Recent Developments of WRF 4D-Var

Xin Zhang, Xiang-Yu Huang, Yong-Run Guo
MMM Division, NCAR, Boulder, CO USA

Nils Gustafsson
Swedish Meteorological and Hydrological Institute, Norrkoping, Sweden

Meng Zhang
Penn State University, PE, USA

Content of talk

- Why is 4D-Var's performance better than 3D-Var's?
- Overview of the WRF 4D-Var
- Observations used by the data assimilation system
- Weak constraint with digital filter
- Multi-incremental 4D-Var
- Lateral boundary control in 4D-Var
- Recent performance of WRF 4D-Var System
- Computational issues
- Further developments

4D-Var versus 3D-Var

(Adopted from ECMWF training Course 2008)

- 4D-Var is comparing observations with background model fields at the correct time
- 4D-Var can use observations from frequently reporting stations
- The dynamics and physics of the forecast model in an integral part of 4D-Var, so observations are used in a meteorologically more consistent way
- 4D-Var combines observations at different times during the 4D-Var window in a way that reduces analysis error
- 4D-Var propagates information horizontally and vertically in a meteorologically more consistent way

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Four-dimensional Variational Approach

The general cost function of the variational formulation:

$$\begin{aligned} J(\mathbf{x}) &= \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}^b)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}^b) + J_x \\ &+ \frac{1}{2}\sum_{k=0}^{K}[\mathbf{h}(\mathbf{x}_k) - \mathbf{y}_k]^T \mathbf{R}_k^{-1}[\mathbf{h}(\mathbf{x}_k) - \mathbf{y}_k] \end{aligned}$$

where

- $\mathbf{x} \equiv [\mathbf{x}_0, \mathbf{x}_1, \cdots, \mathbf{x}_K]^T$ **is a 4-dimensional state vector;**
- h **is the nonlinear observation operator;**
- B **and** R_k **are the background, and observation error covariances, respectively;**
- J_x **represents extra constraint (e.g., balance).**

Strong Constraint Incremental 4DVAR

(Courtier, Thépaut and Hollingsworth (1994))

For simplicity consider now the strong constraint case. In incremental 4DVAR the cost function at the j-th iteration is

$$
\begin{aligned}
J_j(\delta \mathbf{x}_j) &= \frac{1}{2}(\delta \mathbf{x}_j - \delta \mathbf{x}_j^b)^T \mathbf{B}^{-1}(\delta \mathbf{x}_j - \delta \mathbf{x}_j^b) \\
&+ \frac{1}{2}\sum_{k=0}^{K}(\mathbf{H}_{j,k}\mathbf{M}_{j,k}\delta \mathbf{x}_j - \mathbf{d}_{j,k})^T \mathbf{R}^{-1}(\mathbf{H}_{j,k}\mathbf{M}_{j,k}\delta \mathbf{x}_j - \mathbf{d}_{j,k})
\end{aligned}
$$

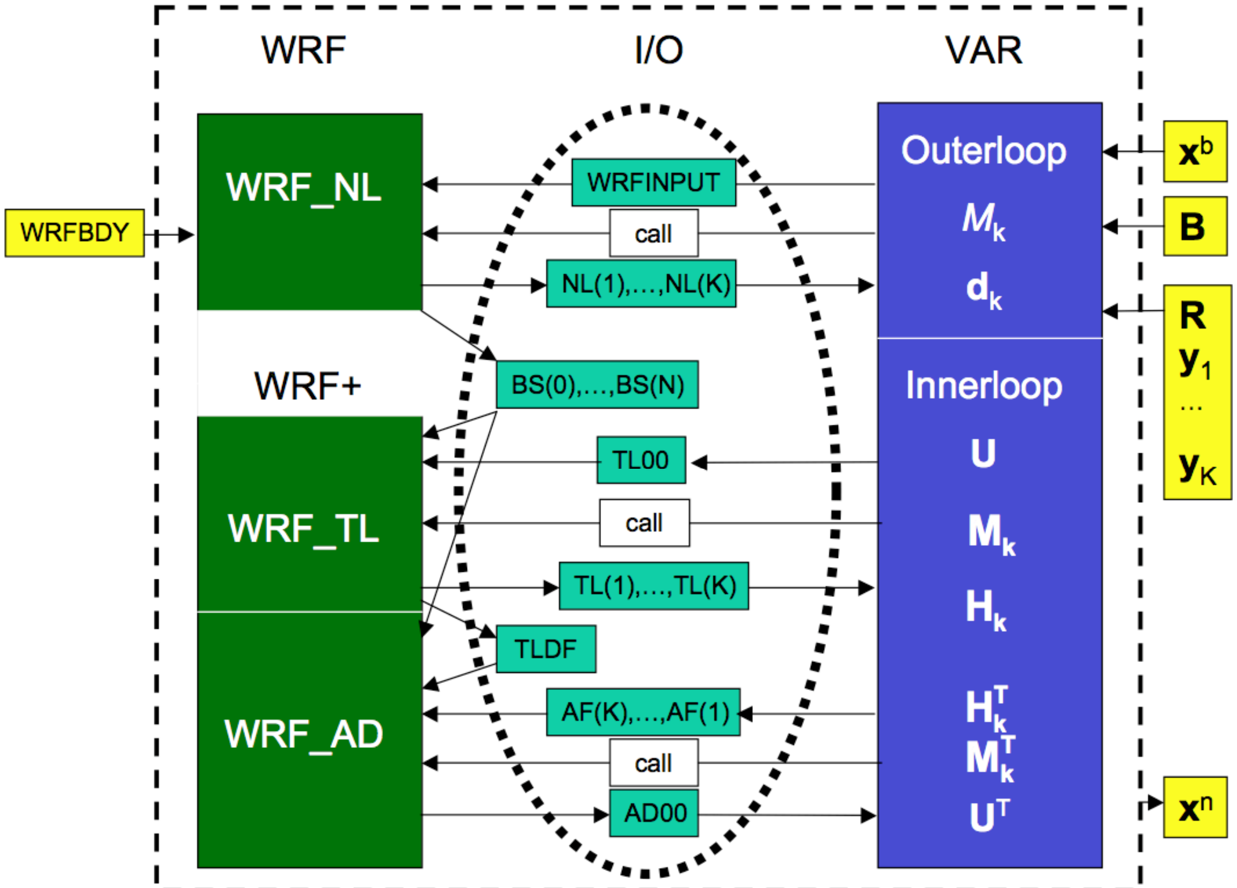
WRF NL

WRF TL (WRF AD)

where $\mathbf{d}_{j,k} \equiv \mathbf{y}_k - \mathbf{h}_k(\mathbf{m}_k(\mathbf{x}^b))$, $\delta \mathbf{x}_j^b \equiv \mathbf{x}^b - \mathbf{x}_{j-1}$ and

