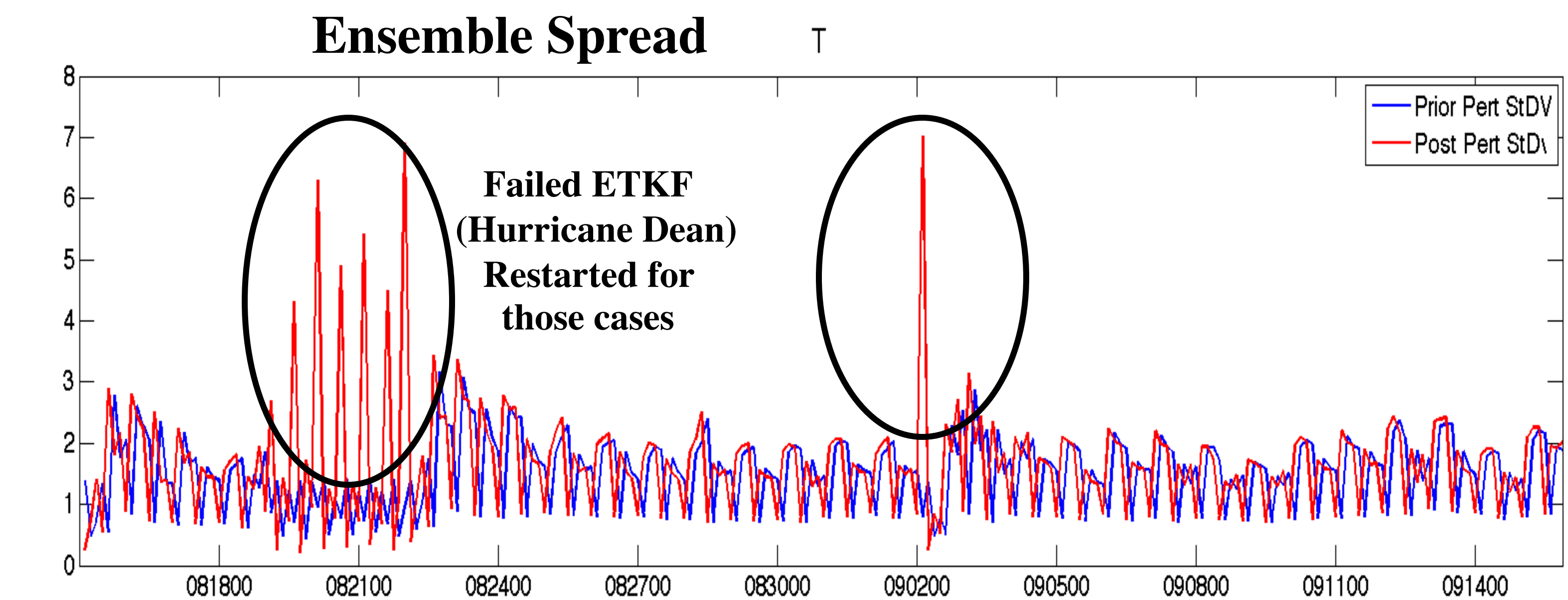
Ensemble Mean and Std. Deviation (spread)