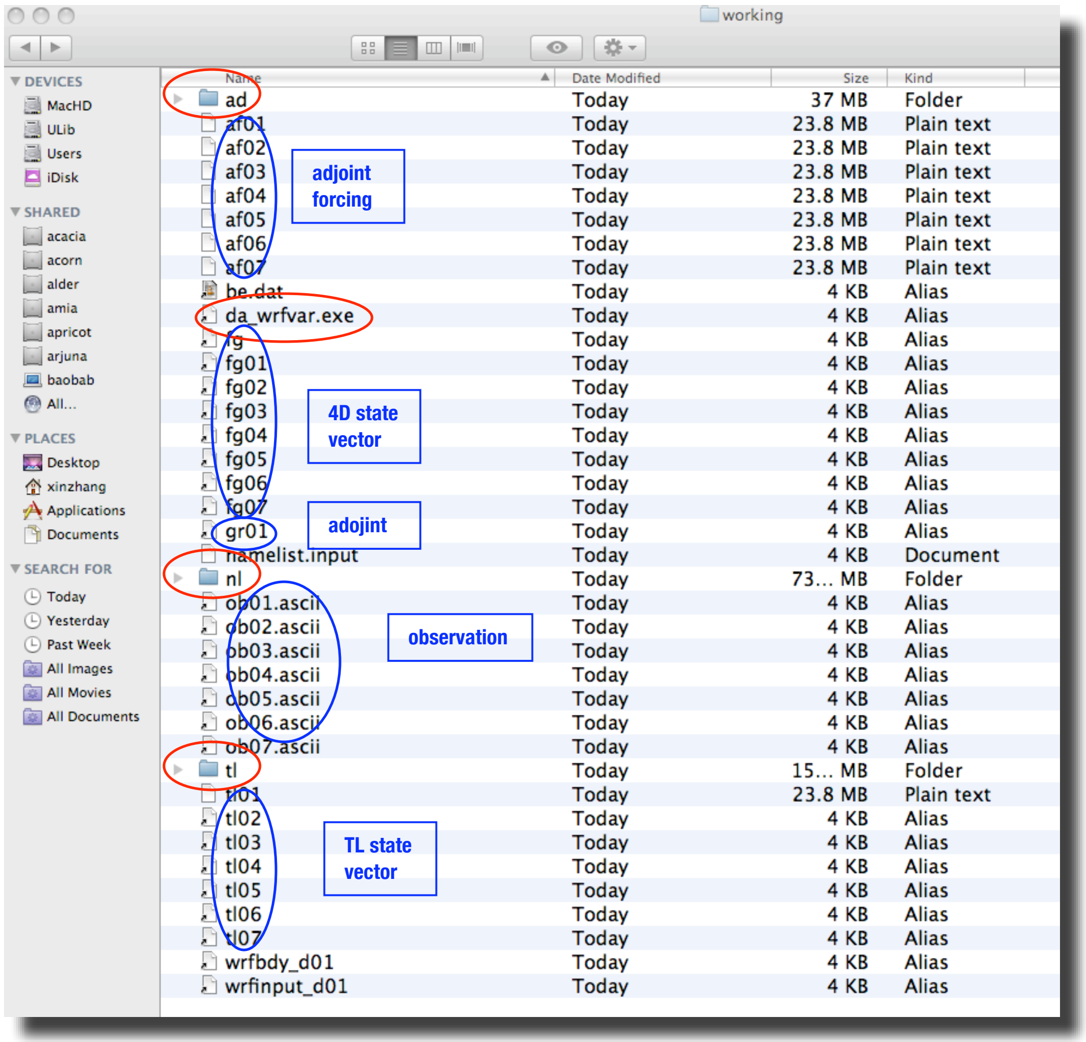
- $\delta \mathbf{x}_j \equiv \mathbf{x}_j - \mathbf{x}_{j-1}$ is the control variable;
- The inner loop minimization of J_j can be solved by Conjugate gradient or Lanczos

Structure of WRF 4D-Var



Hans Huang: WRF 4D-Var MMM Seminar, 13 December 2007

Structure of WRF 4D-Var (Cont'd)



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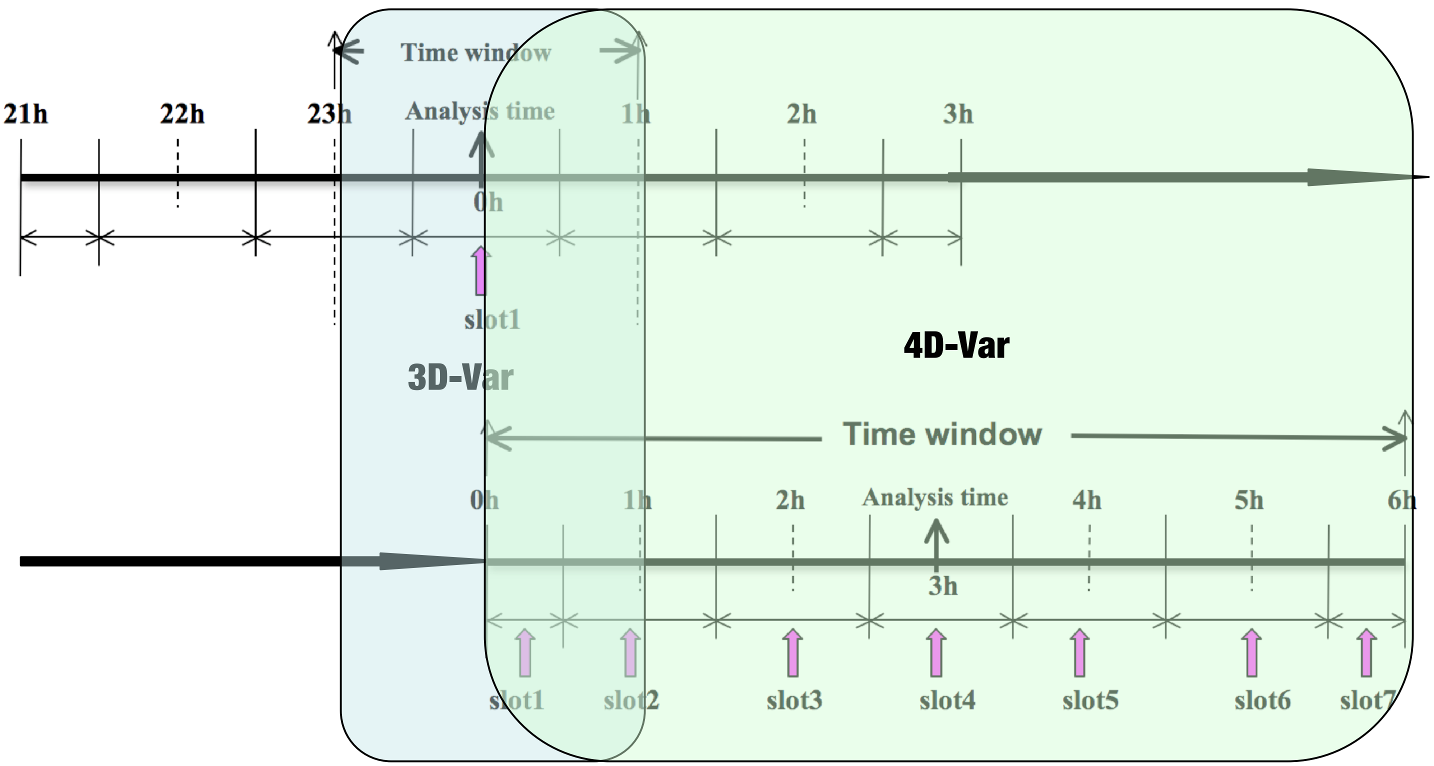
Observations used by the 4D-Var

- Conventional observation data
- Radar radial velocity
- Radiance satellite data (under testing)



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Weak constraint with digital filter

$$J = J_b + J_o + J_c$$

$$J_b(\mathbf{x}_0) = \frac{1}{2}\left[(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_b)\right]$$

$$J_o(\mathbf{x}_0) = \frac{1}{2}\sum_{k=1}^{K}\left[(\mathbf{H}_k\mathbf{x}_k - \mathbf{y}_k)^T \mathbf{R}^{-1}(\mathbf{H}_k\mathbf{x}_k - \mathbf{y}_k)\right]$$

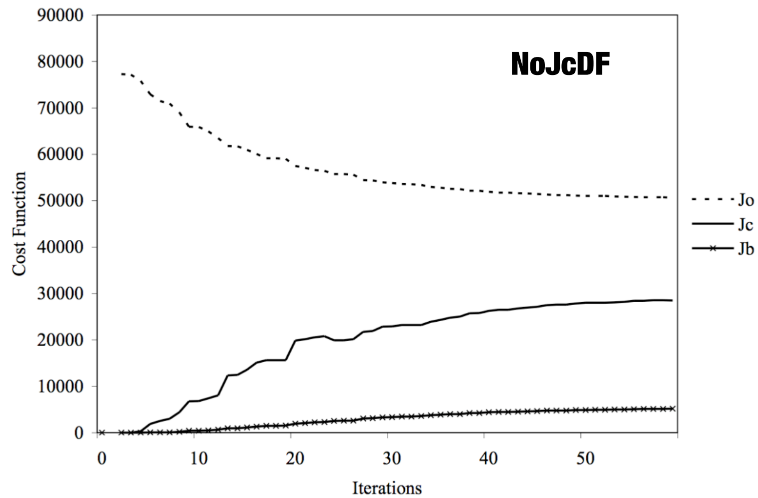
$$J_c(\mathbf{x}_0) = \frac{\gamma_{df}}{2}\left[\left(\delta\mathbf{x}_{N/2} - \delta\mathbf{x}_{N/2}^{df}\right)^T \mathbf{C}^{-1}\left(\delta\mathbf{x}_{N/2} - \delta\mathbf{x}_{N/2}^{df}\right)\right]$$

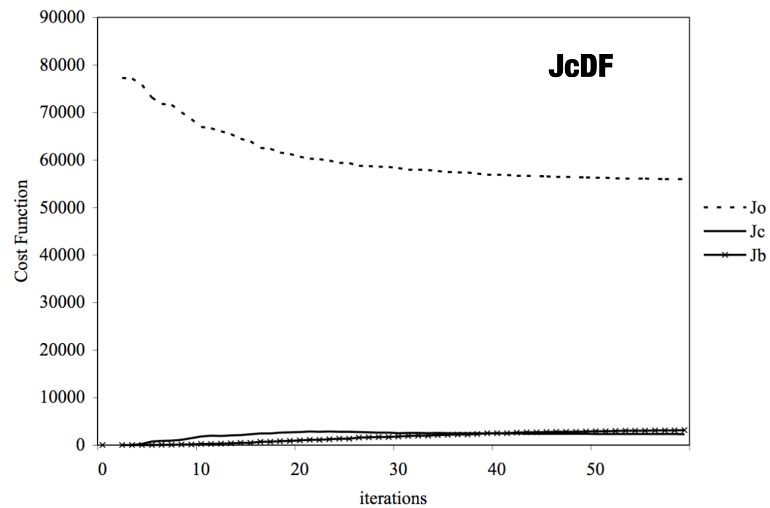
$$= \frac{\gamma_{df}}{2}\left[\left(\delta\mathbf{x}_{N/2} - \sum_{i=0}^{N} f_i\delta\mathbf{x}_i\right)^T \mathbf{C}^{-1}\left(\delta\mathbf{x}_{N/2} - \sum_{i=0}^{N} f_i\delta\mathbf{x}_i\right)\right]$$

$$= \frac{\gamma_{df}}{2}\left[\left(\sum_{i=0}^{N} h_i\delta\mathbf{x}_i\right)^T \mathbf{C}^{-1}\left(\sum_{i=0}^{N} h_i\delta\mathbf{x}_i\right)\right]$$

where:

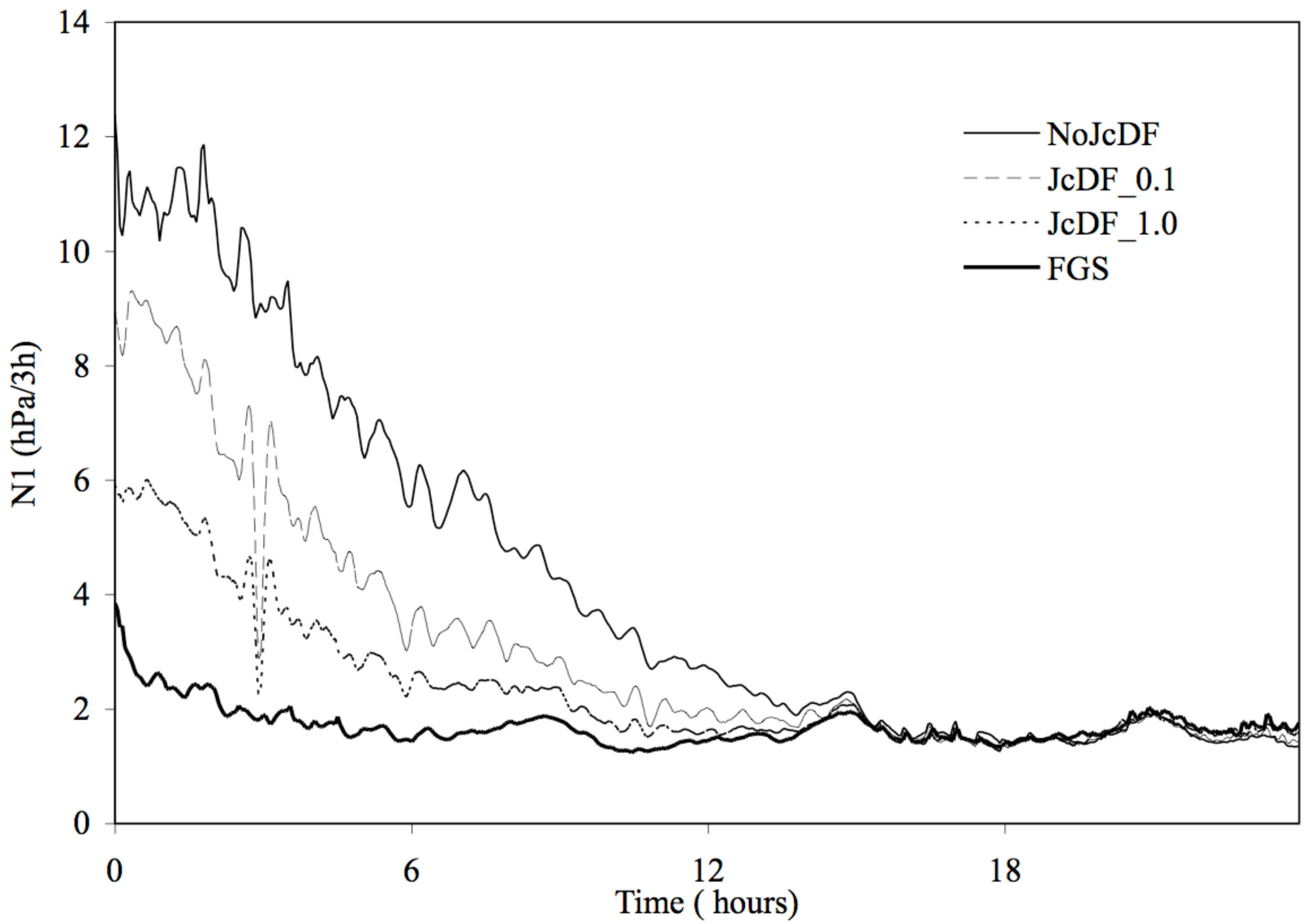
$$h_i = \begin{cases} -f_i, & \text{if } i \neq N/2 \\ 1 - f_i, & \text{if } i = N/2 \end{cases}$$





Weak constraint with digital filter

(domain averaged surface pressure variation)



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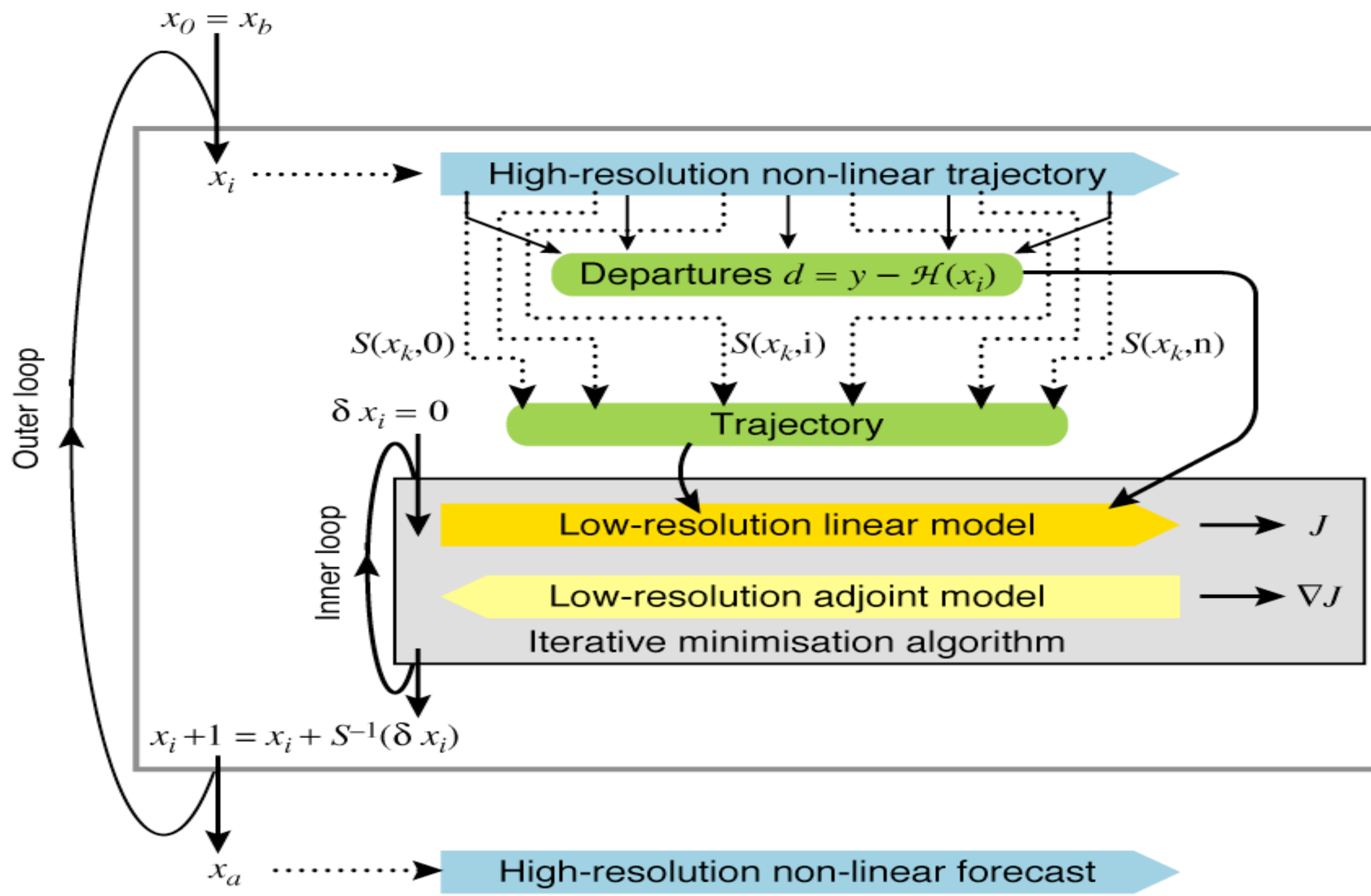
Multi-incremental WRF 4D-Var