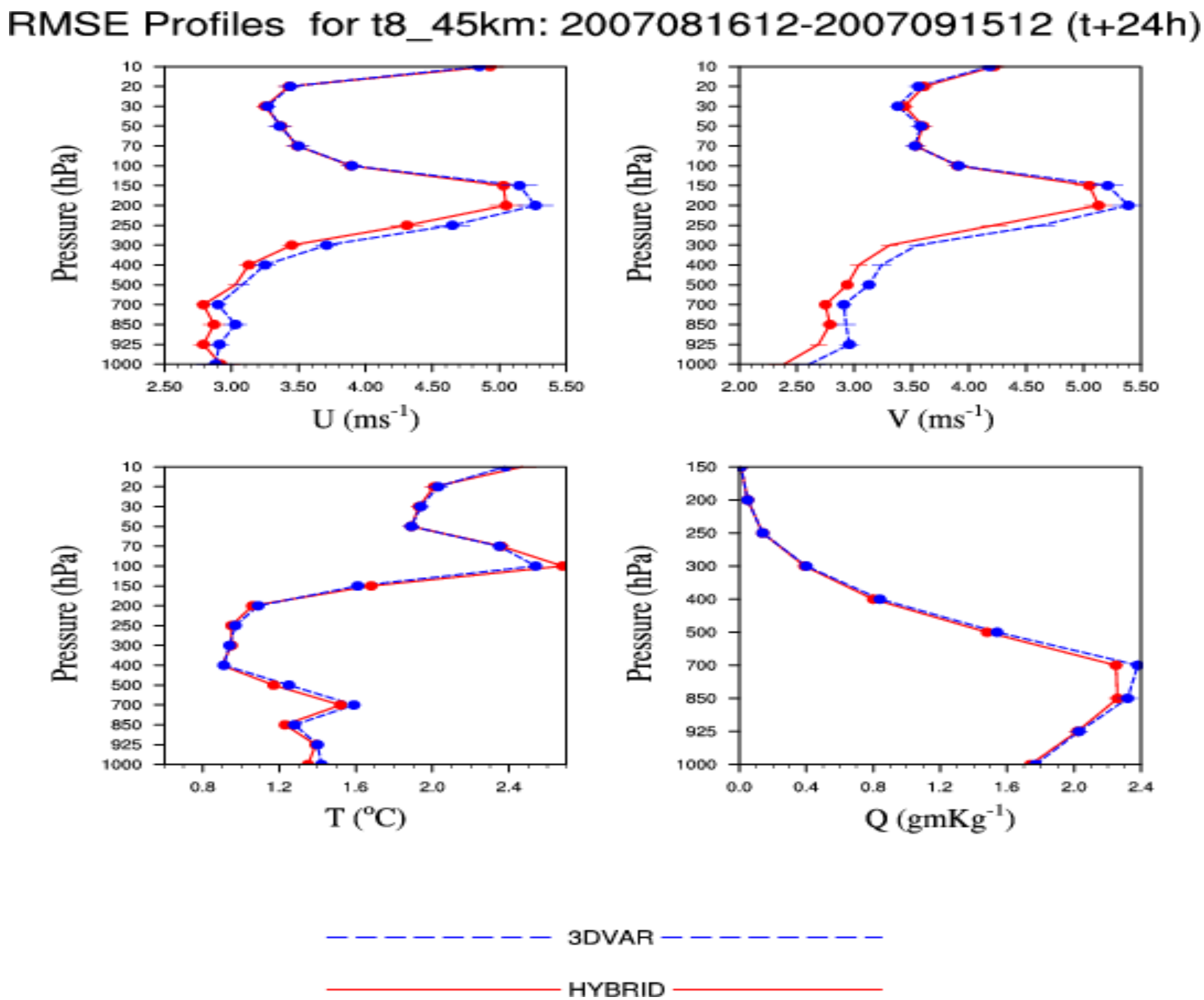


Inflation Factors

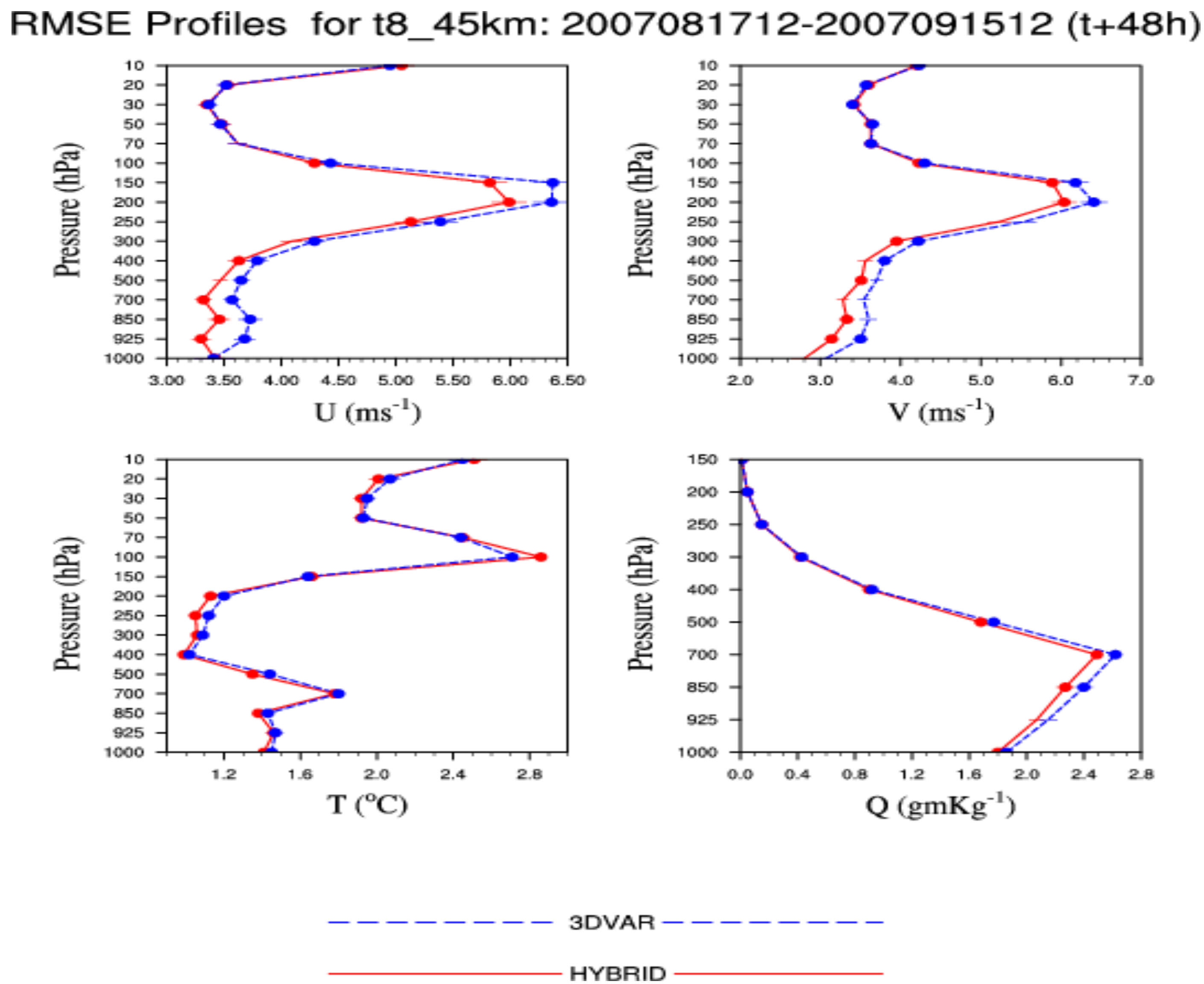



Prior Ensemble Mean
T
ensemble mean
94.5
94
93.5
93
92.5
92
081800
082100
082400
082700
083000
090200
090500
090800
091100
091400


Ensemble Spread
T
8
7
6
5
4
3
2
1
0
Prior Pert StDV
Post Pert StD\
Failed ETKF
(Hurricane Dean)
Restarted for
those cases
081800
082100
082400
082700
083000
090200
090500
090800
091100
091400



Hybrid gives better RMSE scores for wind compared to 3D-VAR.



Hybrid gives better RMSE scores for wind compared to 3D-VAR.

Pros and Cons of ETKF

- Desirable aspects:
  - ETKF is fast (computations are done in model ensemble perturbation subspace).
  - It directly updates perturbations.

- Less desirable aspects:
  - Not localized, therefore it does not represent sampling error efficiently. It may need very high inflation factors. It can even lead failure for Hurricane applications!

- Alternatives for ensemble part
  - EnKF, Perturbed obs, LETKF

Introduction of Hybrid practice session

- **Computation:**
  - Computing ensemble mean.
  - Extracting ensemble perturbations (EP).
  - Running WRFDA in "hybrid" mode.
  - Displaying results for: ens_mean, std_dev, ensemble perturbations, hybrid increments, cost function and, etc.
  - If time permits, tailor your own test by changing hybrid settings; testing different values of "je_factor" and "alpha_corr_scale" parameters.
- **Scripts to use:**
  - Some NCL scripts to display results.
- **ETKF part not included yet in current practice**

Brief information for the chosen case

Ensemble size: 10

Domain info:

- time_step=240,
- e_we=122,
- e_sn=110,
- e_vert=42,
- dx=45000,
- dy=45000,

Input data provided (courtesy of JME Group):

- WRF ensemble forecasts valid at 2006102800
- Observation data (ob.ascii) for 2006102800
- 3D-VAR "be.dat" file

References

Bishop, C. H., B. J. Etherton, S. J. Majumdar, 2001: Adaptive sampling with the ensemble transform Kalman filter. Part I: Theoretical aspects. Mon. Weather Rev., 129, 420–436.

Bowler, N. E., A. Arribas, K. R. Mylne, K. B. Robertson, S. E. Beare, 2008: The MOGREPS short-range ensemble prediction system. Q. J. R. Meteorol. Soc. 134, 703–722.

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