(Adopted from ECMWF training course 2008)



Multi-incremental WRF 4D-Var procedure

(Under testing)

Use all data in a 6-hour window (0900-1500 UTC for 1200 UTC analysis)

1. **Group observations into 1 hour time slots**
2. **Run the (15km) high resolution forecast from the previous analysis and compute "observation"- "model" differences**
3. **Adjust the model fields at the start of assimilation window (0900 UTC) so the 6-hour forecast better fits the observations. This is an iterative process using a lower resolution linearized model (45km) and its adjoint model**
4. **Rerun the (15km) high resolution model from the modified (improved) initial state and calculate new observation departures**
5. **The 3-4 loop is repeated two times to produce a good high resolution estimate of the atmospheric state – the result is the WRF 4D-Var analysis**

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Lateral Boundary in WRF

WRF uses a boundary relaxation applied in a ”nudging” of model tendencies, following Davies and Turner (1977):

$$\frac{\partial \mathbf{x}}{\partial t} = F_1(\mathbf{x}_{\mathrm{lbc}} - \mathbf{x}) - F_2\Delta^2(\mathbf{x}_{\mathrm{lbc}} - \mathbf{x})$$

X_{lbc} is the corresponding boundary value provided by the host model, which is specified in the following form:

$$\mathbf{x}_{\mathrm{lbc}}(time = t) = \mathbf{x}_{\mathrm{lbc}}(time = t_0) + (t - t_0)\frac{\partial \mathbf{x}_{\mathrm{lbc}}}{\partial t}$$

4D-Var LBC control

Considering a data assimilation window from time t_0 until time t_k and having $\delta\mathbf{x}(t_0)$ and $\delta\mathbf{x}_{\mathrm{lbc}}(t_k)$ as the assimilation control variables, the quantities needed for the LBC of the tangent linear WRF model are given by

$$\delta\mathbf{x}_{\mathrm{lbc}}(t_0) = \delta\mathbf{x}(t_0) \tag{4}$$

$$\frac{\partial\delta\mathbf{x}_{\mathrm{lbc}}}{\partial t} = \frac{\delta\mathbf{x}_{\mathrm{lbc}}(t_k) - \delta\mathbf{x}(t_0)}{t_k - t_0} \tag{5}$$

4D-Var LBC control (Cont'd)

The lateral boundary conditions for the adjoint model, $\mathbf{x}_{\rm lbc}^{\rm AD}(t_0)$ and $(\frac{\partial \mathbf{x}_{\rm lbc}}{\partial t})^{\rm AD}$, will be initialized with zeroes at the end of the data assimilation window (time t_k). After the backwards integration of the adjoint model to time t_0 the adjoint control variables (or the error gradients) can be obtained from:

$$\mathbf{x}^{\rm AD}(t_0) = \mathbf{x}_{\rm inner}^{\rm AD}(t_0) + \mathbf{x}_{\rm lbc}^{\rm AD}(t_0) - \frac{1.}{t_k - t_0}(\frac{\partial \mathbf{x}_{\rm lbc}}{\partial t})^{\rm AD} \quad (6)$$

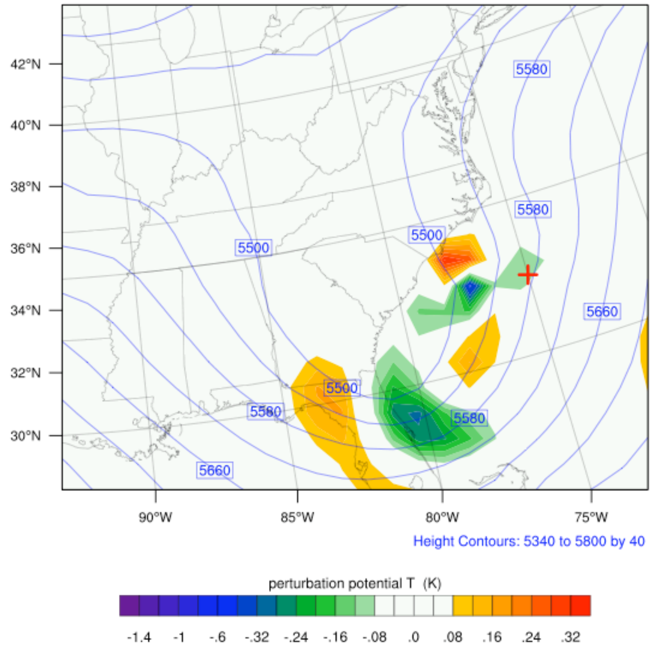
$$\mathbf{x}_{\rm lbc}^{\rm AD}(t_k) = \frac{1.}{t_k - t_0}(\frac{\partial \mathbf{x}_{lbc}}{\partial t})^{\rm AD} \quad (7)$$

where $\mathbf{x}_{\rm inner}^{\rm AD}(t_0)$ denotes the inner domain adjoint model model variable as provided at the initial time t_0.

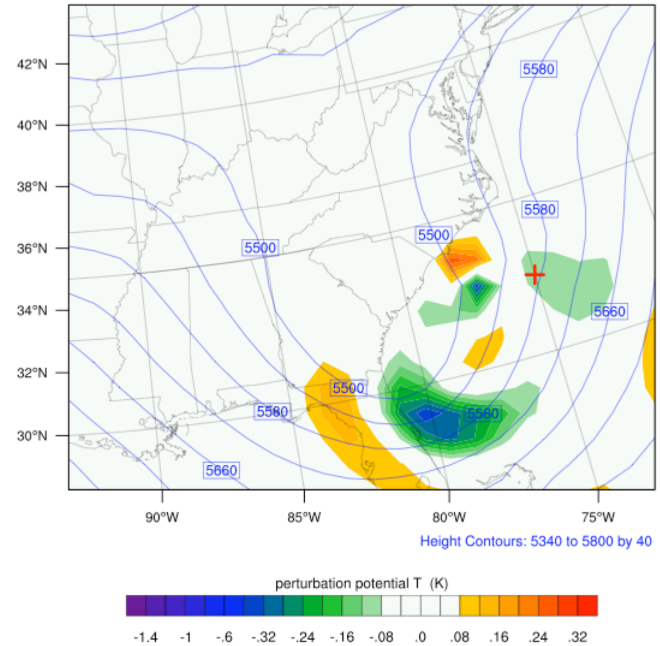
Validation Experiment 1

A single 500hPa Temperature observation at 6 hour which is far from the boundary

Pert. Potential Temperature at 0 hour



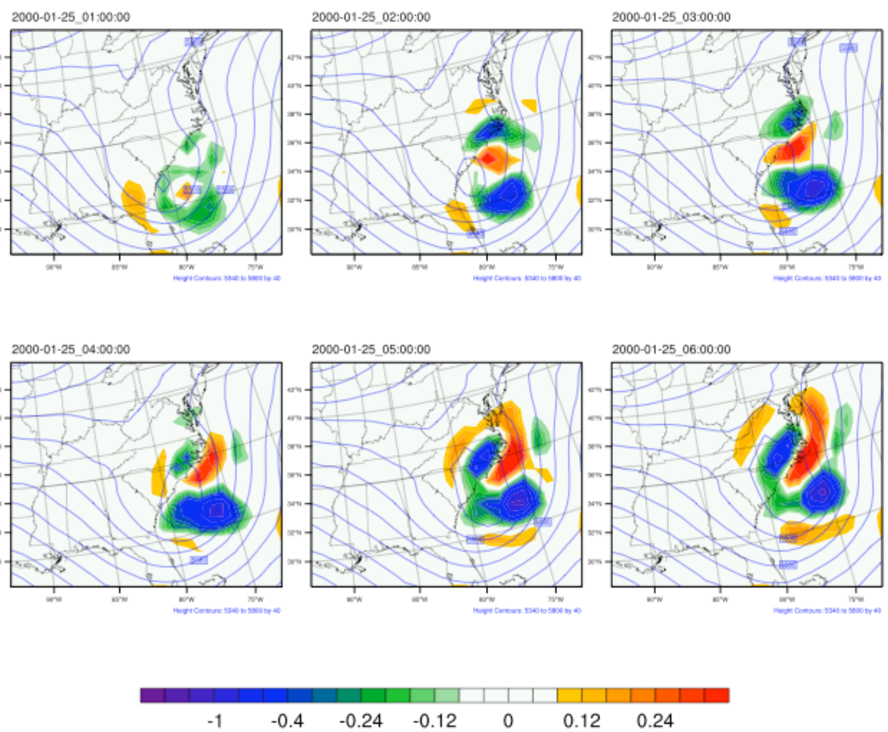
Without LBC control



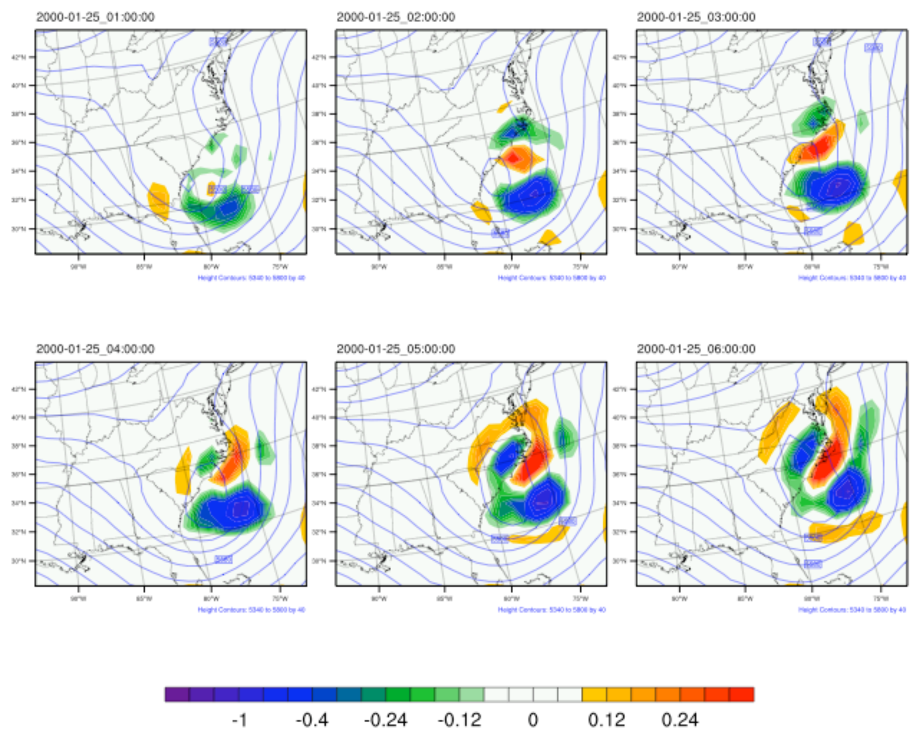
With LBC control

+ *is the location of T observation at 6 hour*

Pert. Potential Temperature at 1-6 hour



Without LBC control

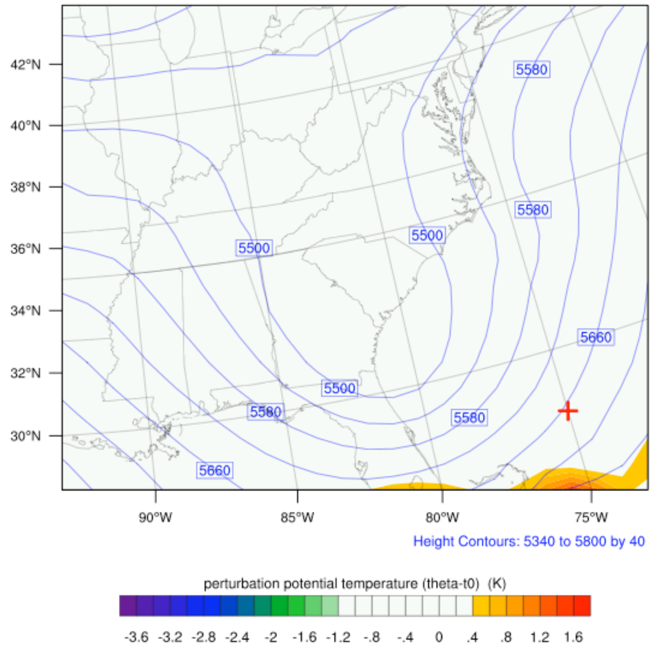


With LBC control

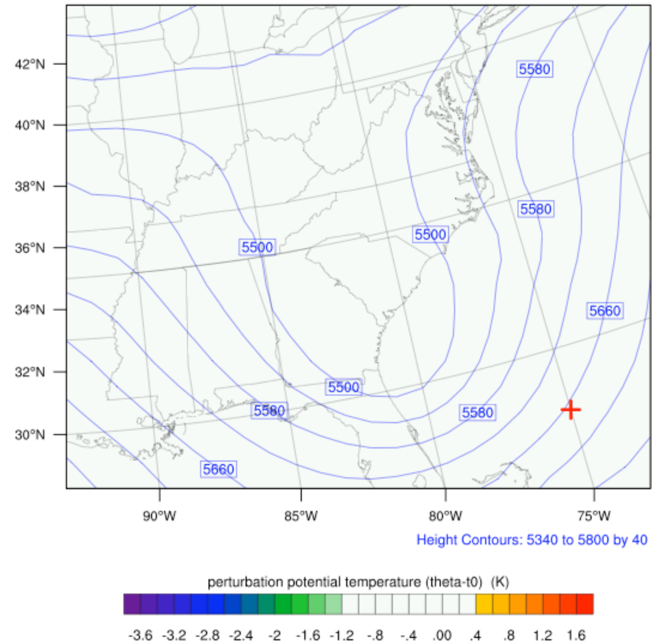
Validation Experiment 2

A single 500hPa Temperature observation at 6 hour which is close to the boundary

Pert. Potential Temperature at 0 hour



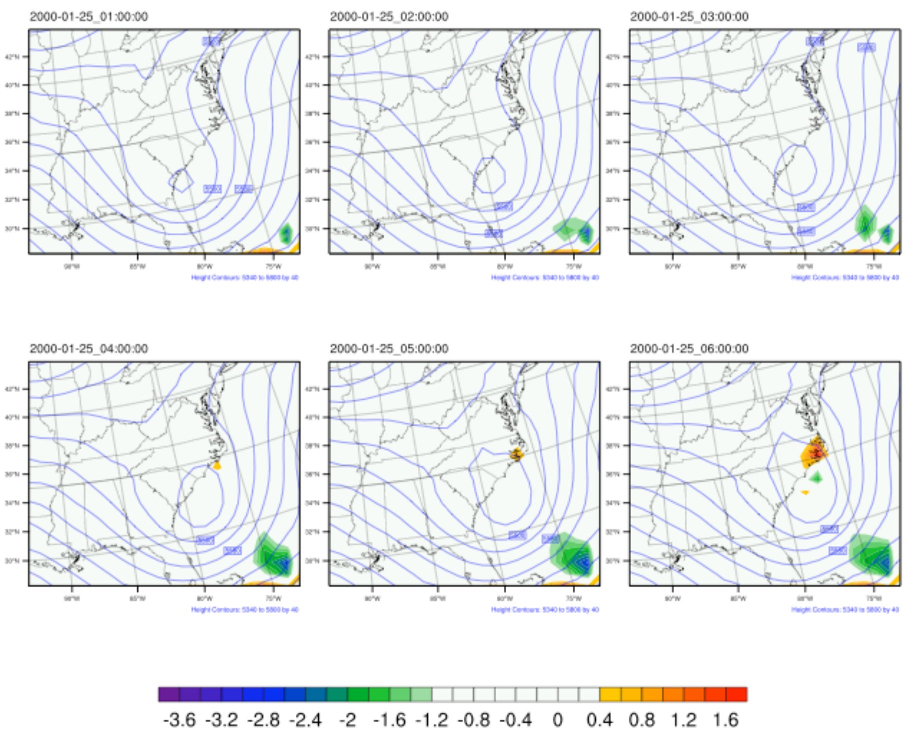
Without LBC control



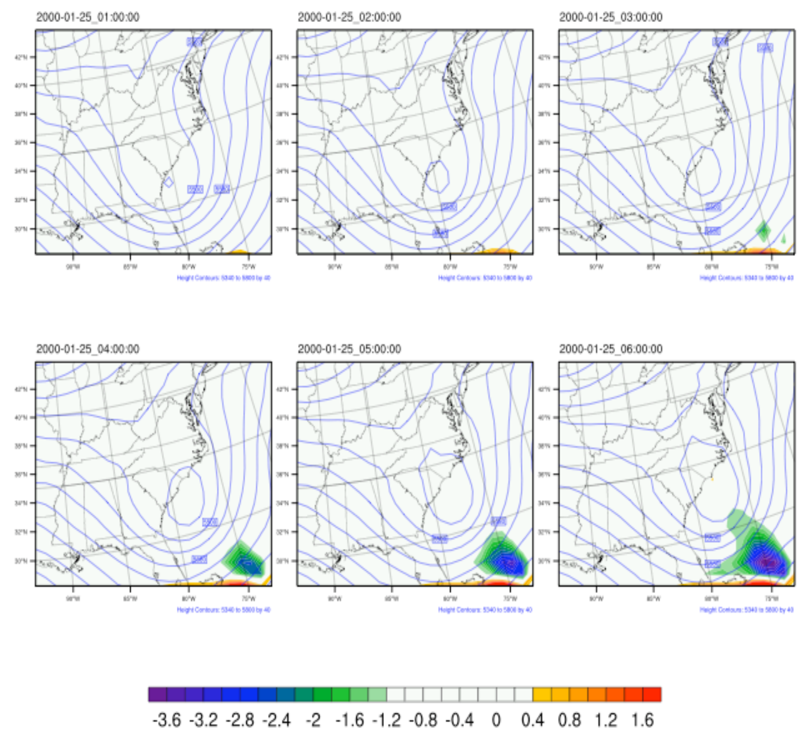
With LBC control

+ *is the location of T observation at 6 hour*

Pert. Potential Temperature at 1-6 hour



Without LBC control



With LBC control

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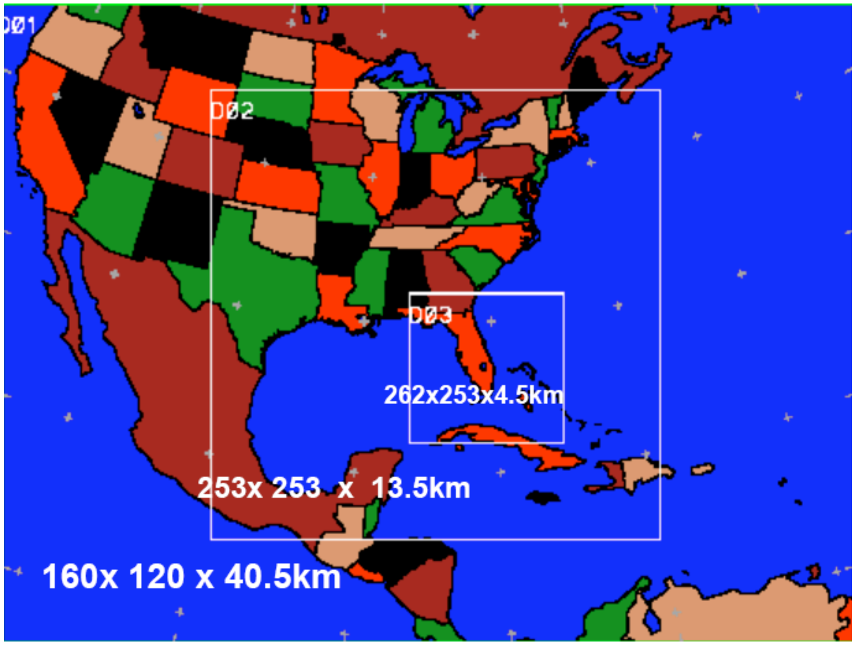
WRF 3D/4D-Var for Katrina 2005

•WRF Domains : D1-D3 with 40.5km, 13.5km, 4.5 km grids and 35 vertical levels; Data assimilations only applied on D1

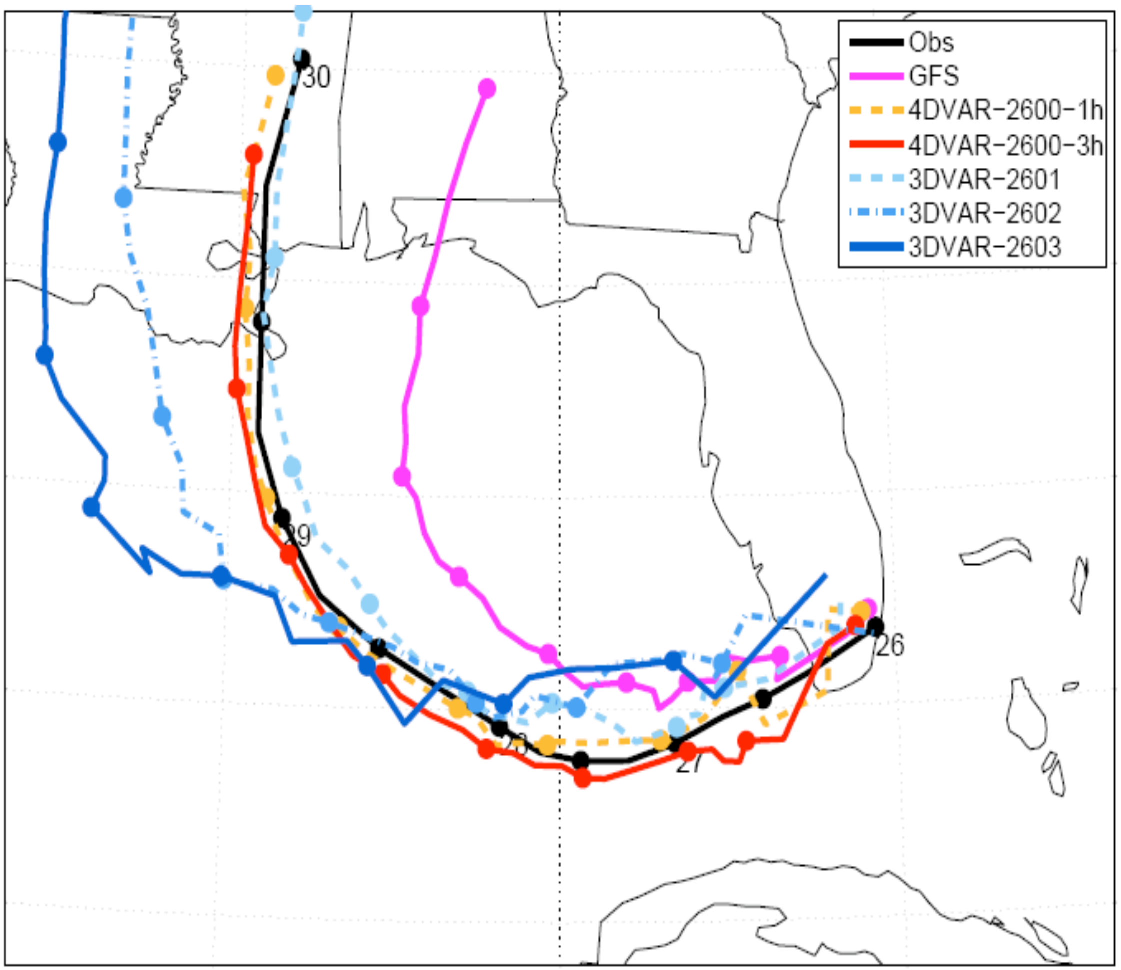
•Forecasts: 96-h deterministic run with D1, D2, D3 (two-way nested and D3 movable) initialized from 00Z August 26 2005 with various ICs

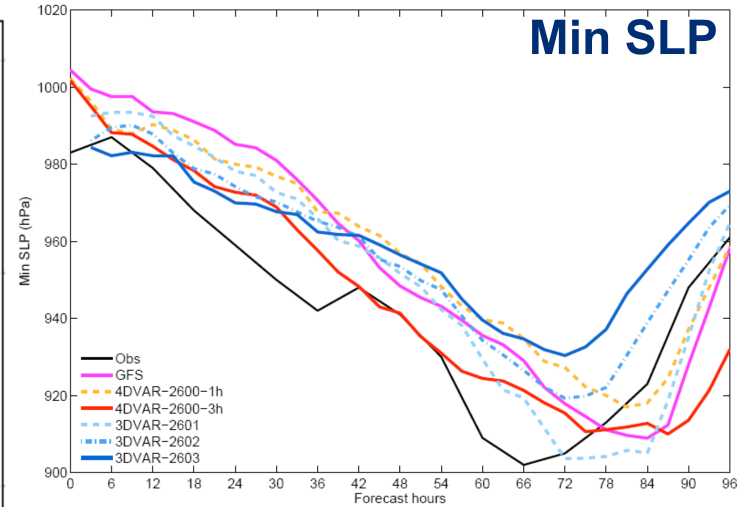
•Data assimilated: Doppler radial velocity (err=3m/s) from KMAX and KBYX during 00~03Z August 26 2005

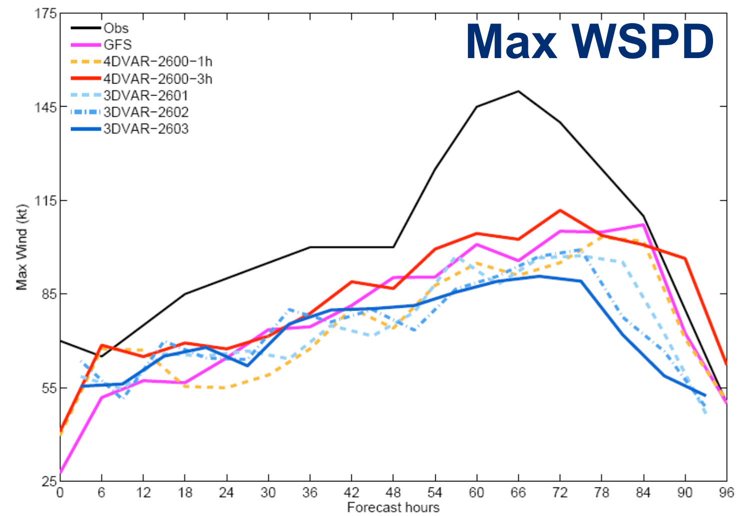
Experiments:
a) Control run with GFS-IC
b) 4DVAR with 1-h and 3-h window
c) Successive 3DVAR with 1-h interval at 01Z, 02Z and 03Z respectively



Track and Intensity forecasts







First radar data assimilation with WRF 4D-Var

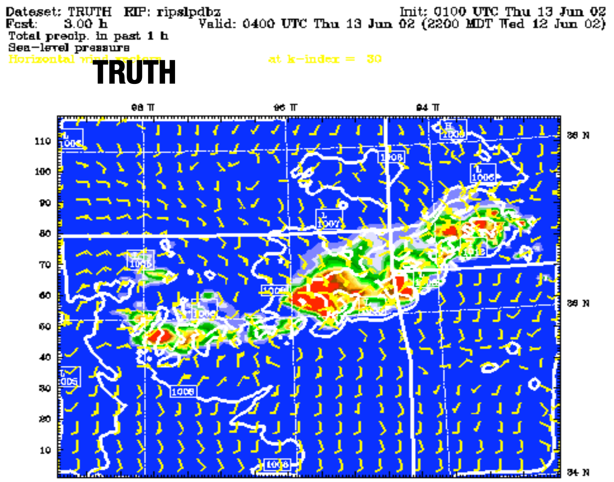
- TRUTH ----- Initial condition from TRUTH (13-h forecast initialized at 2002061212Z from AWIPS 3-h analysis) run cutted by ndown, boundary condition from NCEP GFS data.

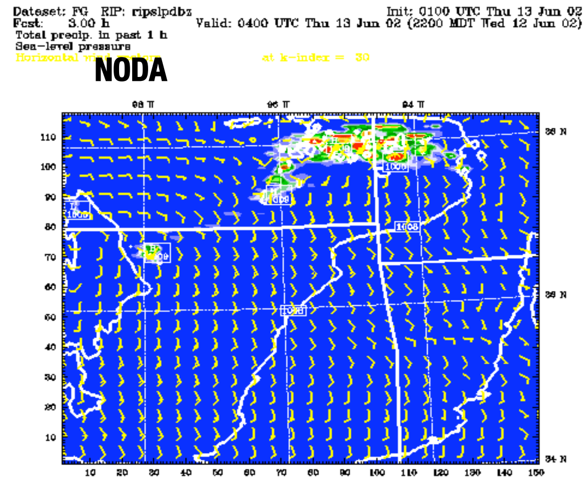
- NODA ----- Both initial condition and boundary condition from NCEP GFS data.

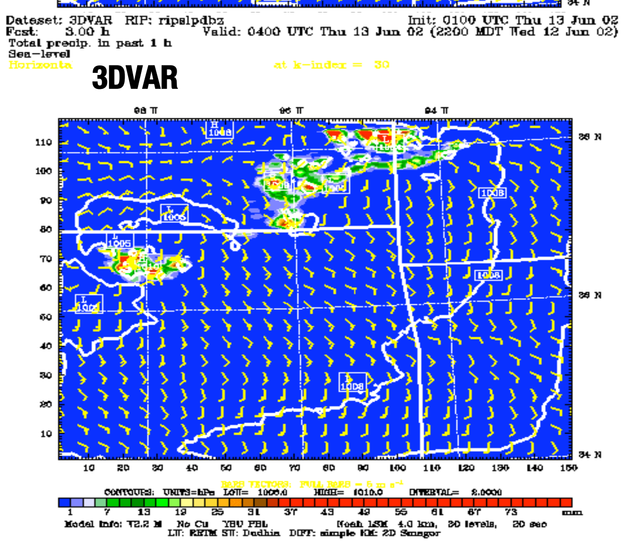
- 3DVAR -----3DVAR analysis at 2002061301Z used as the initial condition, and boundary condition from NCEP GFS. Only Radar radial velocity at 2002061301Z assimilated (total # of data points = 65,195).

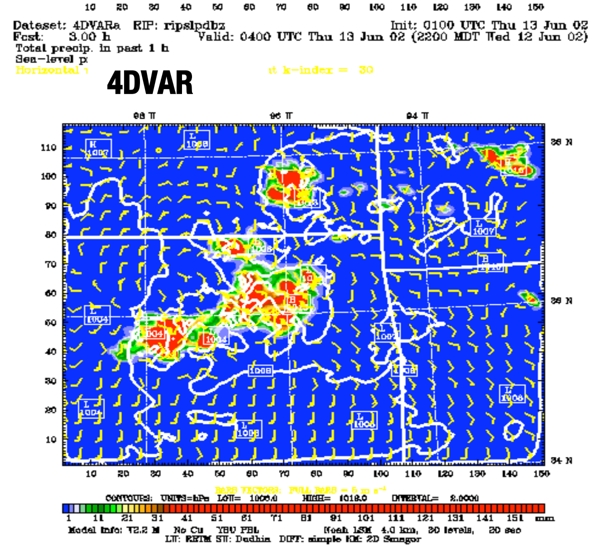
- 4DVAR ----- 4DVAR analysis at 2002061301Z used as initial condition, and boundary condition from NCEP GFS. The radar radial velocity at 4 times: 200206130100, 05, 10, and 15, are assimilated (total # of data points = 262,445).

Hourly precipitation at 03h forecast

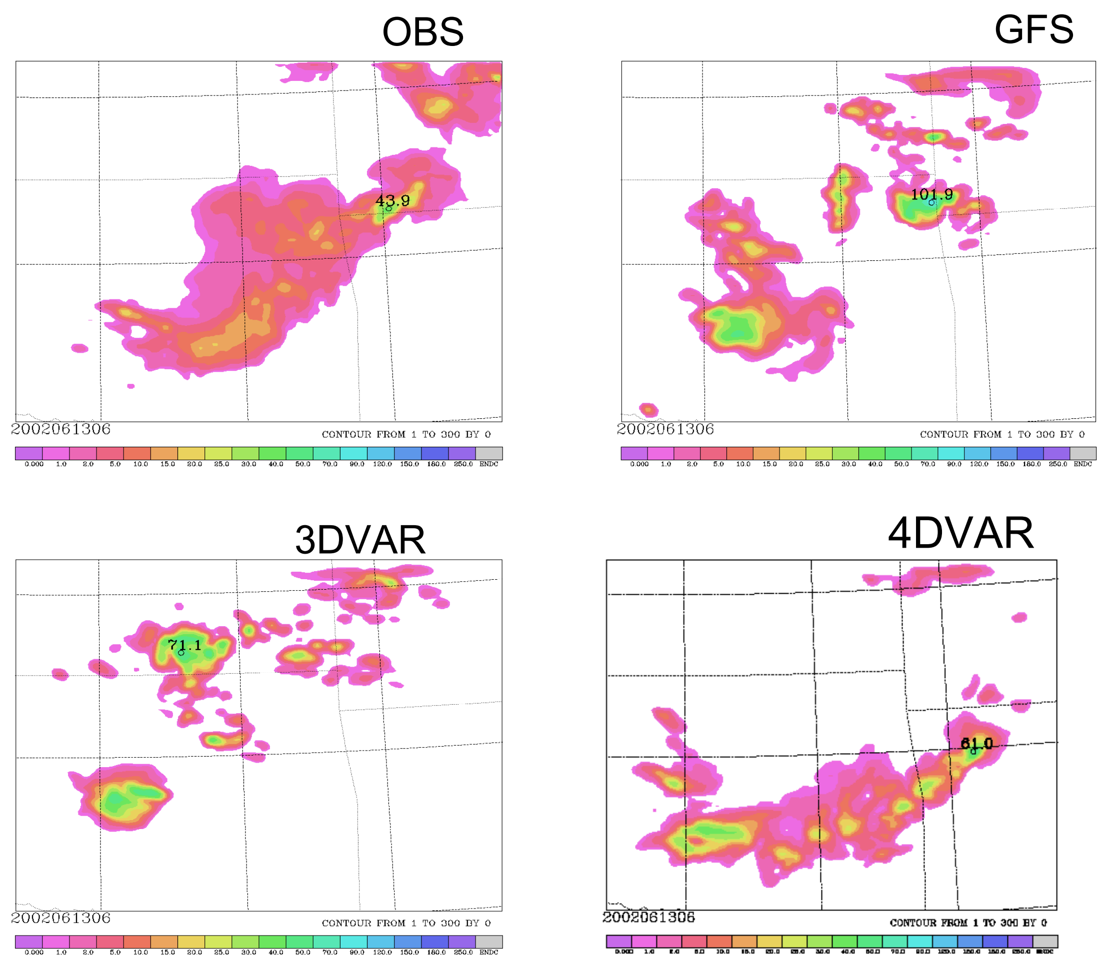








Radar data assimilation (cont'd)
Real data experiments



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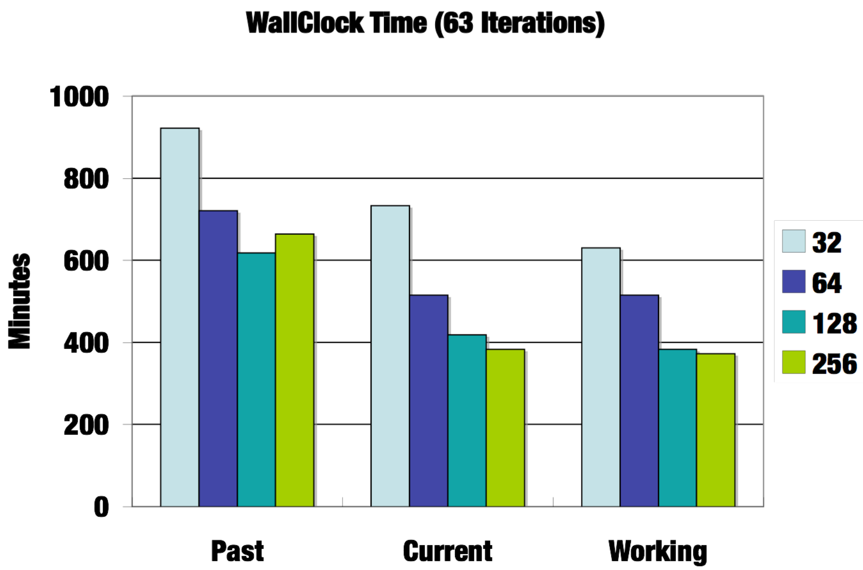
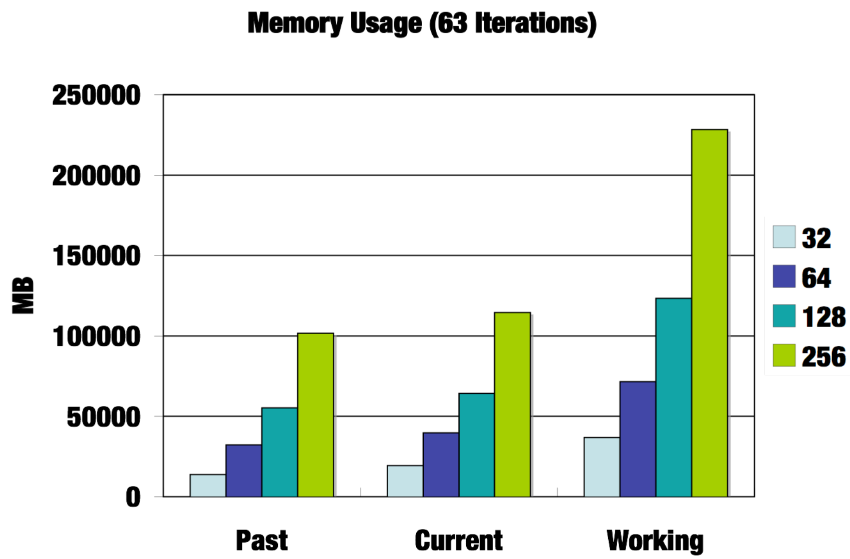
Computational Efficiency of IKE hurricane case on NCAR Bluefire





Past: Before optimization

Current: Eliminate the disk IO for basic states

Working: Reorganizing adjoint codes,reduce re-computation.

Radar Assimilation Case on IBM bluefire

Domain size:151x118x31

Resolution:4km

Time-step: 20s

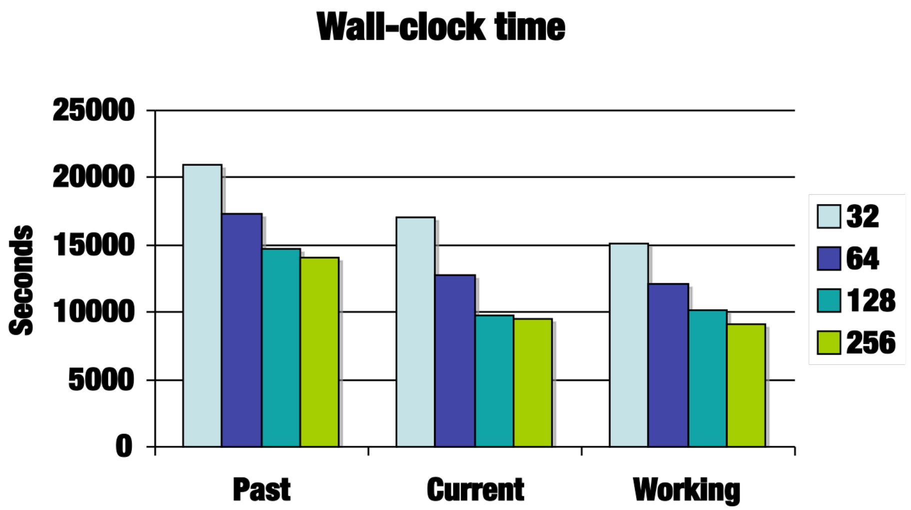
Time window:30m

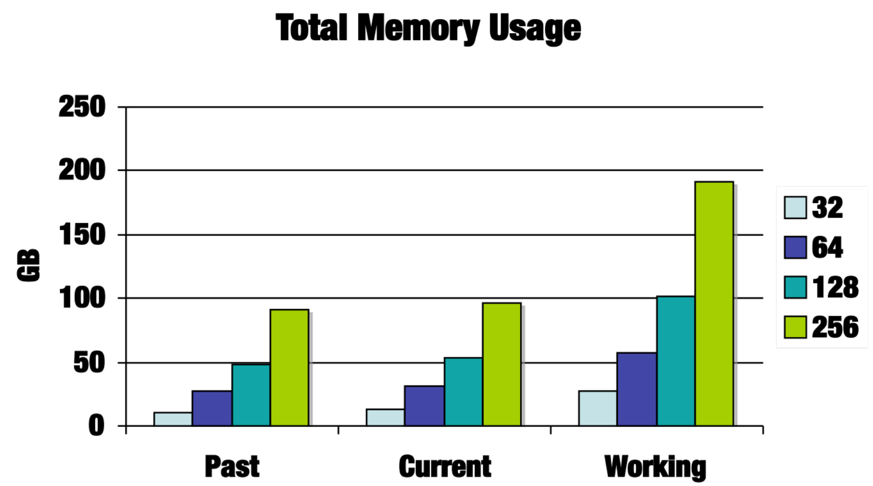
# of iterations: 60

Obs.: OSSE radar wind

# of obs.: 262,517

Obs Freq: 5m





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Further Developments

- ESMF coupling: ESMF will be used to couple WRFNL, WRFPLUS and WRFDA together.

- Improve the current WRF adjoint and tangent Linear codes.

Thank You !